THE β–MEIXNER MODEL

ABSTRACT. We propose to approximate the Meixner model by a member of the β–family introduced in [Kuz10a]. The advantage of such approximations are the *semi–explicit* formulas for the running extrema under the β–family processes which enables us to produce more efficient algorithms for certain path dependent options.

1. INTRODUCTION

Schoutens and Van Damme in [SD10] explore the numerical performance of the β-family introduced by Kuznetsov (see [Kuz10a]) both in the equity and in the credit risk field. Their benchmark is the Variance Gamma (VG) process. Their conclusion is that thanks to the *semi–explicit* formulas for the running extrema under the β–family they are able to produce faster and more accurate results for pricing credit default swaps (CDS). In fact, the formulas for the running extrema are derived from explicit expressions of the Wiener-Hopf factorization. Under the VG process, the CDS are priced using a partial differential integral equation described in Cariboni and Schoutens [CS09]. The aim of this paper is to reproduce the same sort of results with respect to the Meixner model, in this case though the spread of CDS under such model will be compute by an inverse Fourier method. More precisely, the one described by Fang *et al.* in [FHOMS10] and based on the cosine series expansion of the density, which is called COS method (see [FO08] and [FO09]).

The Wiener-Hopf factorization result has lately been receiving an increasing attention for numerical purposes since the papers of [Kuz10a], [Kuz10b] and [KKP10], where a wide range of Lévy processes for which the Wiener-Hopf factorization is known are described. Together with the paper [SD10], the present work shows that there is a potential use of this result for pricing path dependent options as an alternative for the classical approaches, namely the transformation of the problem by the Kolmogorov equations into the deterministic field or use discrete inverse Fourier transform.

The β–family process is a 10 parameter family. We will fix some of their parameters to obtain a 3 parameter member whose Lévy measure has an asymptotic equivalence with the Lévy measure of the Meixner process. This member of the β–family will be called β–M process for obvious reasons. We will show that the asymptotic approximation in [SD10] and the one described here are particular cases of the more general

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technique of approximating generalized hyper-exponential Lévy processes by hyper-exponential jump-diffusion models, which was used for pricing digital options with barriers in Jeannin and Pistorius [JP10].

The paper is organized as follows. In section 2 we present the Meixner model and the β–family, we also construct the β–M process and discus the general framework of Jeannin and Pistorius [JP10]. Section 3 is devoted to derive the expressions used to price vanilla options and CDS. Section 4 will perform the numerical experiments. We will calibrate the Meixner and the β–M process to a surface of vanilla options using the Carr and Madan formula (see [CM99]). With the given optimal parameters we will calibrate the two models to a surface of CDS spreads under the Meixner model - with the COS method - and under the β–M process - with the Wiener-Hopf factorization. Finally, we will conclude the paper with some remarks.

2. THE β–FAMILY AND THE MEIXNER PROCESS

From now on we will consider $X = \{X_t \mid t \geq 0\}$ to be a Lévy process with triplet (μ, σ, ν) and hence characterized by its Lévy exponent

$$\Psi_{X_1}(z) = -i\mu z + \frac{\sigma^2}{2}z^2 - \int_{-\infty}^{\infty}(e^{izx} - 1 - izh(x))\nu(dx) \ , \tag{1}$$

where the cut–off function can be considered to be $h(x) \equiv x$ for the measures we will be looking at. Then the characteristic function for the Lévy process is

$$\varphi_{X_t}(z) = \mathbb{E}[e^{izX_t}] = e^{-t\Psi_{X_1}(z)} \ .$$

The Meixner distribution, see [Sch03], is an infinitely divisible law and thus we can associate to it a Lévy process. The characteristic function of the Meixner distribution is

$$\varphi(u) = \left(\frac{\cos(b/2)}{\cosh((au - ib)/2)}\right)^{2d} \ ,$$

where $a > 0$, $-\pi < b < \pi$ and $d > 0$. It is a process with no Brownian part and thus its Lévy triplet is given by $(\mu, 0, \nu)$ where

$$\mu = ad\tanh(b/2) - 2d\int_1^{\infty}\frac{\sinh(bx/a)}{\sinh(\pi x/a)}dx \tag{2}$$

$$\nu(x) = \frac{\exp(bx/a)}{x\sinh(\pi x/a)} \ . \tag{3}$$

To be precise, (3) is the density of the Lévy measure but we will use the same notation for the density and the measure if there is no confusion. A member of the β–family is a 10 parameter Lévy process (see [Kuz10a]) with triplet given by (μ, σ, ν) where

$$\nu(x) = c_1\frac{e^{-\alpha_1\beta_1 x}}{(1 - e^{-\beta_1 x})^{\lambda_1}}1_{x>0} + c_2\frac{e^{\alpha_2\beta_2 x}}{(1 - e^{\beta_2 x})^{\lambda_2}}1_{x<0} \ , \tag{4}$$

with $\alpha_i > 0$, $\beta_i > 0$, $c_i \geq 0$ and $\lambda_i \in (0,3)$. For the sake of completeness we reproduce here the expression of the characteristic exponent, which satisfies

$$\Psi_{X_1}(z) = -i\mu z + \frac{\sigma^2}{2}z^2 - \left[c_1 I(z;\alpha_1,\beta_1,\lambda_1) + c_2 I(-z;\alpha_2,\beta_2,\lambda_2)\right] , \tag{5}$$

where

$$I(z;\alpha,\beta,\lambda) = \begin{cases} I_1(z;\alpha,\beta,\lambda); \ \lambda \in (0,3)\setminus\{1,2\}; \\ I_2(z;\alpha,\beta,\lambda); \ \lambda = 1; \\ I_3(z;\alpha,\beta,\lambda); \ \lambda = 2 , \end{cases}$$

and

$$\begin{aligned} I_1(z;\alpha,\beta,\lambda) &= \frac{1}{\beta}\mathbb{B}\left[\alpha - \frac{iz}{\beta}, 1\lambda\right] \\ &\qquad - \frac{1}{\beta}\mathbb{B}[\alpha, 1-\lambda]\left(1 + \frac{iz}{\beta}[\psi(1+\alpha-\lambda) - \psi(\alpha)]\right) \\ I_2(z;\alpha,\beta,\lambda) &= -\frac{1}{\beta}\left[\psi\left(\alpha - \frac{iz}{\beta}\right) - \psi(\alpha)\right] - \frac{iz}{\beta^2}\psi'(\alpha) \\ I_3(z;\alpha,\beta,\lambda) &= -\frac{1}{\beta}\left(1 - \alpha + \frac{iz}{\beta}\right)\left[\psi\left(\alpha - \frac{iz}{\beta}\right) - \psi(\alpha)\right] - \frac{iz(1-\alpha)}{\beta^2}\psi'(\alpha) , \end{aligned}$$

and $\mathbb{B}(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ is the Beta function and $\psi(x) = \frac{d}{du}\log(\Gamma(u))\big|_x$ the Digamma function.

Consider a member of the β–family with a 3 parameter Lévy measure given by

$$\nu(x) = c\frac{e^{-\alpha_1 x}}{(1-e^{-x})^2}1_{x>0} + c\frac{e^{\alpha_2 x}}{(1-e^{x})^2}1_{x<0} , \tag{6}$$

where we have set $\lambda_1 = \lambda_2 = 2$, $\beta_1 = \beta_2 = 1$ and $c_1 = c_1 = c$ in the expression (4). Recall the 3 parameter Lévy measure for the Meixner model in (3) given by (a,b,d). If we let $c = ad/\pi$ in the above expression, it turns out that the Lévy measure in (3) and (6) are asymptotically equivalent at 0, in the sense that $(1-e^{-x})^2 \approx x\sinh(x)$ as $x \to 0^+$. The member of the β–family given by the Lévy measure in (6) will be called β–M process. Thanks to the exponential decay outside zero of the Lévy measure of the Meixner and the β–M process, one expects that both processes will perform equivalently when pricing market quotes.

2.1. **Financial framework.** In the equity framework we will consider that the underlying process follows an exponential Lévy process, that is

$$S_t = S_0 e^{(r-q+w)t+X_t} ,$$

where S_0 is the spot at time 0, r is the risk free rate, q is the dividend yield, ω is the mean correcting drift to ensure that the discounted prices are martingales and X_t is a Lévy process - here this will be either the Meixner or the β–M process. A key function in the following will be the characteristic function of the $\log(S_t)$. This can be derived as

$$\varphi_{\log(S_t)}(u) = e^{iu(log(S_0)+(r-q+w)t)}\varphi_{X_t}(u) \tag{7}$$
$$= e^{iu(log(S_0)+(r-q+w)t)-t\Psi_{X_1}(u)}\ , \tag{8}$$

where $\omega = \Psi_{X_1}(-i) = -\log\varphi_{X_1}(-i)$, φ_{X_t} is the characteristic function and Ψ_{X_1} is the Lévy exponent of the process.

In the setting of the credit risk we will follow a firm value approach as done in the equity world. Therefore the total aggregate asset value, V_t, of a firm follows the dynamics given by

$$V_t = V_0 e^{(r-q+w)t+X_t} = V_0 e^{Y_t}\ .$$

Notice that since X is a Lévy process so it is Y, where we only have changed the drift. We will claim that default occurs when the process reaches a certain barrier for the first time, i.e. at time

$$\begin{aligned}\tau_B &= \inf\{0 \le t \le T \mid V_t \le B\}\\ &= \inf\{0 \le t \le T \mid Y_t \le \ln(B/V_0)\}\ ,\end{aligned}$$

where B is the Barrier. Here, the barrier B will be set to $B = RV_0$ for a certain recovery rate $R \in (0,1)$.

2.2. **The running extrema under the $\boldsymbol{\beta}$–M process.** In [Kuz10a], Kuznetsov found the explicit expressions for the Wiener-Hopf factorization of the members of the $\boldsymbol{\beta}$–family and other particular processes. In a posterior work Kuznetsov [Kuz10b] and Kuznetsov *et al.* [KKP10] showed using the same sort of techniques that there exist a wide set of processes from which the Wiener-Hopf factorization is known in explicit form. These are called meromorphic Lévy processes and among other conditions the one that gives them their name is that the Lévy exponent is a meromorphic function. The Wiener-Hopf factors are essentially the characteristic function for the running supremum and infimum of a process at exponential times and for this reason are of great importance in pricing CDS. A related work of one of the authors treat the topic of inverting analytic characteristic functions in [FCU10] - the family of measures described there can be used to construct non trivial examples of meromorphic Lévy processes. Before these studies, only for a few cases the computation of the Wiener-Hopf factors were feasible, see for instance Rogers [Rog00] for one-sided Lévy process or Kou and Wang [KW03], and the references therein, for the case where the jumps are double exponentially distributed.

Let X be a $\boldsymbol{\beta}$–M process, according to [Kuz10a], for a given $q > 0$, the Wiener–Hopf factors for a $\boldsymbol{\beta}$–M process are given by the formulas

$$\Phi_q^-(z) = \frac{1}{1+\frac{iz}{\zeta_0^+(q)}} \prod_{n\geq 1} \frac{1+\frac{iz}{(n-1+\alpha_2)}}{1+\frac{iz}{\zeta_n(q)}}$$

$$\Phi_q^+(z) = \frac{1}{1+\frac{iz}{\zeta_0^-(q)}} \prod_{n\leq -1} \frac{1+\frac{iz}{(n+1-\alpha_1)}}{1+\frac{iz}{\zeta_n(q)}} ,$$

where $\zeta_n(q)$, $\zeta_0^+(q)$ and $\zeta_0^-(q)$ are the zeros of the equation $\Psi_{X_1}(i\zeta) + q = 0$ with Ψ_{X_1} being the Lévy exponent given in (5). The roots of such equation can be localized, which is very desirable result for the numerical implementation, in the intervals

$$\begin{aligned} &\zeta_0^-(q) \in (-\alpha_1, 0) \\ &\zeta_0^+(q) \in (0, \alpha_2) \\ &\zeta_n(q) \in (\alpha_2 + n - 1, \alpha_2 + n) , \quad n \geq 1 \\ &\zeta_n(q) \in (-\alpha_1 + n, -\alpha_1 + n + 1) , \quad n \leq -1 . \end{aligned}$$

It turns out that the expressions $\Phi_q^-(z)$ and $\Phi_q^+(z)$ are invertible, for instance the expression for the running infimum can be written as

$$\mathbb{P}\left[\inf_{0\leq t\leq \tau(q)} X_t > x\right] = 1 - c_0^+(q)e^{\zeta_0^+(q)x} - \sum_{n\geq 1} c_n(q)e^{\zeta_n(q)x} , \tag{9}$$

where $\tau(q)$ is an exponential distributed random variable with parameter q and

$$c_0^+(q) = \prod_{n\geq 1} \frac{1-\frac{\zeta_0^+(q)}{(n-1+\alpha_2)}}{1-\frac{\zeta_0^+(q)}{\zeta_n(q)}} , \quad c_k(q) = \frac{1-\frac{\zeta_k(q)}{(k-1+\alpha_2)}}{1-\frac{\zeta_k(q)}{\zeta_0^+(q)}} \prod_{\substack{n\geq 1\\ n\neq k}} \frac{1-\frac{\zeta_k(q)}{(n-1+\alpha_2)}}{1-\frac{\zeta_k(q)}{\zeta_n(q)}} \quad \text{for } k \geq 1 . \tag{10}$$

The derivations in [Kuz10a] show that $\mathbb{P}[\inf_{0\leq t\leq T} X_t > x]$ is the inverse Laplace transform of $\mathbb{P}[\inf_{0\leq t\leq \tau(q)} X_t > x]$, therefore one can recover the distribution of the running infimum up to a deterministic time T, i.e. we have the equality

$$\frac{d}{dx}\mathbb{P}\left[\inf_{0\leq t\leq \tau(q)} X_t \leq x\right] = \frac{d}{dx}\int_0^\infty q e^{-qt}\mathbb{P}\left[\inf_{0\leq t\leq T} X_t \leq x\right] dt . \tag{11}$$

2.3. **Hyper-exponential framework.** A Lévy process is said to a be generalized hyper-exponential process if its Lévy measure has a density which can be written as

$$k(x) = k_+(x)1_{\{x>0\}} + k_-(-x)1_{\{x<0\}} ,$$

where k_+ and k_- are completely monotone functions on $(0, \infty)$. By Bernstein's theorem on completely monotone functions, $k(x)$ must be a mixture of exponential functions, i.e. of the form

$$k(x) = 1_{x>0}\int_0^\infty e^{-ux}\mu_+(du) + 1_{x<0}\int_{-\infty}^0 e^{-ux}\mu_-(du) , \tag{12}$$

for some measures μ_+ and μ_- on $(0,\infty)$ and $(-\infty,0)$ respectively. Jeannin and Pistorius [JP10] give several examples which belong to this family, some of them are the double exponential model [KW03], the variance gamma, the Meixner process or the normal-inverse Gaussian process. For the double exponential model the Lévy measure can be written as a sum of exponentials because μ_+ and μ_- are point mass measures. The advantage of such observation is that the Wiener-Hopf factors can be computed explicitly. For the rest of the examples though a nice approximation of the density can be computed by a finite Riemann sum of (12) as

$$k(x) \approx 1_{x>0} \sum_{i\in I} \omega_i e^{-\zeta_i x} + 1_{x<0} \sum_{j\in J} \omega_j e^{-\zeta_j x} \ ,$$

where I, J are finite partitions of $(0,\infty)$ and $(-\infty,0)$ respectively, and ω_i, ω_j are weights. For instance, one could choose $\zeta_i \in [t_i, t_{i+1}]$, $\zeta_j \in [t_{j+1}, t_j]$, $\omega_i = \mu_+([t_i, t_{i+1}])$ and $\omega_j = \mu_-([t_{j+1}, t_j])$ for $i \in I$ and $j \in J$. The Wiener-Hopf factorization is known for the above processes, called hyper-exponential jump-diffusion Lévy processes, and thus an approximate price for path dependent options can be generated. This is exactly what Jeannin and Pistorius [JP10] do. A similar study from another point of view can be found in [AMP07]. The only drawback in [JP10] methodology is that the intensities of the approximation are fixed in advance and the computation of the weights are done by minimizing the square error with respect to the original measure. A more systematic approach can be found in Crosby *et al.* [CSM09]. There, the approximation is done at the level of the Lévy exponents but at the end the algorithm also approximate an infinite integral using the Gaussian quadrature. This methodology leads to a ill-posed linear problem, solved using Tikhonov regularization. The work also presents estimates for the discretization error and the truncation error. As opposite to Jeannin and Pistorius, Crosby *et al.* use an inverse Laplace approximation to price barrier options instead of using the Wiener-Hopf factorization. Our purpose is to consider the meromorphic Lévy family [KKP10] as an approximating family, the possible advantages are several. Wiener-Hopf factors seem faster and more accurate than usual approaches, a wide range of approximation techniques by complex analysis methodologies can be used and finally joint distributions for path dependent operators can be computed, see [KKP10] and [KKPvS10]. The theoretical study of this methodology is out of the scope of this paper but, in view of [SD10] and the results presented here, an investigation of how good meromorphic Lévy processes are as an approximating family is of great interest.

Now we show how the approximations of [SD10] and the one here belong to the framework of hyper-exponential jump-diffusion processes. The numerical implementation of the formulas (9) and (10) must be done by a truncation of the infinite sum and the infinite product. This means that essentially we are approximating the Wiener–Hopf factors of the process by a finite product. It turns out that this expressions for the Wiener–Hopf factors generate hyper-exponential jump-diffusion processes. Here though the particular choices of the intensities and the weights for the approximation are given by the way we approximated the Lévy measure. To show that, consider

Newton's generalized binomial theorem which sets the equality

$$(1-e^{-x})^{-n} = \sum_{k\geq 0} \binom{n+k-1}{k} e^{-kx} \qquad x \geq 0,\ n \in \mathbb{N}\,.$$

Therefore, the Lévy measure of the Meixner model is being approximated by the measure

$$\begin{aligned}\nu(x) &= c\frac{e^{-\alpha_1 x}}{(1-e^{-x})^2}1_{x>0} + c\frac{e^{\alpha_2 x}}{(1-e^{x})^2}1_{x<0}\\ &= 1_{x>0}c\sum_{k\geq 0}(k+1)e^{-(k+\alpha_1)x} + 1_{x<0}c\sum_{k\geq 0}(k+1)e^{(k+\alpha_2)x}\ ,\end{aligned}$$

whose infinite sum should be truncated for numerical implementations. The same is valid for the approximation in [SD10].

3. Calibration methods

The models presented here were calibrated to a surface of vanilla options and to a surface of credit default swaps. The calibration will be conducted with respect the mean squared error and thus with respect to the objective function

$$\text{RMSE} = \sqrt{\sum_{\text{options}} \frac{(\text{market price} - \text{model price})^2}{\text{number of options}}}\ .$$

It is worth to remark here that both models have the same number of parameters. Essentially the Meixner model is a three parameter model, since it has a given drift for a given surface of data and it is a pure jump process. For the β–M process we are going to set the volatility equal zero and because again the drift is also given so that the discounted prices are martingales, then it is also a three parameter model.

3.1. Vanilla surface calibration.

Vanilla surface calibration. One way of pricing call options is through the characteristic function of the process by the Carr and Madan [CM99] formula, the main advantage of the formula is the possibility of using the fast Fourier transform (FFT) to invert the transformation. For the sake of completeness we sketch out here the formula. The price of a call option with strike K and maturity T is

$$\begin{aligned}C(K,T) &= e^{-rT}\mathbb{E}[\max((S_T-K),0)]\\ &= \frac{e^{-rT}}{\pi}\int_0^\infty e^{-iuk}\rho(u)du\\ &\approx \frac{e^{-rT}}{\pi}\text{Real}\left(\text{FFT}\left[e^{iu_j b}\rho(u_j)\eta\left(\frac{3+(-1)^j-1_{\{j=1\}}}{3}\right)\right]_{j=1,\dots,n}\right)\ ,\end{aligned}$$

where $\alpha > 0$ is a damping factor, $n \in \mathbb{N}$, $u_j = \eta(j-1)$, $k = -b + \lambda(n-1) = \log(K)$, $\lambda\eta = 2\pi/n$ and

$$\rho(u) = \frac{e^{-rT}\varphi_{\log(S_T)}(u - i(\alpha+1))}{\alpha^2 + \alpha - u^2 + i(2\alpha+1)u} .$$

3.2. **CDS spreads calibration.** Recall the notation of Section 2.1, let us remark here that quoting CDS spreads is very similar to price digital down and out barrier options (DDOB) - or the probability of survival - as showed by the well known relation

$$c(B,T) = (1-R)\left[\frac{1 - e^{rT}\mathbb{P}[\inf_{0\le t\le T} V_t > B]}{\int_0^\infty e^{-rt}\mathbb{P}[\inf_{0\le s\le t} V_s > B]dt} - r\right] , \tag{13}$$

for a CDS spread at maturity T, barrier B and recovery rate R. Therefore, we only need how to price DDOB since the integral part in the above formula can be approximated by multi-step trapezoid rule.

For the β–M process the quantity $\mathbb{P}[\inf_{0\le t\le T} V_t > B]$ is essentially given by the formula (11) where we have to invert a Laplace transform. The general methods to invert a Laplace transform require to evaluate the transformation at complex points, this means evaluating the right hand side of expression (9) at complex points q. This expression essentially depend on $\zeta_0^+(q)$ and $\{\zeta_n(q)\}_{n\ge1}$. Unfortunately, the intervals of localization of such zeros given in Section 2.2 are only valid for $q > 0$. One way to overcome this problem is to use the Gaver-Stehfest algorithm, which was also used in [SD10]. This method only requires to evaluate the transformation at positive points.

Under the Meixner model the computation of $\mathbb{P}[\inf_{0\le t\le T} V_t > B]$ will be given by the COS method. This method is described in Fang *et al.* [FHOMS10] and based in the studies of [FO08] and [FO09]. This method is based on the fact that the Fourier cosine expansion of the conditional density for a Lévy process is close related to its characteristic function.

4. NUMERICAL RESULTS

The data set for the vanilla surface will be the one proposed in [Sch03, p. 6]. Since we already have a calibration of the Meixner model under this surface of call options (see [Sch03, p. 81]). For such data the risk free interest rate is $r = 1.20\%$, the dividend yield is $q = 1.90\%$ and $S_0 = 1124.47$. This data set was taken at the close of the market on 18/04/2002. The CDS spreads are taken from [CS09, p. 70]. We set $r = 2.24\%$, $q = 0$ and the recovery rate $R = 0.5$. This data was taken on 26/10/2004. All computations were carried out in a Intel(R) Core(TM)2 CPU 6300 at 1.86GHz with Octave.

4.1. **Equity results.** The optimal parameters for the calibration of the Meixner model and the β–M model are summarized in Fig. 1. On Fig. 2 and Fig. 3 we depicted the performance of such optimal parameters against the market data. Essentially the two models fail and success on the same regions although the calibration of the β–M model is better with respect to the RMSE error. The results are very similar to the ones in [SD10].

	β–M model (c, α_1, α_2)	Meixner model (a, b, d)
Starting values	$(0.0438,\ 11.6560,\ 4.1428)$	$(0.3977,\ -1.4940,\ 0.3462)$
Optimal parameters	$(0.0538,\ 7.9017,\ 1.7344)$	$(0.4764,\ -1.4723,\ 0.2581)$
RMSE	3.1612	3.3506
CPU(s)	120.24	42.93

Figure 1. Calibration on the vanilla surface.

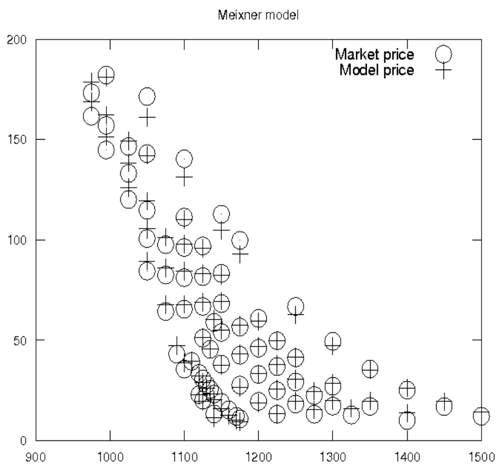

Figure 2. Meixner calibration on the vanilla surface.

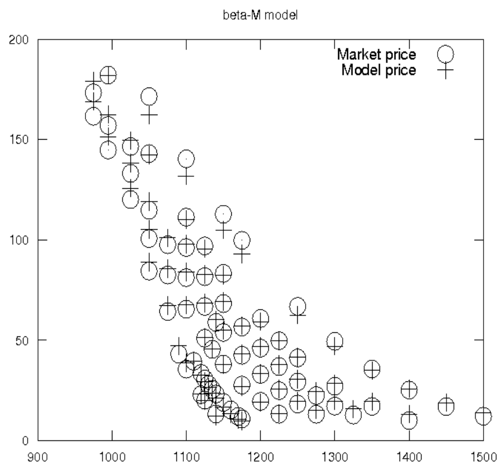

Figure 3. β–M calibration on the vanilla surface.

For the Carr and Madan formula we have set $\eta = 0.25$, $n = 4096$ and $\alpha = 1.5$ and carried out 250 iterations for a minimizing algorithm. The starting points for the Meixner model are the ones given as optimal in [Sch03]. The starting points for the β–M are the ones that make Lévy measures of both models asymptotically equivalent.

4.2. **CDS spreads results.** In this section we start our calibration with obtained parameters of the previous one. For computing the coefficients $c_0^+(q)$, $\zeta_0^+(q)$, $c_n(q)$ and $\zeta_n(q)$ of equation (9) we have computed 100 roots of the equation $\Psi_{X_1}(i\zeta) + q = 0$ and used them to compute 75 coefficients $c_n(q)$, therefore we have discretized (9) by a sum of 75 terms. Finally the integral (11) was discretized following a Gaver-Stehfest algorithm by a sum of 8 terms while the integral in (13) was discretized by the trapezoid rule with 360 steps. Again we have minimize the square error iterating 250 times

Company		1 year	3 years	5 years	7 years	10 years
General Elec.	Market	5	14	25	29	36
	β-M	6	13	25	29	35
	COS	5	15	24	30	36
General Motors	Market	86	157	207	229	242
	β-M	88	162	206	230	239
	COS	80	159	208	229	238
Whirlpool	Market	16	36	66	73	86
	β-M	17	35	67	75	83
	COS	14	40	62	76	85
Walt Disney	Market	6	21	36	45	56
	β-M	6	24	37	47	55
	COS	6	21	36	46	55
Eastman Kodak	Market	54	86	127	143	157
	β-M	50	87	126	142	157
	COS	44	92	126	143	153

Figure 4. Calibration on CDS spreads.

a minimizing function. Fig. 4 shows the spreads of both models in comparison with market data.

In Fig. 5 we summarize the resulting coefficients. The results are again very similar to the ones presented in [SD10]. It turns out that the approximation with the β–M performs better that the Meixner model. The time of computation though is much more greater. As commented in [SD10], the Wiener-Hopf approach algorithm spends most of the time in computing the roots $\zeta_n(q)$. Because these are localized, the computation for a single root is fast and hence allowing a parallel computing implementation of the same algorithm would speed the process by a factor of 100, and therefore outperforming the COS method.

It is worth to remark that our computations show that the square error is lower when the spread curve to be calibrated is flatter. This featured is not observed in the implementation of [SD10]. In Fig. 6 we have depicted the square error of the β-M process and β-VG process with respect to the difference of the spreads at the initial maturity and at the final maturity. In Fig. 7 we depicted the spreads curves.

5. Conclusion

We have showed that the β–M is a good approximation for the Meixner model and derived a fast and accurate algorithm to price CDS based on the Wiener-Hopf factorization of the process. We have showed that the approximations of [SD10] to the variance gamma process and the one made here for the Meixner model are particular cases of the more general framework of hyper-exponential jump-diffusion processes. Together, the results suggest that the Wiener-Hopf approach perform better than the

Company	β-M / COS	c / a	α_1 / b	α_2 / d	RMSE (bps)	CPU (s)
General Elec.	β-M	0.0673	12.1249	6.2399	0.5161	8240.8
	COS	0.2983	-0.4972	0.4299	0.8406	629.2
General Motors	β-M	0.1356	8.0528	4.0011	2.8248	8660.7
	COS	0.9106	0.2355	0.1737	3.2221	633.9
Whirlpool	β-M	0.0728	5.6308	5.5544	1.9191	8152.3
	COS	0.4392	0.0318	0.3507	2.9893	640.1
Walt Disney	β-M	0.0695	6.4666	6.2615	1.6712	8149.5
	COS	0.3597	0.0127	0.4087	0.7459	681.8
Eastman Kodak	β-M	0.1421	12.2455	5.4404	2.1331	8669.7
	COS	0.7093	0.1401	0.2046	5.4497	684.9

Figure 5. Calibration on CDS spreads.

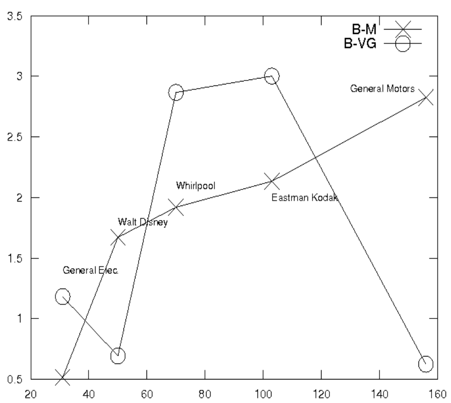

Figure 6. RMSE with respect to difference of the spreads at the initial maturity and at the final maturity.

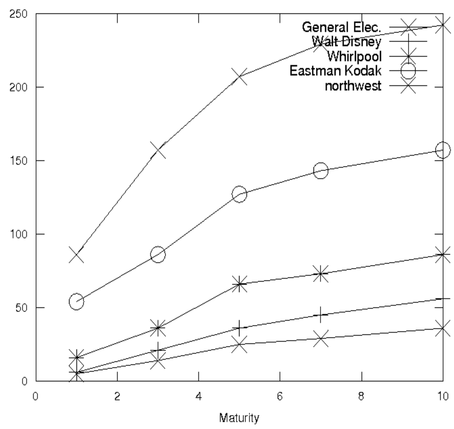

Figure 7. CDS spreads surface.

general methodologies for pricing DDOB options. Despite the two results are based on members of the β–family, what really makes the Wiener-Hopf approach possible is the fact that the β–family belongs to the more general family of meromorphic Lévy processes.

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