Dynamic Matching and Bargaining:

The Role of Deadlines.*

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Abstract

We consider a dynamic model where traders in each period are matched randomly into pairs who then bargain about the division of a fixed surplus. When agreement is reached the traders leave the market. Traders who do not come to an agreement return next period in which they will be matched again, as long as their deadline has not expired yet. New traders enter exogenously in each period. We assume that traders within a pair know each other's deadline. We define and characterize the stationary equilibrium configurations. Traders with longer deadlines fare better than traders with short deadlines. It is shown that the heterogeneity of deadlines may cause delay. It is then shown that a centralized mechanism that controls the matching protocol, but does not interfere with the bargaining, eliminates all delay. Even though this efficient centralized mechanism is not as good for traders with long deadlines, it is shown that in a model where all traders can choose which mechanism to use, no delay will be observed.

Key Words: Bargaining, deadlines, markets.

JEL Classification Numbers: C73; C78.

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1 Introduction.

The driving force of most bargaining models is the assumption or stylized fact that people prefer to realize gains early rather than late. If bargaining partners do not care about the time of agreement, there is no incentive to come to agreements in the first place and bargaining could go on forever. The eagerness to reach early agreements is usually modeled by introducing a discount factor strictly less than one. This approach has of course been very successful in obtaining elegant and intuitive results that yield important insights about which aspects influence bargaining outcomes. Most notably, Rubinstein's (1982) seminal alternating bargaining game shows that players bargaining about the division of a fixed surplus will agree immediately and the more patient a player is, the bigger the share he will obtain. Two very patient players will share the surplus approximately equally with the first proposer having a slight advantage.

In this paper we want to consider deadlines as an alternative or additional way to express a preference for early agreements in bargaining. Deadlines are present in many real bargaining situations: In (pre-)trial negotiations an agreement between prosecutor and defence attorney must be reached before the jury presents the verdict; the re-sale of a concert ticket must be concluded before the concert takes place; a renewal of a contract must often be negotiated before the current contract expires. It is often easier for a person to state by which date an agreement must be reached (say a month from now) than to make precise how much he is willing to pay extra to have an agreement today rather than tomorrow. Moreover, software agents that bargain on behalf of people using the internet must often be programmed with a deadline in order to ensure the termination of the protocol in which they take part.

Our work is motivated by the large growth of person-to-person trade that takes place on the Internet. For example, the leader in this field, eBay, has already over 150 million registered users worldwide, some 60 million of which are defined as "active", having bid or listed items in the past year. In the early stages, eBay was used to selling collectibles by means of auctions. Nowadays, it is an economy of its own, selling all kinds of goods. Used cars, for example, are now the most valuable category on eBay. Although the auction format remains the default option, about 30 percent of the goods sold on eBay are now sold at fixed, so called, "buy-it-now" prices. Currently eBay is planning to introduce "want-it-now" and "best offer" options whereby a buyer can haggle directly with the seller.[1] The recent acquisition of Skype (a global P2P telephony company) by eBay will help facilitate more direct bargaining between buyers and sellers. One to one bargaining over the Internet is likely to become an important part of economic activity as a result of this.

The two assumptions we adopt in our model — exogenous arrival rates of buyers and sellers and existence of deadlines — are particularly suitable for modeling such interactions. In a global market, covering different

[1] Source: The Economist (2005).

time zones, and without opening or closing hours, traders will keep arriving at a steady rate each day. Deadlines are typically used in online auctions[2] and also by software agents that do price comparisons online. In addition, from a user interface perspective, deadlines — rather than discount rates — are the most easily understood and convenient way for people to communicate their degree of impatience to the marketplace. Furthermore deadlines may be part of the good: concert or airline tickets are worthless after the date of the event or flight. The buyer may need to get the good by a specific date, say for a birthday or Christmas present.

This paper studies the interaction of large groups of buyers and sellers who arrive at an exogenous rate to the market. Buyers and sellers are randomly matched into pairs and bargaining takes place in each match about the division of the surplus. We assume that the size of the surplus is fixed in order to focus on the effect of heterogeneous deadlines. If an agreement is reached, the traders disappear with their gains from the market. If there is no agreement, and a trader's deadline has expired, the trader will disappear from the market with no surplus. In the case of disagreement and a non-expiring deadline, the trader returns next period in which he will again be matched, with a different partner. The deadline of this trader has then been reduced by one. We assume that inflowing traders are heterogeneous with respect to deadlines. A trader with deadline i has in total i opportunities to come to an agreement with his assigned partner. After i disagreements the trader receives zero surplus and disappears from the market.

Although entry is assumed to be exogenous and constant over time, the total mass of buyers and sellers present in the market may change over time because exit is endogenous. On top of that, the proportion of traders with short deadlines may change over time if traders with short deadlines are more likely to come to agreements then traders with long deadlines. We will be interested in the stationary state of the model, where the total mass of traders and the relative frequencies of deadlines remains constant over time. This allows us to focus on how the distribution of deadlines affects payoffs and the existence of delays.

We assume that there is perfect information about the deadlines of both traders. We define and show the existence of a stationary equilibrium. We show that in equilibrium, traders with higher deadlines achieve higher payoffs than those who have shorter deadlines. We then show that it is possible that when traders with high deadlines are matched they choose, in equilibrium, not to trade and go back to the market where they could be matched (in the next period) with a trader with a shorter deadline. We characterize, in terms of the distribution of deadlines of traders who flow into the market in every period, whether and when such delays will occur. We then examine the comparative statics of our analysis and show that delay can occur frequently and can have a large negative effect on welfare.

We then propose a simple centralized matching mechanism which eliminates this inefficiency: The mech-

[2] The seller can typically choose between auction formats with a duration of 1, 3, 5, 7, or 10 days.

anism simply matches traders with others with the same deadline (but does not interfere with how traders bargain once they are matched). We show that in equilibrium all trade takes place immediately. The centralized matching mechanism shields traders with short deadlines. We are therefore able to show that if all traders can choose which mechanism to use, then all trade will take place in the centralized mechanism (because liquidity follows the traders with the shorter deadlines).

Although deadlines seem a very natural way of expressing time preferences by people and even though bargaining partners are often tied to personal deadlines, the theory of bargaining has not had much to say about how deadlines affect bargaining. In contrast to the current paper, this literature considers two person bargaining with a common deadline. This starts with the finite horizon version of the Rubinstein bargaining model (Ståhl, 1972), which shows that the first mover advantage decreases with the number of bargaining rounds. In some experimental set-ups participants have a limited time to come to agreements. In most cases agreements occur close to the deadline or deadlines may even be missed. (See Roth, Murnighan and Schoumaker, 1988). Yildiz (2004) explains why agreements are often reached close to the deadline when the deadline is fixed, but that agreement is immediate in case of stochastic deadlines. Ma and Manove (1993) consider a model of bargaining with a known deadline in which players can make proposals and can delay their proposals, but in which the offers are received with some random delay after having been made. They show that players will start by delaying making proposals, then make proposals that are sometimes rejected. Agreements tend to be agreed upon near the deadline and sometimes no agreement is reached before the deadline expires. Ponsati (1995) analyzes a bargaining game between two players over two outcomes with a deadline. The players have opposed preferences about the outcomes but the exact utility the players experience from the outcomes is private information. Players basically have to decide how long to wait before giving in to the opponent's preferred outcome. Ponsati (1995) shows that many concessions are made exactly at the deadline but not just before (but possible much earlier). The deadline may also be missed altogether.

Our paper also relates to the large literature on dynamic matching and bargaining, beginning with Rubinstein and Wolinski (1985) and summarized in Gale (2000). A dynamic matching and bargaining game constitutes a natural model of bargaining when there are many traders on both sides of the market. Most closely related to our paper is Samuelson (1992), who also incorporates bargaining into a market with matching. He considers buyers and sellers with heterogenous valuations and shows that two traders who could realize a positive surplus from trading, may decide to break up negotiations and look for alternative partners, with which they can make even more profitable agreements. The current paper has in common with Samuelson (1992) that the disagreement point of a pair of traders is endogenously determined by the outside options generated by the market, and that this is different from the one of the bargaining problem studied in isolation.

Finally, our paper considers what will happen when traders can choose between two market institutions. Existing literature, for the most part, has confined its attention to the analysis of different market mechanisms in isolation. Comparisons between different market mechanisms are usually done from the perspective of the seller, asking which mechanism a single seller would prefer under the assumption that buyers have no choice but to participate in the chosen mechanism (as in, e.g., Milgrom and Weber, 1982). A small number of papers has considered the endogenous distribution of mechanisms (see, e.g., McAfee, 1993 and Peters, 1994) but in models where competing sellers choose a type of auction through which to sell and buyers select in which auction to participate. That is, the implication of the assumption that traders are free to choose the exchange mechanism through which to trade on the outcome of competition between different mechanisms was mostly studied in asymmetric models that favor sellers over buyers. More recently, Neeman and Vulkan (2005) studied competition between a decentralized bargaining mechanism and a centralized market where traders are privately informed about their valuation. They show that, in equilibrium, all trade will take place via the centralized market.

The rest of the paper is organized in the following way: Section 2 presents the general model. In Section 3 we define and show existence of stationary equilibria and analyze their properties. Finally, in section 4 we discuss a number of extensions of the model, notably for the case where the distributions of deadlines for buyers and sellers are not the same. Proofs are collected in the Appendix.

2 The Model

We consider a model with a continuum of sellers and buyers (of mass 1 each) flowing into the market every period. All sellers have one unit of a good they produced at zero cost and all buyers have unitary demands for this good, which they all value at one. The only difference between different traders is their *deadline*. The deadline of a trader is an integer number from $\{1, 2, ..., N\}$ that indicates how many periods are remaining for this trader to conclude a deal. If a trader fails to conclude a deal at the last opportunity he misses his deadline and his utility is zero. That is, a trader with deadline 1 will have to make a deal immediately or his opportunity will be lost. Such a trader will be willing to accept any deal that gives him a positive utility. On the other hand, traders with a high deadline will be able and willing to reject certain deals and wait for better opportunities in the future.

We assume that proportion p_i of the sellers (buyers) that flow into the market place every period has deadline i. The procedure for closing trades is as follows: in each period $t \in Z$ each buyer is matched with a seller. One trader in each pair is chosen at random and becomes the proposer (with probability one half). This trader makes a proposal which can be accepted or rejected. In the first case trade takes place and traders disappear from the market. In the second case no trade takes place and both traders go back to the market and become matched next period (with different partners), as long as their deadlines have not expired. Of course, their deadline will then be reduced by one.

We will be interested in the steady state or stationary equilibrium, which will be defined formally below. A stationary equilibrium is an equilibrium where all buyers (sellers) with the same deadline make and accept the same proposals (independent of the time period t) and where the mass of traders in the market place and the distribution of deadlines among the buyers (sellers) (denoted by q) remains constant over time. There are two different scenarios possible. In the first scenario, which we will refer to as the *no delay case*, trade occurs in each matching. In this case the stationary distribution q of deadline types is simply given by p,. In the second case, which we will refer to as the *delay case*, there is no trade taking place in some matches. In this case the stationary distribution q will be different from the inflow distribution p.

We will assume that traders discount late trades by a factor $\delta \leq 1$. It will become clear later on that the role of the discount factor is not as important as in standard bargaining models. The reason is that, as will be shown, traders with longer deadlines will close better deals than traders with shorter ones. This gives traders an incentive to make deals early, even if the discount rate is equal to one. However, if we do not allow for discounting of utilities, there is no cost of having delay, as long as deadlines are never missed.

We assume throughout this paper that traders that are matched know each other's deadline. The proposal made by one of the traders may thus depend on his deadline and that of his partner. In a companion paper we analyze the case where deadlines are private information.

3 Equilibrium Analysis

A pure strategy for a trader specifies the offers he makes when chosen as a proposer and the offers he accepts as a responder, both as a function of his own deadline, as well as of his trading partner's deadline. In its full generality, the strategy could also depend on the time period t and on the received and rejected offers in the past by this trader. When some traders do not accept the proposal received, the total mass of buyers and sellers around in the next period will be strictly more than one. Also, the distribution of deadlines may change. In principle, the optimal strategy for a trader depends on the current and future distributions of deadlines. If these distributions change over time, the optimal strategy of a trader does not only depend on his deadline or the one his current trading partner, but also on the exact moment that he enters the market. That obviously complicates our (and the trader's) task.

However, since all traders that enter will leave within N periods, the total mass of traders present in the market at any time will never explode. The total mass of traders in any period t remains bounded above

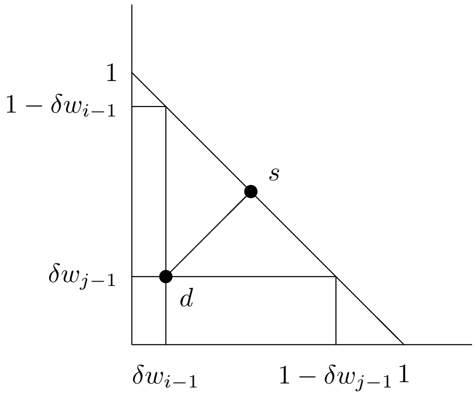

Fig. 1a: Feasible disagreement point

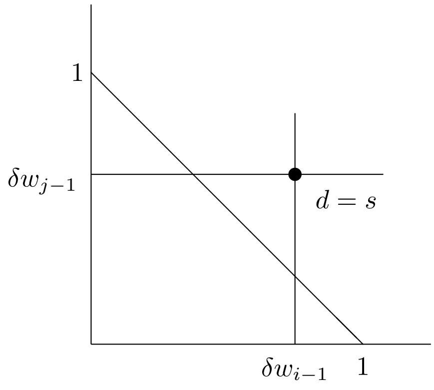

Fig. 1b: Unfeasible disagreement point

Figure 1: Feasible and unfeasible disagreement points

by N through time and (at least a subsequence) will in fact converge and eventually the distribution of deadlines will settle down on a stationary distribution. We will be interested only in how traders behave in this stationary state, and we will not be worried over how and how fast the distribution settles down.

A stationary equilibrium is almost completely characterized by the expected equilibrium payoff w_i of a trader with deadline i. (For convenience we denote $w_0 = 0$.) From the vector of expected payoffs w one can almost completely reconstruct the associated stationary equilibrium strategies. One needs to distinguish three cases in the bargaining process between two traders with deadlines i and j. Namely, is the disagreement point in the interior of the feasible set, outside the feasible set, or on the Pareto frontier. The first two cases are illustrated in Figure 1 below.

A responder with deadline i must accept any proposal that gives her strictly more than δw_{i-1} and reject any proposal that gives her strictly less than δw_{i-1}. In equilibrium she must also accept a proposal of exactly δw_{i-1} whenever it is in the strict interest of the proposer to do so. (That is, when $1 - \delta w_{i-1} > \delta w_{j-1}$, where j is the deadline of the proposer. This corresponds to Fig. 1a.) The reason is that the proposer could guarantee acceptance with probability one by just offering slightly more to the responder. This implies that, in equilibrium, the responder must accept a proposal of exactly δw_{i-1}. Only in the peculiar case where $1 - \delta w_{i-1} = \delta w_{j-1}$ a responder may accept the proposal of δw_{i-1} with any probability between zero and one. This is the case if the disagreement point is exactly on the Pareto frontier.

Similarly, in a stationary equilibrium, a proposer with deadline i (when matched with a trader with deadline j) must offer exactly δw_{j-1} to his trading partner when $\delta(w_{i-1} + w_{j-1}) < 1$. Offering strictly more cannot be optimal while offering strictly less would result in rejection and a strictly lower payoff. If

$\delta(w_{i-1} + w_{j-1}) > 1$, as is indicated in Fig. 1b, any acceptable proposal would yield a payoff strictly less than δw_{i-1}, in case of acceptance. Hence, in this case the proposer must make sure that the proposal will be rejected. He can offer anything up to (but not including) δw_{j-1}. Again, in the peculiar case that $\delta(w_{i-1} + w_{j-1}) = 1$ the proposer has many options of offering deals that will be rejected for sure as well as offering exactly δw_{j-1}. What is important for the equilibrium outcome is the *probability* that trade will take place when i is matched with j. We will denote this probability by E_{ij}. From these probabilities one can then calculate the mass z_i of sellers (buyers) with deadline i.

3.1 Definition and existence of equilibrium

We now formally define a stationary subgame perfect equilibrium configuration.[3]

Definition 1 *We call* $(z, w, E) = ((z_1, ..., z_N), (w_1, ..., w_N), E) \in \Re^N_+ \times \Re^N_+ \times \Re^{N \times N}$ *a stationary subgame perfect equilibrium configuration (with T-delay) if the following holds:*

1. $\sum_{i=1}^N z_i = 1 + T$
2. *E is an $N \times N$ symmetric matrix with $E_{ij} = 1$ if $\delta(w_{i-1} + w_{j-1}) < 1$, $E_{ij} = 0$ if $\delta(w_{i-1} + w_{j-1}) > 1$ and $E_{ij} \in [0, 1]$ otherwise.*
3. $z_N = p_N$; $z_i = p_i + z_{i+1}(\sum_j q_j(1 - E_{i+1j}))$ *where* $q_i = z_i/(1 + T)$
4. $w_i = \frac{1}{2}\delta w_{i-1} + \frac{1}{2}(\sum_{j=1}^N q_j(\max\{\delta w_{i-1}, 1 - \delta w_{j-1}\})$ *for all i.*

This definition requires some further explanation. The mass of traders with deadline i is denoted by z_i. Condition 1 says that the total mass of traders equals $1 + T$. Given that per period a mass of 1 of new traders enters, one can interpret T as the amount of delay: $T/(T + 1)$ is the fraction of traders that will postpone their trade by (at least) 1 period. The matrix E indicates the pair of traders (i, j) that will come to an immediate agreement. If $E_{ij} = 1$, proposer i will offer responder j δw_{j-1} and keep the rest $1 - \delta w_{j-1} \geq \delta w_{i-1}$ and this will be accepted. If $E_{ij} = 0$ then $1 - \delta w_{j-1} < \delta w_{i-1}$ and trader i will not want to make an acceptable proposer to j. For example, he may offer at most $1 - \delta w_{i-1}$ to j but j will then not accept. Condition 3 is the stationarity condition. The total mass of traders with deadline i in any period $t+1$ equals the mass of new traders (p_i) plus the mass of traders with deadline $i+1$ present in period t who decide to postpone their trade and wait for better times (these either reject proposals or make unacceptable proposals). Finally, condition 4 describes the relation between the expected equilibrium payoffs for traders with different deadlines, given the stationary distribution.

[3]We do not use the term stationary subgame perfect equilibrium, as that would implicitly refer to strategies. As argued above, the strategies cannot always be pinned down exactly.

We first show the existence of a stationary subgame perfect equilibrium configuration.

Theorem 2 *For any inflow distribution* p *and any discount factor* $\delta \in (0, 1]$ *there exists a stationary subgame perfect equilibrium configuration.*

3.2 Properties of the equilibrium

We next show that in any stationary subgame perfect equilibrium configuration, the expected equilibrium payoff is strictly increasing in the deadline. Although this result seems intuitive, it is not trivial. In particular, it is not true that traders with deadline $i + 1$ can simply adopt the strategy of a trader with deadline i i order to guarantee at least the same payoff as that trader, since the offers received may differ. The proof uses induction with respect to deadlines and condition 4 of Definition 1.

Theorem 3 *In any stationary subgame perfect equilibrium configuration we have, for all* i*,* $w_i < w_{i+1}$*.*

The theorem illustrates that having a relatively distant deadline is good in terms of expected payoffs. Since acceptable equilibrium proposals to a trader with deadline j equal δw_{j-1}, these traders receive better offers than traders with a near deadline. When making an acceptable proposal, only the deadline of the responder matters (as each pair is split up immediately after disagreement and no counteroffer can be made). The probability of making acceptable proposals is both weakly decreasing in one's own deadline and in the partner's deadline. Agreements are immediate when at least one of the partners has a near deadline, but delay occurs when both traders have a distant deadline.

Corollary 4 *In a stationary subgame perfect equilibrium configuration, if traders with deadlines* i *and* j *trade with probability strictly less than one, then any pair of traders* $(i', j') \neq (i, j)$ *with* $i' \geq i$ *and* $j' \geq j$ *does trade with probability* 0*. Similarly, if traders with deadlines* i *and* j *trade with probability one, then any pair of traders* (i', j') *with* $i' \leq i$ *and* $j' \leq j$ *does trade with probability* 1*.*

3.3 (In)existence of delay

In this subsection we investigate the necessary and sufficient conditions for the existence of an equilibrium without delay. We also characterize the equilibrium strategies precisely for such an equilibrium configuration, and we perform comparative statics exercises. It is immediate from conditions (1) and (3) in Definition 1 that $T = 0$ implies that $z_i = p_i$. That is, the stationary distribution of deadlines coincides with the inflow distribution. This is rather intuitive as the no delay assumption implies that no trader remains in the market for more than one period in such an equilibrium.

Suppose there exists a stationary subgame perfect equilibrium configuration with 0-delay. To emphasize the distinct case of no delay, we let $v_i(= w_i)$ denote the expected payoff a trader with deadline i obtains in this equilibrium. For convenience we denote $v_0 = 0$.

A responder with deadline j will accept any trade that yields him $x > \delta v_{j-1}$ and reject any proposal that yields $x < \delta v_{j-1}$. In the equilibrium, a proposer of type i will offer to a trader of type j exactly δv_{j-1} which will be accepted with probability one. Hence, a proposer of type i keeps $1 - \delta v_{j-1}$ for himself when meeting a type j. Note that this is independent of i. Also observe that proposer of type i could have made an unacceptable proposal, in which case his payoff would equal δv_{i-1}. In any equilibrium without delay, we must thus have $\delta v_{i-1} \leq 1 - \delta v_{j-1}$. It follows immediately that for $i > 1$

$$v_i = v_1 + \frac{1}{2}\delta v_{i-1} \tag{1}$$

since with probability 1/2 trader i is offered δv_{i-1} and with probability 1/2 he obtains the same payoff as a trader with deadline 1 gets, conditional on being the proposer. It follows that

$$v_i = v_1(1 + \frac{1}{2}\delta + ... + (\frac{1}{2}\delta)^{i-1}) = v_1(1 - (\frac{1}{2}\delta)^i)/(1 - \frac{1}{2}\delta). \tag{2}$$

Note that $v_1 > 0$ since otherwise we must have $v_i = 0$ for all i, which is impossible. It follows thus from (2) that $v_{i+1} > v_i$ for all i. Before knowing one's type the expected payoff in this equilibrium is one half (given that there is no delay), so

$$\frac{1}{2} = \sum_{i=1}^{N} p_i v_i = v_1 \sum_{i=1}^{N} p_i(1 + \frac{1}{2}\delta + ... + (\frac{1}{2}\delta)^{i-1}). \tag{3}$$

Hence,

$$v_1 = \frac{1/2}{\sum_{i=1}^{N} p_i(1 + \frac{1}{2}\delta + ... + (\frac{1}{2}\delta)^{i-1})} = \frac{(1 - \frac{1}{2}\delta)}{2\sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)}. \tag{4}$$

We summarize our findings thus far in the following proposition.

Proposition 5 *In an equilibrium without delay, the expected equilibrium payoff of a trader with deadline j equals*

$$v_j = \frac{1 - (\frac{1}{2}\delta)^j}{2\sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)} \tag{5}$$

If it is optimal for the N type to make an acceptable proposal to another N type, then it is optimal for all types to make acceptable proposals all the time. Namely, for the highest type to be willing to make an acceptable proposal, it must hold that

$$1 - \delta v_{N-1} \geq \delta vN - 1 \Leftrightarrow vN - 1 \leq 1/(2\delta)$$

while for i to be willing to make an acceptable proposal to j it must hold that

$$1 - \delta v_{j-1} \geq \delta v_{i-1} \Leftrightarrow v_{i-1} + v_{j-1} \leq 1/\delta$$

which is satisfied since we know that $v_{i-1} \leq v_{N-1}$ for all i. From (5) we can verify that

$$v_{N-1} = \frac{1 - (\frac{1}{2}\delta)^{N-1}}{2\sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)}.$$

Hence, the existence of delay depends on the parameters of the model as the following result summarizes.

Theorem 6 *There exists an equilibrium without delay if and only if*

$$\frac{1 - (\frac{1}{2}\delta)^{N-1}}{2\sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)} \leq \frac{1}{2\delta} \tag{6}$$

In this case there is exactly one equilibrium without delay. In this equilibrium, a responder of type j accepts any offer $x \geq \delta v_{j-1}$ (and rejects any other offer) while a proposer of type i proposes exactly δv_{j-1} to a trader of type j.

It follows immediately from our equilibrium existence result in Theorem 2 and the previous theorem that an equilibrium with delay must exist whenever no equilibrium without delay exists.

Corollary 7 *In case the inequality in (6) is not satisfied, there exists an equilibrium with delay.*

3.4 Comparative statics and special cases

From the necessary conditions for the existence of an equilibrium without delay in Theorem 6 we obtain immediately

Corollary 8

1. *For $N = 2$, and for any inflow distribution p, there exists a unique stationary subgame perfect equilibrium configuration, and there is no delay in it.*

2. *Suppose there exists a stationary subgame perfect equilibrium configuration without delay with inflow distribution p, and let p' first-order stochastically dominate p. That is, $\sum_1^i p_j \geq \sum_1^i p'_j$ for all i. Then there also exists a stationary subgame perfect equilibrium configuration without delay with inflow distribution p'.*

3. *Suppose there exists a stationary subgame perfect equilibrium configuration without delay when the discount factor is equal to δ. Then there exists also a stationary subgame perfect equilibrium configuration without delay for any lower discount factor $\delta' \leq \delta$ (for the same inflow distribution p).*

These results are quite intuitive. First, when the highest deadline is equal to 2, delaying an agreement is not very attractive since then one becomes a trader with deadline 1. With probability one half one becomes a responder in which case one is forced to accept the zero offer. Hence, it is not possible to get an expected payoff above one half by delaying. Hence, two traders with deadline 2 will agree immediately.

The second result states that shifting the distribution towards longer deadlines will not increase the likelihood of delay. Even though such a shift increases the probability that two traders with long deadlines meet, it reduces the incentive to delay as it becomes less likely to be matched in future periods with traders with short deadlines.

Finally, the third result states the existence of a lower bound on the discount factor to make delay a possible equilibrium phenomenon. For lower discount factors the cost of delay is so high that it never pays off to delay, even when it is very likely that next period one is matched with a trader with very low deadline. Of course, in the extreme case of a discount factor equal to zero, the situation is equivalent to one where all traders have to agree immediately, that is, where all traders effectively have a deadline equal to 1.

Remark 9 *In the special case of the uniform distribution ($p_i = 1/N$) one obtains*

$$v_{N-1} = \frac{N(1-(\frac{1}{2}\delta)^{N-1})}{2\sum_{i=1}^{N}(1-(\frac{1}{2}\delta)^i)} = \frac{N(1-(\frac{1}{2}\delta)^{N-1})}{2(N-\frac{\delta/2}{1-\delta/2}(1-(\frac{1}{2}\delta)^N)}$$

Whether $v_{N-1} \leq 1/(2\delta)$ depends on the parameters δ and N. For example, for $\delta = 0.9$ this is true unless $N \in \{4, 5, 6, 7\}$. On the other hand, for $\delta = 0.99$ this is true only for $N = 2$ and for $N > 98$. For $\delta = 0.999$ this is true only for $N = 2$ and for $N > 998$.

Remark 10 *For a different project we collected data from eBay on the distribution of durations of online auctions, which range from 1 to 10 days. These data are presented in Figure 2 below. Using these data as a proxy for the distribution of deadlines we find that each of these these distributions lead to stationary subgame perfect equilibria with delay.*

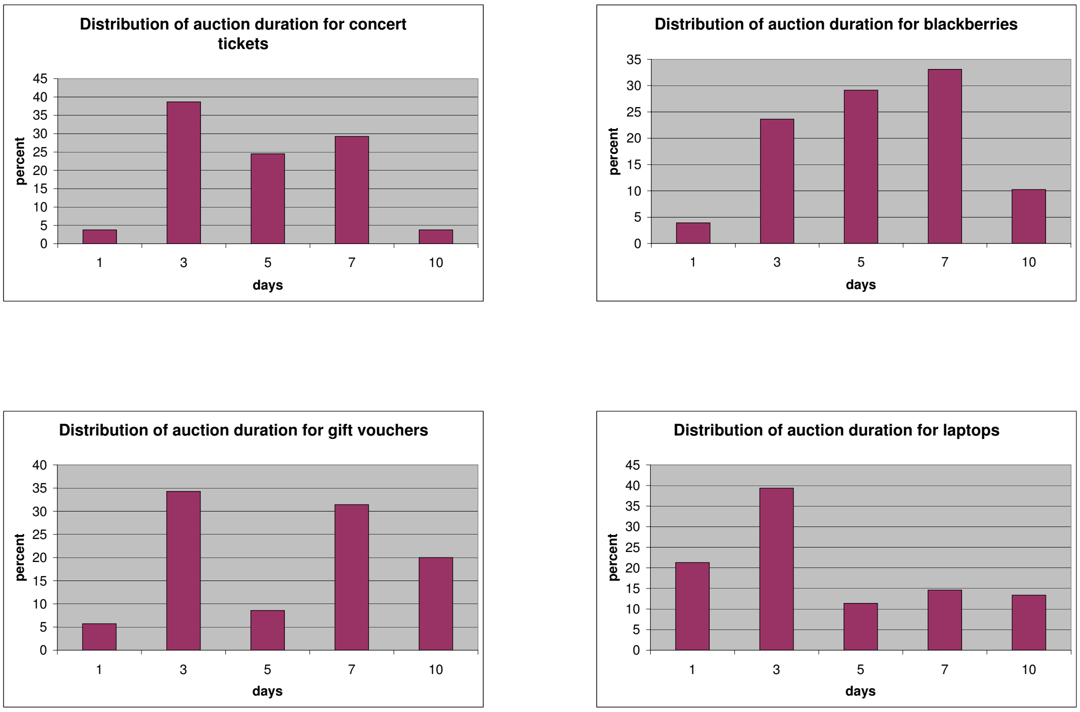

Figure 2: Auction duration for various goods on eBay.

Example 11 *In order to illustrate how much delay may occur, we calculate (approximately) a stationary subgame perfect equilibrium configuration with delay for an example with deadlines 1 through 6. The inflow distribution is assumed to be* $p = (0.235, 0.108, 0.077, 0.090, 0.090, 0.400)$ *and the discount factor is chosen to be* $\delta = 0.995$. *It can be shown that there exists a stationary subgame perfect equilibrium configuration with* T*-delay where* $T = 0.45$ *and the stationary distribution equals* $q = (0.167, 0.1, 0.124, 0.132, 0.194, 0.283)$. *The expected payoffs realized by the traders is given by* $w = (0.287, 0.430, 0.505, 0.559, 0.602, 0.638)$. *The pairs of traders who do not come to an agreement are in the set* $\{(3,6),(4,4),(4,5),(4,6),(5,5),(5,6),(6,6)\}$. *Clearly, since some trades are delayed and there is discounting, there are inefficiencies.*

4 Centralized deterministic matching

We have seen that delay will occur in many instances, and the examples have also demonstrated that the amount and cost of delay may be substantial. In this subsection we introduce a new market or matching mechanism that has the property that no delay will occur. Moreover, we subsequently show that when all traders can choose whether to participate in this mechanism or in the random matching mechanism discussed before, all traders will choose the mechanism where no delay occurs, even though this implies that traders with longer deadlines cannot benefit from this.

Our mechanism works as follows. We order the sellers and the buyers in increasing deadlines and then we match the j-th seller with the j-th buyer. For the case where the distribution of deadlines for buyers and sellers is the same, this simply means that each buyer (seller) with deadline i will be matched with a seller (buyer) with deadline i.

We claim that when traders are matched in this way, all trading partners will immediately agree. Moreover, each trader's expected payoff is equal to 1/2. Namely, consider first the case of traders with deadline 1. Since they are matched with a trader with the same short deadline, they will obtain 1 when they are chosen as proposer and 0 otherwise. Now consider any trader with deadline $i > 1$ and suppose that traders with deadline $i' < i$ expect a payoff of 1/2. Then a responder with deadline i will accept any offer above $\delta v_{i-1} = \delta/2$. A proposer with deadline i will offer exactly $\delta/2$ to its partner and keep $1 - \delta/2$ for himself. In expectation a trader with deadline i thus obtains a payoff of 1/2.

Theorem 12 *The mechanism where traders with the lowest deadlines are matched yields no delay and an expected payoff of 1/2 to each trader, independent of his deadline.*

4.1 Endogenous choice of mechanism: unraveling

Above we have argued that the centralized mechanism is more efficient than the random matching mechanism whenever there exists delay in the latter. This would seem to suggest that the centralized mechanism should be used from a welfare point of view. On the other hand, traders with long deadlines may make higher profits under the random matching scheme and are therefore reluctant to participate in the efficient mechanism. It is in their interest to have trade taking place through the random matching model. Of course, the opposite is true for the traders with short deadlines. In this subsection we analyze what happens when traders are given the choice to participate in one or the other mechanism. Can both mechanisms co-exist or will one dominate the other?

In each period there is an inflow of new traders. Each of those chooses between the random matching mechanism (RM) and the efficient centralized mechanism (CM). Then matching takes place according the chosen mechanism amongst the traders that have chosen the same mechanism. We assume that traders choose once and for all one of the mechanisms. Hence, traders that do not conclude a trade return next period (if their deadline has not expired yet) and will be matched in the same mechanism.[4] We will again be interested in the steady state equilibrium.

[4] If traders could choose in each period between mechanisms and could thus switch from one to the other, the analysis is altered, specifically in the random matching mechanism. Namely, each trader here would have the possibility to switch next period to the centralized mechanism and obtain a payoff of 1/2. Hence, none of such traders (except the one with expiring deadlines) would accept proposals below $\delta/2$.

Note that if all traders coordinate on the same mechanism, nobody has an incentive to deviate as no trading partner would be found in the rival mechanism. To avoid these artificial corner solutions we assume that each new trader with small positive probability ε chooses a mechanism at random. This implies that a positive mass of traders is present in each mechanism, so that a trading partner is found with positive probability. We denote by Γ^{ε} the game where traders choose consciously between mechanisms with probability $1-2\varepsilon$ and choose each mechanism with probability ε unconsciously. We will be interested in the choices of the traders in the limit when ε approaches zero.

Note that the endogenous choice by traders may be such that in one of the mechanisms more buyers than sellers enter, and vice versa in the other mechanism. We have not dealt with asymmetric distributions.[5] If in RM more buyers than sellers participate, some buyers will remain unmatched. If in CM more buyers than sellers participate, buyers with the highest deadlines will remain unmatched. Instead of dealing explicitly with asymmetric distributions, we will focus on symmetric equilibria. Assuming that the inflow distribution for sellers and buyers is the same, as we have done throughout the paper, the situation for buyers and sellers with the same deadline is exactly the same, and symmetric equilibria will exist. We will show that the only symmetric equilibrium is where, in the limit, all traders go to CM.

Theorem 13 *In a symmetric equilibrium (where buyers and sellers with the same deadline behave identically) when traders can choose between the efficient centralized mechanism and the random matching mechanism, all traders with deadline strictly less than N will choose the efficient one. Only traders with deadline N may choose RM, but in any case they obtain the same payoff of 1/2 as they would obtain in CM.*

The intuition for this result is that traders with the lowest deadlines in RM will make the lowest expected payoffs. This expected payoff is strictly below 1/2. No trader would consciously choose the RM option given the guaranteed payoff of 1/2 in CM. Hence, only a few traders with deadline N may choose to do so in order to take advantage of the few traders who by mistake choose RM.

5 Extensions and Conclusions

Since the focus of the present paper is on the effect of deadlines on bargaining outcomes, we adopted some simplifying assumptions, mainly for ease of exposition. In this section we discuss briefly some possible extensions and generalizations.

First, we adopted the bargaining protocol of a single take-it-or-leave-it offer by a randomly chosen proposer. The resulting outcome of this protocol when traders with deadlines i and j are matched is in fact the

[5]It is possible to extend the analysis to asymmetric distributions and masses. See section 5 for details.

Nash Bargaining Solution of the division of a unit with disagreement points δv_{i-1} and δv_{j-1}. It is well known that many other bargaining procedures lead to the same solution. For example, we could have assumed a Rubinstein type of model with very frequent alternating offers.

Second, instead of assuming that the negotiation ends and the pair is broken up after a rejected offer, we could also have allowed traders to decide whether to keep negotiating with the same trader. However, since traders have perfect information about their deadlines, this would not make any difference. If traders agree not to close a deal now, because of better outside options for both of them, they will surely agree not to postpone looking for those outside options.

Third, we assumed that traders within a pair know each other's deadline. In a companion paper we examine the case where deadlines are private information. It is shown that not only delay can take place, but that deadlines can be missed altogether.

Fourth, we assumed throughout the paper that the number of sellers and buyers flowing into the market is the same. We also assumed that the distribution of deadlines is the same for buyers and sellers. Both of these assumptions can be relaxed. Let p_i denote the mass of sellers with deadline i flowing into the market, and let $(1+b)p'_i$ denote the mass of buyers with deadline i flowing into the market. Without loss of generality we may assume that $\sum_{i=1}^{N} p_i = 1$ and $\sum_{i=1}^{N'} p'_i = 1$. If $b > 0$ then the buyers form the long side of the market. Obviously, in this case there will always be buyers who will not get matched. That means that there will certainly be delay among the buyers, and also that some buyers may miss their deadline. However, we will be interested in the possibility of delay on the (short) side of the sellers.

The definition of a stationary subgame perfect equilibrium configuration can be generalized in a straightforward manner. It can be shown that on each side of the market the payoffs are strictly increasing in deadlines. The payoffs will typically be lower on the long side of the market because of the positive probability of not being matched. When the ratio of buyers per seller is very high, buyer's payoffs will be close to zero. In this extreme case there is no reason for sellers to delay since they can appropriate almost all of the surplus, even when matched with a high deadline buyer. For such extreme cases there is thus no need to discuss an alternative matching mechanism. In more moderate settings, the occurrence of delay on the shorter side of the market will depend on the distribution of deadlines of inflowing buyers and sellers in a way that is very similar to what we have described in section 3. In particular, assuming the existence of an equilibrium without delay on the short side of the market, equations (1) and (2) hold for the payoffs of the sellers. For the payoffs of the buyers similar expressions can be written. These will include the probability of not being matched, $b/(b+1)$. From these equations and the assumption of no delay, one can get explicit expressions for the payoffs of all traders. As in section 3, the condition for no delay occurring in equilibrium is that when two traders with the highest deadlines meet, they prefer to agree, that is when $\delta v^S_{N-1} + \delta v^B_{N'-1} \leq 1$, where the superindices S and B refer to the seller's and buyer's payoff, respectively.

If delay occurs in the random matching mechanism, then again a centralized mechanism of matching can eliminate the delay. Moreover, when traders endogenously choose which mechanism to use, most of the trade takes place in the centralized mechanism and no delay occurs on the short side of the market. The crucial idea of the centralized mechanism is as before: We order the traders on both sides of the market by increasing deadlines and match the n-th seller with the n-th buyer. It is beyond the scope of the present paper to analyze the centralized mechanism in its full generality. To illustrate how the centralized mechanism works, let us now consider different distributions but equal mass (that is, $b = 0$). Let us also assume that $p_1 = p'_1$.

Under these assumptions traders with deadlines 1 are matched among each other. Each of them will obtain a payoff of one half. Traders with deadlines above 1 will be matched with traders with deadlines above 1, but the deadlines of two bargaining partners need not be the same. Suppose $p_2 \leq p'_2$. In this case there are less sellers than buyers among the traders with deadline 2. This in turn implies that such sellers are matched, in our centralized mechanism, with probability one to buyers with deadline 2. Such a match results in immediate agreement with expected payoff of one half for each trader, since the proposer will always offer $\delta/2$. It is not difficult to verify that when in all matches with traders with deadlines above 1 such offers are made, no trader has an incentive to deviate and all traders obtain an expected payoff of one half.

When traders can choose between the random matching mechanism and the centralized one described above, all traders will obtain a payoff of one half. Namely, if some were to obtain a higher payoff, others must obtain less. But each trader can guarantee a payoff of one half by choosing CM. Only if a few traders choose RM by mistake, some other traders with the highest deadline may be willing to choose RM and take advantage. However, only very few will be able to do this and the resulting payoff will be equal to one half, even for these traders.

Finally, we assumed that the surplus to be divided between any two traders is fixed. In this respect we followed Rubinstein and Wolinski (1985) and Binmore and Herrero (1988). It would be clearly more realistic to have buyers with a distribution of valuations and sellers with a distribution of costs. Samuelson (1992) shows that delay may occur, even without the presence of deadlines and even when all pairs have a positive gain from trade. The reason is simply that traders may expect even larger gains of trade in future matches. Gale (2000) and Satterthwaite and Shneyerov (2003) allow for distributions of buyer's valuations and seller's costs in a dynamic matching and bargaining model. Of course, their focus is on the convergence of equilibria of such model to Walrasian equilibria as the time period between consecutive rounds goes to zero. In such a model delay could occur whenever low valuation buyers are matched with high cost sellers. However, since the prices at which trade occurs converge to the Walrasian equilibrium price, delay is avoided by having buyers with valuation below this price and sellers with costs above this price leave the market. When deadlines are introduced, however, it is possible for high valuation buyers and low cost sellers to delay when they have both long deadlines. Also, high cost sellers and low valuation buyers may enter the market as they can find profitable trades when matched with short deadline traders. Convergence to Walresian equilibrium prices will require some additional assumption about the distribution of deadlines. It should be noted that in our model in some sense traders with different deadlines do have different valuations and costs. Namely, the offers they are willing to accept depend crucially on their individual deadline.

It should also be mentioned that although our model deals with buyers and sellers, it also applies to other settings of two-sided matching markets, such as workers and firms, authors and co-authors, or men and women. (See, e.g., Shimer and Smith, 2000.) In these situations one usually assumes that the surplus is not fixed but depends on the characteristics of the partners. The subject of study is then to determine who matches up with whom. For example, will there be positive assortive matching whenever this is efficient? Clearly, the model we have employed in this paper needs to be enriched in order to deal with these type of questions. In any case, it seems plausible that the introduction of heterogeneous deadlines will affect the results of these matching models in similar ways. Delay may occur and inefficient matches may form whenever the partners are pressed by time.

Appendix: Proofs

Proof of Theorem 2

Let $Z = \{z \in \Re^N_+ : \sum_i z_i \geq 1 \text{ and } z_i \leq N + 1 - i\}$ and let $W = \{w \in \Re^N_+ : w_1 \leq w_2 \leq \ldots \leq w_N\}$. Let $\mathcal{M}$ denote the set of all symmetric $N \times N$ matrices with entries in the interval $[0, 1]$. Consider the following correspondence $G : Z \times W \to Z \times W \times \mathcal{M}$:

$$\begin{aligned} G(z, w) \quad = \quad & \{(z, w, E) : E_{ij} = 0 \text{ if } \delta(w_{i-1} + w_{j-1}) > 1 \text{ and} \\ & E_{ij} = 1 \text{ if } \delta(w_{i-1} + w_{j-1}) < 1\} \end{aligned}$$

and the following mapping $H : Z \times W \times \mathcal{M} \to Z \times W$:

$$H(z, w, E) = (\tilde{z}, \tilde{w})$$

where

$$\tilde{z}_N = p_N,\ \tilde{z}_i = p_i + \frac{z_{i+1}}{\sum_k z_k}(\sum_{j=1}^{N} z_j(1 - E_{i+1j}))\text{ for } i < N$$

and

$$\tilde{w}_i = \frac{1}{2}\delta w_{i-1} + \frac{1}{2\sum_k z_k}(\sum_{j=1}^{N} z_j(\max\{\delta w_{i-1}, 1 - \delta w_{j-1}\}).$$

Note that $\tilde{z}_i \leq p_i + z_{i+1} \leq 1 + N + 1 - (i+1) = N + 1 - i$ and that $\tilde{w}_i \leq \tilde{w}_{i+1}$ when $w_{i-1} \leq w_i$ so that H really maps into $Z \times W$.

We now combine G and H to construct a correspondence $F : Z \times W \to Z \times W$ as follows:

$$F(z, w) = \{H(z, w, E) : (z, w, E) \in G(z, w)\}.$$

F is an upper semi-continuous correspondence from a non-empty, compact, convex set $Z \times W$ into itself such that for all $(z, w) \in Z \times W$, the set $F(z, w)$ is convex and non-empty. Convexity of $F(z, w)$ is of course immediate in the case of a singleton set. Suppose $(\tilde{z}, \tilde{w}) = H(z, w, E)$ and $(\tilde{z}', \tilde{w}') = H(z, w, E')$ are two different elements of $F(z, w)$ and let $\alpha \in [0, 1]$. By the definition it follows immediately that $\tilde{w} = \tilde{w}' = \alpha\tilde{w} + (1 - \alpha)\tilde{w}'$. On the other hand,

$$\alpha\tilde{z}_i + (1 - \alpha)\tilde{z}'_i = p_i + \frac{z_{i+1}}{\sum_k z_k}(\sum_j z_j(1 - (\alpha E_{i+1j} + (1 - \alpha)E'_{i+1,j})).$$

We conclude that $\alpha H(z, w, E) + (1 - \alpha)H(z, w, E')) = H(z, w, \alpha E + (1 - \alpha)E') \in G(z, w)$.

Then Applying Kakutani's fixed point theorem delivers the required result. □

Proof of Theorem 3

Obviously, $0 = w_0 < w_1$. Assume that $w_0 < w_1 < \ldots < w_i$ for some $i \geq 1$. It is immediate from condition 4 that then $w_{i+1} > w_i$ since (by the induction step) $w_i > w_{i-1}$ and $\max\{\delta w_i, 1 - \delta w_{j-1}\} \geq \max\{\delta w_{i-1}, 1 - \delta w_{j-1}\}$ for all j. □

Proof of Corollary 8

1. For $N = 2$ we have

$$\begin{aligned} v_1 &= \frac{1 - \frac{1}{2}\delta}{2(p_1(1 - \frac{1}{2}\delta) + (1 - p_1)(1 - (\frac{1}{2}\delta)^2))} \\ &= \frac{1}{2 + \delta(1 - p_1)} \leq \frac{1}{2} \leq \frac{1}{2\delta} \end{aligned}$$

2. Define $S(p) = \sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)$. Let p' first-order stochastically dominate p. Define

$$\begin{aligned}
p^1 &= p \\
p^2 &= (p'_1, p_2 + p_1 - p'_1, p_3, ..., p_N) \\
p^3 &= (p'_1, p'_2, p_3 + p_1 + p_2 - p'_1 - p'_2, p_4, ..., p_N) \\
... &= ... \\
p^N &= p'
\end{aligned}$$

Clearly, $0 < S(p^i) \leq S(p^{i+1})$ for all i and therefore $0 < S(p) \leq S(p')$. Therefore

$$\frac{1}{2\delta} \geq \frac{1 - (\frac{1}{2}\delta)^{N-1}}{2S(p)} \geq \frac{1 - (\frac{1}{2}\delta)^{N-1}}{2S(p')}$$

3. Let $L(\delta) = \delta - (\frac{1}{2})^{N-1}\delta^N - \sum_{i=1}^{N} p_i(1 - (\frac{1}{2}\delta)^i)$. We will show that $L(\delta)$ is increasing for $\delta < 1$ which proves the claim as Theorem 3 states that there exists an equilibrium without delay if and only if $L(\delta) \leq 0$. Observe that for all natural numbers N and any $\delta \in [0, 1)$

$$2^{N-1} - N\delta^{N-1} > 2^{N-1} - N \geq 0$$

so that

$$L'(\delta) = 1 - \frac{N\delta^{N-1}}{2^{N-1}} + \sum_{i=1}^{N} p_i \frac{i\delta^{i-1}}{2^i} > 0.$$

□

Proof of Theorem 13

Let $\varepsilon > 0$ be small. Consider a symmetric equilibrium of Γ^ε and let v_i^{CM} and v_i^{RM} denote the equilibrium payoffs of the traders with deadline i in CM and RM, respectively. (Recall that because of the "unconscious traders" there are always traders of any type present in any mechanism, although the total mass of such traders may be small.) We know that $v_i^{CM} = 1/2$, for any trader with deadline i. In particular, $v_1^{CM} = 1/2$.

On the other hand, $v_1^{RM} < 1/2$ as we have established in Theorem 3. So traders with deadline 1 will certainly choose CM in equilibrium. In fact, all traders with deadline i where $v_i^{RM} < 1/2$ must choose CM. That means that all traders with deadline j who voluntarily choose RM must have $v_j^{RM} \geq 1/2$. These will be the traders with the higher deadlines, according to our result in Theorem 3. That is, for some $j > 1$, only traders with deadline $j, j+1, ..., N$ will voluntarily choose RM and $v_j^{RM} \geq 1/2$. Let us assume for the moment that $j < N$. Note that a trader with deadline j can at most expect to obtain 1/2 when matched with a similar trader, and strictly less than 1/2 when matched with a trader with a higher deadline. Namely, he will be offered $\delta v_{j-1}^{RM} < 1/2$ and must offer more than $\delta v_j^{RM} > \delta v_{j-1}^{RM}$. The payoff obtained conditional on being matched with a trader with deadline above or equal to j is thus bounded away from 1/2. He can make a payoff strictly above 1/2 when matched with a trader with a lower deadline, but the probability of this occurring is of the order ε. Hence, the trader with deadline j will obtain $v_j^{RM} < 1/2$, which means he will prefer to choose the mechanism CM.

Thus the only possibility for traders voluntarily choosing RM is when only traders with deadline N do so. This means that $v_{N-1}^{RM} \leq 1/2$, and thus, $\delta v_{N-1}^{RM} < 1/2$. This means that when two traders with deadline N are matched they will agree to trade, i.e., there will be no delay. It follows that $v_N^{RM} > 1/2$. All traders with the longest deadline will choose RM. When they get matched among each other, which is very likely (as it occurs with probability at least $1 - \varepsilon$), they will obtain, in expectation, exactly 1/2. When matched with traders with lower deadlines, they will get a higher payoff, but of course this happens with probability close to zero. In the limit, all traders with deadline less than N choose CM and obtain a payoff equal to 1/2. Traders with deadline N choose RM and also get a payoff equal to 1/2. $\square$

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