Mathematics II 2014/2015

Code: 102344
ECTS Credits: 6

Degree	Type	Year	Semester
2501572 Administració i Direcció d'Empreses	FB	1	2
2501573 Economia	FB	1	2

Contact

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Use of languages

Principal working language: català (cat)
Some groups entirely in English: Yes
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: Yes

Prerequisites

To follow properly this course, it is necessary a correct handle of the basic mathematical concepts and tools, as well as of the fundamental notions of continuity, differentiability, and graphical representation of real functions of one real variable studied in the course of Mathematics I.

Objectives and Contextualisation

This course introduces students to the study of linear algebra and functions of several variables, with emphasis on their applications in economics. Students should not only acquire and assimilate new mathematical knowledge, but also be able to apply them in quantitative analysis in economics and business.
Therefore, the purpose of the course is that students become familiar with basic mathematical concepts to be used in the study of economic theory and analysis.

Specifically the objectives are intended to achieve are:

1. To familiarize students with the n-dimensional Euclidean space.
2. Working with determinants and matrices.
3. Solving systems of linear equations.
4. Understanding the functions of several variables and their role in more complex economic models. 5. Geometric representation of functions of two variables using contour maps.

6. Understand the concepts of limit of a function at a point and of a continuous function.
7. Understanding the Weierstrass theorem.
8. To familiarize students with the partial derivatives of functions of several variables and the concept of differentiability. 9. Using partial derivatives to obtain the slope of the contour at one point and to perform comparative statics exercises. 10. Solving optimization problems without constraints and with equality constraints.

Skills

Administració i Direcció d'Empreses

- Apply mathematical instruments to synthesise complex economic-business situations.
- Capacity for oral and written communication in Catalan, Spanish and English, which enables synthesis and oral and written presentation of the work carried out.

- Demonstrate an understanding of mathematical language and some methods of demonstration.
- Organise the work in terms of good time management, organisation and planning.
- Use of the available information technology and adaptation to new technological environments.

Economia

- Apply mathematical instruments to synthesise complex economic-business situations.
- Capacity for independent learning in the future, gaining more profound knowledge of previous areas or learning new topics.
- Demonstrate an understanding of mathematical language and some methods of demonstration.
- Demonstrate initiative and work individually when the situation requires it.
- Organise the work in terms of good time management, organisation and planning.
- Select and generate the information necessary for each problem, analyse it and take decisions based on that information.
- Use of the available information technology and adaptation to new technological environments.

Learning outcomes

1. A capacity of oral and written communication in Catalan, Spanish and English, which allows them to summarise and present the work conducted both orally and in writing.
2. Analytically consider and solve optimisation problems in the context of the economy.
3. Apply the inverse function and implicit function theorems to specific problems.
4. Calculate and study the extrema of functions.
5. Calculate derivatives of functions using the chain rule, the implicit function theorem, etc.
6. Calculate determinants and decompositions of matrices.
7. Capacity to continue future learning independently, acquiring further knowledge and exploring new areas of knowledge.
8. Classify matrices and linear applications according to different criteria (rank, diagonal and Jordan).
9. Demonstrate initiative and work independently when required.
10. Know the basic results of differential calculus on different real variables.
11. Organise work, in terms of good time management and organisation and planning.
12. Solve and discuss linear equation systems.
13. Use available information technology and be able to adapt to new technological settings.
14. Work with different finite-dimensional bases of vector spaces.

Content

Topic 1: The n-dimensional Euclidean space

Objective: To explain the n-dimensional Euclidean space and the basic operations on vectors (sum of vectors, multiplication by scalars, scalar product of two vectors), the notions of norm and Euclidean distance and the basic properties of sets (open, closed, bounded, compact and convex).

1.1. Vectors.
1.2. Sum of vectors.
1.3. Multiplication by scalars.
1.4. Scalar product of two vectors.
1.5. Properties of basic operations and their geometric interpretations. 1.6. Euclidean norm and its properties.
1.7. Euclidean distance and its properties.
1.8. Open and closed sets.
1.9. Bounded and compact sets.
1.10. Convex sets.

Case study: Budget constraints. Textbook: Sections 12.2, 12.3 and 12.4.

Topic 2: Linear algebra

Objective: To introduce the basic notions of linear algebra: linear dependence and independence on vectors, matrices, determinants, inverse of a matrix and systems of linear equations.

2.1. Linear dependence and independence on vectors.
2.2. Matrices, determinants, inverse of a matrix and rank.
2.3. Rouché-Frobenius theorem.
2.4. Solving systems of linear equations: Cramer's rule and Gauss' system.

Case study: Calculation of determinants, inverse of a matrix and solutions of systems of linear equations.

Textbook: Chapters 12, 13 and 14.

Topic 3: Functions of several variables, geometric representation, and contour maps

Objective: Introduce the functions of several variables as a tool to describe relationships among economic variables. Geometric representation of functions of several variables as level curves and contour maps.

3.1. Functions of several variables.
3.2. Component functions.
3.3. Domain of the function.
3.4. Linear functions and associated matrices. 3.5. Level curves.

3.6. Contour maps.

Case study: Production functions, utility functions and wage functions. Cobb-Douglas functions, linear, quasi-linear, and Leontiev functions.

Textbook: Chapter 15.

Topic 4: Limit of a function at a point and continuity. Weierstrass theorem

Objective: Define the limit of a function at a point (and study their basic properties), the directional limits and continuous

functions (and study their properties). Present the Weierstrass theorem and provide the geometric properties of the constrained optima from the graphical representations of the constraint and the contour map of the function.

4.1. Limit of a function at a point.
4.2. Properties of the limit of a function at a point. 4.3. Directional limits.
4.4. Concept of a continuous function.
4.5. Continuous functions and directional limits. 4.6. Weierstrass theorem.

Case study: Obtention through geometric methods of the maxima of several functions of interest in economics and business.

Textbook: Sections 4.4, 6.1, 6.2 and 17.3.

Topic 5: Partial derivatives and differentiable functions

Objective: Define the directional and partial derivatives of functions of several variables, interpretation of their meaning. Introduce the gradient vector of a function at a point and its geometric interpretation as the growth of the function at the point. Define the Jacobian matrix. Define the differential of a function at a point and the differentiable functions. Introduce the chain rule, higher order derivatives, the Hessian matrix, and Schwartz's theorem.

5.1. Derivative of a function at a point in the direction of a unit vector.
5.2. Partial derivatives at one point.
5.3. Partial derivative functions.
5.4. Gradient of the function at one point. Geometrical interpretation and directional derivatives. 5.5. Jacobian of the function at one point.

5.6. Differentiable functions. Continuity of partial derivative functions. 5.7. Chain rule.
5.8. Higher order derivatives, Hessian matrix and Schwartz's theorem.

Case study: Obtention and geometric interpretation of the gradients of different functions of interest in economics and business.

Textbook: Chapters 15 and 16.

Topic 6: Implicit function theorem and inverse function theorem

Objective: Introduce the implicit function theorem and the inverse function theorem as instruments to obtain the linear approximation to the level curve of a function at a point and to perform comparative statics exercises respectively.

6.1. Implicit function theorem.
6.2. Inverse function theorem.
6.3. Applications and geometric intuition.

Case study: Obtaining the slope of the level curve of several functions of interest in economics and business. Exercises in comparative statics in several basic economic models.

Textbook: Section 16.3.

Topic 7: Unconstraint optimization

Objective: Introduce the fundamental techniques to optimize functions of several variables and introduce the concepts of concave and convex functions.

7.1. Quadratic forms.
7.2. Local and global optima.
7.3. First and second order conditions for local optima. 7.4. Global optima of concave and convex functions.

Case study: Identification of extreme points. Study of the concavity and convexity of several functions of interest in economics and business.

Textbook: Chapter 17.

Topic 8: Constrained optimization

Objective: Introduce the fundamental techniques for optimizing functions of several variables with equality constraints. Introduce the Lagrange theorem.

8.1. Maximization and minimization problems with equality constraints. 8.2. Constrained local optima. Lagrange's Theorem.
8.3. Constrained global optima of concave and convex functions.
8.4. Introduction to inequality constraints.

Case study: Solving optimization problems in economics and business. Textbook: Chapter 18.

Methodology

To achieve the objectives of the course, the following taxonomy of activities will be used:

1. Theory classes where teachers develop the main concepts.

The objective of this activity is to present the fundamental notions of course, and to facilitate their learning through the analysis of examples illustrating the intuitions and economic applications.

2. Exercises sessions devoted to the resolution of problems.

This activity aims to discuss and answer any questions that students may have in solving the problem sets, and at the same time to correct mistakes. These sessions will also stimulate the participation of students presenting the solutions of the problem sets either orally or in written form.

3. Organized supervised activities, to apply the concepts studied to economic situations

The objective of this activity is to encourage the student to establish links between the mathematical tools and their use in economics. When possible, these sessions will be organized in small groups of students.

4. Problem solving by students

Each topic will have a list of associated problems that must be solved independently by students.
The objective of this activity is two-fold: on the one hand it aims at the reinforcement of the theoretical concepts and tools exposed in the theory sessions; on the other hand it aims at the acquisition of the skills required to solve exercises and problems.
We promote the cooperative resolution of problems in stable working groups of 3 or 4 students throughout the semester, to stimulate team work to overcome the difficulties that may arise to their components.

5. Tutorial attendance

Students have several hours where the teachers of the course may help them to resolve any doubts that may arise in the study of the course and in the solution of the problem sets. These sessions cannot be on-line, but face-to-face between the teacher and the students.

Activities

Title	Hours	ECTS	Learning outcomes
Type: Directed			
Preparing and solving exercises	15	0.6	3, 5, 6, 4, 1, 7, 8, 10, 9, 11, 2, 12, 14, 13
Theory classes	30	1.2	3, 5, 6, 4, 8, 10, 2, 12, 14
Type: Supervised			
Follow-up of homework	3	0.12	3, 5, 6, 4, 1, 7, 8, 10, 9, 11, 2, 12, 14, 13
Tutorships	7	0.28	3, 5, 6, 4, 1, 7, 8, 10, 9, 11, 2, 12, 14, 13
Type: Autonomous			
Study	90	3.6	3, 5, 6, 4, 7, 8, 10, 9, 11, 2, 12, 14, 13

Evaluation

The course evaluation will be done continuously through a series of partial evaluation activities, and a final exam. The weight of each of the above components in the calculation of the final grade is as follows: - At least 60% for the final exam - At least 20% for the remaining part of assessment activities.

Partial evaluation activities: At least there will be a partial exam. If there is only one activity, it will be a compulsory exam that will have a maximum time resolution of 90 minutes. Other evaluation activities can be envisaged. None of the assessment activities will liberate topics for the final exam.

Final Exam: It is a comprehensive exam of all the topics of the course. The exam is designed to force students to make a last effort of learning. Such effort is necessary to consolidate previously acquired knowledge. The

maximum resolution time is 3 hours. All students are required to take the final exam and participate in all the other assessment activities on the dates indicated in the academic calendar. No evaluation activities will be programmed outside of the dates indicated.

If using the weights mentioned above a student's grade is 5 or higher, the course will be considered as passed and it can not be the subject of a new assessment. In the case of a grade below 4, the student must retake the course the following year. For those students who have obtained a grade equal to or greater than 4 and less than 5 there will be a re-evaluation. Teachers will decide the design of the re-evaluation. The re-evaluation is scheduled in the last week of the semester. Its grading will be qualitative and only admit two possible outcomes: pass or fail. A student obtaining a grade of PASS is considered to have passed the course with a maximum numerical grade equal to 5. If the student receives a grade of fail, fail the course and the final grade will be equal to the grade obtained before the re-evaluation.

A student is considered "no-show" inthe subject only if has not participated in any of the evaluation activities. Therefore, the participation in a single of the several activities eliminates the no-show outcome.

Code of honor: Without prejudice to other disciplinary action deemed appropriate and in accordance with current academic standards, any irregularity committed by the student that may lead to a change in the qualification of an act of assessment will convey a grade of zero. Therefore, copying or allowing to copy in any assessment activity will involve suspending it with a zero. Also, if passing such activity is necessary to pass the course, the entire course will be graded as fail. The activities failed due to violations of the code of honor will not be recoverable by the assessments described and the course will be graded as fail directly without the opportunity to recover in the same academic year.

Evaluation activities

Title	Weighting	Hours	ECTS	Learning outcomes
Final exam	at least 60%	3	0.12	3, 5, 6, 4, 8, 10, 2, 12, 14
Partial assessments	at least 20%	2	0.08	3, 5, 6, 4, 1, 7, 8, 10, 9, 11, 2, 12, 14, 13

Bibliography

Main textbook:
Sydsaeter, K. and P.J. Hammond, 1995, Mathematics for Economic Analysis. London, Prentice Hall.

This is a textbook of great tradition and acceptance. In addition, this book also covers the subjects of Mathematics I. It is a complete and friendly text, including economic applications in all its chapters.

Complementary textbooks:

The textbooks listed below can be helpful to complement the explanations contained in the main textbook and also to students wishing to enlarge their knowledge.

Alegre, P., L. Jorba, F.J. Orti, G. Rodriguez, J.B. Saez, T. Sancho and A. Terceño, 2000, Ejercicios Resueltos de Matemáticas Empresariales II, Madrid, Alfacentauro.

Besada, M., F.J. García, M.A. Mirás and M.C. Vázquez, 2001, Cálculo de varias variables. Cuestiones y ejercicios resueltos, Madrid, Ed. Prentice Hall.

Chiang, A.C., 2005, Fundamental Methods of Mathematical Economics, McGraw-Hill. Larson, R., R. Hostetler, and B. Edwards, 1994, Calculus with Analytic Geometry, Lexington,

D.C. Heath.
Sydsaeter, K. and P.J. Hammond, 2002, Essential Mathematics for Economic Analysis. London, Prentice Hall.
Other complementary material will be uploaded in the webpage of the course.