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# Stochastic Optimal Bid to Electricity Markets with Environmental Risk Constraints

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MSc Thesis

# Stochastic Optimal Bid to Electricity Markets with Environmental Risk Constraints

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[...] I salute the efforts and agreements being made (in favor of sustainable development) [...] But we must realize that the water crisis and the aggression to the environment is not the cause. The cause is the model of civilization that we have created. And the thing we have to re-examine is our way of life [...] Development cannot go against happiness. It has to work in favor of human happiness, of love on Earth, human relationships, caring for children, having friends, having the basics. Precisely because this is the most precious treasure we have: happiness. When we fight for the environment, we must remember that the first element of the environment is called human happiness. [a]

[a]Speech given by the current President of Uruguay, José Mujica for the "RIO+20 United Nation Conference on Sustainable Development" held in Rio de Janeiro, Brazil, on June 20-22, 2012.

# Abstract

**Keywords:** CEaR · CVaR · Electricity market · Emission allowances · Emissions Trading Scheme · Optimal bid · Stochastic Programming · Risk

There are many factors that influence the day-ahead market bidding strategies of a generation company (GenCo) in the current energy market framework. Environmental policy issues have become more and more important for fossil-fuelled power plants and they have to be considered in their management, giving rise to emission limitations. This work allows to investigate the influence of both the allowances and emission reduction plan, and the incorporation of the derivatives medium-term commitments in the optimal generation bidding strategy to the day-ahead electricity market. Two different technologies have been considered: the coal thermal units, high-emission technology, and the combined cycle gas turbine units, low-emission technology. The Iberian Electricity Market and the Spanish National Emissions and Allocation Plans are the framework to deal with the environmental issues in the day-ahead market bidding strategies. To address emission limitations, some of the standard risk management methodologies developed for financial markets, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), have been extended. This study offers to electricity generation utilities a mathematical model to determinate the individual optimal generation bid to the wholesale electricity market, for each one of their generation units that maximizes the long-run profits of the utility abiding by the Iberian Electricity Market rules, the environmental restrictions set by the EU Emission Trading Scheme, as well as the restrictions set by the Spanish National Emissions Reduction Plan. The economic implications for a GenCo of including the environmental restrictions of these National Plans are analyzed and the most remarkable results will be presented.

The work in this MSc Thesis was carried out in the Research Group GNOM (Group on Numerical Optimization and Modeling) in the Department of Statistics and Operations Research, UPC, Barcelona, under MICINN Project DPI2008-02153, *Short- and Medium-Term Multimarket Optimal Electricity Generation Planning with Risk and Environmental Constraints* of the Spanish Ministry of Science and Innovation. The data of the generation units used in this project was gently provided by the Spanish utility Gas Natural Fenosa.

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# Contents

# Chapter 1
# Introduction

Due to the constantly increasing pollution of earth's atmosphere, in recent years emission control has become a matter of paramount importance. Nowadays world energy systems are highly dependent on fossil fuels (such as coal, oil and gas formed from the organic remains of prehistoric plants and animals). Fossil fuels share in world energy production is more than 85% and in electricity generation more than 60% [**1**]. Although they provide a reliable and affordable source of energy, the use of fossil-fuelled power plants harm the global ecosystem by emitting into the atmosphere noxious gases and toxic substances, causing the greenhouse effect, which is thought to be responsible for climate change.

One of the major international instruments to address this problem is the Kyoto Protocol, which seeks ways to stabilize the greenhouse concentration in the atmosphere. The Kyoto Protocol provides a framework for combating climate change: sets legally binding limits on greenhouse gas emissions and envisages novel market-based mechanisms for achieving cost-effective emission reduction. The European Community, abiding with the Kyoto Protocol, was committed to reduce the aggregated emissions of Greenhouse Gasses by 8 %, compared to 1990 levels, in the period 2008-2012, with different targets set among Member States [**2**]. The European Union (EU) directive for greenhouse gas emission allowance trading (2003/87/CE) [**3**] establishes that each member has to elaborate the National Allocation Plan (NAP) to determine the total quantity of carbon dioxide ($CO_2$) emissions that Member States grant to their companies, which can then be sold or bought by the companies themselves. Emissions trading, as set out in Article 17 of the Kyoto Protocol, allows countries that have emission units to spare - emissions permitted but not "used" - to sell this excess capacity to countries that are over their targets.

The EU Emission Trading Scheme [**3**] (EU ETS) is a cornerstone in the fight against climate change and the first international trading system for $CO_2$ emissions in the world. The aim of the EU ETS is to help EU Member States achieve compliance with their commitments under the Kyoto Protocol. Emissions trading does not imply new environmental targets, but allows for cheaper compliance with existing targets under the Kyoto Protocol.

Generation companies are subject to other environmental limitations besides the aforementioned $CO_2$ emission allowances. The EU also sets limits for emissions of pollutants from large combustion plants (Directive 2001/80/EC [**4**]). This Directive applies to combustion plants (technical apparatus in which fuels are oxidized in order to use the heat thus generated) with a rated thermal input equal to or greater than 50 MW, irrespective of the type of fuel used (solid, liquid or gaseous). Its purpose is to limit the amount of sulphur dioxide ($SO_2$), nitrogen oxides ($NO_x$) and dust emitted form large combustion plants each year. Following this commitments, the Spanish public administration launched in 2004 the Spanish National Emissions Reduction Plan (NERP, Real Decreto 430/2004 [**2**]).

The impact of both the National Allocation and the Emissions Plans on the power industry appears to be very significant and whether these new restrictions are an opportunity or a threat for the power industry depends on several factors, especially the strategies set by power companies to integrate these new restrictions in their energy's market bid strategy. In this regard, NAP and NERP has to be necessarily considered in the elaboration of the generation units optimal sale bid to the wholesale electricity market.

This MSc thesis develops several topics related with the research project DPI2008-02153, *Short- and Medium-Term Multimarket Optimal Electricity Generation Planning with Risk and Environmental Constraints*[1] of the *Spanish Ministry of Science and Innovation*. The main objective of this project is to study and develop stochastic optimization models and algorithms that help electrical utilities to optimize the physical and financial electricity transactions decisions in a multimarket context, taken into account hedging and environmental constraints.

## 1. Objectives and Contribution of the Thesis.

The scientific aim of this thesis is to investigate the impact of NAP and NERP in the optimal operation of a generation company (GenCo) that operates in the day-ahead Iberian Electricity Market. This aim is met through accomplishing the following two specific objectives:

- Regarding the NAP, the first purpose of this thesis is to find the generation scheduling and sales bid of each one of the generators that maximize the expected value of the net profit of a Genco including the incomes/costs of the $CO_2$ allowances in the emissions' rights market.

- Regarding the NERP, this thesis aim to assess how emission limits may influence the generation scheduling and profits. The motivation for this second objective has been derived from the efforts to reduce negative trends in climate change.

The tool to achieve this two objectives is the stochastic programming model for the optimal generation bid in electricity markets developed in [**5**] . This model allowed to find the optimal generation bid to the electricity market for a set of thermal and combined cycle generation units, but without any environmental consideration. In order to extend this model with the environmental issues of the NERP and NAP regulation, the following methodology was adopted:

- Emission-bounded model: a first model was developed where amount of the NERP emission limits were introduced as a set of emission bounding constraints to the $SO_2$ and $NO_x$ released by the generation units. Also, in this model, the NAP regulation was considered as a new financial term in the profit function. This is the

[1] `gnom.upc.edu/projects/energy/dpi2008-02153`

model described in Chapter 3 of this thesis and it has been published in [**6**].

- Risk-constrained model: secondly, the precedent emission bounded model was improved through the explicit consideration of both the financial risk of the profit function as well as the risk of the NERP emissions limit violation, giving rise to the risk-constrained model presented in Chapter 4 of this thesis.

The specific contribution of this thesis to each one of the two precedent models is:

- Regarding the emission-bounded model [**6**], the reformulation of the unit commitment constraints of the CC units (Section 4.4. and 4.5 of Chapter 3). This reformulation was yet introduced in [**5**] for the case of the thermal units but it has been extended in this thesis to the case of the Combined Cycle units, which are much more difficult to formulate than the classical thermal generation units. It is known that this formulation improves the computational performance of the unit commitment problems [**7**].

- Regarding the risk-constrained model, the incorporation to the model of:

  - The NERP emission limits through a new measure of risk called Conditional Emission-at-Risk (CEaR) and
  - Risk-aversion constraints to limit the financial risk of the market through a classical CVaR formulation.

- The computational implementation and solution of the precedent models for a set of real-case electricity market problems.

- The analysis of the results for both the emission-bounded and risk-constrained models.

Besides the scientific objectives and contributions of this thesis, there is also several personal reasons that motivated the author to undertake this project:

- To understand the organization and operations of electricity and $CO_2$ rights markets and their mathematical modelization as stochastic programming problems.

- To explore several very important topic related to the inclusion of environmental restrictions into the stochastic optimal generation bid models, in order to understand how to best accommodate and environmental friendly compromise of the electrical utilities with the economical objectives of the electricity industry.

- To understand the environmental issues related with the electricity generation and their mathematical modelization within stochastic programming problems.

- To understand the basis of the risk analysis methodology based on VaR and CVaR measurements and their application to real energy market problems.

- To get introduced in the research methodology supporting the elaboration of scientific publications.

- To gain experience in the numerical solution of real-life large scale stochastic optimization problems (with real data from the Iberian Electricity Market) and in the analysis of its solutions.

## 2. Structure of the thesis

The thesis is organized as follows:

- In Chapter 2 the Iberian Electricity Market, and the allowances and emission reduction plan are described. Their special characteristics, which are needed as background for the comprehension of the further analysis, are highlighted.

- Chapter 3 presents a detailed problem description, as well as the stochastic programming model proposed to cope with the optimal generation bid to the next day auctions of the Iberian Electricity Market (IEM) day-ahead market (DAM) taking into account, according to both National Allocation and Emissions Plans, the $CO_2$ allowances and the $SO_2$ and $NO_x$ emission constraints. This is the called emission-bounded model.

- The model developed in the precedent chapter satisfies the emission limits by imposing explicitly the NERP limits to each one of the scenarios. This methodology, although valid, may seem to be quite restrictive as it forces the optimal bid to abide by the NERP rules even in the most extremes (less likely) scenarios. Regarding the modelization of the $CO_2$ allowances, the simple maximization of the incomes from the $CO_2$ rights market is not taking full advantage of the probabilistic information that the scenario tree contains. In Chapter 4 some general risk management ideas are presented, as well as a detailed description of how to incorporate, using the risk management methodology, the $SO_2$, $NO_x$ and $CO_2$ risk emission and financial constraints into the model for the optimal electricity market bid problem. A detail case study is solved and analyzed.

- In Chapter 5 the final conclusions of this work are presented.

- Chapter 6 offers some possible further developments.

- Finally the appendix provides a glossary where the acronyms and symbols used in this project are described.

It is worth mentioning that the emission-bounded model presented in Chapter 3, developed in collaboration with prof. F.-Javier Heredia and prof. Cristina Corchero, was presented in the 9th International Conference on the European Energy Market (EEM12, Florence, Italy, May 2012, `http://eem12.org/` ). This work has also been published as a full length paper in the proceedings of the conference published by the Institute of Electrical and Electronics Engineers, IEEE ([**6**], `http://dx.doi.org/10.1109/EEM.2012.6254676`).

# Chapter 2
# Context: Energy Markets and Environment

In recent years electricity markets in Europe experienced a big transformation. From being a regulated market with no or very low uncertainty in future earnings, the market is now liberalized and deregulated. Electricity prices are no longer determined by the regulator, but by the market. As a consequence the price of electricity has become a significant risk factor because is unknown at the moment when the generation companies has to take the operational decisions. Although the liberalization of the electricity markets brings along a lot of risks for the generation companies, also plenty of possibilities and chances. The uncertainty is however not necessarily negative for electricity producers. Flexible generation companies can take advantage of volatile prices. One of the keys to success in the liberalized market is the ability to manage these new risks.

This chapter introduces the general framework for this project and presents the special characteristics of Iberian Electricity Market, along with a summery of the current regulation imposed in the allowances and emission reduction plans. Also sets out a detailed description of the problem and introduce the main operational characteristics of the two kind of generating technologies considered in this study: thermal units and combined cycle gas turbine units.

## 1. The Iberian Electricity Market (IEM)

An important factor to determine the efficiency of electricity markets is the specific market structure and trading rules, such as regulations, applied in each specific market. The Iberian Electricity Market (IEM) is the result of a joint initiative of the Governments of Portugal and Spain to integrate their markets. The generation companies have to make daily bids to sell its electricity through the wholesale market, while distribution companies perform an energy demand. This market is organized by the Electricity Market Operator (OMEL, by its Spanish initials) who has to match supply with demand in real time. Nowadays the day-ahead market (DAM) (short-term mechanism) is the market where the most important part of the electricity demand is negotiated (78% in the case of the IEM), explaining why finding the optimal bid to the DAM is of utmost significance in the daily operation of any GenCo. However DAM is not only the main physical energy market of the IEM, in terms of the amount of traded energy, but also the mechanism thought which other energy

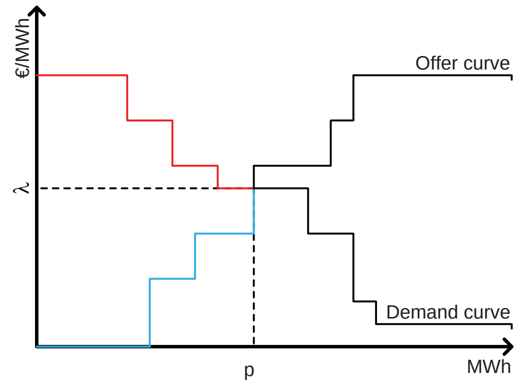


FIG. 1. Market clearing for a certain hour: intersection of the aggregated offer and demand curves.

products, as bilateral and futures contracts (medium term mechanisms), are integrated into the energy production system.

The **DAM** of day D consists of a series of twenty-four hourly auctions which are cleared simultaneously between 10:00h and 10:30h of the previous day (D-1). Selling and buying agents must submit their sale/purchase bids to each auction before 10:00h of D-1. Both sale and purchase bids are composed of up to 24 price-energy pairs with non-increasing price values, and each agent is unaware of the bids of the other agents. The clearing price $\lambda_t^D$ of each hourly auction for time $t$ is determined by the intersection of the aggregated offer and demand curves: Fig. 1 . All the sale/purchase bids with a lower/greater bid price are matched and will be remunerated at the same clearing price $\lambda_t^D$, whichever the original bid price.

**Bilateral contracts** (BC) are agreements between a GenCo and a qualified consumer to provide a given amount of electrical energy at a stipulated price along with a delivering period. The agreements terms, namely: the energy, the price and the delivered period, are negotiated several days before the DAM, and the energy that is destined to the BC cannot be included in the DAM. Moreover, accordingly to the IEM rules, the DAM bid of each unit must include the whole available energy not allocated to the BC. This fact makes the optimal sale bid and the optimal BC's dispatching mutually dependents, coupling both problems. From the point of view of the GenCo, a BC represents a scheduled load curve to be delivered, chargeable at a fixed price, that has to be optimally dispatched among the GenCo's units.

A **future contract** (FC) is an exchange-traded derivative that represents agreements to buy/sell some underlying asset in the future at a specified price [**8**].The DAM's operator demands every GenCo to commit the quantity designed to each FC through the DAM bidding of a given sets of generation units. This commitment is done through a sale offer with a bid price of 0€/MWh, the so called *price acceptance offer*. All instrumental price offers will be matched (i.e. accepted) in the clearing process, i.e., the energy shall be produced and will be remunerated at the DAM spot price.

Some medium-term products, as for instance the futures or the bilateral contracts, are used for hedging the market-price risk mentioned above. Therefore, a generation company operating in such a complex market can no longer find its optimal bid, without considering the relationship between the short-term bid and the medium-term physical products.

# 2. European Emission Trading Scheme (UE ETS) and Spanish National Allocation Plan (NAP)

The threat of large-scale climate change and global warming has led to the Kyoto Protocol [**9**]. According to its guidelines, industrialized countries must reduce emissions of six greenhouse gases, among which $CO_2$ is the most important. In order to facilities developed countries to achieve parts of their emission reduction commitments, Kyoto protocol envisages three market-based mechanisms, one of them being Emissions Trading Scheme ETS.

The ETS is divided in two phases. The first phase runs from 2005 to 2007 and the second from 2008 to 2012. During each phase, installations have to measure and report their emissions at the end of each year. Each allowance corresponds to the right to emit one ton of $CO_2$ equivalent. If a company under-uses its allowances in one year, it can save, i.e. bank, them and use in the future (either to emit $CO_2$ or to sell then on the market). The banked allowances, however, have to be used within the compliance period, that have been set to 3 and 5 years (to help companies to smooth out eventual annual imbalances due to weather, maintenance,and other factors), respectively for each one of the two phases. Otherwise, if the company has over-used its annual quota during a year, it can balance it either by reducing emissions during other years in the compliance periods, or by buying them on the market. There is no possibility to carry over unused allowance from one to another compliance period, or to borrow them from a future compliance period.

Each member state in the EU decides on its allocation policy, this means that each country can decide, through its National Allocation Plan, which of its industry sectors will have to curtail $CO_2$ emissions and by what amount. The electricity sector is one of the major sources of $CO_2$ emissions, explaining why generating units are included in the EU ETS mechanism. Under this emission trading scheme a specified amount of emission allowances are allocated to various industrial installations, including generators. These allowances can be used either for producing corresponding amounts of $CO_2$ or traded in the market [**10**]. In the case that the total emission over a monitored period exceeds defined emission allowances, the GenCo has an option to either buy additional allowances on the market or pay a relatively high penalty. Although the allocation procedures are specific for each EU country, all of them need to reduce emissions and will consequently have to limit the amount of allowances that can be allocated.

The Spanish NAP for the period 2008-12 (approved on 2007 (RD 1402/2007 [**11**])) imposed to the electricity generation sector a reduction in the $CO_2$ emissions for the period 2008-12 of almost a 60% with respect to the emissions in the period 2000-05. Since generation power outputs are bounded by the $CO_2$ allowances, it is becoming increasingly important for generation companies to manage their allocations in the most profitable way and decide when and how much of permissions to spent to produce electricity.

# 3. Spanish National Emission Reduction Plan (NERP)

In addition to developing United Nations commitments in relation to protection of atmospheric environment, the EU envisages a Community Strategy to combat acidification within the EU. One of the main objectives of the strategy is ”not to exceed, at any time, critical loads and levels” of certain acidifying pollutants such as $SO_2$ and $NO_x$, so that both, people and ecosystems are protected effectively against the risks of air pollution. In line with this strategy the Spanish NERP [**2**] imposes, for the period 2008-15, a global reduction of 81% and 15% for the $SO_2$ and $NO_x$ emissions respectively, compared to the emissions in 2001. These measures are intended to improve the environmental conditions under atmospheric international commitments on pollution and the EU Strategy to combat acidification.

It is worth mentioning that technological improvements made to reduce the emission of pollutants involved in NERP, were taken into account for the allocation of NAP 2008-2012. This shows the interrelationship between the different measures taken to control emissions.

# 4. Generation Units

The development of an optimal bidding strategy based on hourly unit commitment in a GenCo that participates in energy markets, consists of deciding which electricity generation units should be running in each period so as to satisfy the bilateral and future contracts agreements abiding by legal the emission bounds. The objective is to design the optimal bidding strategies to the next day auctions of the day-ahead market of each one of the generators in order to maximize the expected value of the net profit of a GenCo, expressed as the difference between the incomes from the electricity market and the generation costs, and taking into account the trading of the $CO_2$ emissions rights. Environmental regulation enforced in recent years has turned the emission control a very important operation objective. This point out the need of incorporating the emission constraints in the formulation of the nit commitment problem.

In a typical electrical system there are a variety of units available for generating electricity, and each has its own characteristics. This work considers a price taker GenCo (an electrical utility with no capability of altering the market prices) with a set of coal thermal units, high emission technology, and combined cycle gas turbines (CCGT) generation units, low emission technology.

**Thermal Units** Thermal generation has been part of the energy story for nearly fifty years. Thermal energy is generated by burning coal, natural gas, oil, or the combustion of diesel. In thermal power plants electricity is generated by burning fossil fuels such as coal, natural gas, or petroleum (oil). Fuel (coal for this study) and compressed air are mixed in a combustion chamber and ignited. This combustion produces heat that is used to heat water, which turns into steam, and spins a steam turbine rotor which drives an electrical generator (G1) to produce electricity. Unfortunately, complete conversion of fuel into energy is no possible. Consequently, flue gas and cooling water from combustion of the fossil fuels are discharged to the air.

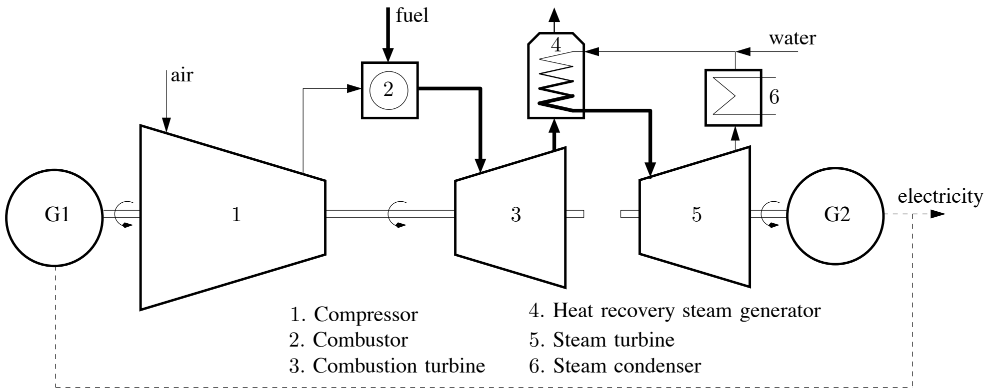


FIG. 2. Combined cycle unit

**Combined Cycle Units** The combined cycle (CC) units represent a combination of combustion and steam turbines within a power plant. The CC plants employ more than one thermodynamic cycle. Water is heated and turns into steam, the heat captured from the exhaust gas of combustion turbine (CT), is used in the heat recovery steam generator (HRSG) and spins a steam turbine (ST) which consequently drives an electrical generator (G2) to produce electricity (Fig. 2). This additional electricity improves the efficiency of electricity generation [**12**].

Currently, in Europe most of the new generating unit installations are combined cycle (CC) units. They are between 20 and 30 % more efficient than thermal power plants, and can reach upto 60 % of efficiency. They are fast response units, which can be a quite decisive feature in facing fluctuations in power plants. Moreover, they are less climate-damaging as the $CO_2$ production of a CC plant is much lower than that of other technologies.

# Chapter 3
# Emission Bounded Model for the Optimal Electricity Generation Bid

On current competitive and environmentally constrained electricity supply industry, a GenCo faces the optimal trade-off problem of how to achieve the maximum profit while minimizing the environmental impact by the management of the energy available in fossil fuels for power generation.

This chapter presents the complete characterization of the mathematical model: parameters, variables, constraints and objective function.

## 1. Model Description

As it has been mentioned before, the price of the electricity is unknown at the moment that generation companies have to take operational decisions. This means that the market price can be considered as random variable whose realization is only known once the market has been cleared. Generation companies would need to predict the unknown price in order to decide its strategies and maximize profits. Such problem can be modeled using a stochastic optimization model which can provide a framework for decision making. Stochastic programming is a powerful optimization technique that allows to incorporate in a single mathematical optimization model the same statistical information on the relevant random variables handled in simultaneous studies. Doing so, stochastic programming models are able to provide, in a single run, the best possible here-and-now decisions taking into account the most complete and available statistical information.

This project extends the electricity bid stochastic optimization models developed in [**5**] and [**13**], through the consideration of several environmental issues, as the $CO_2$ emission trading market, the $SO_2$ and $NO_x$ emission restrictions and the specificities of the combined-cycle low emission technology. Bilateral and future contracts are merged into a single model and at the same time two types of generating units are considered: the coal thermal units and the combined cycle gas turbine units. The proposed model includes modifications related to the formulation of the operational characteristics of thermal and the CC units operations, improving the formulation in [**5**] and [**13**]. Modifications are specified in sections 4.4 and 4.5.

The two-stage stochastic optimization model developed allows a GenCo to optimally decide the unit commitment of its thermal and CC units, the economic dispatch of the bilateral and futures contracts between all the programming units, the optimal generation bid of the committed units operating in the IEM and to decide when and how much of permissions to spent to produces electricity, in order to manage their allocations in the most profitable way.

The objective function of the model represent the expected benefits of the GenCo obtained with the participation in the DAM and the incomes/costs associated with $CO_2$ allowances. The constraints ensures that IEM's rules for the included market mechanisms are defined, and that all the operational restrictions of the units, and the environmental limitations, according to the NERP directives, are respected. The main decision variables are the ones that model the start-up and shut down of the units, the quantity that will be bid at instrumental price and the scheduled energy committed to the bilateral and the futures settlement.

## 2. Parameters

Following [**13**], the model is built for a price-taker GenCo owning a set of thermal generation units $\mathcal{I}$ and a set CC units that bid to the $t \in \mathcal{T} = \{1, 2, .., 24\}$ hourly auctions of the DAM. The pseudo-units $\mathcal{P}$ represents the different generation configurations of the CC units, therefore, the total set of generation units considered is $\mathcal{U} = \mathcal{I} \cup \mathcal{P}$.

The parameters for the $i^{th}$ generation unit are:

- Initial state of each thermal unit: $u_{0i}$. Its value is 1 if the unit is on and 0 otherwise.
- Constant, linear and quadratic coefficients generation costs: $c_i^b$, $c_i^l$ and $c_i^q$ ([€], [€/MWh] and [€/MWh$^2$] respectively).
- Upper and lower bounds on the energy generation: $\overline{P}_i$ and $\underline{P}_i$ [MWh].
- Start-up and shut-down costs: $c_i^{on}$ and $c_i^{off}$ [€].
- Minimum operation and minimum idle time: $t_i^{on}$ and $t_i^{off}$ [h].
- Number of periods that unit $i$ must be initially online, $G_i$, due to its minimum uptime $t_i^{on}$.
- Number of periods that unit $i$ must be initially offline, $H_i$, due to its minimum downtime $t_i^{off}$.

A base load physical futures contract $j \in \mathcal{F}$ is defined by:

- The set of generation units allowed to cover the FC $j$: $\mathcal{I}_j \in \mathcal{U}$.
- The amount of energy [MWh] to be procured each interval of the delivery period by the set $\mathcal{I}_j$ of generation units to cover contract $j$: $L_j^F$.
- The price of contract $j$: $\lambda_j^F$ [€/MWh].

A base load bilateral contract $k \in \mathcal{B}$ is defined by:

- The amount of energy [MWh] to be procured during hour $t$ of the delivery period by the set of available generation units to cover the BC $k$: $L_{tk}^B$.
- The price of the contract $k$: $\lambda_k^B$ [€/MWh].

The random variable $\lambda_t^D$, the clearing price of the $t^{th}$ hourly auction of the DAM, is represented in the two-stage stochastic model by a set of scenarios $s \in \mathcal{S}$, each one with its associated clearing price for each DAM auction $t \in \mathcal{T}$:

- Clearing price for auction $t$ at scenario $s$: $\lambda_t^{D,s}$ [€/MWh].
- Probability of scenario $s$: $P^s$ .

$\lambda_t^{D,s}$ and $P^s$ used in this thesis have been obtained from the works [**14, 15**].

The emission of a thermal unit is assumed to be linear with respect to the amount of energy generated by this unit. Then the relevant parameters needed to formulate the $SO_2$ and $NO_x$ emissions of the thermal units are the coefficients of the aforementioned linear relations and the total amount of pollutants allowed to be emitted by the NERP:

- Limits to the joint $SO_2$ and $NO_x$ emissions of the thermal units: $\overline{SO_2}$ [kg/day] and $\overline{NO_x}$ [kg/day].
- The $SO_2$ and $NO_x$ emission coefficients of the generation-to-emission linear function: $e_i^{SO_2}$ [kg/ MWh], $e_i^{NO_2}$ [kg/ MWh], $i \in \mathcal{I}$.

According to the Spanish NERP the CC units are exempt from compliance with emissions requirements because they are committed not to run for more than 20,000 operational hours starting from 1 January 2008 until, at the latest, on December 31, 2015.

The parameters that defines the electricity generation's $CO_2$ emissions rate and allowances are:

- The GenCo's aggregated free emission allowances: $\overline{CO_2}$ [$tCO_2$]. The enforcement of the constraints related to the $CO_2$ emission is based on the presumption that the generating system is constrained by a predefined emission bound. This bound correspond to the GenCo's aggregated free emission allowance.
- The estimated $CO_2$ -emission's price in the emission trading market [**3**]: $\lambda^{CO_2}$ [€/$tCO_2$]. Trading price of emission allowances are estimated based on prices in emissions markets that operates separately from electricity markets.
- The emission conversion factor: $e_i^{CO_2}$ [$tCO_2$/ €]. The power production by thermal power plants is always accompanied by the release of a given amount of $CO_2$ that depends on consumed amount of fuel and the the type of generating unit. A further explication on how the produced $CO_2$ can be calculated, can be found in Section 4 of Chapter 4.

## 3. Variables

In stochastic programming models, those decision variables that doesn't depend on the scenarios $s \in \mathcal{S}$ are called first stage variables. In this model these variables are for every time period $t \in \mathcal{T}$ and generation unit $i \in \mathcal{U}$:

- The unit commitment binary variables: $u_{ti} \in \{0, 1\}$ expressing the off-on operating status of the $i^{th}$ unit.
- The start-up/shut-down costs variables: $c_{ti}^u$, $c_{ti}^d$ [€].
- The price acceptant offer bid: $q_{ti}$ [€].

- The scheduled energy for futures contract $j \in \mathcal{F}$: $f_{tij}$ [MWh].
- The scheduled energy for bilateral contract: $b_{ti}$ [MWh].

Decision variables that can adopt different values depending on the scenario are called second stage variables. In this formulation these variables are for each $t \in \mathcal{T}$, generation unit $i \in \mathcal{U}$ and scenario $s \in \mathcal{S}$.:

- The total generation: $g_{ti}^s$ [MWh].
- The matched energy in the day-ahead market: $p_{ti}^s$ [MWh].

## 4. Constraints

The maximization of the objective function is done subject to a set of constraints associated with the IEM's market rules, the operational characteristics of the generation units and the $CO_2$, $SO_2$ and $NO_x$ emission control.

**4.1. Futures and bilateral contracts covering constraints.** The coverage of both the physical futures and bilateral contracts obligations must be guaranteed. The constraints for each FC are:

$$(1)\qquad \begin{cases} \displaystyle\sum_{i \in \mathcal{I}_j} f_{tij} = L_j^F & \forall j \in \mathcal{F},\ \forall t \in \mathcal{T} \quad (a) \\ f_{tij} \geq 0 & \forall i \in \mathcal{U},\ \forall j \in \mathcal{F},\ \forall t \in \mathcal{T} \quad (b) \end{cases}$$

where $\mathcal{I}_j \subset \mathcal{U}$ stand for the set of thermal and pseudo-units allowed to cover the FC $j$. The BC constraints are:

$$(2)\qquad \begin{cases} \displaystyle\sum_{i \in \mathcal{U}} b_{ti} = \sum_{k \in B} L_{tk}^B & \forall t \in \mathcal{T} \quad (a) \\ 0 \leq b_{ti} \leq \overline{P}_i u_{ti} & \forall i \in \mathcal{U},\ \forall t \in \mathcal{T} \quad (b) \end{cases}$$

**4.2. Day-ahead market bid constraints.** The IEM establishes the following rules to integrate energies $L_j^F$ and $L_{tk}^B$ in the day-ahead market bid of a generation unit:

- If generator $i \in \mathcal{U}$ contributes with $f_{tij}$ MWh at period $t$ to the coverage of the FC $j$, then the energy $f_{tij}$ must be offered to the pool for free (*price acceptance sale bid*).
- If generator $i \in \mathcal{U}$ contributes with $b_{ti}$ MWh at period $t$ to the coverage of any of the BCs, then the remaining production capacity $\overline{P}_i - b_{ti}$ must be bid to the DAM.

These market rules can be included in the model by means of the following set of constraints:

$$p_{ti}^s \leq \overline{P_i} u_{ti} - b_{ti} \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T},\ \forall s \in S \tag{3}$$

$$p_{ti}^s \geq q_{ti} \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T},\ \forall s \in S \tag{4}$$

$$q_{ti} \geq \underline{P_i} u_{ti} - b_{ti} \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T} \tag{5}$$

$$q_{ti} \geq \sum_{j \mid i \in \mathcal{I}_j} f_{tij} \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T} \tag{6}$$

where:

- (3) and (4) ensures that if a unit is on, the matched energy $p_{ti}^s$ will be between the instrumental price bid $q_{ti}$ and the total available energy not allocated to a BC. $\overline{P_i}$, $\underline{P_i}$ represent upper and lower bounds on the energy generation (MWh).
- (5) and (6) guarantee respectively that the minimum generation output of the committed units will be matched, and that the contribution of the unit to the FC coverage will be included in the instrumental price bid.

The analytical expression of the optimal generation bid of the generations units can derived from this market constraints [**5**].

**4.3. Total generation constraints.** The total generation level of a given unit $i$, $g_{ti}^s$, is defined as the addition of the allocated energy to the BC, plus the matched energy in the DAM:

$$g_{ti}^s = b_{ti} + p_{ti}^s \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T},\ \forall s \in S \tag{7}$$

The generation output of a any generation unit $g_{ti}^s$ are restricted to $g_{ti}^s \in \{0\} \cup \underline{P_i}, \overline{P}_i$, that is:

$$\underline{P_i} u_{ti} \leq g_{ti}^s \leq \overline{P}_i u_{ti} \qquad \forall i \in \mathcal{U},\ \forall t \in \mathcal{T},\ \forall s \in S \tag{8}$$

**4.4. Thermal unit commitment constraints.** The following set of constraints conveniently models the start-up and shut-down costs, and the minimum operation and idle time for each unit. Contrary to formulation in [**5**] that does not consider the case $t = 0$, in this formulation, the parameter $u_{0i}$ allows to include, the initial state of each thermal unit $i \in \mathcal{I}$, into the constraints:

$$c_{ti}^u \geq c_i^{on}[u_{ti} - u_{(t-1),i}] \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \tag{9}$$

$$c_{ti}^d \geq c_i^{off}[u_{(t-1)i} - u_{ti}] \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \tag{10}$$

$$\sum_{j=n}^{G_i} (1 - u_{ji}) = 0 \qquad \forall i \in \mathcal{I} \tag{11}$$

$$\sum_{j=1}^{H_i} u_{ji} = 0 \qquad \forall i \in \mathcal{I} \tag{12}$$

(13)
$$\sum_{n=t}^{\min\{t+t_i^{on}-1,|\mathcal{T}|\}} u_{ni} \geq \alpha_{ti}^{on}[u_{ti} - u_{(t-1)i}] \qquad t = G_i + 1, \ldots, |\mathcal{T}|, \ \forall i \in \mathcal{I}$$

(14)
$$\sum_{n=t}^{\min\{t+t_i^{off}-1,|\mathcal{T}|\}} (1 - u_{ni}) \geq \alpha_{ti}^{off}[u_{(t-1)i} - u_{ti}] \qquad t = H_i + 1, \ldots, |\mathcal{T}|, \ \forall i \in \mathcal{I}$$

where the parameters $\alpha_{ti}^{on}$ and $\alpha_{ti}^{off}$ are defined as:

(15)
$$\alpha_{ti}^{on} = \min\{t_i^{on}, |\mathcal{T}| - t + 1\}$$

(16)
$$\alpha_{ti}^{off} = \min\{t_i^{off}, |\mathcal{T}| - t + 1\}$$

The last two expressions (15) and (16) allow the regulation of the minimum up and down time which are imposed by the parameters ($t_i^{on}$) and ($t_i^{off}$), respectively, or at a minimum, the number of periods that are needed to complete the optimization period: $t \in \mathcal{T} = \{1, 2, \ldots, 24\}$. For instance, if for and specific unit $i \in \mathcal{I}$, $t_i^{off} = 5$ and $t = 22$, therefore $\alpha_{ti}^{on} = \min\{5, 24 - 22 + 1\}$ which equals 3, i.e. although the unit must should be on for 5 hours, as to finish the optimization period, remaining only three hours, the unit must be on at instants: $t = 22$, $t = 23$ and $t = 24$.

It is worth mentioning that the above two constraints 13 and 14 simplify equations formulated in [**5**] whose expressions impose the minimum up and down time, through the following set of constraints, respectively:

$$\sum_{n=t}^{t+t_i^{on}-1} u_{in} \geq t_i^{on}[u_{ti} - u_{(t-1)i}] \qquad t = G_i + 1, \ldots, |\mathcal{T}| - t_i^{on} + 1, \ i \in \mathcal{I}$$
$$\sum_{n=t}^{|\mathcal{T}|}(u_{ni} - [u_{ti} - u_{(t-1)i}]) \geq 0 \qquad t = |\mathcal{T}| - t_i^{on} + 2, \ldots, |\mathcal{T}|, \ i \in \mathcal{I}$$

and

$$\sum_{n=t}^{t+t_i^{off}-1} (1 - u_{ni}) \geq t_i^{off}[u_{(t-1)i} - u_{ti}] \qquad t = H_i + 1, \ldots, |\mathcal{T}| - t_i^{off} + 1, \ i \in \mathcal{I}$$
$$\sum_{n=t}^{|\mathcal{T}|}(1 - u_{ni} - [u_{(t-1)i} - u_{ti}]) \geq 0 \qquad t = |\mathcal{T}| - t_i^{off} + 2, \ldots, |\mathcal{T}|, \ i \in \mathcal{I}$$

The above expressions are divided into two periods, one up to $|\mathcal{T}| - (t_i^{on} - 1)$ and $|\mathcal{T}| - (t_i^{off} - 1)$ and another for the last $t_i^{on} - 1$ and $t_i^{off} - 1$ time periods respectively for $t^{on}$ and $t^{off}$. Thus, although the number of constraints formulated following [**5**] coincides with the number of constraints introduced in this work, eq. (13) and (14), through the definition of parameters $\alpha_{ti}^{on}$ and $\alpha_{ti}^{off}$, provides a more compact presentation of the minimum up and down time constraints.

TABLE 1. States of the CC unit and its associated Pseudo units

| **CC unit with a CT and HRSG/ST** | | | | | |
|---|---|---|---|---|---|
| State | Composition | Pseudo unit 1 | $u_{\mathcal{P}_c(1)t}$ | Pseudo unit 2 | $u_{\mathcal{P}_c(2)t}$ |
| 0 | 0CT+0HRSG/ST | off | 0 | off | 0 |
| 1 | 1CT+0HRSG/ST | on | 1 | off | 0 |
| 2 | 1CT+1HRSG/ST | off | 0 | on | 1 |

**4.5. Combined cycle unit commitment constraints.** A CC unit consists of several CTs and an HRSG/ST set. Can operate at multiple states or configurations, based on the different combinations of CTs and HRSG/ST. The first two columns of Table 1 show the states of a CC unit with a CT and an HRSG/ST. Operational rules of a typical CC unit were formulated in [**16**] with the help of the so-called pseudo units (PUs). The PUs of each CC unit can be viewed as a special set of non-independent or coupling thermal units. As the thermal units, the PUs have their own unique characteristics: start up cost, real power generation limits and minimum down time limits. The formulation presented in this work only considers two PUs, each one associated with states 1 and 2 of the CC. Columns 3 and 5 of Table 1 shows how the on/off state of these two PUs uniquely determines the state of the CC.

Let $\mathcal{P}_c$ the set of PUs of the CC unit $c \in \mathcal{C}$, and $\mathcal{P} = \cup_{c \in \mathcal{C}} \mathcal{P}_c$, the complete set of PUs. By $\mathcal{P}_c(j)$, we denote the PU associated with the state $j \in \{1,\ 2\}$ of the CC unit $c$. Columns 4 and 6 of Table 1 illustrate the relation of the commitment binary variables of the PUs, $u_{t\mathcal{P}_c(1)}$ and $u_{t\mathcal{P}_c(2)}$, with the state of the associated CC unit. The start-up cost and minimum on/off time formulation for both PU and CC units has been made following the formulation in [**13**]. This reformulation represents an improvement of the original formulation presented in [**13**]:

$$-\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \Big[ c^{on}_{\mathcal{P}_c(1)} \left( e_{\mathcal{P}_c(1)t} - a_{\mathcal{P}_c(2)t} \right) + c^{on}_{\mathcal{P}_c(2)} e_{\mathcal{P}_c(2)t} \Big]$$

where the auxiliary binary variables $a_{it}$ and $e_{it}$ are defined to be $a_{it} = 1$ iff $u_{i(t-1)} = 1$ and $u_{it} = 0$, and $e_{it} = 1$ iff $u_{i(t-1)} = 0$ and $u_{it} = 1$. Instead of the two auxiliary binary variables $a_{it}$ and $e_{it}$, that increase the execution time of the model, in the formulation of this study is necessary to consider only the start-up costs continuous variables: $c^u_{t\mathcal{P}_c(1)}$ and $c^u_{t\mathcal{P}_c(2)}$ for each PU $i \in \mathcal{P}$. There are not shut-down costs associated to the PU, and neither cost associated to the transition from state 2 to state 1:

$$c^u_{t\mathcal{P}_c(1)} \geq \quad c^{on}_{\mathcal{P}_c(1)} \left[ [u_{t\mathcal{P}_c(1)} - u_{(t-1)\mathcal{P}_c(1)}] - [u_{(t-1)\mathcal{P}_c(2)} - u_{t\mathcal{P}_c(2)}] \right] \quad \forall t \in \mathcal{T},\ \forall c \in\ \mathcal{C} \tag{17}$$

$$c^u_{t\mathcal{P}_c(2)} \geq c^{on}_{\mathcal{P}_c(2)} [u_{t\mathcal{P}_c(2)} - u_{(t-1)\mathcal{P}_c(2)}] \qquad \forall t \in \mathcal{T},\ \forall c \in\ \mathcal{C} \tag{18}$$

Tables 2, and 3 shows all the possible values of the expression: $[u_{\mathcal{P}_c(1)t} - u_{\mathcal{P}_c(1)(t-1)}] - [u_{\mathcal{P}_c(2)(t-1)} - u_{\mathcal{P}_c(2)t}]$. Note that it's value is 1 only when CC unit $c$ is in state 0 at period

TABLE 2. Start-up Cost of the PUs

| $u_{t\mathcal{P}_c(1)}$ | $u_{(t-1)\mathcal{P}_c(1)}$ | $u_{(t-1)\mathcal{P}_c(2)}$ | $u_{t\mathcal{P}_c(2)}$ | $[u_{t\mathcal{P}_c(1)} - u_{(t-1)\mathcal{P}_c(1)}] - [u_{(t-1)\mathcal{P}_c(2)} - u_{t\mathcal{P}_c(2)}]$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | -1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |

TABLE 3. Start-up Cost of the PUs (Combinations excluded because of the feasible transitions rules)

| $u_{t\mathcal{P}_c(1)}$ | $u_{(t-1)\mathcal{P}_c(1)}$ | $u_{(t-1)\mathcal{P}_c(2)}$ | $u_{t\mathcal{P}_c(2)}$ | Does no satisfy the feasible transitions rules because |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | if the CC unit $c$ is in state 2 at period $(t-1)$ , it can not be in sate 0 at period $t$ |
| 0 | 0 | 0 | 1 | if the CC unit $c$ is in state 0 at period $(t-1)$ , it can not be in sate 2 at period $t$ |
| 0 | 1 | 1 | 0 | only one of the PUs can be committed at a given period |
| 0 | 1 | 1 | 1 | ” |
| 1 | 0 | 0 | 1 | ” |
| 1 | 0 | 1 | 1 | ” |
| 1 | 1 | 1 | 1 | ” |
| 1 | 1 | 0 | 1 | ” |
| 1 | 1 | 1 | 0 | ” |

$t-1$ (i.e. when $u_{\mathcal{P}_c(1)(t-1)} = 0 + u_{\mathcal{P}_c(2)(t-1)} = 0$), and in state 1 at period $t$ (i.e. when $u_{\mathcal{P}_c(1)t} = 1$ and $u_{\mathcal{P}_c(2)t} = 0$).

Each PU $i \in \mathcal{P}$ has its own minimum up time, $t_i^{on}$:

$$\sum_{n=t}^{\min\{t+t_i^{on}-1,|\mathcal{T}|\}} u_{ni} \geq \alpha_{ti}^{on}[u_{ti} - u_{(t-1)i}] \qquad t = G_i+1,\ldots,|\mathcal{T}|,\ \forall i \in \mathcal{P} \tag{19}$$

where again $u_{0i}$ represents the initial state of each pseudo unit $i \in \mathcal{P}$ and $G_i$ is the number of the initial time periods along which the pseudo unit must remain on. So as in Eq. (11):

$$\sum_{t=1}^{G_i}(1-u_{ti}) = 0 \qquad i \in \mathcal{P}\ ,\ t \in \mathcal{T} \tag{20}$$

Equation (19) is again a simplification of the constraint considered in [**13**]:

$$\left.\begin{array}{ll} u_{it} - u_{i(t-1)} - e_{it} + a_{it} = 0 & (a) \\ e_{it} + \displaystyle\sum_{j=t}^{\min\{t+t_i^{on},|\mathcal{T}|\}} a_{ij} \leq 1 & (b) \\ u_{it}, a_{it}, e_{it} \in \{0,1\} \cap \mathcal{K}_i & (c) \end{array}\right\} \quad i \in \mathcal{P}\ , t \in \mathcal{T}$$

where the set $\mathcal{K}_i$ stands for the initial state of each unit, and the auxiliary binary variables $a_{it}$ and $e_{it}$ are defined as above. Eq.(19) is clearly easy to understand and computationally more efficient.

In turn each CC unit also has a minimum down time, i.e., once shut down, the CC unit cannot be started up before $(t_c^{off})^C$ periods. As in the case of the thermal and pseudo

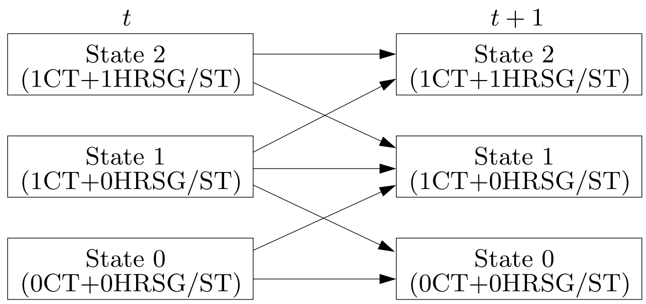


FIG. 1. Feasible transitions of the CC unit with a CT and HRSG/ST

units, the following constraints formulate the minimum down time condition for the CC units:

(21)
$$\begin{aligned} \sum_{n=t}^{\min\{t+(t_c^{off})^C-1,|\mathcal{T}|\}} & [1-(u_{n\mathcal{P}_c(1)}+u_{n\mathcal{P}_c(2)})] \geq \\ \alpha_{tc}^{off} & \Big[(u_{(t-1)\mathcal{P}_c(1)}+u_{(t-1)\mathcal{P}_c(2)})-(u_{t\mathcal{P}_c(1)}+u_{t\mathcal{P}_c(2)})\Big] \\ & t = H_c^C+1,\ldots,|\mathcal{T}|,\ \forall c\in\mathcal{C} \end{aligned}$$

where

$$\alpha_{tc}^{off} = \min\{(t_c^{off})^C, |\mathcal{T}|-t+1\}$$

and $H_c^C$ represents the number of the initial time periods along which the CC unit must remain off. So as in Eq. (12):

(22)
$$\sum_{t=1}^{H_c^C} u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} = 0 \qquad \forall c\in\mathcal{C}\,,\ \forall t\in\mathcal{T}$$

Equation (21)represents a simpler expression of the minimum down time constraint of each unit introduced in [**13**]:

$$\left.\begin{array}{lcll} \big(u_{\mathcal{P}_c(1)t}+u_{\mathcal{P}_c(2)t}\big)-\big(u_{\mathcal{P}_c(1)(t-1)}+u_{\mathcal{P}_c(2)(t-1)}\big)+ & & & \\ +\big(a_{\mathcal{P}_c(1)t}-e_{\mathcal{P}_c(1)t}\big)-\big(e_{\mathcal{P}_c(2)t}-a_{\mathcal{P}_C(2)t}\big) & = & 0 & (a) \\ \big(a_{\mathcal{P}_c(1)t}-e_{\mathcal{P}_c(2)t}\big)+\displaystyle\sum_{j=t}^{\min\{t+t_c^C,|\mathcal{T}|\}}\big(e_{\mathcal{P}_c(1)j}-a_{\mathcal{P}_c(2)j}\big) & \leq & 1 & (b) \end{array}\right\} \quad \begin{array}{l} c\in\mathcal{C} \\ t\in\mathcal{T} \end{array}$$

where $t_c^C$ represent the minimum down time of each CC unit.

Furthermore, the satisfaction of the feasible transitions rules (Fig. 1) impose additional constraints to the operation of the PUs associated to the same CC unit, $c \in \mathcal{C}$. First, the PUs in $\mathcal{P}_c$ are mutually exclusive Eq. (23)($a$), i.e., only one of them can be committed at a given period (a CC can only be in one state simultaneously). Second, the change of the commitment of the PUs in $\mathcal{P}_c$ between periods $t-1$ and $t$ are limited to the feasible transitions depicted in Fig. 1. These feasible transitions impose that, if the CC unit $c$ is in state 0 at period $t-1$ (i.e $u_{(t-1)\mathcal{P}_c(1))} + u_{(t-1)\mathcal{P}_c(2)} = 0$), it cannot be in state 2 at period $t$ (i.e. $u_{t\mathcal{P}_c(2)} = 0$) (Eq. (23)$(b)$). Conversely, if $u_{(t-1)\mathcal{P}_c(2)} = 1$, then $u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} \geq 1$ (Eq. (23)$(c)$). The following set of constraints formulates the specific operation rules of the CC units:

$$
(23) \quad \left.\begin{array}{llll}
\sum_{m\in\mathcal{P}_c} u_{tm} & \leq & 1 & (a) \\
u_{t\mathcal{P}_c(2)} & \leq & u_{(t-1)\mathcal{P}_c(1))} + u_{(t-1)\mathcal{P}_c(2)} & (b) \\
u_{(t-1)\mathcal{P}_c(2)} & \leq & u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} & (c)
\end{array}\right\} \begin{array}{l}\forall c \in \mathcal{C}, \\ \forall t \in \mathcal{T}\end{array}
$$

**4.6. $SO_2$ and $NO_x$ emissions constraints.** The Spanish National Emission Reduction Plan imposes limits $\overline{SO_2}$ and $\overline{NO_x}$ to the joint emission of the thermal units (CC units are excluded). These limitations can be included in the model by imposing an emission limit at every scenario $s$ through the following set of constraints:

$$
(24) \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s \leq \overline{SO_2} \qquad \forall s \in \mathcal{S}
$$

$$
(25) \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{NO_x} g_{ti}^s \leq \overline{NO_x} \qquad \forall s \in \mathcal{S}
$$

where the emission coefficients $e_i^{SO_2}$ and $e_i^{NO_x}$ depend on the generation technology.

**4.7. $CO_2$ emission constraints.** Following the method proposed in [**17**], the incorporation of the $CO_2$ emission limits in the solution of the unit commitment problem have been made integrating into the objective function the costs and revenues associated with the potential purchases/sales of $CO_2$ allowances in the market. This method allows, according to the EU ETS mechanism, the acceptance of solutions which violate the enforced emission limits. Therefore, emission allowances can be purchased in order to overcome the shortfall in the existing emission rights. The expression that represent the cost/incomes of the emission allowances is:

$$
\lambda^{CO_2}\left[\sum_{t\in\mathcal{T}}\sum_{s\in S} P^s \left[\sum_{i\in\mathcal{U}} e_i^{CO_2}(c_i^b u_{ti} + c_i^l g_{ti}^s + c_i^q (g_{ti}^s)^2)\right] - \overline{CO_2}\right]
$$

where $e_i^{CO_2}$ is the emission conversion factor (Section 4 of Chapter 4 shows how to calculate its value), $\overline{CO_2}$ corresponds to the GenCo's aggregated free emission allowances ($tCO_2$) and $\lambda^{CO_2}$ is the estimated $CO_2$ -emission price (€/$tCO_2$) in the emission trading market [**3**]. The model for the $CO_2$ emission follows the assumption in [**10**] and [**17**] that the nonlinear emission function is proportional to the quadratic generation cost function of each unit.

## 5. Objective Function

The expected value of the profit function of the GenCo with respect to the spot market price random variable $\lambda^D$ can expressed as:

$$E_{\lambda^D}\left[h(u,c^u,c^d,g,p,b,f;\lambda^D)\right] =$$

$$|\mathcal{T}|\left[\sum_{k\in\mathcal{F}}\lambda_k^F L_k^F\right]+\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{B}}\lambda_{tj}^{BC}L_{tj}^{BC} \quad (26)$$

$$-\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left[c_{ti}^u + c_{ti}^d + c_i^b u_{ti}\right] \quad (27)$$

$$-\sum_{t\in\mathcal{T}}\sum_{c\in\mathcal{C}}\left[c_{t\mathcal{P}_c(1)}^u + c_{t\mathcal{P}_c(2)}^u + \sum_{i\in\mathcal{P}_c} c_i^b u_{ti}\right] \quad (28)$$

$$+\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{U}}\sum_{s\in S}P^s\left[\lambda_t^{D,s}p_{ti}^s-(c_i^l g_{ti}^s+c_i^q(g_{ti}^s)^2)\right] \quad (29)$$

$$-\lambda^{CO_2}\left[\sum_{t\in\mathcal{T}}\sum_{s\in S}P^s\left[\sum_{i\in\mathcal{U}}e_i^{CO_2}(c_i^b u_{ti}+c_i^l g_{ti}^s+c_i^q(g_{ti}^s)^2)\right]-\overline{CO_2}\right] \quad (30)$$

where:

(26) corresponds to the incomes of the FCs and the BCs and is a constant term. $\lambda_k^F$ and $\lambda_{tj}^{BC}$ are the prices of FCs and BCs respectively.
(27) accounts for the on/off fixed cost of the unit commitment of the thermal units. It is independent of the realization of the random variable $\lambda_t^D$. $c_i^b$ are the constant coefficients of the generation costs (€).
(28) CC's start-up and fixed generation costs. Only start-up costs are associated to the PU, and no cost is associated to the transition from state 2 to state 1. This term does not depend on the realization of the random variable $\lambda^D$.
(29) represents the expected value of the benefits from the day-ahead market, where $P^s$ is the probability of scenario $s$. The term between brackets corresponds to the expression of the quadratic generation costs with respect to the total generation of the unit, $g_{ti}^s$.
(30) at it has mentioned above this term accounts for cost/incomes associated to the purchase/sale of the $CO_2$ emissions rights [**18**].

## 6. Final Model

The proposed market clearing problem with emission trading can be mathematically expressed as the following optimization problem:

$$
(31) \quad \begin{cases} \max & E_{\lambda^D}\left[h(u, c^u, c^d, g, p, b, f; \lambda^D)\right] \\ \text{s.t. :} & \\ & \text{Eq. } (1) - (2) \qquad \text{FC and BC} \\ & \text{Eq. } (3) - (6) \qquad \text{Day-ahead market} \\ & \text{Eq. } (7) - (8) \qquad \text{Total generation} \\ & \text{Eq. } (9) - (14) \qquad \text{Thermal unit commitment} \\ & \text{Eq. } (17) - (23) \qquad \text{CC unit commitment} \\ & \text{Eq. } (24) - (25) \qquad \text{Emission} \end{cases}
$$

The deterministic equivalent of the two-stage stochastic problem (31) is a mixed, continuous-binary concave quadratic maximization problem with linear constraints and with a well defined global optimal solution.

As it has been mentioned in the introduction the model described above (the emission-bounded model) was accepted for presentation to the $9^{th}$ International Conference on the European Electricity Market (`http://eem12.org/`) and has been published by the Institute of Electrical and Electronics Engineers (IEEE) as a full paper proceeding [**6**]. Three case studies was used to evaluate the impact of the $CO_2$ allowances and emission constraints in the optimal scheduling and bid of the generation units:

- BASE: optimal bid problem without neither emission constraints nor $CO_2$ allowances. Corresponds to problem (31) excluding both the $CO_2$ rights incomes/cost term (30) and emission constraints (24)-(25).
- EC: optimal bid problem with only emission constraints. Corresponds to problem (31) excluding the $CO_2$ rights incomes/cost term (30) but retaining emission constraints (24)-(25).
- CO2EC: The complete model (31) with both $CO_2$ allowances and emission constraints.

All the cases were implemented with the AMPL modeling language [**19**] and solved with CPLEX 12.0 [**20**] (`mipgap=0.05, threads=20`) over a SunFire X2200 with 32 Gb of RAM memory and two dual core processors AMD Opteron 2222 at 3 GHz. The number of continuous and binary variables was $20.448$ and $200$ respectively and the number of linear constraints was $49.458$ for the BASE case and $49.558$ for the other two cases. The execution time was below one minute in all the cases.

In this study, the numerical test provided a reduction of the overall $CO_2$ , $SO_2$ and $NO_x$ emissions by 70% together with 30% increase in the expected total profit, availing of the $CO_2$ emissions rights market. The results obtained suggest that, with the day-ahead market and CO2 allowances prices used in this study, the $CO_2$ rights market can be a valid tool for utilities to reduce their emissions without any loss in their overall profits. Table 4 depicts the expected value of the $CO_2$, $SO_2$ and $NO_x$ emissions, at the optimal solution of the three cases considered, while Table 5 shows the optimal value of the expected profits obtained. Fig. 2 shows the percentage share of the different cost and income sources in the objective function value for the four cases.

It should be recalled that the main interest of this thesis is in the risk-constrained models that will be developed in the next chapter, where a complete analysis of the results will be

TABLE 4. Daily emissions at optimal solution

| | $E[CO_2]$ | $E[SO_2]$ | $E[NO_x]$ |
|---|---|---|---|
| BASE | 2.761 t | 15.381 kg | 26.811 kg |
| EC | 835 t | 3.900 kg | 6.798 kg |
| CO2EC | 801 t | 3.898 kg | 6.796 kg |

TABLE 5. Optimal incomes and costs

| | BASE | EC | CO2EC |
|---|---|---|---|
| $E$[Profit] | 554.479 € | 472.677 € | 734.052 € |
| BC Incomes | 419.122 € | 419.122 € | 419.122 € |
| FC Incomes | 374.543 € | 374.543 € | 374.543 € |
| $E$[market incomes] | 1.101.440 € | 482.469 € | 398.955 € |
| $E$[Generation Costs] | -1.334.540 € | -799.772 € | -720.998 € |
| Start-up/shut-down Costs | -6.088 € | -3.683 € | -3.683 € |
| $E$[$CO_2$ rights cost/incomes] | - | - | 266.114 € |

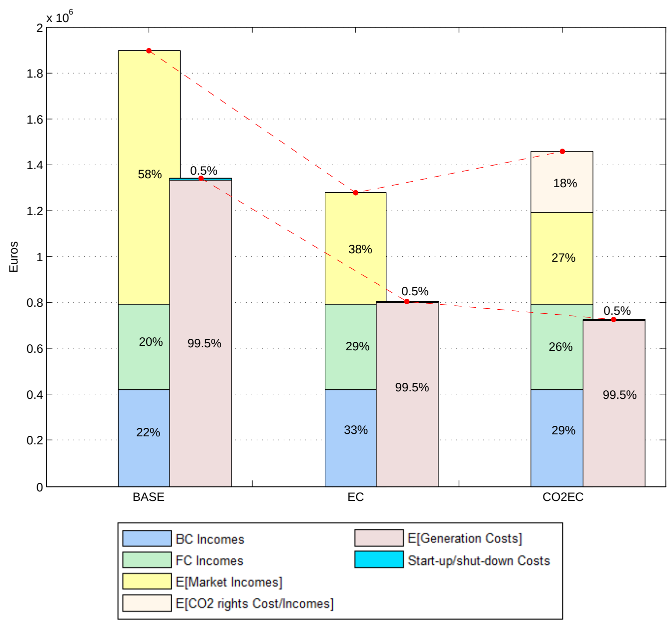


FIG. 2. Optimal Incomes and Costs. The graph shows the overall percentage of participation of the different income and cost sources as to the total value of the profit function.

presented. For this reason only a summary of the results and conclusions for the emission-bounded model developed in this chapter has been included in this section. The detailed analysis can be found in [**6**].

# Chapter 4
# Emission Risk Constrained Model for the Optimal Electricity Generation Bid

Risk Management is the theory about how to handle risks. A risk measure is a measure of how much one could lose or how uncertain a profit or loss is within a given time-period. In portfolio optimization theory, methods of risk management include *Value-at-Risk* (VaR) and *Conditional Value-at-Risk* (CVaR). VaR is a risk assessment tool to measure the minimum occasional loss expected in a given portfolio within a stated time period. The VaR determines the monetary risk associated with a generation schedule . It is an estimate that shows how much, for a given probability of occurrence, the power company could lose due to fluctuations in prices. As an alternative risk assessment tool, CVaR does quantify the losses associated with the tail of the profit distribution [**21**]. For the same confidence level used for VaR, CVaR provides an estimate of the average loss exceeding the VaR value. That is why it gives a better indication of risk than VaR.

The risk management ideas developed for the financial markets and the CVaR concept have been conveniently adapted to give a new approach to address the $SO_2$ and $NO_x$ emission constraints. Based on the CVaR concept developed in the finance industry to monitor losses within pre-specified tolerances, it has been developed a new concept, Conditional Emission-at-Risk (CEaR) to measure the risk of violation of the emission limits.

This chapter sets out a brief introduction to the risk management process as a framework to reformulate the emission constraints. Section 1 introduce the mathematical description of VaR and CVaR as risk measure, and gives, following the formulation in [**22**], the basics for using the CVaR concept in optimization problems. Section 2 present, using the CVaR concept, a detail description of a new $SO_2$ and $NO_x$ emission constraints formulation. Section 3 explain how to formulate a CVaR constraint to control possible low-profit outcomes through a set of linear constraints. The proposed method has been successfully tested using real data of a typical generation company operating in the IEM, and a set of scenarios for both the Spanish day-ahead and emission trading market. Section 4 describe all data used to validate the model. Finally Section 7 present and discuses the numerical results.

# 1. Risk Management

A typical risk measuring methodology used to guide the risk management process is Value-at-Risk (VaR). It is a very popular measure of risk, but it is difficult to optimize when calculated using scenarios. In this case, VaR is non-convex and it has multiple local extrema. An alternative measure of risk, which more desirable mathematical characteristics, is conditional Value-at-Risk (CVaR), also called the mean excess loss or tail VaR. It is convex and it can be optimized using linear programming and optimization algorithms which allow handling large numbers of scenarios [**22**]. In fact, whilst CVaR is the mean excess loss, the VaR provides no indication on the extent of losses that might be suffered beyond the amount indicated by this measure.

In portfolio optimization theory the VaR and CVaR concepts are used to measure the expected loss, for a given time horizon, through a loss function. This concepts can be used also for maximizing expected returns functions, as opposed to minimizing the expected loss. The fact that the objective function of the optimal bid model develop in this thesis represent the expected profits, leads to reformulate the VaR and CVaR concepts to use them in the study of the problem address in this project.

Let $h(x, \lambda^D)$ be the profit function associated with the decision vector $x$ , and the random vector $\lambda^D$. The decision vector $x$ represents the decision variables $u,\ c^u,\ c^d,\ g,\ p,\ b,\ f$ of the problem (31), and belongs to a certain set of generation bids belonging to the feasible set $\mathcal{X} \in \mathbb{R}^n$, while the vector $\lambda^D$ in $\mathbb{R}^{|\mathcal{S}|}$ stands for the uncertainties that can affect the profit (the spot price scenarios in the case of this work).

For each $x$, the profit $h(x, \lambda^D)$ is a random variable having a distribution in $\mathbb{R}$ induced by that of $\lambda^D$. For convenience, it is assumed that the random vector $\lambda^D$ has a probability density function $p(\lambda^D)$.

The probability that the profit $h(x, \lambda^D)$ does not fall bellow a threshold $\zeta$ is given by:

$$\Psi(x, \zeta) = \int\limits_{h(x,\lambda^D) \geq \zeta} p(\lambda^D) d\lambda^D$$

By definition, the VaR$(1-\alpha)$ value, denoted by $\zeta_{(1-\alpha)}(x)$, for the profit random variable associated with the decision vector $x$ and any specified probability level $\alpha$ in (0, 1), is the lowest amount $\zeta$ such that with probability $\alpha$, the profit will not fall bellow $\zeta$. Therefore, the probability that the monetary profit falls bellow VaR$(1-\alpha)$ is $\alpha$. Its value is given by:

$$\zeta_{(1-\alpha)}(x) = \min\{\alpha \in \mathbb{R} : \Psi(x, \zeta) \geq 1 - \alpha\}$$

The given probability is called a *confidence level*, which represents the degree of certainty of the VaR estimate. Three values of the confidence level are commonly considered: 0.90, 0.95, and 0.99. For instance, a confidence level of 0.95 means that 95% of the time, the power companys revenues will be more than VaR, and 5% of the time the revenues will be less than VaR.

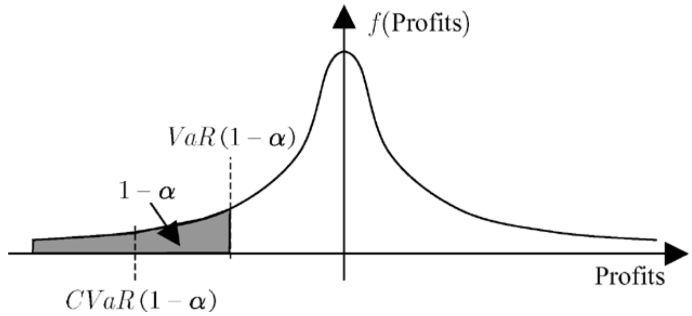


FIG. 1. Graphical representation of CVaR concept. From [**23**].

The CVaR$(1-\alpha)$ value, denoted by $\phi_{(1-\alpha)}(x)$, for the profit random variable associated with the decision vector $x$ and any specified probability level $\alpha$ in (0, 1) is defined as the conditional expectation of the profit above $\zeta_{(1-\alpha)}(x)$ . Its value is given by:

$$\phi_{(1-\alpha)}(x) = (1-\alpha)^{-1} \int\limits_{h(x,\lambda^D)\leq\zeta_{(1-\alpha)}(a)} h(x,\lambda^D)p(\lambda^D)d\lambda^D \tag{32}$$

For instance, the CVaR(1 - 0.95) is the average of the expected monetary profit for potential revenue values that fall bellow the VaR(1 - 0.95) threshold. The concept of CVaR is illustrated in Fig.1.

The main idea of the formulation in [**22**] is a characterization of $\phi_{(1-\alpha)}(x)$ in terms of a far simpler function $F_{(1-\alpha)}$ , called the *CVaR function*, defined as follows:

$$F_{(1-\alpha)}(x,\zeta) = \zeta + (1-\alpha)^{-1} \int\limits_{b\in\mathbb{R}^m} [\zeta - h(x,\lambda^D)]^+ p(\lambda^D)d\lambda^D \tag{33}$$

where $[t]^+ = \max\{0,\ t\}$

This function can be used instead of $\phi_{(1-\alpha)}(x)$ which is difficult to handle because of the VaR$(1-\alpha)$ function $\zeta_{(1-\alpha)}(a)$ involved in its definition. Moreover the most relevant properties of the function $F_{(1-\alpha)}$ are as follows [**21**]:

- $F_{(1-\alpha)}(x,\zeta)$ is convex, which is a key property in optimization that eliminates the possibility of a local maximum being different from a global maximum.
- The VaR$(1-\alpha)$ function $\zeta_{(1-\alpha)}(x)$ is a maximum point of $F_{(1-\alpha)}$ with respect to $\zeta$: $\zeta_{(1-\alpha)} = \text{argmax}_{\zeta\in\mathbb{R}}\ F_{(1-\alpha)}$.
- The maximum of $F_\gamma(x,\zeta)$ with respect to $\zeta$ gives the CVaR$(1-\alpha)$, (function $\phi_{(1-\alpha)}(x)$).

These properties are summarized in the following expression:

$$\phi_{(1-\alpha)}(x) = F_{(1-\alpha)}(x, \zeta_{(1-\alpha)}(x)) = \max_{\zeta \in \mathbb{R}} F_{(1-\alpha)}(x, \zeta)$$

This formula reveals the fact that the CVaR$(1-\alpha)$ can be calculated without the need to calculate first the VaR$(1-\alpha)$ value on which its definition formally depends (Eq.(32)). Another advantage of the function $F_{(1-\alpha)}$ is that it can be used for the calculation of VaR$(1-\alpha)$ and the optimization of CVaR$(1-\alpha)$ at the same time:

$$\max_{x \in \mathcal{X}} \phi_{(1-\alpha)}(x) = \max_{(x,\zeta) \in (\mathcal{X}, \mathbb{R})} F_{(1-\alpha)}(x, \zeta)$$

In Eq. (33) the integral in the function $F_{(1-\alpha)}$ can be approximated by sampling the probability distribution of $\lambda^D$ according to its density $p(\lambda^D)$. If the sampling generates a collection of vectors $\lambda^{D,s}$, $s = 1, \ldots, S$ , then the corresponding approximation can be calculated as follows:

$$\tilde{F}_{(1-\alpha)}(x, \zeta) = \zeta - (1-\alpha)^{-1} \sum_{s=1}^{S} P^s [\zeta - h(x, \lambda^{D,s})]^+ \tag{34}$$

where $P^s$ are probabilities of scenarios $\lambda^{D,s}$.

Further, CVaR can be modeled by means of linear expressions. The work in [**22**] describe a linear programming formulation to optimize the value of CVaR and, at the same time, to calculate the value of VaR. By using auxiliary variables $a_s$, $s = 1, \ldots, S$, the function $\tilde{F}_{(1-\alpha)}(x, \zeta)$ can be replaced by the function:

$$\zeta - (1-\alpha)^{-1} \sum_{s=1}^{S} P^s a_s \tag{35}$$

and the set of linear constraints:

$$a_s \geq \zeta - h(x, \lambda^{D,s}), \quad a_s \geq 0 \qquad s = 1, \ldots, S \tag{36}$$

Thus maximization of the approximation of the CVaR function $\tilde{F}_{(1-\alpha)}(x, \zeta)$ can be reduced to the following linear programming problem:

$$\begin{cases} \max & \zeta - (1-\alpha)^{-1} \sum_{s=1}^{S} P^s a_s \\ \text{s.t. :} & \\ & a_s \geq \zeta - h(x, \lambda^{D,s}) \qquad \forall s \in \mathcal{S} \\ & a_s \geq 0 \qquad \forall s \in \mathcal{S} \end{cases} \tag{37}$$

This formulation provides a practical technique as to solve problems with a large number of scenarios.

The description just introduced in this section establishes the foundations for applying the CVaR concept to reformulate the $SO_2$ and $NO_x$ emission constraints, and extend the optimal bid generation model of the precedent chapter with CVaR-like risk constraints.

## 2. Conditional Emission at Risk (CEaR)

In the previous chapter, the NERP limits for the $SO_2$ and $NO_x$ emissions control, have been addressed in a very-restricting way. Constraints (24) and (25) imposed an upper limit on emissions that can never be violated. To enrich the analysis referred to the emission control, and following the CVaR risk notions presented above, some flexibility will be introduced in the emission limits control through a new risk tool called *Conditional Emission-at-risk* (CEaR).

In order to evaluate the consequences of exceeding emission limits, the $SO_2$ emission limit constraint (24) can be reformulated as a probabilistic constraint in the following way:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \le M^{SO_2} y^s \qquad \forall s\in\mathcal{S} \tag{38}$$

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \ge M^{SO_2} (y^s - 1) \qquad \forall s\in\mathcal{S} \tag{39}$$

$$\sum_{s\in S} P^s y^s \le \gamma \tag{40}$$

The first two equations (38) and (39) conveniently classify the scenarios in which the $SO_2$ emission exceed the limit. $y^s$, $s\in\mathcal{S}$ is a binary variable that takes value 1 if the emissions are higher than $\overline{SO_2}$ and 0 otherwise, and parameter $M^{SO_2} \gg 1$ is an upper bound of the emission violation, that is:

$$M^{SO_2} \ge \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \ge -M^{SO_2}$$

Equation (40), in turn , limits joint probability of those of scenarios that may exceed the $SO_2$ emission upper bound $\overline{SO_2}$. Thus, instead of imposing an emission limit at every scenario $s$, this approach allows with some probability $\gamma$ that some of the scenarios exceeds the limit. It is worth mentioning that when $\gamma$ takes value 0 constraints (38)- (40) are equivalent to constraint (24) in which no scenario can exceed the emission limit, while taking $\gamma = 1$ is equivalent to not to impose any limit at all.

The above three constraints: (38) - (40) can be complemented imposing certain limit on the average amount in which the emissions exceed the limit. Taking the risk ideas presented in Section 1, but applied to the study of the emission function $\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s$ instead of the profit function $h(x,\lambda^D)$, it is possible to propose, by analogy of the CVaR function, the so-called Conditional Emission-at-Risk (CEaR) in order to establish a new measure of risk associated with the expected value of $\overline{SO_2}$ excess.

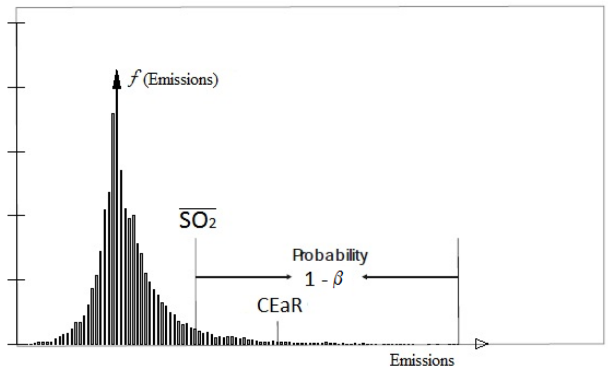


FIG. 2. Graphical representation of CEaR concept. Based on [**22**]

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To do this it's necessary to define, for every scenario $s$, a new set of variables $emi^{SO_2,s}$ whose value will be equal the $SO_2$ emissions ($\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s$) if the emission of scenario $s$ exceed the limit, i.e. $y^s = 1$; or 0 if the emission of scenario $s$ are below the limit i.e. $y^s = 0$. The following constraints ensure the correctness definition of $emi^{SO_2,s}$:

$$emi^{SO_2,s} - \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s \leq M^{SO_2}(1-y^s) \qquad \forall s\in\mathcal{S} \tag{41}$$

$$emi^{SO_2,s} - \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s \geq -M^{SO_2}(1-y^s) \qquad \forall s\in\mathcal{S} \tag{42}$$

$$emi^{SO_2,s} \leq M^{SO_2} y^s \qquad \forall s\in\mathcal{S} \tag{43}$$

In this way Eq. (41) - (43) guarantee that:

$$emi^{SO_2,s} = \begin{cases} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{SO_2} g_{ti}^s & \text{if } y^s = 1 \\ 0 & \text{if } y^s = 0 \end{cases} \qquad \forall s\in\mathcal{S}$$

With these set of variables, the CEaR definition can be now formalized as follows: the $\overline{SO_2}$-CEaR value, for the emission random variable associated with the decision vector $g_{ti}^s$ and any specified probability level $\beta$ is the conditional expectation of the emissions, given that $SO_2$ emission is beyond the $SO_2$ emission limit $\overline{SO_2}$. The following constraint may conveniently control the average amount of the emissions excess over the limit $\overline{SO_2}$:

$$\frac{1}{\sum_{s\in S} P^s y^s} \sum_{s\in S} P^s emi^{SO_2,s} \leq (1+\beta)\overline{SO_2} \tag{44}$$

where $\beta \geq 0$ is the maximum permitted violation as a fraction of the emission limit $\overline{SO_2}$. Eq. (44) then ensures, that in case that emissions are above the limit, the deviation will be on the average less than a fraction $\beta$ of $\overline{SO_2}$. Note that when $\beta = 0$ no scenario can exceed the emission limit, and when $\beta \gg 0$ this constraint does not impose any limit.

It is worth mentioning that known value $\overline{SO_2}$ term plays the role of the VaR level in the CVaR definition, which as mentioned above, is difficult to handle. This is why the CEaR concept is much easier to implement than the CVaR concept. The definition of CEaR($\beta$) is illustrated graphically in Fig. 2.

In this way, to include in the model the $SO_2$ emission limit through Eq. (38)-(44) gives the opportunity to relax the problem. Using different values for parameters $\gamma$ and $\beta$ it is possible to evaluate the impact of excess emissions in the unit commitment of the generation units and in the objective function value.

Similarly to what has been done in the case of $SO_2$ emissions, it is possible to reformulate the $NO_x$ emission limit constraint (25) through the following set of constraints:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \leq M^{NO_x} z^s \qquad \forall s \in \mathcal{S} \tag{45}$$

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \geq M^{NO_x} (z^s - 1) \qquad \forall s \in \mathcal{S} \tag{46}$$

$$\sum_{s\in S} P^s z^s \leq \gamma \tag{47}$$

$$emi^{NO_x,s} - \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{NO_x} g_{ti}^s \leq M^{NO_x}(1 - z^s) \qquad \forall s \in \mathcal{S} \tag{48}$$

$$emi^{NO_x,s} - \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} e_i^{NO_x} g_{ti}^s \geq -M^{NO_x}(1 - z^s) \qquad \forall s \in \mathcal{S} \tag{49}$$

$$emi^{NO_x,s} \leq M^{NO_x} z^s \qquad \forall s \in \mathcal{S} \tag{50}$$

$$\frac{1}{\sum_{s\in S} P^s z^s} \sum_{s\in S} P^s emi^{NO_x,s} \leq (1+\beta)\overline{NO_x} \tag{51}$$

The set of variables $z^s$ and $emi^{NO_x,s}$, and parameter $M^{NO_x}$ are defined analogously to $y^s$, $emi^{So_2,s}$ and $M^{SO_2}$ respectively. Of course, both constraints (44) and (51) can be linearized by multiplying both terms by the non-negative expression $\sum_{s\in S} P^s y^s$ and $\sum_{s\in S} P^s z^s$ respectively.

## 3. CVaR of Total Profits Including CO2 Emission Allowances

A generation company typically prepares budgets which include an estimate of the profits obtained during a given period, in order to be able to evaluate whether the probability of facing a low-profit outcome is higher than desired. This requires analyzing the operating profit of the Genco. Focusing on the probability distribution function for the profits obtained during a given period, one could consider the possibility of finding a strategy to improve the worst scenarios. This strategy can be approached with the CVaR concept through CVaR constraints. CVaR establishes a hard limit on the expected value of revenues for the subset of scenarios in which the profit falls below the VaR at a given confidence level. In this regard, using the CVaR concept, the aim of this section is to extend the proposed model for the optimal electricity market bid problem, with a new constraint to impose a lower limit to the worst-case expected profits value.

Following [**22**] and [**23**] a CVaR constraint, that is, a constraint that imposes a lower bound to the value of the CVaR associated to the optimal generation bid, can be approximated by including in the optimization problem a new set of linear constraints. Let $S$ be a set of scenarios, and let $\lambda^D$, $\forall s \in \mathcal{S}$ be a sampled from the density $\lambda^D$. The CVaR constraint, $\phi_\gamma(x) \geq \omega$ can be equivalently represented, by the following set of constraints:

$$\left\{\begin{array}{ll} \zeta - (1-\alpha)^{-1} \sum_{s=1}^{S} P^s a_s \geq \omega & \\ a_s \geq \zeta - E_{\lambda^D}\left[h(u, c^u, c^d, g, p, b, f; \lambda^D)\right] & \forall s \in \mathcal{S} \\ a_s \geq 0 & \forall s \in \mathcal{S} \end{array}\right. \tag{52}$$

where $\omega$ is some constant constraining CVaR, $a_s$ are the auxiliary variables used to maximize CVaR and to simultaneously calculate the VaR value $\zeta$, described in Section 1 above and $E_{\lambda^D}\left[h(u, c^u, c^d, g, p, b, f; \lambda^D)\right]$ is the objective function described in Section 5 of Chapter 3 (i.e. the expected profits of the GenCo obtained with the participation in the DAM and the incomes/costs associated with CO2 allowances).

This set of constraints ensures that in the worst case scenario, the benefit will be, at least, the fixed $\omega$ value. For instance setting the lower bound $\omega$ and the confidence level $\alpha = 0.95$ implies that the average profit in the 5% worst scenario cases will not be below the lower bound $\omega$ value.

In the next sections the effect of the CEaR and CVaR risk constraints on the optimal generation bid will be analyzed.

## 4. Case Study

The data for the day-ahead market prices has been downloaded from the website of the Independent Iberian Market Operator OMEL [**24**]. This study uses the same set of 50 scenarios generated in [**25**] for the random day-ahead market spot prices $\lambda^D$ as the result of the application of a scenario reduction algorithm [**26**] to the complete set of historic data available from June 2007 to May 2010 [**25**]. The generation units of this study corresponds to the same two combined cycle units considered in [**13**]. They correspond to actual generation units currently operating in the IEM. The operational characteristics of the thermal units have been updated using the generation cost of two different kinds of coal thermal units: national and imported. The electricity production cost from both national and imported coal has been updated according to the Book of Energy in Spain 2010 published by The Ministry of Industry, Energy and Tourism of Spain Government [**27**]. The values found ($42.16$ [€/tCoal] for national coal and $65.08$ [€/tCoal] for imported coal) correspond to the average price of coal during the year 2010.

In order to recalculate, accordingly with the updated price of coal, the constant, linear and quadratic coefficients, $c_i^b$, $c_i^l$ and $c_i^q$, of the generation costs, the price of the coal ([€/ton]) have been transformed to [c€/kWh], based on the information found in *Energy*

TABLE 1. Operational Characteristics of the Thermal Units

| $i$ | $c_i^b$ € | $c_i^l$ €/MWh | $c_i^q$ €/MWh$^2$ | $\underline{p}_i$ MW | $\overline{p}_i$ MW | $st_i^0$ hr | $c_i^{on}$ € | $c_i^{off}$ € | $t_i^{on}$ hr | $t_i^{off}$ hr |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 159.24 | 42.55 | 0.016 | 160.0 | 350.0 | +3 | 435.09 | 435.09 | 3 | 3 |
| 2 | 901.70 | 59.38 | 0.038 | 250.0 | 563.2 | +3 | 1307.70 | 1307.70 | 3 | 3 |
| 3 | 344.68 | 30.41 | 0.038 | 160.0 | 370.7 | +3 | 462.07 | 462.07 | 3 | 3 |
| 4 | 322.04 | 60.04 | 0.032 | 160.0 | 364.1 | +3 | 682.04 | 682.04 | 3 | 3 |

TABLE 2. Operational Characteristics of the Combined Cycle Units

| $c$ | $\mathcal{P}_c$ | $c_i^b$ € | $c_i^l$ €/MWh | $c_i^q$ €/MWh$^2$ | $\underline{p}_i$ MW | $\overline{p}_i$ MW | $st_i^0$ hr | $c_i^{on}$ € | $t_i^{on}$ hr |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 151.08 | 50.37 | 0.023 | 160.0 | 350.0 | –2 | 803.75 | 2 |
| 1 | 6 | 224.21 | 32.50 | 0.035 | 250.0 | 563.2 | –2 | 412.80 | 2 |
| 2 | 7 | 163.11 | 55.58 | 0.019 | 90.0 | 350.0 | –2 | 320.50 | 2 |
| 2 | 8 | 245.32 | 31.10 | 0.022 | 220.0 | 700.0 | –2 | 510.83 | 2 |

*in a Ton of Coal* [**28**]that ensures that each ton of coal consumed at an electric power plant produces about $2.000$ kilowatt hours (or 2 [MWh]) of electricity. According to this, for example, $40$ [€/tCoal] corresponds to $20$ [€/MWh]. Once the costs are expressed in [c€/kWh] the coefficients from the generation costs and the start-up and shut-down costs, $c^{on}$ and $c^{off}$ have been re-calculated proportionally to the respective values in [**13**]. Table 1 shows the updated operational characteristics of the thermal units. For instance, in [**13**], the constant coefficient from generation cost for the first thermal unit is $151.08$ [€], and its fuel supply cost is $20$ [€/MWh], then as the updated fuel supply cost is $21.08$ [€/MWh] (or $42.16$ [€/tCoal]) the updated constant coefficient from generation cost can be calculated as follows:

$$\frac{151.08\ [€] * 21.08\ [€/\text{MWh}]}{20\ [€/\text{MWh}]} = 159.24\ [€]$$

The remaining data have been calculated in a similar way. The values of $\underline{p}_i$, $\overline{p}_i$, $st_i^0$, $t_i^{on}$ and $t_i^{off}$ are the same that were considered in [**13**]. The characteristics of the CC units and bilateral and future contracts are shown in Table 2 and 3, respectively. The parameter $st_i^0$ stands for the number of hours the unit has been on ($st_i^0 > 0$) or off ($st_i^0 < 0$) previous to the first optimization period. The minimum off time for both CC units (parameter $(t_c^{off})^C$ in Eq.21) was set to 3 hours, and both CC units was considered shut-down for one hour previous to the first optimization period.

All data related with $SO_2$ and $NO_x$ can be obtained from Tables 4 and 5. The emission limits $\overline{SO_2}$ and $\overline{NO_x}$ derives from the National Emission Reduction Plan [**2**].

The generation unit's emission data shown in Table 5 are adapted from [**17**] while the SO2 and NOx emissions rates correspond to the values published by the Intergovernmental Panel on Climate Change Emission [**29**] for coal thermal units.

TABLE 3. Characteristics of Futures and Bilateral Contracts

| $j$ | $L^{BC}_{j,t=1\dots24}$ MW | $\lambda^{BC}_{j,t=1\dots24}$ €/MWh | $L^{FC}_{j,t=1\dots24}$ MW | $\lambda^{FC}_{j,t=1\dots24}$ €/MWh |
|---|---|---|---|---|
| 1 | 164 | 43,35 | 120 | 45,6 |
| 2 | 50 | 43,35 | 120 | 46,1 |
| 3 | 150 | 43,35 | 120 | 51,2 |

TABLE 4. Daily emission limits data

| $\lambda^{CO_2}$ €/t | $\overline{CO_2}$ $tCO_2$/day | $\overline{SO_2}$ kg/day | $\overline{NO_x}$ kg/day |
|---|---|---|---|
| 15.28 | 1.527 | 3.900 | 17.651 |

TABLE 5. Generation unit's emissions data

| | $e_i^{CO_2}$ $tCO_2$/€ | $e_i^{SO_2}$ kg/MWh | $e_i^{NO_x}$ kg/MWh |
|---|---|---|---|
| Thermal units (National Coal) | 0.06765 | 0.7848 | 1.368 |
| Thermal units (Imported Coal) | 0.04383 | 0.7848 | 1.368 |
| CC units (Gas) | 0.0104 | - | - |

Emission conversion factors for $CO_2$ have been recalculated according to section: *Calculation of Carbon Dioxide Emissions* of [**17**]. As the value of the emission coefficient for each generator $i$, $e_i^{CO_2}$ depends on the type of a generating unit, as well as on the quality of the used fuel, its value needs to be calculated in a way that accounts for these variations. Columns 3-5 of Table 6 shows the net calorific value (NCV), the emission factor (EF) and the oxidation factor (OF) respectively. Column 6 shows the fuel supply cost (FSC) whose values have been taken from Book of Energy in Spain 2010 [**27**] (national coal form p. 121 and imported coal from: p. 120), and finally column 7 show the emission conversion factors whose values can calculated according to equation (16) in [**17**] by the following expression:

$$e_i^{CO_2} = \frac{NCV \cdot EF \cdot OF}{FSC} \tag{53}$$

Net calorific value and emission factor depend on the particular type of a fuel used, and have to be regularly measured. The emission factor is based on the carbon content of a fuel. Finally, the oxidation factor accounts for the fact that a portion of carbon content remains unburned or partly oxidized and is therefore not emitted into the atmosphere.

TABLE 6. $CO_2$ Emission Conversion Factors

| $i$ | Unit Type | NCV kJ/kg or kJ/$m^3$ | EF $tCO_2$ | OF % | FSC €/t | $e_i^{CO_2}$ $tCO_2$/€ |
|---|---|---|---|---|---|---|
| 1 and 3 | National Coal | 29308 | 98.3 | 0.995 | 42.16 | 0.06765 |
| 2 and 4 | Imported Coal | 29308 | 98.3 | 0.995 | $65,08$ | 0.04383 |

# 5. Numerical Tests

Four case studies was used to evaluate the impact of the emission constraints in the optimal scheduling and bid of the generation units:

- **BASE:** optimal bid problem without neither emission constraints nor $CO_2$ allowances. Corresponds to problem (31) excluding both the $CO_2$ rights cost/incomes term (30) and emission constraints (24)-(25).

- **EC:** optimal bid problem with only emission constraints. Corresponds to problem (31) retaining emission constraints (24)-(25).

- **EC($\gamma$, $\beta$):** optimal bid problem with only emission constraints, but formulated using the risk concepts. Corresponds to problem (31) excluding the $CO_2$ rights cost/incomes term (30) and replacing emission constraints (24) and (25) by (38)-(44) and (45)-(51) respectively. Parameters $\gamma$ and $\beta$ are those described in the Section 2 of this chapter. Different values of $\gamma$ and $\beta$ have been used in order to assess the possibility of exceeding emission limits. In particular, as an example, the results highlights the case where $\gamma = 0.3$ and $\beta = 0.15$. The problem with emission constraints using the risk concept can be mathematically expressed as the following optimization problem:

$$
(54) \quad \left\{ \begin{array}{lll} \max & E_{\lambda^D}\left[h(u, c^u, c^d, g, p, b, f; \lambda^D)\right] & \\ \text{s.t. :} & & \\ & \text{Eq. } (1)-(2) & \text{FC and BC} \\ & \text{Eq. } (3)-(6) & \text{Day-ahead market} \\ & \text{Eq. } (7)-(8) & \text{Total generation} \\ & \text{Eq. } (9)-(14) & \text{Thermal unit commitment} \\ & \text{Eq. } (17)-(23) & \text{CC unit commitment} \\ & \text{Eq. } (38)-(44) \text{ and } (45)-(51) & \text{Emission with risk} \end{array} \right.
$$

- **CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$):** the complete model (31) with both $CO_2$ allowances and $SO_2$ and $NO_x$ emission constraints formulated with the risk management methodology. Corresponds to full model (31) replacing emission constraints (24) and (25) by (38)-(44) and (45)-(51) respectively, and including the CVaR constraint associated with the total profits including $CO_2$2 emission allowances (52). The parameter value $\gamma$ and $\beta$ have been set to 0.3 and 0.15, respectively. $\alpha$ is the confidence level, its value has been set at 0.95, $\omega$ the minimum CVaR. To calculate its value, first the optimal bid problem with only $CO_2$ allowances was solved (CO2 case). Then the CVaR of the optimal solution of problem CO2 is calculated with the help of problem (37). To this

TABLE 7. Case studies, number of variables and constraints

| Cases | Variables Continuous | Variables Binary | Constraints |
|---|---|---|---|
| BASE | 20.160 | 200 | 49.458 |
| EC | 20.160 | 200 | 49.558 |
| EC($\gamma$, $\beta$) | 20.260 | 300 | 49.962 |
| CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) | 20.311 | 300 | 50.013 |

$\alpha = 0.95$, $\gamma = 0.3$, $\beta = 0.15$ and $\omega = 669.000$

end, the optimal value for both the first and the second stage variables is fixed in the formulation of problem (37) and the resulting problem is optimized with respect to variable $\zeta$. The optimal value of this problem (704.862 €) corresponds to the CVaR value associated with the case CO2, as explained previously in Section 2. Finally, the lower bound to the CVaR value has been fixed to a 95% of the CVaR associated with the case CO2: 669.000 €. The problem with emission constraints using the risk concept and with the CVaR constraint can be mathematically expressed as follows:

$$
(55) \quad \left\{ \begin{array}{lll}
\max & E_{\lambda^D}\left[h(u, c^u, c^d, g, p, b, f; \lambda^D)\right] & \\
\text{s.t. :} & & \\
& \text{Eq. } (1)-(2) & \text{FC and BC} \\
& \text{Eq. } (3)-(6) & \text{Day-ahead market} \\
& \text{Eq. } (7)-(8) & \text{Total generation} \\
& \text{Eq. } (9)-(14) & \text{Thermal unit commitment} \\
& \text{Eq. } (17)-(23) & \text{CC unit commitment} \\
& \text{Eq. } (38)-(44) \text{ and } (45)-(51) & \text{Emission with risk} \\
& \text{Eq. } (52) & \text{CVaR constraint}
\end{array} \right.
$$

All these cases have been implemented with the AMPL modeling language [**19**] and solved with CPLEX 12.4 [**20**] (`mipgap=0.05, threads=20`) over a SunFire X2200 with 32 Gb of RAM memory and two dual core processors AMD Opteron 2222 at 3 GHz, taking advantage of the multithreading capabilities of CPLEX.

The risk constrained models EC($\gamma$, $\beta$) and CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) are, in fact, a family of models parameterized by the risk factors $\alpha$, $\omega$,$\gamma$ and $\beta$. In order to illustrate the effect of the risk constraints in the solution of the optimal generation bid problem, the detailed solution for the instance with $\alpha = 0.95$, $\omega =$ 669.000, $\gamma = 0.3$, $\beta = 0.15$ and will be presented.

Table 7 shows the number of continuous and binary variables as well as the number of constraints for the four cases. Execution time is below one minute for cases: BASE, EC, and CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) with $\alpha = 0.95$, $\omega = 669.000$, $\gamma = 0.3$ and $\beta = 0.15$. Execution time is also below one minute for case EC($\gamma$, $\beta$) with $\gamma = 0$, but when $\gamma$ and $\beta$ take values the execution time increases considerably, for example when $\gamma = 0.9$ and $\beta = 1.10$ the execution time is 269.051 s, about 3h45min.

TABLE 8. Daily emissions at optimal solution

| | $E[CO_2]$ t/day | $E[SO_2]$ kg/day | $E[NO_x]$ kg/day |
|---|---|---|---|
| BASE | 1.561 | 8.413 | 14.665 |
| EC | 832 | 3.900 | 6.798 |
| EC($\gamma$, $\beta$) | 859 | 4.074 | 7.101 |
| CO2($\alpha$, $\omega$)EC($\alpha$, $\beta$) | 796 | 3.899 | 6.797 |

$\alpha = 0.95$, $\gamma = 0.3$, $\beta = 0.15$ and $\omega = 669.000$

TABLE 9. Optimal Incomes and Costs

| | BASE | EC | EC($\gamma$, $\beta$) | CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) |
|---|---|---|---|---|
| $E$[Profit] | 469.663 € | 448.766 € | 455.451 € | 710.828 € |
| BC Incomes | 419.122 € | 419.122 € | 419.122 € | 419.122 € |
| FC Incomes | 374.543 € | 374.543 € | 374.543 € | 374.543 € |
| $E$[Market Incomes] | 691.628 € | 482.469 € | 494.382 € | 395.160 € |
| $E$[Generation Costs] | -1.011.020 € | -823.194 € | -828.422€ | -741.428 € |
| Start-up/shut-down Costs | -4.608 € | -4.173 € | -4.173 € | -4586 € |
| $E$[$CO_2$ Rights Cost/Incomes] | - | - | - | 268.017 € |

$\alpha = 0.95$, $\gamma = 0.3$, $\beta = 0.15$ and $\omega = 669.000$

Table 8 depicts the expected value of the $CO_2$, $SO_2$ and $NO_x$ emissions, at the optimal solution of the four cases. The table clearly shows the impact of considering the emission constraints on the level of emissions. Focusing on the CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) case, the $CO_2$, $SO_2$ and $NO_x$ emissions has been reduced by 49%. 54% and 54% respectively, compared to the BASE case.

Table 9 shows the optimal value of the expected profits (objective function of problem (31) for the four cases together with the value of the different terms (26)-(30)). Although the reduction in the total generation forced by the $SO_2$ and $NO_x$ limits (case EC(0.3, 0.15)) causes a decrease of 3% in the total profit, the expected incomes due to the $CO_2$ rights: 268.017 € compensates this loss increasing the total expected profits in a 34%. Fig. 3 illustrates the structure of the benefits and the costs in each one of the cases-studies showing the individual contribution of the bilateral contracts, the future contracts, the market revenue, and the $CO_2$ right incomes (if any) to the expected value of total profit. Total cost has been also splitted in generation costs and Start-up/shut-down costs. While the distribution of the benefits and costs is quite similar in the EC and EC($\gamma$, $\beta$) cases , the 6,685 €/day difference in the optimal value of the two objective functions, turn out to be quite significant. For instance considering Gas Natural Fenosa with a total of 19 generating units (11 thermal units and 8 combined cycle units) the equivalent increase in the expected profits will be 21.170 € daily which results in nearly 7.621.000 € over one year. This, in turn, shows how flexibility in the emission limits satisfaction introduced by the CEaR constraints affects (increases) the total profits.

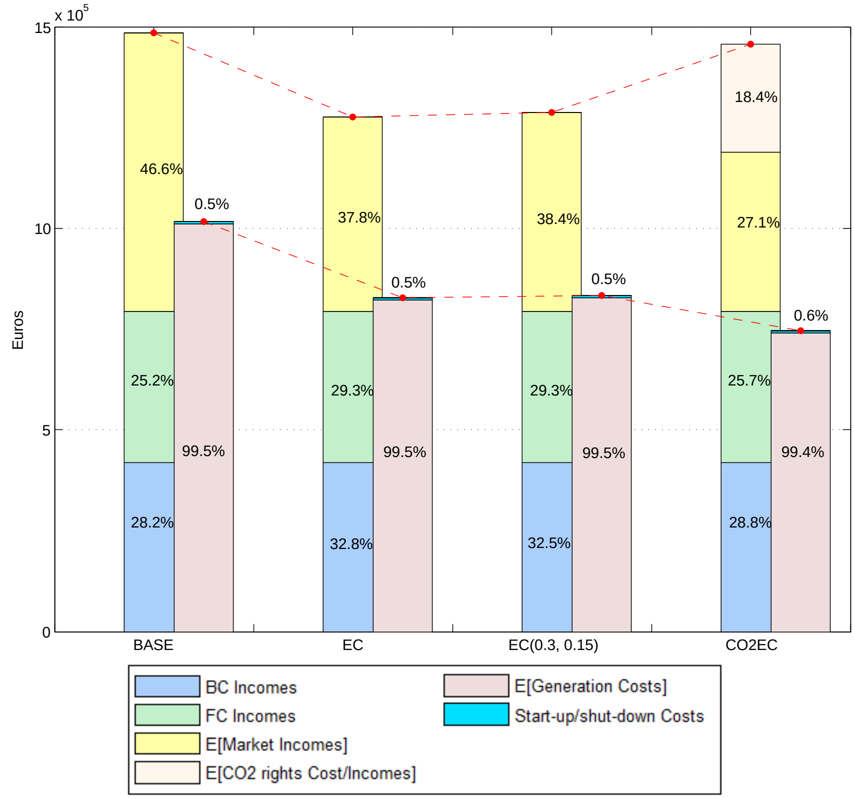



FIG. 3. Optimal Incomes and Costs. The graph shows the overall percentage of participation of the different income and cost sources as to the total value of the profit function.

The impact of the emission constraints over the individual units commitment of each generation unit, together with the optimal dispatch of the bilateral and future contracts, i.e., the quantity each unit commits to the bilateral contracts for each interval $t$, and the energy used to cover the futures contracts, can be judged from Fig. 4, 5 and 6. Fig. 4 depicts the optimal unit commitment for the BASE case. The blue area corresponds to the energy allocated to the bilateral contracts (variable $b_{ti}$); the green area is the energy of the price acceptance bid $q_{ti}$ that includes the energy allocated to the futures contracts $f_{tij}$. Finally, the yellow area is, for each generation $i$ and period $t$, the expected value of the matched energy in the day-ahead market $\sum_{s\in\mathcal{S}} P^s p^s_{ti}$. Comparing the generation profiles in Fig. 4 and 5 it is clear how the environmental constraints are affecting the unit commitment: all the high-emission coal thermal generators are shut-down as soon as $t_i^{off}$ allows, except thermal unit 3, which is maintained on to satisfy future contract 3. This effect is due mainly to the opportunity of profits in the $CO_2$ market by reducing the emission below the Genco's free emission allowances. Although the energy matched in the day-ahead market (the addition of the green plus yellow areas) is reduced from the BASE to the CO2EC cases, the overall profits increases due to the $CO_2$ rights incomes.

The basic premise, that higher expected returns can only be achieved by taking a greater risk, leads naturally to the concept of an efficient frontier. The *efficient frontier* defines the maximum profit that can be achieved for a given confidence level of risk. One way to assess risk is to consider different expected profits varying the confidence levels and generate the efficient frontier. In Fig. 7 it is possible to assess how affects the value of the objective function to overcome the $SO_2$ and $NO_x$ emissions limits. The fewer limitations there are in the value of emissions, the greater the value of the objective function will be, i.e. as $\gamma$ takes larger values, emissions may exceed the limit with greater probability and consequently the objective function value increases. Moreover, if the average percentage at which emissions exceeds the limit is increased (i.e. when $\beta$ increases), the expected value of the profits increase accordingly. Figure 7(b) has highlighted the case EC(0.3, 0.15) to emphasize the result in one of the case studies. As an example of an improvement in profits in an extreme case, it has also highlighted the case $\gamma = 0.9$ and $\beta = 0.3$ whose objective function value is 459,779 €. In this case, the violation of emission restrictions would represent for a company like Gas Natural an increase in profits at the end of a year of about 12,564,000 €. It is worth mentioning that given the proximity of the plotted points in Fig.7, it was necessary to use a smaller optimality gap (`mipgap=0.01`) to build it. For these reason the value represented of the EC case (i.e. $\gamma = 0$ and $\beta = 0$) does not match the value presented in Table 9.

Figure 8 depicts, for each generation unit, the total expected matched energy (yellow bar), the total price-acceptant offer (green bar) and the total energy assigned to bilateral contracts, excluded from the market (blue bar). It can be observed how the CEaR and CVaR risk constraints affects the overall participation of the units in the market. It is specially relevant how heavily the CEaR risk constraints are affecting the results in the case of thermal units 3 and CC unit 2.

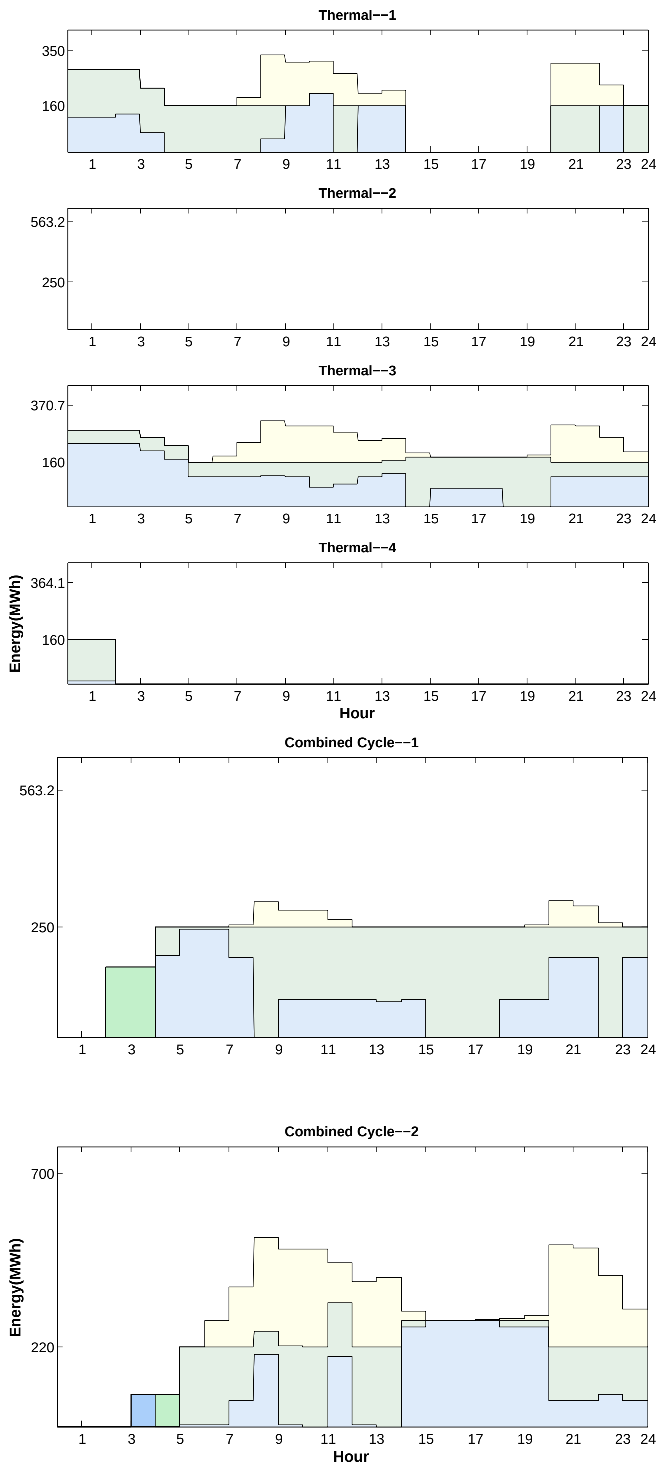



FIG. 4. Unit commitment of the generation units for the BASE case: $b_{ti}$ (scheduled energy for bilateral contract, blue), $q_{ti}$ (price acceptant bid, green). In yellow the expected value of matched energy. For the CC units, dark colors are for state 1 and light colors for state 2.

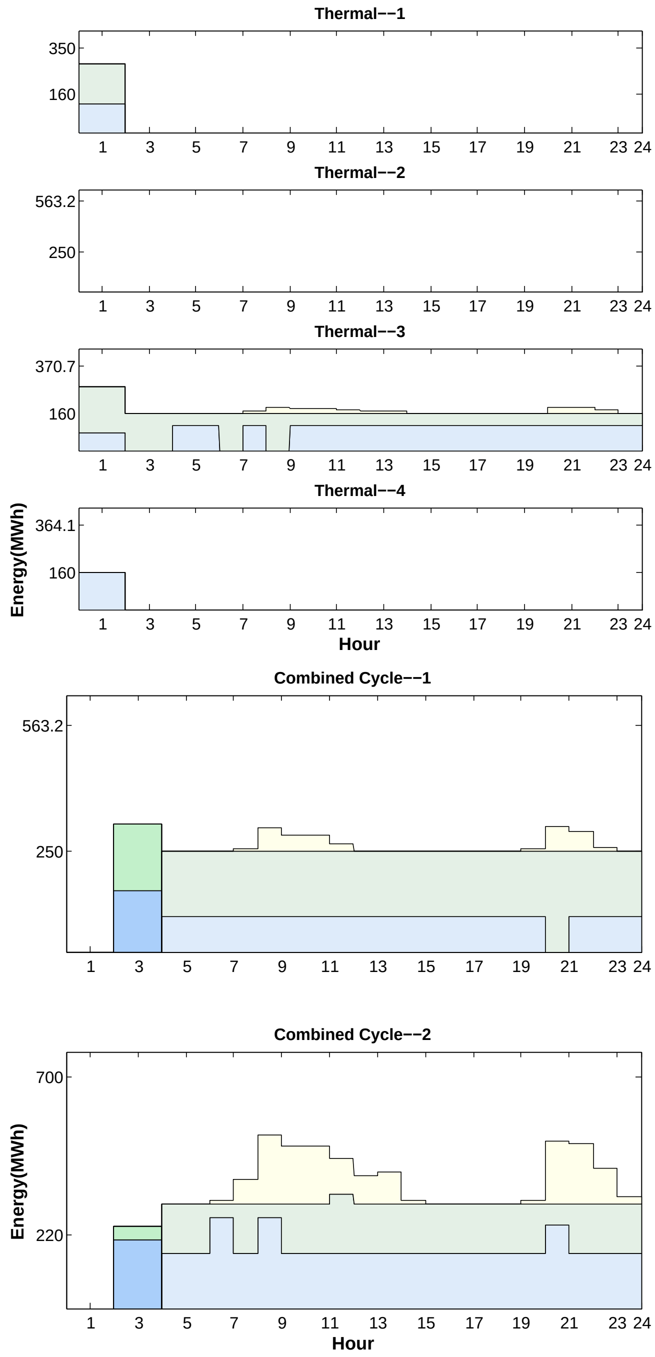



FIG. 5. Unit commitment of the generation units for the EC($\gamma$, $\beta$) case with $\gamma = 0.3$ and $\beta = 0.15$ : $b_{ti}$ (scheduled energy for bilateral contract, blue), $q_{ti}$ (price acceptant bid, green). In yellow the expected value of matched energy. For the CC units, dark colors are for state 1 and light colors for state 2.

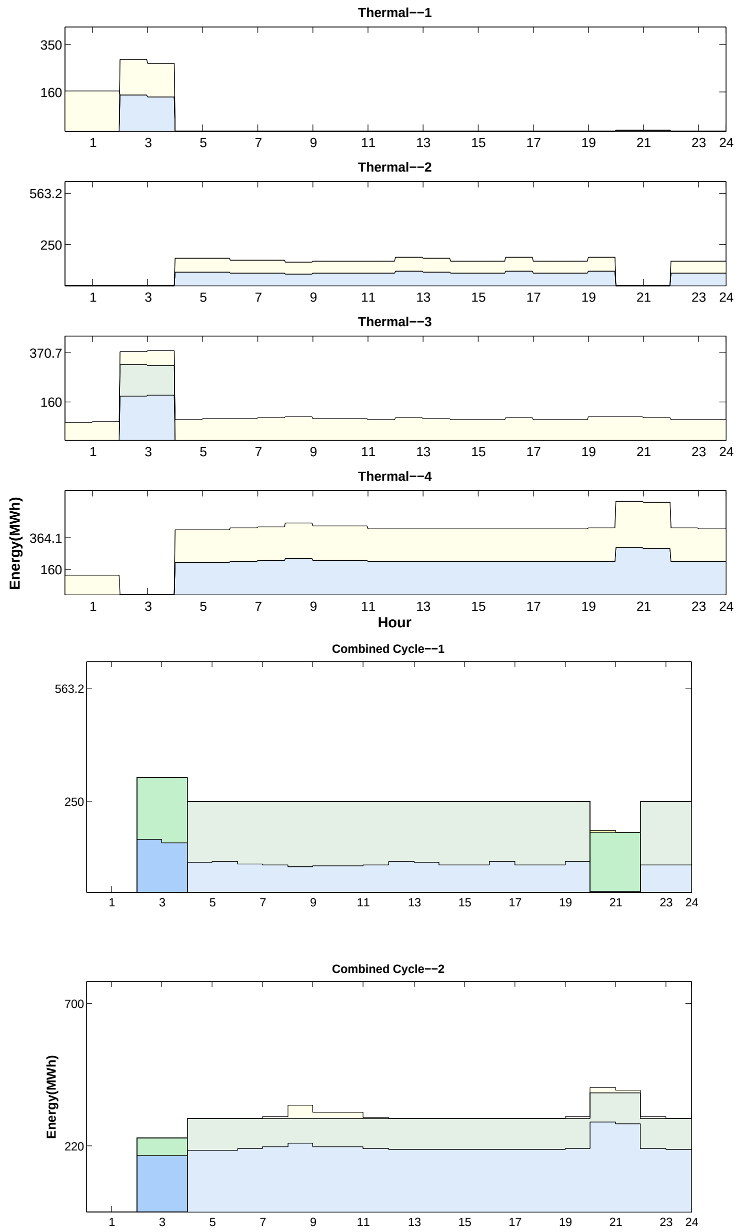
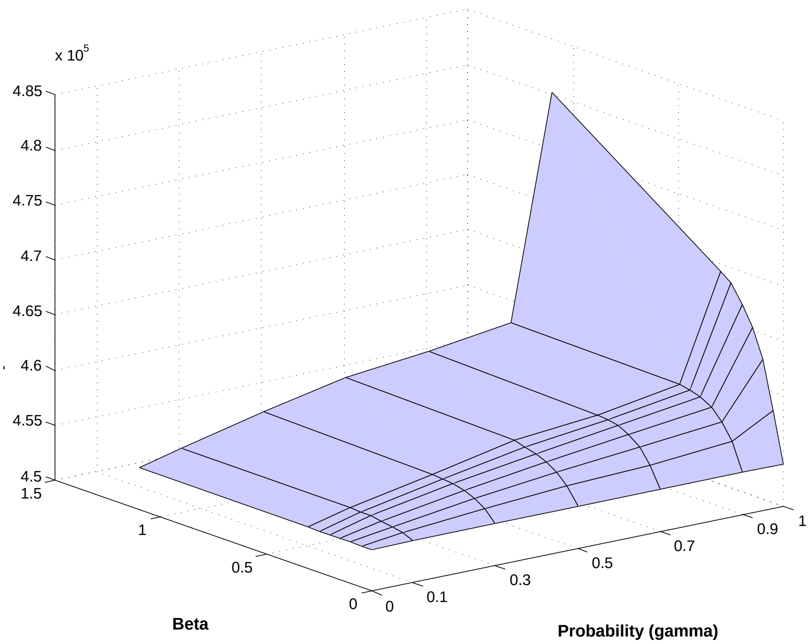
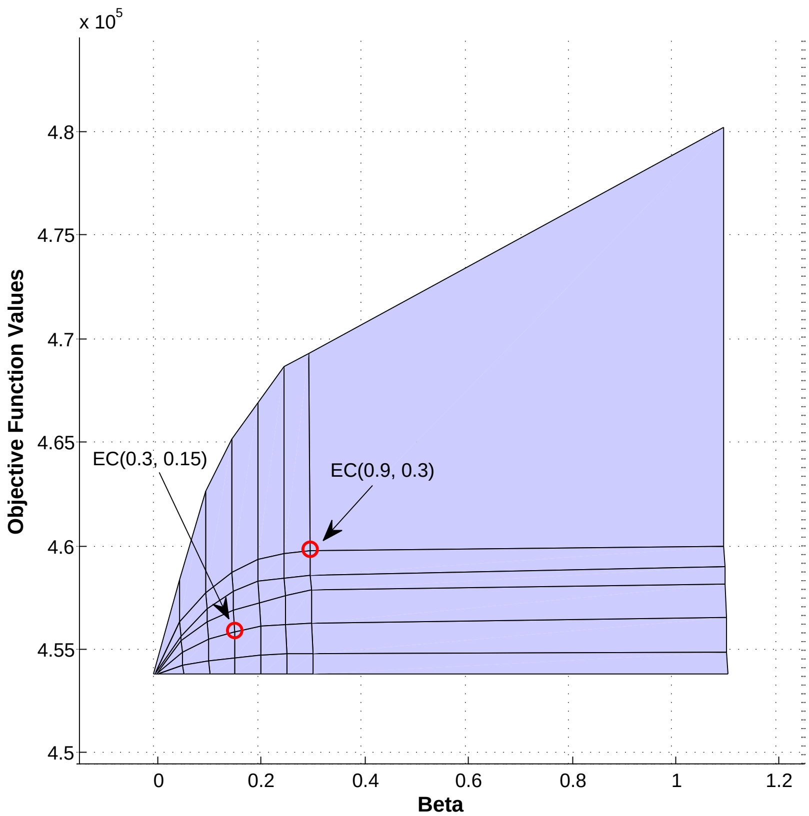



FIG. 6. Unit commitment of the generation units for the CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) case with $\alpha = 0.95$, $\gamma = 0.3$, $\beta = 0.15$ and $\omega = 669.000$: $b_{ti}$ (scheduled energy for bilateral contract, blue), $q_{ti}$ (price acceptant bid, green). In yellow the expected value of matched energy. For the CC units, dark colors are for state 1 and light colors for state 2.




(a)




(b)

FIG. 7. Efficient CEaR frontier of the problem EC($\gamma$, $\beta$): change in the value of the expected profits depending on parameters $\gamma$ and $\beta$. $\gamma \in \{0,\ 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9,\ 1\}$, $\beta \in \{0,\ 0.05,\ 0.1,\ 0.15,\ 0.2,\ 0.25,\ 0.30,\ 1.10\}$.

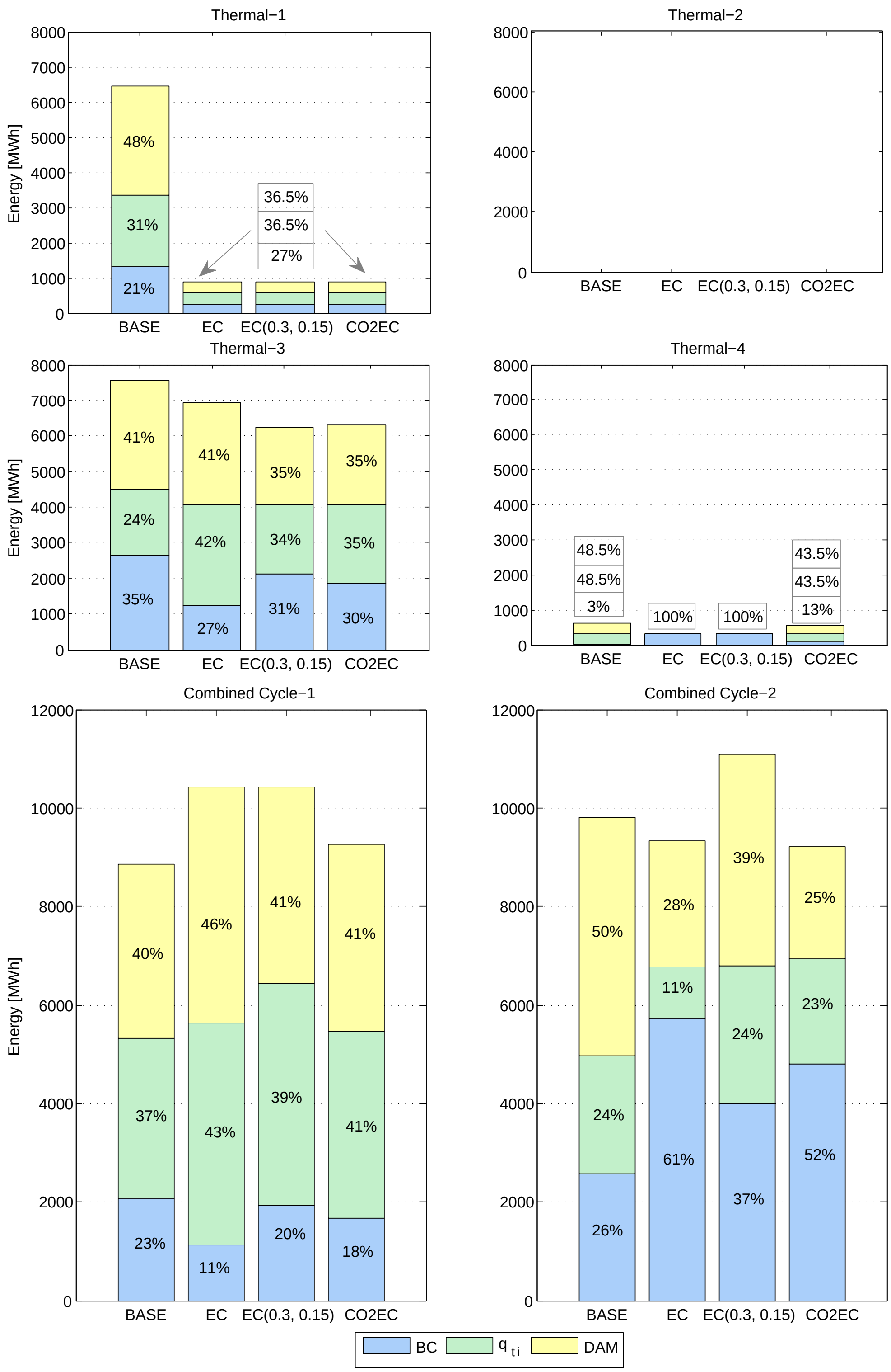



FIG. 8. Comparison of the market participation of each thermal and CC units.

# Chapter 5
# Conclusions

In this thesis we addressed one of the key challenges faced by electricity generation companies. On a daily base, generation companies have to decide the generation bid to be submitted to the day-ahead electricity market[1], where a total of more than 30 millions of Euros are negotiated daily. Management decision are aimed reduce cost and increase income. Hence, economic efficiency is of utmost importance for generating companies, but new constraints should be taken into account to ensure admissible levels of emission in the environment. The new competitive and environmentally constrained electricity supply industry requires new computing tools to ensure both competitiveness with other generating companies in the electricity market and secondly, environmental protection by limiting the emission of greenhouse gases into the atmosphere.

Generally, the impact of the environmental restrictions in the optimal electricity generation has been studied through simulation techniques ([**30, 31**]) or deterministic models that neglect stochasticity [**10**]. The approach of this thesis is to cope with stochasticity using stochastic programming techniques [**32**]. Stochastic programming is a powerful optimization technique that allows to incorporate in a single mathematical optimization model the same statistical information on the relevant random variables handled in the simulation studies performed by other authors, with the advantage that stochastic programming provides, in a single run, the optimal decisions with respect to all possible outcomes of the random variables.

This work provides a mixed-integer stochastic programming model for the integration of both bilateral and futures contracts and the emission restrictions into the day-ahead bidding problem of a Genco operating in the IEM, taking into account the most recent environmental policy. The optimal bid of the model determines, not only the optimal bidding strategy and the optimal operation of the generating units, but also the optimal economic dispatch for BCs and the committed FCs for all thermal and combined cycle units each hour, as well as the influence of the emission constraints on the generation scheduling. The model was implemented in the AMPL language and solved with the commercial optimization package CPLEX. The applicability of the results is supported by the computational test preformed with real data of the Spanish wholesale electricity market in addition to data of actual generation units of Gas Natural Fenosa, one of the major Spanish electrical utilities which is collaborating actively with the GNOM-Energy research team. Results indicate that the

[1] `http://www.omie.es/files/flash/ResultadosMercado.swf`

formulation of the problem can be used to find good-quality solutions in a relatively short time frame.

This thesis has successfully achieved the list of proposed objectives, namely:

- The improvement of the emission-bounded model (Chapter 3) with a new formulation of the unit commitment constraints for Combined Cycle units and the computational implementation and solution of the model, together with the analysis of the achieved results.

- The formulation of a new risk-constrained model, a more elaborated alternative to the initial emission-bounded model, where the risk of both financial profits and violation of emission limits are formulated in terms of CVaR and CEaR (a new CVaR-like risk measurement tool introduced in this thesis) constraints.

- The computational resolution of four case-studies:

  - Base case model disregarding emissions limits,
  - Emission-bounded model: EC case,
  - CEaR risk-constrained model: EC($\gamma$, $\beta$) case and
  - CEaR+CVaR risk constrained model: CO2($\alpha$, $\omega$)EC($\gamma$, $\beta$) case.

- The detailed analysis of the four aforementioned models showing the impact of the environmental constraints in both the generation unit's optimal bid and scheduling as well as the impact in the expected profits' structure, together with the economical consequences of different levels of risk-aversion.

# Chapter 6
# Further developments

The work developed in this project can be extended in three ways.

- From the point of view of the modelization, it would be interesting to include in the stochastic programming model emission-free technologies, as wind-power, solar and hydro-generation units, taking advantage of the experience of the GNOM group in the mathematical modellization of such generation systems. Also, the actual multi-market structure of the electricity markets, where the electricity generation can be bid not only to the day-ahead market but to other subsequent markets (mainly system reserve and intraday markets), should be incorporated in the model, following the most recent developments of the GNOM-Energy team [**25**].

- From the point of view of the theoretical study of the mathematical properties of the optimization problem, it would be interesting to try to find specific valid inequalities of the feasible polytope of problem (31) to be aggregated to the Branch and Cut algorithm of CPLEX. This study will contribute to the characterization of the polytope of (31) [**33**].

- The execution times observed in the computational tests performed so far are below one minute, which is acceptable for this kind of problems with a day-ahead horizon. But with the addition of the wind, solar and hydro-generation units and the inclusion of the reserve and intraday markets, execution times are envisaged to strongly increase. This will lead to the necessity of substituting the general purpose optimization package CPLEX with specialized algorithm for large-scale mixed integer nonlinear programming (MINLP). The GNOM-Energy group has been engaged in the last years in the development of such specialized procedures that could be applied to this end: outer polyhedral approximations of the quadratic objective function $h(u, c^u, c^d, g, p, b, f)$ through perspective cuts [**5**]; Branch and Fix Coordination [**34**].

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# Notation

### Acronyms and symbols

BC: Bilateral Contracts.
CEaR: Conditional Emission-at-risk.
CVaR: Conditional Value-at-Risk.
CC: Combined Cycle.
CCGT: Combined Cycle Gas Turbines.
$CO_2$: Carbon Dioxide.
CT: Combustion turbine.
DAM: Day-ahead Market.
€: Euro.
ETS: Emission Trading Scheme.
EF: Emission Factor.
EU: European Union.
FC: Future Contracts.
FSC: Fuel Supply Cost.
GenCo: Generation Company.
HRSG: Heat Recovery Steam Generator.
IEM: Iberian Electricity Market.
kWh Kilowatt hour.
MWh: Megawatt hour.
NAP: National Allocation Plan.
NCV: Net Calorific Value.
NERP: National Emission Reduction Plan.
$NO_x$: Nitrogen Oxides.
OF: Oxidation Factor.
OMEL: Electricity Market Operator.
RD: Real Decreto.
$SO_2$: Sulphur Dioxide.
ST: Steam Turbine.
PUs: Pseudo Units.
VaR: Value-at-Risk.

## Sets

- $\mathcal{B}$: set of bilateral contracts.
- $\mathcal{F}$: set of futures contracts.
- $\mathcal{I}$: set of thermal units.
- $\mathcal{I}_j$: set of generation units allowed to cover the FC $j$.
- $\mathcal{K}_i$: set of the initial states of each unit $i$ (formulation presented in [**13**]).
- $\mathcal{P}_c$: set of pseudo-units of the CC unit $c \in \mathcal{C}$.
- $\mathcal{P}$: the complete set of pseudo-units. $\mathcal{P} = \cup_{c\in\mathcal{C}}\mathcal{P}_c$.
- $\mathcal{S}$: set of scenarios.
- $\mathcal{X} \in \mathbb{R}^n$: feasible set of generation bids.
- $\mathcal{T}$: set of hourly auctions of the DAM.
- $\mathcal{U}$: total set of generation units.

## Parameters

- $c_i^b$, $c_i^l$, $c_i^q$: constant, linear and quadratic coefficients of the generation cost function of unit $i$.
- $c_i^{on}$, $c_i^{off}$ shut-down and start-up cost of unit $t$.
- $\overline{CO_2}$: the GenCo's aggregated free emission allowances.
- $e_i^{SO_2}$ , $e_i^{NO_2}$: the $SO_2$ and $NO_x$ emission coefficients of the generation-to-emission linear function of thermal unit $i \in \mathcal{I}$.
- $e_i^{CO_2}$: the emission conversion factor of unit $i$
- $G_i$: number of periods that unit $i$ must be initially online, due to its minimum up-time $t_i^{on}$.
- $H_i$: number of periods that unit $i$ must be initially offline, due to its minimum down-time $t_i^{off}$.
- $H_c^C$: represents the number of the initial time periods along which the CC unit $c$ must remain off.
- $L_{tk}^B$: amount of energy to be procured during hour $t$ of the delivery period by the set of available generation units to cover the BC $k$.
- $L_j^F$: amount of energy to be procured each interval of the delivery period by the set $\mathcal{I}_j$ of generation units to cover contract $j$.
- $M^{SO_2}$: upper bound of the $SO_2$ emission violation.
- $M^{NO_x}$: upper bound of the $NO_x$ emission violation.
- $P^s$: probability of scenario s.
- $\overline{P}_i$ , $\underline{P}_i$: upper and lower bounds on the energy generation.
- $st_i^0$: the number of hours the unit $i$ has been on ($st_i^0 > 0$) or off ($st_i^0 < 0$) previous to the first optimization period.
- $\overline{SO_2}$, $\overline{NO_x}$: limits to the joint $SO_2$ and $NO_x$ emissions of the thermal units.
- $t_i^{on}$, $t_i^{off}$: operational minimum idle and in service time of unit $i$.
- $(t_c^{off})^C$ minimum down time of the CC unit $c$ .
- $t_c^C$: the minimum down time of each CC unit.
- $u_{0i}$: initial state of each thermal unit $i$.

$\alpha$: confidence level of the CVaR constraint.

$\alpha_{ti}^{on}$, $\alpha_{ti}^{off}$: allow the regulation of the minimum up and down time which are imposed by the parameters $(t_i^{on})$ and $(t_i^{off})$, respectively, or at a minimum, the number of periods that are needed to complete the optimization period: $t \in \mathcal{T} = \{1, 2, \ldots, 24\}$.

$\beta$: the maximum permitted violation as a fraction of the emission limit.

$\gamma$: probability that some of the scenarios exceeds the emission limit.

$\lambda_k^B$: settlement price of futures contract $j$

$\lambda^{CO_2}$: the estimated $CO_2$-emission price in the emission trading market.

$\lambda_t^{D,s}$: clearing price for auction $t$ at scenario $s$.

$\lambda_j^F$: settlement price of futures contract $j$.

$\omega$: the minimum CVaR for the CVaR constraint.

**Variables**

$a_s$: auxiliary variables to optimize the value of CVaR and, at the same time, to calculate the value of VaR.

$a_{it}$: binary variable indicating the shutting-down of unit $i$ at interval $t$ (formulation presented in [**13**]).

$b_{ti}$ scheduled energy for bilateral contract allocated to thermal unit $i$ at interval $t$.

$c_{ti}^u$, $c_{ti}^d$: start-up/shut-down costs variables of unit $i$.

$c_{t\mathcal{P}_c(1)}^u$, $c_{t\mathcal{P}_c(2)}^u$: the start-up costs continuous variables for each PU $i \in \mathcal{P}$.

$e_{it}$: binary variable to indicate the turning-on of unit $i$ at interval $t$ (formulation presented in [**13**]).

$emi^{SO_2,s}$,
$emi^{NO_x,s}$: auxiliary continuous variables to control the average amount in which the emissions exceed the $SO_2$ and $NO_x$ emissions respectively.

$f_{tij}$: continuous variable representing the energy of the future contract $j$ allocated to thermal unit $i$ at interval $t$.

$g_{ti}^s$: total generation.

$p_{ti}^s$: matched energy in the day-ahead market.

$q_{ti}$: continuous variable standing for the energy of the instrumental price offer of unit $i$ at interval $t$.

$u_{ti}$ : unit commitment binary variables expressing the off-on operating status of the unit $i$ .

$y^s$, $z^s$: auxiliary binary variables to classify the scenarios in which the $SO_2$ and $NO_x$ emissions exceed the limit.

$\zeta_{(1-\alpha)}(x)$: the VaR$(1-\alpha)$ value.

$\phi_{(1-\alpha)}(x)$: the CVaR$(1-\alpha)$ value.

### Functions

$F_{(1-\alpha)}(x, \zeta)$: the CVaR function.

$\tilde{F}_{(1-\alpha)}(x, \zeta)$: approximation of the CVaR function.

$h(x, \lambda^D)$: the profit function associated with the decision vector $x$ and the random vector $\lambda^D$.

$\Psi(x, \zeta)$: probability that the profit $h(x, \lambda^D)$ does not fall bellow a threshold $\zeta$.

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