Rivalry, Exclusion and Coalitions

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Abstract

We analyze a situation where individuals and coalitions can obtain effective property rights over a resource by means of an exclusion contest. Coalitions face a trade-off when they decide to incorporate new members: Big groups control the resource more likely but individual property rights are more diluted. Under cooperative explotation of the resource the grand coalition is the efficient partition. It is also stable if players are committed to minimize deviators' payoffs. This is not the case when players play best responses and the conflict technology is sufficently effective with respect to the concavity of the production function: Then there is a strong tendency towards bi-partisan conflicts. Moreover, under non-cooperative exploitation of the resource, conflict may be socially efficient and Pareto dominate free access.

JEL classification codes: C71, D62, D74

Keywords: coalition formation, exlusion contest, tragedy of the commons

1 Introduction

> Suppose a society to fall into such want of all common necessaries that the utmost frugality and industry cannot preserve the greater number from perishing, and the whole from extreme misery: It will readily be admitted that the strict laws of justice are suspended in such pressing emergence, and give place to the motives of necessity and self-preservation..."
>
> David Hume (1751), *An Enquiry Concerning the Principles of Morals.*

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1.1 Motivation and overview

Individuals often face situations in which to interact with many other agents provokes a decline in individual payments (the exploitation of natural resources or markets for instance.) In these cases, agents perceive the presence of others as potentially dangerous or harmful; They are *rivals in nature*. But what if they can anticipate this "tragic" result? Will not they be tempted to invest effort in non-economic means in order to avoid such ending? In these settings, diverting part of the productive endowments into *appropriative* activities aiming to reduce the number of rivals or competitors arises as a natural option.

Historically, fights for the control over or access to resources have been a main root of conflict among individuals and states: During the English Enclosure on the 18th century, land, traditionally of common property, was privatized through the political initiative of the upper classes[1]; in 1998, the Project on Environmental Scarcities, State Capacity and Civil Violence of the University of Toronto concluded that resource scarcity has triggered predative behavior by elite groups in Indonesia, China and India (among other countries). These groups aim to change property rights in order to obtain monopolistic access to the resources. The immediate consequence is the defensive reaction of excluded groups[2].

Rivalry leads to competition, but competition may lead to cooperation: Even if individuals are rivals in nature, it is not obvious that they will remain in the state of "the war of all against all". Sooner or later they realize that by joining with others individuals and agreeing on a peaceful arrangement, groups may face external hostilities in a much better position. A natural question is: Does this clustering process eventually lead to universal agreement, or to social fragmentation?

This paper investigates the formation of groups or coalitions when individuals may engage in activities aiming to exclude others from a resource of common ownership. With that purpose we explore a general equilibrium model where, once the population is partitioned into a coalition structure, members of coalitions allocate their endowments into conflict effort (*effort* henceforth) and productive activities (*labor* henceforth). Agents are identical and budget constrained by their initial endowments. For each coalition,

[1] "Where enclosure involved significant redistribution of wealth it led to widespread rioting and even open rebellion" (North and Thomas, 1973).

[2] This was the case of the events in the Senegal River valley in 1989: Anticipating the construction of a dam that increased land values, the Moor elite in Mauritania rewrote legislation governing land ownership, effectively abrogating the rights the black Africans to continue their economic activities on that lands. After the subsequent explosion of violence in response, the black Mauritanians were stripped of their citizenship, expelled from the country and their properties seized. For more examples of recent conflicts over water or land supply see Homer-Dixon (1994).

a *conflict technology* maps the profile of total coalitional efforts to the probability of winning the *exclusion contest* that follows the group formation stage. According to these probabilities, Nature selects one coalition as the winner of the contest. Therefore, *agents use conflict to create effective property rights*[3]. Once control is granted, members of the winning coalition exploit the resource with the supplied labor. The sharing rule employed is a convex combination between equal sharing and proportional to labor contributions. This specification not only satisfies some desirable properties, but also encompasses as specific cases *joint production*, where agents can sign binding agreements over labor contributions, and *individual production* case, where members of the winning coalition exploit the resource non-cooperatively and the 'tragedy of the commons' arises.

In this model the coalition formation process presents two particular features. First, coalitions face a trade-off when they decide to incorporate a new member: Given that output is shared among the members of the winning coalition, the more players join in a coalition the more likely is that it obtains control but the more diluted the control rights are. Second, group formation induces externalities in non-members: When two individuals merge they agree not to fight each other. Consequently, their exclusion effort changes. This affects the winning probabilities across coalitions, and thus payoffs.

We are interested on what coalition structures arise in this game and their impact on efficiency.

We first analyze the basic properties of the non-cooperative game played after coalitions have formed: For any partition of the set of players and any sharing rule considered, there exists a unique interior Nash Equilibrium. This result is very important because it allows us to associate a unique vector of individual payoffs to each possible coalition structure. We then explore comparative statics and show that low-elasticity production technologies and more egalitarian sharing rules lead to higher total levels of conflict.

Next, we focus on the coalition formation stage for the two polar cases described above. There is not a unique approach to this issue. We consider the following procedures. First, we simplify the existence of externalities by assuming a fixed pattern of behavior on outsiders. This approach defines game in characteristic form where the coalitional worth is independent of outsiders' movements. In the second approach we assume that coalitions play only best responses and form sequentially following the game proposed by Bloch (1996). In the game ensuing payoffs depend on the entire coalition structure. Unfortunately, general closed forms are not possible and results are thus limited. However, the qualitative features can be displayed by examples.

[3]Following Grossman (2001), to say that an agent has an effective property right over an object means that this agent controls its allocation and distribution.

We first analyze the joint production case. We show that sufficiently effective conflict technologies make coarser coalition structures induce higher levels of overall conflict. Under the assumption that players are committed to inflict as much harm as possible to possible deviators the grand coalition is stable. When only best responses are employed a conflict among coalitions of the same size cannot improve upon universal agreement. The game shows a strong tendency towards (at most) bi-partisan conflicts, the incentives to open conflict depending upon the relation between effectivity of conflict and rivalry/congestion: Under constant returns to scale of conflict effort, coalitions do not find profitable to open conflict if they believe that the complementary coalition will break up into singletons; but it may be profitable for small coalitions to break the grand coalition if their deviation will not be followed by any other. Increasing returns to effort reverse these results. In any case the size of the population turns out to be critical in the generation of deviations.

We continue addressing the case of individual production in which, once a coalition obtains control over the resource, its members exploit it non-cooperatively. In this case, the production stage takes the form of the 'tragedy of the commons', one of the most clear examples of economic rivalry. Recall that this phenomenon occurs when, due to negative externalities, a resource of common use becomes overexploited. For instance, when herdsmen put the individually optimal amount of cattle in the pasture they do not take into account that this decreases the available pasture for other herdsmen's cattle. Inefficiency increases as the number of individuals who exploit the resource grows.

Normative approaches to the problem of the commons are perhaps too naive if appropriative activities are available. In fact, we show that conflict may be *socially efficient.* It acts as a discipline device that deters players from devoting too much labor in the exploitation of the resource: Under these circumstances, the formation of the grand coalition is much more difficult than under joint production because coalitions may prefer to expel members and put up with higher hostilities in order to avoid overexploitation of the resource. Moreover, contrary to the joint production case, it is very likely that a conflict among coalitions of the same size Pareto dominates free access.

The paper is structured as follows: In the subsection below the present work is related with the literature. In Section 2 we give some basic notation and assumptions. In Section 3 we show the uniqueness of the Nash equilibrium for any coalition structure and do some comparative statics. Section 4 and 5 address coalition formation for the polar cases of joint and individual production respectively. In Section 6 we conclude and discuss questions opened for further research. All proofs are in the Appendix.

1.2 Related literature

This paper is related with three different strands of the economic literature: Economic models of conflict, coalition formation games with externalities and common property resources.

Economic models of conflict date back to Bush and Meyer (1974) and have received important contributions by Skaperdas (1992), Hirshleifer (1995) and Neary (1996)[4]. The basic idea underlying this literature is that if property rights are not properly defined individuals face a trade-off between undertaking productive and non-economic or appropriative activities. The main consequence is that the allocations resulting from economic interactions may not be exclusively those derived from productivity but also from relative performance in a conflict stage: Agents engage first in productive activities -labor is transformed into output- and output is redistributed by force in the second stage when players devote effort to appropriation. The probability of winning the conflict can thus be identified as the proportion of the total output allocated to each agent.

This canonical model however has been criticized because it can be interpreted only as a theory of the right of access to common property, but fails to account for the creation of private property rights[5]. Grossman and Kim (1995) and Muthoo (2002) deal with the enforcement of the right to enjoy the fruits of one's labor. Rather than over some aggregate, the contest is over individual productions. In any case, all these models render conflict activities as socially wasteful because resources are diverted away from productive uses. On the contrary, we show that when players fight for the right to exploit a common good, conflict activities may be socially efficient precisely because of that.

All the mentioned models ignore as well the issue of coalition formation[6]. Moreover, they share the unsatisfactory feature of focusing on struggles over objects rather than over *rights*: In the former case agents produce in the shadow of expropriation of the common output so they might be appropriate to discuss pre-modern conflicts. However, victory in present-day contests (as the cases above show) implies that winners control (or have effective property rights) some contested object that enables them to produce without further opposition[7]. Under this broader view the role of

[4] These models are closely related to rent-seeking models. In fact, they belong to the more general class of models of rivalry. We refer the reader to Neary (1997) for a nice exposition of these issues.

[5] See Grossman and Kim (1995), Neary (1996) and Muthoo (2002)

[6] The only exception is the recent paper by Noh (2002) who extends the canonical conflict model to the case of three heterogeneous players.

[7] To the best of our knowledge, the only model of coflict that makes this distinction is Skaperdas and Syropoulos (1998), where two agents fight for the right to access to some fixed factor that they can use in production in case of victory.

coalitions in conflicts is more sensitive and clearer[8]. Then, the main trade-off is no longer between productive and unproductive activities but between a higher chance of success and shared property rights.

The issue of coalition formation in common-pool resources in the absence of conflict for control has been explored by Funaki and Yamato (1999) and Meinhardt (1999). If players can communicate, they can form groups in order to exploit the common. This (partially) solves the externality problem because members internalize the negative effects on other members. Although the grand coalition is the most desirable outcome, the presence of external effects may prevent its formation because all players prefer others to form coalitions: Funaki and Yamato (1999), in a partition function approach, show that the core of their game is non-empty if players have the most pessimistic expectations but not if they have the most optimistic ones. Meinhardt (1999) addresses the issue through a characteristic function approach that turns out to be convex and whose core coincides with the Von Neumann-Morgenstern's stable set.

Our model would be therefore complementary to these two because we all address, from a positive point of view, situations in which the tragedy of the commons may not be an irreversible outcome.

Third, the present work adds up to the existing literature on coalition formation games with externalities, surveyed in Yi (1999), that departs from traditional characteristic form games in that coalitional payoffs depend on outsiders actions. With the exception of Tan and Wang (2000) and Noh (2002), appropriative activities have been ignored as a source of externalities. In this context, we also try to provide foundations to conflict models by analyzing what coalition structures arise in our game.

Finally, our model is somehow related with some works in the field of sociobiology as an instance of the *competitive exclusion* principle[9] that states that two species cannot coexist indefinitely under a limited amount of resource. Anyway, we assume implicitly this principle rather than proving it.

2 The model

Consider a set $N = \{1, 2, ..., n\}$ of identical players. Each of them owns one unit of endowment that can be transformed into *effort* in the exclusion

[8] On the models of enforcement of private property rights, the simple existence of more than two players may lead to inconsitencies: If an agent challenges two outsiders he may loose two times his individual production!

[9] See Carneiro (1970) and (1978).

contest (effort henceforth) or in *labor*. We denote these investments by r_i and l_i respectively, subject to the constraint $r_i + l_i \leq 1$.

Players may form coalitions. A *coalition structure* π is a partition of N in a collection of disjoint coalitions $\{S_k\}_{k\in K}$. Let us denote by s_k the cardinality of S_k. We say that a coalition structure is *symmetric* when all coalitions in it are of the same size. Finally, the structure π is said to be a *coarsening* of π' if π can be obtained from π' by merging coalitions in π'.

Once a coalition structure π has formed an *exclusion contest* takes place: Denote by $\mathbf{r}(\pi) = (r^{S_1}, r^{S_2}, ..., r^{S_K})$, where $r^{S_k} = \sum_{i\in S_k} r_i$, the vector of coalitional efforts (we will denote individuals by subscripts and coalitions by superscripts). The result of the contest among coalitions is driven by the *conflict technology* that maps $\mathbf{r}(\pi)$ to a vector $\mathbf{p} =\{p^{S_k}\}_{k\in K}$ of coalitional winning probabilities (with probability p^{S_k} the coalition S_k attains the control of the resource and so on). We adopt a simple functional form where a generic element of π denoted, with some abuse of notation, by S accesses to the resource with probability

$$p^S(\mathbf{r}) = \frac{(r^S)^m}{(r^S)^m + r^{-S}} \tag{1}$$

where $(r^S)^m$ is the *coalitional outlay*, $r^{-S} = \sum_{S_k\in\pi\backslash\{S\}} (r^{S_k})^m$ is the sum of all coalitional outlays outside S, and m represents the returns to scale or *effectivity* of conflict effort. It is assumed that $m \geq 1$. Notice that S cares only about the supply of effort r^{-S} and not about the exact composition of π. However, the particular π we are considering makes a difference: The *total* of coalitional efforts may be the same for two different coalition structures but, for $m > 1$, they lead to different levels of *total* coalitional outlays; the limit case when $m = \infty$ is formally equivalent to a first-price auction where the coalition with the highest coalitional effort wins the contest with probability 1.

Exploitation of the resource is carried through the production function $f(L)$, where $L = \sum_{i\in S} l_i$, satisfying $f(0) = 0$. This technology is continuous and concave in labor and satisfies that $f'(0) > n\omega$, where ω is the unit cost of labor, in order to ensure the existence of an interior solution to the production problem of all coalitions.

The elasticity of production with respect to labor

$$\varepsilon = \frac{f'(L)L}{f(L)},$$

is a useful proxy for scarcity; by concavity $\varepsilon \leq 1$. We establish a partial ordering: technology f is said to *dominate* technology g if and only if $\varepsilon_f > \varepsilon_g$ for any L.

Each member of the winning coalition receives a share

$$\alpha_i = \frac{\lambda}{s} + (1-\lambda)\frac{l_i}{L} \tag{2}$$

of the output generated so the individual payoff in the production stage is

$$\alpha_i f(L) - \omega l_i.$$

This family of sharing rules presents some advantages: The parameter λ can be related with the *enforceability* of the contracts over labor that members of a coalition can sign. If $\lambda = 0$ we are in a case of joint or cooperative production in which players are pre-committed to share the final output equally. However, if $\lambda = 1$ production is totally individual or non-cooperative; sharing is proportional to labor contributions and members of S play the 'tragedy of the commons'[10]. Second, it is the only family that satisfies the axioms of Additivity and Non Advantageous Reallocation (NAR)[11]; the latter ensures that no sub-coalition in S can benefit from redistributing labor contributions among its members; The total dividend for any subgroup depends only upon its contribution and the total labor contribution[12].

3 The Exclusion game

In this Section we explore the game agents play once a particular coalition structure π has formed and analyze its properties.

The individual payoff for an individual $i \in S$ is

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}[\alpha_i f(L) - \omega l_i]. \tag{3}$$

Players in N are identical: All of them are equally efficient when transforming their endowments in effort or labor and the constant marginal cost of labor ω is also the same for all players.

It is easy to see that at any optimal decision, individuals will employ their entire endowments in both activities. Consequently, $L = s - r^S$ and one can rewrite (3) as

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}[\alpha_i f(s - r^S) - \omega(1 - r_i)]. \tag{4}$$

[10] This formulation of the tragedy of the commons can be found, among others, in Cornes and Sandler (1983). With it we try to focus only on the trade-off between exclusionary and productive activities rather than between labor and leisure.

[11] See Moulin (1987).

[12] This is equivalent to a decentralization property because in order to compute his payoff an agent does not need to know who contributed by how much. Other rules like $\alpha_i = \frac{l_i^\lambda}{\sum_{i\epsilon N} l_i^\lambda}$ does not satisfy it.

where α_i is now equal to $\frac{\lambda}{s} + (1-\lambda)\frac{1-r_i}{s-r^S}$. Then, the strategy space of all individuals is $R_i = [0,1]$. They make their choice of r_i simultaneously and non-cooperatively.

Definition 1 *The* ***Exclusion game*** $\Gamma = (N, \{X_i, u_i^S\}_{i \in S \in \pi}, f, \omega, \lambda, m)$ *induced by the coalition structure* π *is defined by the payoff function in (4)*

The profile of individual choices yields both the vector $\mathbf{r}(\pi)$ of coalitional effort and individual payoff in the production stage. If π is the grand coalition, all players accede to the resource without contest.

Let us now define the best reply of an agent: Denote by $\mathbf{r}(\pi)\backslash r_i$ the strategy profile under the unilateral deviation of player i from the strategy profile $\mathbf{r}(\pi)$.

Definition 2 (Individual Best reply) *Given a coalition structure* π, *the set of individual best replies, denoted by* $B_i^S(r_{-i})$, *of agent* $i \in S$ *to the strategy profile* $r_{-i} = \{r_j\}_{i \neq j}$, *chosen by the rest of members of* S *(if any) and the outsiders is*

$$B_i^S(r_{-i}) = \left\{ r_i \in [0,1] \ / \ u_i^S(\mathbf{r}(\pi)) \geq u_i^S(\mathbf{r}(\pi)\backslash r_i) \right\}.$$

In the Nash Equilibrium of Γ all players are playing their best response $r_i(r_{-i})$ to the strategy profile r_{-i}. More formally:

Definition 3 (Nash Equilibrium of the Exclusion game) *A profile of effort choices* $(r_1, ..., r_n)$ *is a Nash Equilibrium of the Exclusion game* Γ *induced by* π *if and only if* $u_i^S(\mathbf{r}(\pi)) \geq u_i^S(\mathbf{r}(\pi)\backslash r_i)$ $\forall i \in N$.

Proposition 4 *The Exclusion game* Γ *induced by any coalition structure* π *has a unique interior Nash Equilibrium. Moreover, it is symmetric, i.e.* $r_i = r_j$ $\forall i, j \in S$, $\forall S \in \pi$, *and it is given by the following system of equations*

$$\frac{m}{sr^S}\frac{r^{-S}}{(r^S)^m + r^{-S}}[f(s-r^S) - \omega(s-r^S)] = \tag{5}$$

$$(1-\lambda)\frac{s-1}{s}\frac{f(s-r^S)}{s-r^S} + \frac{1}{s}f'(s-r^S) - \omega \qquad \forall S \in \pi$$

When we introduce the exclusion contest the supply of effort becomes a "rat race": Every unit spent by outsiders in excluding me reduces the opportunity cost of investing one more unit in excluding others, i.e. best response functions are increasing in r^{-S}. On the other hand effort of the members of a given coalitions are strategic substitutes. The reason is that whereas the winning probability is a public good, α_i is non increasing in r_i. So there are always incentives to free ride on other members' effort.

Our target now is to investigate the effect of different productive and conflict technologies, parametrized by ε and m respectively and the particular sharing rule employed (parametrized by λ) on the agents' optimal and equilibrium choices.

Proposition 5 *In the Exclusion game* Γ *the equilibrium level of total effort* $\sum_{s_k \in \pi}(r^{S_k})^m$

(i) *is higher under* g *than under* f *provided that* f *dominates* g;

(ii) *is increasing in* λ;

(ii) *is increasing in* m *if*

$$\sum_{k \in \pi \setminus S} (r^{S_k})^m (\ln \frac{r^{S_k}}{r^S}) \geq 0.$$

Conflict is linked to scarcity: In a world of constant returns to labor conflict makes less sense. As the opportunity cost of labor increases exclusion effort may be advantageous. In the same fashion, more egalitarian groups behave more aggressively because they can overcome free-riding in effort contributions. If the coalitions in π would differ in λ, these groups would have an advantage in the exclusion contest.

Unfortunately, the last part of the Proposition allow us to extract partial conclusions only: We can ensure that symmetric coalition structures induce higher conflict expenditures in equilibrium when the conflict technology improves.

For the rest of the paper we will consider two families of production functions:

Linear quadratic: with $f(L) = aL - bL^2$ that can be parametrized through $\theta = \frac{a}{b}$ as measure of linearity

Exponential: with $f(L) = L^\alpha$ where $\alpha \leq 1$ that satisfies constant elasticity of labor, i.e. $\varepsilon = \alpha$.

Example 1: Let us illustrate Proposition 5 with the following example. Take $N = \{a, b, c, d, e\}$. Initially, let $f(L) = \sqrt{L}$, $m = 1$, $\omega = 0.1$ and $\lambda = 0$. The coalition structure we assume that has previously form is $\pi = \{\{a, b, c\}, \{d, e\}\}$.

In Figure 1 we plot the effect of different sharing rules and conflict technologies. In the vertical axis we project the coalitional effort of $\{a, b, c\}$ and of $\{d, e\}$ in the horizontal one. The intersection of these reaction functions constitutes the Nash equilibrium of the coalitional game.

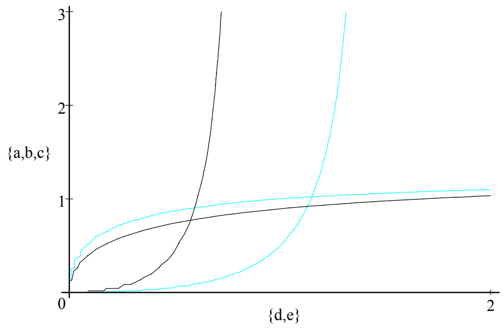

Fig. 1a: The effect of λ.

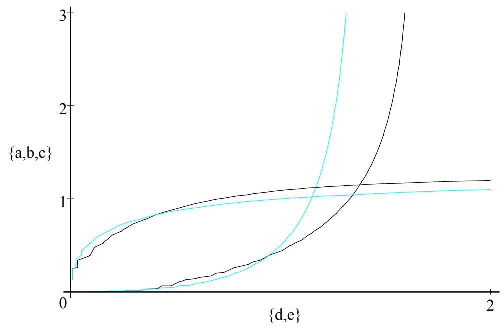

Fig 1b: The effect of m.

The lighter lines represent the baseline case. In panel 1a, the darker line depict the case $\lambda = 0.5$. As stated above, players put less effort as λ decreases because free-riding. In panel 1b, the dark lines correspond to the case when $m = 2$. As shown in the last Proposition the effect of this change is ambiguous: For low values of outsiders' effort, best replies below the ones for the baseline. However, the equilibrium takes place at higher investments of effort.

4 Coalition formation under Joint production

When $\lambda = 1$, agents in the winning coalition share the output equally. This is equivalent to say that they can sign binding agreements on labor contributions. As a consequence the resource is never overexploited, that is the total labor supply always satisfies $f'(L) > \omega$ (this can be seen in the RHS of expression (5).) Hence, for simplicity and throughout this Section we will assume that $\omega = 0 :$ An Exclusion game with a production function $h(L) = f(L) + \omega L$ would yield the same equilibrium[13].

Then, the payoff function can be rewritten as

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}} \frac{1}{s} f(L). \tag{6}$$

Let us now state some basic properties of the coalition formation game in this case. First. we identify the sign of the spillovers that coalition formation generates: Games with positive externalities are those where mergers of coalitions produce positive effects on non-members; with negative externalities, the effect is the opposite. It turns out that in our case this depends on the effectivity of conflict effort

[13]The only relevant effect of doing this is that the Exclusion game has multiple payoff equivalent equilibria: A game where players are coalitions instead of individuals can be defined. It yields a unique Nash equilibrium profile $\mathbf{r}^*(\pi)$. Given that there is no personal cost of contributing labor, any sharing of the equilibrium coalitional effort among its members is an equilibrium at individual level.

Proposition 6 *The coalition formation game under joint production (i) is of negative externalities if $m \geq 2$ (ii) and of positive externalities under linear quadratic and exponential technologies if $m = 1$.*

Proposition 6 and the fact that coalitional effort is increasing in r^{-S} allow us to state the following Corollary

Corollary 7 *When $m \geq 2$ coarser coalition structures induce higher levels of total effort.*

Another basic property of the coalition formation game is the following

Proposition 8 *Under joint production and productive linear quadratic and exponential technologies, in any symmetric coalition structure π, $u_i^S < u_i^N$.*

It immediately implies that the 'war of all against all' will never be the outcome of a coalition formation process; universal agreement will be broken only in favor of an asymmetric coalition structure.

4.1 Characteristic function approach

The main difficulty of the non-orthogonal games of coalition formation, in contrast with standard characteristic form games, is that outsiders' actions affect coalitional payoffs. Static approaches simplify this issue by assuming a specific pattern of behavior for the rest of players that pins down only one coalitional payoff. In this context there are two alternatives: One can make assumptions over the particular *strategies* that outsiders will choose and about the *coalition structure* that they will form.

4.1.1 The α and β characteristic functions

The α and β concepts, introduced by Aumann (1959), assume that players are committed to punish deviations as much as they can. It implies as well the formation of a particular coalition structure in order to achieve that goal. Notice that this behavior will not be rational most of the times.

For our purposes, it will be important to define an indirect payoff function. Denote by $r^S(r^{-S})$ the maximizer of expression (6).

Definition 9 *The* ***indirect payoff function*** *is*

$$u^*(r^{-S}) = u^S(r^S(r^{-S}), r^{-S}) = \underset{r^S}{Max}\ u^S(r^S, r^{-S}) \tag{7}$$

Now we can define the α and β characteristic functions:

Definition 10 *The α-characteristic function, v_α, in the Exclusion game is defined by:*

$$v_\alpha(S) = \underset{r^S}{Max}\ \underset{r^{-S}}{Min}\ u^S(r^S, r^{-S}) = \underset{r^S}{Max}\ u^S(r^S, \widehat{r}^{-S}) = u^S(r^S(\widehat{r}^{-S}), \widehat{r}^{-S}) = u^*(\widehat{r}^{-S})$$

where $\widehat{r}^{-S}$ is the minimizer of the coalitional payoff. This expression corresponds to the indirect payoff function (7) when the outsiders have chosen the action (and therefore a partition) that minimizes the coalitional payoff: It is the minimum payoff that coalition S can guarantee to itself.

The beta notion defines the payoff coalition cannot prevented from for any choice of outsiders:

Definition 11 *The β-characteristic function v_β in the Exclusion game is defined by:*

$$v_\beta(S) = \underset{r^{-S}}{Min}\ \underset{r^S}{Max}\ u^S(r^S, r^{-S}) = \underset{r^{-S}}{Min}\ u^S(r^S(r^{-S}), r^{-S}) = \underset{r^{-S}}{Min}\ u^*(r^{-S}).$$

Notice that both characteristics functions coincide if $\widehat{r}^{-S} = Min_{r^{-S}}\ u^*(r^{-S})$. Under joint production this holds.

Proposition 12 *Under joint production, the indirect payoff function is decreasing in r^{-S}. Therefore the α and β characteristic functions coincide, i.e. $v_\alpha(S) = v_\beta(S)$.*

The minimizer of the coalitional payoff is the same regardless of wether players react passively (after) or actively (before) to outsider's "best" punishment: We obtain the coincidence result also obtained for Common-Pool games (Meinhardt (1999)) and Cournot games (Zhao (1999)).

Now we ask: Can the grand coalition be blocked by some coalition $S \subset N$ under these assumptions about outsiders' behavior? Is there any room for cooperation?

Definition 13 *The α-core (β-core) is nonempty if there is no coalition $S \subset N$ such that $v_\alpha(S) > v_\alpha(N)$ ($v_\beta(S) > v_\beta(N)$).*

Scarf (1971) showed that the α-core of a NTU game is non-empty if the strategy space for each player is compact and convex and payoff functions are all continuous and quasiconcave. This conditions are satisfied by our game . Then, Proposition 12 implies the next result.

Proposition 14 *Under joint production, the $\alpha-$core and $\beta-$core are nonempty. Moreover, they coincide.*

Given that Proposition 14 is important we briefly outline its proof: Simple inspection of (6) show us that the worst case scenario for S when they are waiting for the choice of their rivals occurs when r^{-S} attains its maximum. When $m \geq 1$ the coalition $N \backslash S$ must form and all its members must put their entire endowment; then $\widehat{r}^{-S} = (n-s)^m$. Then, the alpha characteristic function is just the best response to $(n-s)^m$.

We know by Proposition 12 that the indirect characteristic function is decreasing in r^{-S}. This ensures that for any coalition $u^*(r^{-S})$ attains its minimum when $r^{-S} = (n-s)^m$. So finally we have:

$$v_\alpha(S) = u^S(r^S((n-s)^m), (n-s)^m) = u^*((n-s)^m) = \underset{r^{-S}}{Min}\; u^*(r^{-S}) = v_\beta(S).$$

Hence, if individuals are committed to inflict as much harm as possible to potential deviators universal agreement prevails.

4.1.2 The γ and δ characteristic functions

The static approach to coalition formation allows for non optimal reactions because it is never optimal for them to invest the entire endowment in conflict. Agents in the complement coalition may not be able thus to commit to total warfare in case of deviation. Hence, the next step would be to exogenously impose a coalition structure but allow players to use best responses. Coalitions will still be associated with a single payoff.

The idea, first introduced by Hart and Kurtz (1983) is to model the coalition formation process as a normal form game where the strategy space of the players is the set $S_i = \{S \subseteq N \ / \ i \in S\}$. They define two possible games: In the γ game, a coalition forms if and only if all its members announced that coalition; in the δ game a coalition forms among those that announced the same coalition even though some of its prospective members announced something else.

Given that in the Joint production case the grand coalition is the efficient coalition structure, in the sense that the sum of individual payoffs is the maximum[14], a natural question is if the universal agreement is stable, that is, if it can be supported as a (Strong) Nash equilibrium of these games.

When analyzing the stability of the grand coalition the γ and δ concepts can be easily interpreted as expectations of players about the coalition structure that outsiders will form after an individual or group decides to open hostilities: In the γ case they believe that the deviation will trigger a chain reaction until all remaining players form singletons; in the δ case they believe that remaining players will stick together. This allow us to define again two characteristic functions.

[14]Notice that the total output is maximized under the grand coalition: Any other coalition structure yields a convex combination among lower total productions.

Definition 15 *The grand coalition is $\gamma-$immune or γ stable [$\delta-$immune or δ stable] if there is no coalition $S \subset N$ such that $v_\gamma(S)$ $[v_\delta(S)] > \sum_{i\in S} u_i^N$.*

Proposition 16 *Under joint production, constant returns to scale of effort and an exponential production function*

(i) *the grand coalition is γ stable*;

(ii) *the grand coalition is δ immune to deviations by coalitions with $s \geq \frac{n}{2}$ but it is not δ immune to deviations by smaller coalitions.*

In particular for $\alpha = 0.1$ the grand coalition is not δ immune to the deviation os a single player when $n \geq 15$ and to the deviation of a two players coalition when $n \geq 31$.

When players hold "optimistic" expectations about the behavior of outsiders is hard to get the stability of the grand coalition. To have optimistic expectations means different regarding m : When $m = 1$ the coalition formation presents, optimism happens under the δ concept and, as Proposition 16 shows, universal agreement breaks up. However, it is in the other way around when $m \geq 2$. In such case we show with the following example that the previous result get reversed

Example 3: Suppose that $n = 5$ and $\alpha = 0.1$. Then the γ and δ characteristic functions are

m	$v_\delta(1)$	$v_\delta(2)$	$v_\delta(3)$	$v_\delta(4)$	$v_\delta(5)$
1	0.160	0.183	0.185	0.193	0.234
2	0.080	0.147	0.210	0.228	0.234
3	0.039	0.105	0.218	0.246	0.234

Table 1: δ characteristic function when $n = 5$

m	$v_\gamma(1)$	$v_\gamma(2)$	$v_\gamma(3)$	$v_\gamma(4)$	$v_\gamma(5)$
1	0.160	0.172	0.182	0.193	0.234
2	0.142	0.233	0.241	0.228	0.234
3	0.138	0.291	0.280	0.246	0.234

Table 2: γ characteristic function when $n = 5$

Notice first that $v_\delta(s) \leq v_\gamma(s)$ only when $m \geq 2$. In that case, incentives to deviate are in the hands of big coalitions: When $m = 2$ the grand coalition is not γ-immune and it is neither γ or δ-immune for $m = 3$. It seems that the stability of the grand coalition m depends critically on the relationship between m and α : Given a productive technology one only needs to have a sufficiently effective technology of conflict to attack N successfully.

4.2 Sequential coalition formation approach

We now assume that players rationally predict the coalition structure that outsiders will form after a deviation. There is no a unique approach to tackle this issue. Here, we will follow Bloch (1996), where coalitions form if and only if all members agree to do it *à la* Rubinstein: The first player in a pre-determined protocol makes a proposal for a coalition; the players in this proposed coalition decide sequentially to accept or not. The process stops when all members accept or one rejects. In the former case, the coalition finally forms; in the latter, the rejector must make another proposal. Bloch (1996) shows that this game yields the same stationary subgame perfect equilibrium coalition structure as the much simpler "Size Announcement game": First player proposes a coalition of size s_1 that immediately forms. Then the $(s_1 + 1)$-th player in the protocol proposes a coalition s_2 and so on, until the player set is exhausted. The game is solved through backward induction and has generally a unique subgame perfect equilibrium.

However, we face a new difficulty: Contrary to the existing literature on coalition formation games with externalities, the payoff function cannot be characterized uniquely by the number of coalitions in π. So we can only provide some partial results that show the importance of the relationship between the productive and conflict technology for the stability of the grand coalition.

Proposition 17 *Under joint production and exponential production functions, the Bloch's stable coalition structure*

(*i*) *is the grand coalition under constant returns of labor* ($\alpha = 1$).

(*ii*) *is not the grand coalition if labor is unproductive* ($\alpha = 0$) *and under increasing returns of effort. In particular it is of the form* $\{s, n-s\}$ *when* $m \geq 2$ *and where* s *satisfies*

$$(\frac{n}{s} - 1)^{m-1}((m-1)\frac{n}{s} + 1) = 1. \tag{8}$$

(*iii*) *is of the form* $\{s, n-s\}$ *with* $s \geq \min\{\frac{n}{2(1-\alpha)}, n\}$ *in the first-price auction-like case if the coarsest stable partition is selected.*

With constant returns of labor, conflict activities are wasteful from an individual point of view: The first player in the protocol announces the coalition that maximizes his probability of access because the cost of absorbing one rival is zero.

The case of total congestion coincides with Tan and Wang (2000) for the case of n identical players: Given that the prize is unaffected by labor investments, players invest all their endowments in the exclusion contest and this allows to derive a closed form for u_i^S. Under increasing returns to scale of effort there is a strong tendency towards bi-partisan conflicts because the formation of an outside group reduces the cost of joining with others. The first coalition formed is less inclusive as m increases: It is of size $\frac{\sqrt{2}}{2}n$ (ignoring the integer problem) when $m = 2$, $\frac{2}{3}n$ when $m = 3$, and $0.54n$ when $m = 20$.

Finally, in the first-price-auction-like any coalition structure in which the first coalition is greater or equal than $\frac{n}{2}$ is Bloch's stable. In order to be consistent with the previous result, we take the criterion of selecting the coarsest stable partition (because then the first coalition formed when $\alpha = 0$ is of size $\frac{n}{2}$.) In that case, the grand coalition if and only if $\alpha \geq \frac{1}{2}$. Again, universal agreement can only be supported as a Bloch stable coalition structure if returns to scale of labor are not too decreasing with respect to the effectivity of effort.

Let us illustrate the points made so far with an example for a linear quadratic production function.

Example 3: Assume that $N = 4$. In order to obtain reader friendly figures we assume that players have 35 units of initial endowment. Let $f(l) = 20L - \frac{1}{8}L^2$. We allow m to be 1 or 2. Payoffs are displayed in the following tables.

	$m=1$				$m=2$			
π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	83	83	83	83	60	60	60	60
$ab \mid c \mid d$	144	144	85	85	151	151	50	50
$abc \mid d$	184	184	184	90	202	202	202	42
$ab \mid cd$	150	150	150	150	123	123	123	123
$abcd$	200	200	200	200	200	200	200	200

It is easy to see what the Bloch stable coalitions structures are: For player a (the first in the protocol) it is dominant to announce $\{N\}$ when $m = 1$ and to announce $\{3\}$ when $m = 2$. In the latter case a deviation is possible because increasing returns to effort make exclusion cheap for big coalitions. In that case, the possibility of conflict breaks up the efficiency of universal agreement.

5 Coalition formation and commons

A common good is an object that is owned by nobody or, equivalently, by everybody: a fishery, a pasture... In this Section we will be concerned with how the existence of an exclusion contest and the possibility of coalition formation affect the creation of effective property rights over common goods: Will the tragedy of the commons "remorselessly" occur?

If a good is of common property, one would think that it cannot be owned by a few![15] However, as pointed out by Grossman (2001), there is a clear difference between *effective* and *formal* property rights: The former entail control, the latter are those stated by legal ownership and may not confer control rights by themselves. In fact, sufficiently strong control rights are the main step for the recognition of formal ones if they were previously undefined. For example, in the 1960s, oil and gas were found under the North Sea. Several countries contested for the exploitation rights. Although the United Nations' Law of the Sea claims that resources in the seabed are "the common heritage of all mankind", Britain and Norway finally obtained such rights because they were able to impose the "smallest distance to the coast" criterion.

Let us briefly described the basic analysis of the 'tragedy of the commons': In the unique symmetric Nash Equilibrium of the game the total labor input l^F when s players has free access to the common

$$\frac{1}{s}f'(l^F) + \frac{s-1}{s}\frac{f(l^F)}{l^F} = \omega. \tag{9}$$

Efficiency would require that $f'(l^S) = \omega$. However the total labor input yields a weighted average between the efficiency level (achieved only when one agent entries) and the equalization to the average productivity, where the resource is overexploited. Moreover, the equilibrium payoff is decreasing in s because inefficiency becomes more severe as s grows; as $s \to \infty$ individual payoff approaches zero.

As we know the Exclusion game in the presence of a common pool resource corresponds to the case of $\lambda = 0$. In that case condition (5) becomes

$$\frac{m}{s-l^E}\frac{r^{-S}}{(r^S)^m + r^{-S}}\frac{1}{s}[f(l^E) - \omega l^E] = \frac{s-1}{s}\frac{f(l^E)}{l^E} + \frac{1}{s}f'(l^E) - \omega \tag{10}$$

The RHS of this expression is precisely the difference between the terms in (9) that now is positive instead of zero. It implies that $l^F > l^E$. Hence *conflict acts as a discipline device* because it deters players from contributing too much labor. This result opens the door to the *social efficiency of conflict*, because exclusion activities (partially) alleviate the tragedy of the commons. However, it would be totally trivial and vacuous if the stable coalition structures, according to the concepts employed above, yielded always free access or other inefficient partitions. Next, we analyze this issue by using linear quadratic production functions.

The first result is in sharp contrast with the Joint production case: Players in a symmetric coalition structure may be better off than in the grand coalition and hence conflict may Pareto dominate free access.

[15] I thank Carmen Beviá for this point.

Proposition 18 *Suppose that the free access problem case has an interior solution under a linear quadratic technology. Then there exist a treshold* $\widetilde{m}(n, k, \omega, \theta)$ *such that a k-sided symmetric conflict Pareto dominates free access if and only if* $m \leq \widetilde{m}(n, k, \omega, \theta)$.

As expected the threshold $\widetilde{m}$ is increasing in θ : If conflict technology is too effective and players underexploit the common free access can be again dominated by rising θ and making production technology sufficiently concave.

The second observation is that the α and β characteristic functions may not coincide, as the following Lemma illustrates.

Lemma 19 *The indirect payoff function of a player* $i \in S$ *is strictly decreasing in* r^{-S} *if*

$$s \geq (m+1)\frac{1-\varepsilon}{1-(\omega(s-r^S)/f(s-r^S))} - m. \tag{11}$$

Why can we only state a sufficient condition? Notice first that if a coalition is underexploiting, $f'(L) > \omega$, condition (11) holds because the right hand side is negative. However this is not necessarily true when the coalition is overexploiting the resource. In such cases, in increment of the effort by outsiders reduce the total labor contribution of the coalition and payoff in the production stage increases. One can only ensure that $v_\alpha(s) = v_b(s)$ as long as coalition S cannot overexploit, i.e. $f'(s) > \omega$. As a consequence only when no coalition can overexploit the common[16].

Now we illustrate Proposition 18 and Lemma 19 by means of the following example where we also show that the α characteristic function is not convex contrary to what happens in common pool games (see Meinhardt (1999)).

Example 4: The initial data of this game are taken from Meinhardt (1999). Note that, by symmetry, they are equivalent to those used in Example 3. This allows to compare the three models, Meinhardt's and joint and individual production.

Let $n = 4$, players have 35 units of initial endowment, the unit cost of labor is 3 and let $f(L) = 23L - \frac{1}{8}L^2$. Again, we consider the cases of m equal to 1 and 2.

First, we compare the alpha (and beta, because they coincide for the individual production case too) characteristic functions. The characteristic

[16] This does not imply that the β core is empty. In fact, if $\widehat{r}^{-S} = (n-s)^m$ is the actual minimizer of u_i^*, the exclusion game satisfies all the conditions posed in Theorem 1 in Zhao (1999) for the non-emptiness of the β core. The real problem is that it is not possible in this framework to compare a corner solution with possible interior minimizers.

form game of Meinhardt (1999) is convex, that is, individual contributions to coalitional worth are greater the bigger the coalition the player joins with. However, this does not hold for our exclusion games

	$v_\alpha(\{1\})$	$v_\alpha(\{2\})$	$v_\alpha(\{3\})$	$v_\alpha(\{4\})$
Meinhardt (1999)	95	253	488	800
Joint production ($m = 2$)	10.2	123.4	454.5	800
Individual production ($m = 2$)	10.2	106.5	380.3	512

where $v_\alpha(\{s\})$ is the value generated by a coalition of size s when outsiders behave in the α fashion. Note that the joint production case is intermediate between Meinhardt's and individual production. Anyway, values for the exclusion games are always below Meinhardt's ones. Furthermore, Lemma 19 does not apply: Suppose that $n > 4$. Then, the indirect payoff function for the member of coalition of four players attains a maximum when $r^{-S} \approx 0.8$.

Let us now assume that coalitions play best responses. We compute the partition function for Meinhardt (1999):

π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	128	128	128	128
$ab \mid c \mid d$	100	100	200	200
$abc \mid d$	118	118	118	335
$ab \mid cd$	177	177	177	177
$abcd$	200	200	200	200

Notice that this game is of positive externalities: When players merge they reduce their labor input because they internalize part of the social costs. Outsiders take advantage from it: They have now a bigger share of a higher overall production. Then, players are reluctant to form coalitions: They want others to do so. Knowing this, the best announcement for the first player in the protocol can do is the grand coalition.

Things change dramatically for the individual production case:

	$m = 1$				$m = 2$			
π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	83	83	83	83	61	61	61	61
$ab \mid c \mid d$	134	134	102	102	142	142	67	67
$abc \mid d$	152	152	152	156	171	171	171	94
$ab \mid cd$	177	177	177	177	158	158	158	158
$abcd$	128	128	128	128	128	128	128	128

Note first that, in contrast with the joint production case, this game is of positive externalities for both values of m. When $m = 1$, it is dominant for a to announce a two-player coalition because for c it will optimal to form

$\{cd\}$. On the other side, when $m = 2$ it is dominant for a to form $\{abc\}$. The reason for this difference lies at the fact that when $m = 1$ the three players coalition is overexploiting and the winning probability does not decrease too much by expelling c although it joined d.

Observe that, as pointed out in Proposition 18, for both values of m the structure $\{ab \mid cd\}$ is the most efficient one and Pareto dominates free access. The consequence is that the latter is neither δ or γ immune. The same happens with $\{abc \mid d\}$ when $m = 1$. However, when $m = 2$ although players in $\{abc\}$ no longer overexploit the resource because their effort has increased, d is in a too weak position. So big coalitions "need" the presence of outsiders. Not too many, as in the latter case for $\{ab\}$, but not too few, as in the former for $\{abc\}$.

Finally let us compare the stable structures of the three models:

	$m = 1$	$m = 2$
Meinhardt (1999)	$abcd$	$abcd$
Joint exploitation	$abcd$	$abc \mid d$
Separate Exploitation	$ab \mid cd$	$abc \mid d$

This results suggest that even if agents can communicate, very effective conflict technologies make a difference. On the other side, by accepting the possibility of conflict in non-cooperative environments, the 'tragedy of the commons' is partially alleviated: The expected production is closer to the joint production of the resource, the best case scenario.

6 Conclusion

We have presented an economic model of conflict where agents reduce the rivalry or congestion over some resource by excluding others. We have considered also the possibility that these agents may form coalitions in order to be more successful in that endeavour. Effective property rights are created through contests that determine what coalition gains access to the resource. One important feature is that coalitions face a trade off: As they incorporate more members they attain control more likely, but individual property rights within it dilute. Moreover, individual and coalitional payoffs depend on the entire coalition structure.

We show that the more concave the technology of production is the more likely a sufficiently effective conflict technology will break up universal agreement. Under increasing returns to scale of exclusion efforts, the formation of other groups reduces the costs of sharing property rights for the rest of agents, leading to bi-partisan coalition structures. This would support the standard two-player models of conflict. However, the main result under individual production is completely new in this literature: Conflict may be socially efficient because deters individuals from using to much labor in the exploitation of the resource. Moreover, it is relatively easy to generate coalition structures that Pareto dominate free access and to support them as the outcome of some well-know coalition formation games. Our results must be interpreted with caution: We do not advocate that property rights over commons should be allocated through conflict. Our contribution should be regarded from a purely positive perspective; as an alternative that would emerge when agents are not able to contract for welfare enhancing arrangements nor communicate.

Some other comments are in order. First, our game is one shot: Once a coalition has won the contest, its members agree not to fight again. Other models, like Tan and Wang (2000), consider the scenario of *continuing conflict* where conflict is assumed to be fought until a conflict-proof coalition (that is, one immune to the re-opening of conflict) prevails. We think that this is a very relevant question with a population of identical agents where only size matter: Why should a group of agents that fought mercilessly with others cooperate for ever once conflict is solved?[17]. Continuing conflict seems to be very well suited to overcome this problem for instance in rent-seeking setups where the value of the prize is fixed (rather than cooperative bargaining or fixed sharing rules). However, it seems less valid in a setting like ours where the after-conflict stage is a production stage with its own structure.

Another objection would be that our players are identical. It can be argued that conflicts many times often because agents are different. Beyond the problem of tractability, Noh (2002) points out how complex is to consider just three heterogenous players, we can answer that assuming inequality of endowments or strengths may be self-explanatory of the presumable unequal allocations resting on conflict (or power relationships). Us, we are mainly interested on exploring the validity of conflict as a *mechanism* that generates such inequality as a by-product when agents use it to attains their goals. Nevertheless, this option is worth considering: Given the complexity of the model the easiest way would be to assume the existence of two types of players.

Another possible extension of the model would be to relax the assumption that the losing players face "death". This may be a source of additional conflict investments. In Skaperdas and Syropoulos (1996) victory means that the winners trade with the losers in a dominant position. In these line, It could be assumed that the winning coalition also gains the power to hire labor (or take it freely) from the losers. Then, if the payoff after exclusion is not zero or the winners care about the left over endowments of the losers exclusion races might be alleviated and conflict less fierce.

[17] We thank Serge Kolm for pointing out this question.

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A Appendix

Proof of Proposition 4. Let us denote $f(s - r^S)$ by f and so on. Moreover, let us denote $R = (sr)^m + r^{-S}$. Then, the first order condition for the maximization problem faced by $i \in S$ states

$$m(r^S)^{m-1}\frac{r^{-S}}{R^2}[\alpha_i f - \omega(1 - r_i)] - \frac{(r^S)^m}{R}[(1 - \lambda)\frac{s - 1 - r^{S\backslash i}}{s - r^S}\frac{f}{s - r^S} + \alpha_i f' - \omega] = 0 \tag{12}$$

Now we show that all optimal decisions inside any coalition S must be the same across its members: Expression (12) can be rewritten as

$$m\frac{r^{-S}}{R}[\alpha_i f - \omega(1 - r_i)] = r^S[(1 - \lambda)\frac{1 - r_i}{s - r^S}(f' - \frac{f}{s - r^S}) + (1 - \lambda)\frac{f}{s - r^S} - \omega] \tag{13}$$

Dividing (13) by the analogous expression a member j of S and rearranging yields

$$\frac{\alpha_i f - \omega(1 - r_i)}{a_j f - \omega(1 - r_j)} = 1 + \frac{(1 - \lambda)\frac{r_j - r_i}{s - r^S}(f' - \frac{f}{s - r^S})}{(1 - \lambda)\frac{1 - r_j}{s - r^S}(f' - \frac{f}{s - r^S}) + (1 - \lambda)\frac{f}{s - r^S} - \omega} \tag{14}$$

Suppose that contrary to our conjecture $r_i > r_j$. Then the LHS of (14) must be smaller than one. Otherwise r_i would not be optimal for i because by decreasing his effort would get a higher share of a higher total output. However, the RHS of (14) is clearly greater than one because by concavity $f' < \frac{f}{s - r^S}$. Therefore, expression (14) only holds true when $r_i = r_j$.

Denote by r the individual level of effort inside S. Now we show that there is a unique value of $r > 0$ that satisfies (12). Let

$$g(r) = m\frac{r^{-S}}{R}[\frac{1}{s}f - \omega(1 - r)] - r[(1 - \lambda)(s - 1)\frac{f}{s - sr} + f' - s\omega].$$

Nest we show that whenever $g(r) \leq 0$ then $g'(r) < 0$.

$$\begin{aligned} g'(r) &= -m\frac{r^{-S}}{R^2}[f - \omega(s - sr)] - m\frac{r^{-S}}{R}[f' - \omega] \\ &\quad -[(1-\lambda)(s-1)\frac{f}{s - sr} + f' - s\omega] \\ &\quad -sr[(1-\lambda)\frac{(s-1)}{s-sr}(\frac{f}{s-sr} - f') - f''] \\ &\leq m\frac{r^{-S}}{R}[\omega - f'] + sr(1-\lambda)\frac{(s-1)}{s-sr}(f' - \frac{f}{s-sr}) \\ &\leq (f' - \frac{f}{s-sr})[sr(1-\lambda)\frac{(s-1)}{s-sr} - m\frac{r^{-S}}{R}] \leq 0. \end{aligned}$$

where the last inequality follows from the fact that when $g(r) \leq 0$

$$\frac{m}{sr}\frac{r^{-S}}{R} \leq \frac{(1-\lambda)(s-1)\frac{f}{s-sr} + f' - s\omega}{f - \omega(s - sr)} < (1-\lambda)\frac{(s-1)}{s-sr}, \tag{15}$$

because $f' < \omega \leq (s(1-\lambda)+1)\omega$. Therefore, if $g(r)$ has a critical point or it is decreasing, it is concave. This result implies that there exist at most one r that makes $g(r) = 0$. Now we show that this r exists: $\lim_{r\to 0} g(r) = m[\frac{1}{s}f - \omega] > 0$[18] whereas by L'Hôpital rule $\lim_{r\to 1} g(r) = -[(s(1-\lambda) + \lambda)f'(0) - s\omega] < 0$ by assumption. ■

Proof of Proposition 5. Let us denote by γ the parameter of interest. By total differentiation

$$\frac{dr^S}{d\gamma} = \frac{\partial r^S}{\partial \gamma} + \frac{\partial r^S}{\partial r^{-S}}\frac{dr^{-S}}{d\gamma},$$

Define

$$H(r^S, r^{-S}) = m\frac{r^{-S}}{R}[\alpha_i f - \omega(1 - r_i)] - r^S[(1-\lambda)\frac{s - 1 - r^{S\backslash i}}{s - r^S}\frac{f}{s - r^S} + \alpha_i f' - \omega] \tag{16}$$

One can repeat easily the procedure of the previous proposition and show that $\partial H(r^S, r^{-S})/\partial r_i < 0$ when $H(r^S, r^{-S}) = 0$. First we show that the best reply effort is increasing in r^{-S} : By the Implicit Function Theorem, the sign of derivative is simply given by

$$\frac{\partial H(r^S, r^{-S})}{\partial r^{-S}} = m\frac{(r^S)^m}{R^2}[\alpha_i f - \omega(1 - r_i)] > 0.$$

[18]If $\frac{1}{s} f < \omega$ then $g(r)$ is clearly bounded from below by a level $\underline{r}$ such that when $r \to \underline{r}$, $g(r) \to 0^+$.

Therefore, for any parameter γ the sign of $\frac{dr^S}{d\gamma}$ and thus the direction in which R moves when γ increases is totally described by the sign of $\partial H(r^S, r^{-S})/\partial\gamma$.

In the case of the elasticity. Condition (16) can be rewritten as

$$H(r^S, r^{-S}) = m\frac{r^{-S}}{R}(1-r_i)[\frac{s-r^S}{1-r_i}\frac{\alpha_i}{\varepsilon} - \frac{\omega}{f'}] - r^S[(1-\lambda)\frac{s-1-r^{S\backslash i}}{s-r^S}\varepsilon + \alpha_i - \frac{\omega}{f'}]$$

and the sign of that derivative is clearly negative, so the effort is inversely related with the elasticity of output with respect to labor. Following the same procedure, the sign of the derivative of the best reply with respect to λ is given by the sign of

$$\frac{\partial H(r^S, r^{-S})}{\partial\lambda} = r^S\frac{s-1-r^{S\backslash i}}{s-r^S}\frac{f}{s-r^S} > 0.$$

so more egalitarian groups are more aggressive. Finally, in the case of m

$$\frac{\partial H(r^S, r^{-S})}{\partial m} > m\frac{(r^S)^m\sum_{S_k\in\pi\backslash S}(r^{S_k})^m[\ln r^{S_k} - \ln r^S]}{R^2}[\frac{1}{s}f - \omega(1-r)].$$

Hence, the condition stated in the text is enough to obtain the desired result. For symmetric coalition structures $\ln\frac{r^{S_k}}{r^S} = 0$ for any $S_k \in \pi$ and the equilibrium level of effort is increasing in m for sure. ■

Proposition 7. It is shown in Proposition 12 (proved below) that the indirect payoff function in the joint production case is decreasing in r^{-S} then the coalition formation game is of positive (negative) externalities if given two coalition structures π and π' it happens that $r^{-S}(\pi') < (>)\ r^{-S}(\pi)$, where $\pi'\backslash\{S\}$ can be obtained by merging coalitions in $\pi\backslash\{S\}$.

The next step is to look to the convexity or concavity of the coalitional outlay with respect to size in order to discern how the overall level of hostility changes when coalition structures become coarser. Let us define $r = r^S/s$. Then $\partial r^S/\partial s = r + s\frac{\partial r}{\partial s}$. Monotone average coalitional outlay is sufficient to ensure that $(r^{S\cup T})^m \geq (r^S)^m + (r^T)^m$. One can rewrite the average coalitional outlay as a function of r, i.e. $\overline{r} = s^{m-1}r^m$, and its derivative:

$$\frac{\partial\overline{r}}{\partial s} = s^{m-2}r^m[(m-1) + \frac{s}{r}\frac{\partial r}{\partial s}].$$

Let us first investigate the sign of $\frac{\partial r}{\partial s} = -\frac{\partial g(r)/\partial s}{\partial g(r)/\partial r}$ where

$$g(r) = m\frac{r^{-S}}{R}\frac{1}{s}f - rf'.$$

Then

$$\frac{\partial g(r)}{\partial s} = -m^2\frac{(sr)^{m-1}r^{-S}}{R^2}\frac{r}{s}f - m\frac{r^{-S}}{R}\frac{1}{s^2}f + m\frac{r^{-S}}{R}\frac{1-r}{s}f' - (1-r)rf''$$
$$= m\frac{r^{-S}}{R}\frac{1}{s^2}f[\frac{1-p^S}{r} - (m+1)] - (1-r)rf''$$

when $g(r) = 0$. And

$$g'(r) = -m^2\frac{r^{-S}(sr)^{m-1}}{R^2}f - m\frac{r^{-S}}{R}f' - f' + srf''.$$

But when $g(r) = 0$

$$m\frac{r^{-S}}{(sr)^m + r^{-S}}f' = m^2\frac{(r^{-S})^2}{[(sr)^m + r^{-S}]^2}\frac{1}{sr}f.$$

Then

$$\frac{\partial r}{\partial s} = \frac{m\frac{r^{-S}}{R}\frac{1}{s^2}f[\frac{1-p^S}{r} - (m+1)] - (1-r)rf''}{m\frac{(m+1)r^{-S}(sr)^{-1}}{R}f - srf''},$$

and it can be easily checked that $\frac{\partial r}{\partial s} > -\frac{r}{s}$. Then $\frac{\partial \bar{r}}{\partial s} > s^{m-2}r^m(m-2)$ and hence that is positive for any $m{\geq}2$. On the other side, concavity of the coalitional outlay implies that it is sub-additive.

$$\frac{\partial (r^S)^m}{\partial^2 s} = m(r^S)^{m-1}[\frac{m-1}{r^S}(\frac{\partial r^S}{\partial s})^2 + \frac{\partial^2 r^S}{\partial^2 s}].$$

For linear quadratic and exponential production functions second derivatives are:

$$\frac{\partial^2 r^S}{\partial^2 s} = \frac{2\frac{m^2(r^S)^2(r^{-S})^2}{R^2}[\theta(s-r^S) - (s-r^S)^2][\frac{mr^{-S}}{R} - (m+1)]}{[\frac{m(m+1)(r^S)^{-2}r^{-S}}{R}[\theta(s-r^S) - (s-r^S)^2] + 2]^2} \leq 0,$$

$$\frac{\partial^2 r^S}{\partial^2 s} = \frac{2\frac{m(r^S)^{-2}r^{-S}}{R}[\frac{mr^{-S}}{R} - (m+1)]\alpha(1-\alpha)(s-r^S)^{2\alpha-3}}{[\frac{m(m+1)(r^S)^{-2}r^{-S}}{R}(s-r^S)^\alpha + \alpha(1-\alpha)(s-r^S)^{\alpha-2}]^2} \leq 0.$$

Then, $m \leq 1$ is sufficient for concavity in size. ■

Proof of Proposition 8. Under a symmetric coalition structure all coalitions exert the same effort so $p^S = \frac{1}{k}$, where k is the number of coalitions in π. Therefore, the FOC can be rewritten as

$$m\frac{k-1}{k}(s - r^S) = \alpha r^S,$$

Hence the equilibrium level of coalitional effort and the payoff for all individuals in N are

$$\begin{aligned} r^S &= \frac{n}{k}\frac{m(k-1)}{m(k-1)+\alpha k}, \\ u_i^S &= n^{\alpha-1}(\frac{1}{k}\frac{\alpha k}{m(k-1)+\alpha k})^{\alpha}, \end{aligned}$$

and it is easy to see that $u_i^S < n^{\alpha-1} = u_i^N$.

Now, under the linear quadratic production technologies we compute the best case scenario for a coalition and show that the resulting payoff is lower than the one received under the grand coalition:

By the results on comparative statics we know that in symmetric coalition structures r^S is increasing in m and therefore payoffs are decreasing. So let us fix $m = 1$. Then the FOC states:

$$(k-1)(s-r^S-\theta(s-r^S)^2)-(kr^S-2\theta kr^S(s-r^S))=0,$$

and then the equilibrium level of effort as a function of the k and the parameters of the game is:

$$r^S(k) = \frac{2\theta n-k}{2k\theta} + \frac{k-2\theta n+\sqrt{\left(-4k\theta n-4k+4\theta^2n^2+1+4k^2\right)}}{2\theta\left(3k-1\right)}.$$

The next step is to obtain the $k \in [2, n]$ for which individual payoff is maximized. Given $p^S = \frac{1}{k}$, and $s = \frac{n}{k}$ this reduces to know when total level of labor is maximum. The derivative of $s - r^S$ w.r.t k is

$$\frac{\partial(s-r^S)}{\partial k} = -\frac{n}{k^2} - \frac{\partial r^S}{\partial k},$$

where

$$\frac{\partial r^S}{\partial k} = \frac{(6\theta n-1)\sqrt{\left(-4k\theta n-4k+4\theta^2n^2+1+4k^2\right)}+6k\theta n+2\theta n+2k-1-12\theta^2n^2}{2\sqrt{\left(-4k\theta n-4k+4\theta^2n^2+1+4k^2\right)}\theta(3k-1)^2}.$$

The latter expression only equals zero at $k = \frac{1}{3}$, so r^S is either increasing or decreasing in k. Now we evaluate the derivative at $k = 2$ in order to obtain its sign; It turns out that

$$\left.\frac{\partial r^S}{\partial k}\right|_{k=2} = \frac{(6\theta n-1)\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}+14\theta n+3-12\theta^2n^2}{50\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}\theta} > 0,$$

where the last inequality follows after some tedious algebra; the derivative is thus always positive and r^S is increasing in k. Finally $\frac{\partial(s-r^S)}{\partial k} < 0$, so the total level of labor is decreasing in k. Therefore, it remains to check that a symmetric two sided conflict is worse than universal peace

$$\begin{aligned} r^S(2) &= \frac{3\theta n - 3 + \sqrt{(-8\theta n + 9 + 4\theta^2 n^2)}}{10\theta}, \\ u_i^S &= \frac{4\theta n + 3 - 2\theta^2 n^2 + (\theta n - 1)\sqrt{-8\theta n + 9 + 4\theta^2 n^2}}{25 n\theta}. \end{aligned}$$

Next we compute the payoff under universal peace: For interior solution we need $2\theta n > 1$. Then

$$u_i^N = \frac{1}{4\theta n}.$$

If $2\theta n \leq 1$ we have a corner solution and

$$u_i^N = 1 - \theta n.$$

Now we show for each of these cases that this payoffs improve u_i^S:

(*i*) $2\theta n \leq 1$. The inequality $\frac{4\theta n+3-2\theta^2 n^2+(\theta n-1)\sqrt{(-8\theta n+9+4\theta^2 n^2)}}{25n\theta} < 1 - \theta n$ holds whenever θn is positive and smaller than 0.775, which is satisfied by assumption

(*ii*) $2\theta n > 1$. Simple algebra shows that the inequality $\frac{4\theta n+3-2\theta^2 n^2+(\theta n-1)\sqrt{(-8\theta n+9+4\theta^2 n^2)}}{25n\theta} < \frac{1}{4\theta n}$ holds for any value of θn.

Hence, $u_i^S < u_i^N$ in any symmetric coalition structure. ■

Proof of Proposition 11. As stated in the text, both characteristic functions coincide if and only if $\widehat{r^{-S}} = Min_{r^{-S}}\ u^*(r^{-S})$. Then, to prove that the indirect payoff function is decreasing in r^{-S} is a sufficient condition.

$$u^* = \frac{(r^S(r^{-S}))^m}{(r^S(r^{-S}))^m + r^{-S}}[f(s - r^S(r^{-S}))] \tag{17}$$

Let r' be the short hand notation of $\frac{\partial r(r^{-S})}{\partial r^{-S}}$. Then, by the envelope theorem

$$\begin{aligned} \frac{\partial u^*}{\partial r^{-S}} &= -\frac{(sr)^m}{R^2}f + sr'[m\frac{(sr)^{m-1}r^{-S}}{R^2}f - \frac{(sr)^m}{R}f'] \\ &= -\frac{(sr)^m}{R^2}f < 0. \end{aligned}$$

where the last equality follows from the fact that the terms in brackets is exactly the first order condition for the joint production problem. ■

Proof of Proposition 16. The first order conditions for the problem of a coalition of size s against $t(= n - s)$ individual players are

$$\begin{aligned} \frac{tr^1}{tr^1 + r^S}(s - r^S) &= \alpha r^S \\ \frac{(t-1)r^1 + r^S}{tr^1 + r^S}(1 - r^1) &= \alpha r^1 \end{aligned}$$

where r^1 is the effort exerted by each singleton. Now

$$\frac{tr^1}{r^S}(s - r^S) = \frac{(t-1)r^1 + r^S}{r^1}(1 - r^1),$$

yielding that

$$r^1 = r^S \frac{t - 1 - r^S + \sqrt{(t-1-r^S)^2 + 4(ts - r^S)}}{2\,(st - r^S)}.$$

Now we can rewrite the equilibrium winning probability for coalition S as a function of the equilibrium level of r^S :

$p^S(r^S) = \frac{2\left(ts-r^S\right)}{2(ts-r^S)+t(t-1-r^S)+t\sqrt{(t-1-r^S)^2+4(ts-r^S)}}$

Now we establish the bounds for this probability. When $\frac{2s}{2s+t-1+\sqrt{(t-1)^2+4ts}}$ $r^S = 0$

$$p^S(0) = \frac{2s}{s + n - 1 + \sqrt{n - s - 1)^2 + 4(n-s)s}}.$$

whereas when $r^S = s$, $p^S(s) = \frac{s}{n}$. It is easy to check that $p^S(s) > p^S(0)$. Finally,

$$\frac{\partial p^S}{\partial r^S} = \frac{2t(ts-t+1)(1+\frac{\frac{2(r-ts)}{(ts-t+1)}+t-r^S-1}{\sqrt{(n-s)(n-s-2-2r^S)+1-2r^S+(r^S)^2+4st}})}{\left(2ts-2r^S+t(t-1-r^S)+t\sqrt{t(t-2-2r^S)+1-2r^S+(r^S)^2+4st}\right)^2}.$$

Some algebra shows that this derivative does not equal zero in the interval $[0, s]$ and hence the winning probability in equilibrium will lie for sure in the interval $[p^S(0), p^S(s)]$. Therefore

$$u_i^S = p^S \frac{1}{s}(s - r^S)^\alpha \leq \frac{1}{n}(s - r^S)^\alpha < n^{\alpha - 1} = u_i^n.$$

So the grand coalition is γ stable.

For δ stability we follow the same procedure. First order conditions are

$$\frac{r^T}{r^T + r^S}(s - r^S) = \alpha r^S, \quad (18)$$

$$\frac{r^S}{r^T + r^S}(t - r^T) = \alpha r^T. \quad (19)$$

where r^T and t are the coalitional effort and cardinality respectively of the complement coalition of S. Then

$$\frac{r^T}{r^S}(s - r^S) = \frac{r^S}{r^T}(t - r^T)$$

and

$$r^T = r^S \frac{-r^S + \sqrt{(r^S)^2 + 4t(s - r^S)}}{2(s - r^S)}..$$

Again the equilibrium winning probability as a function of r^S is

$$p^S(r^S) = \frac{2(s - r^S)}{2s - 3r^S + \sqrt{(r^S)^2 + 4t(s - r^S)}}$$

Evaluated at the extremes $p^S(0) = \frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}$ and $p^S(s) = \frac{s}{n}$. It turns out that $p^S(0) \lesseqgtr p^S(s)$ if and only if $s \gtreqless \frac{n}{2}$.

Again, the derivative

$\frac{\partial p^S}{\partial r^S} = \frac{2}{\left(2s-3r^S+\sqrt{(r^S)^2+4t(s-r^S)}\right)^2}(s - \frac{2t(s-r^S)+r^S s}{\sqrt{(r^S)^2+4t(s-r^S)}})$

shows that $p^S(r^S)$ has no critical point in $(0, s)$. Therefore the equilibrium winning probability will lie in $[\frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}, \frac{s}{n}]$ if $s > \frac{n}{2}$ and in $[\frac{s}{n}, \frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}]$ otherwise. It immediately implies that no coalition greater or equal than half of the population will deviate. Now we show that this is not the case for small coalitions.

Manipulation of (18) shows that in equilibrium

$\alpha\frac{\left(r^S\right)^2}{s-(1+\alpha)r^S} = r^T = \frac{\sqrt{(1+\alpha)^2(r^S)^2+4\alpha t r^S}-(1+\alpha)r^S}{2\alpha}.$

Therefore we can be sure that in equilibrium $r^S < \frac{s}{1+\alpha}$. This, together with the fact that when $s < \frac{n}{2}$ $p^S(r^S)$ is decreasing in r^S implies that

$$u_i^S \geq p^S(\frac{s}{1+\alpha})(\frac{\alpha}{1+\alpha})^\alpha s^{\alpha-1} = \frac{2\alpha(\frac{\alpha}{1+\alpha})^\alpha s^{\alpha-1}}{2\alpha - 1 + \sqrt{1 + 4\alpha(1+\alpha)\frac{n-s}{s}}}$$

and it is easy to generate examples where the latter term is greater than $n^{\alpha-1}$, the individual payoff under the grand coalition. ■

Proof of Proposition 17. When $\alpha = 1$ the individual payoff is

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}(1 - \frac{r^S}{s}) < 1 = u_i^N.$$

So the first player in the protocol will announce $\{n\}$. When $\alpha = 0$ the game is exactly the one of Tan and Wang (2000) with n identical rivals. For the (lengthy) proof of the statement for $m \geq 2$ we refer the reader to their paper. We briefly describe where the condition stated comes from: Once one proves that at most two coalitions will form the first player in the protocol announces the coalition

$$s_1 = \arg\max_{1\leq s_1\leq n} \frac{s_1^{m-1}}{s_1^{m-1} + (n-s_1)^{m-1}}.$$

Condition (8) is the FOC of this problem. For the case of $m \in (1,2)$ notice that in that case the game is of negative externalities. In that case we only need to find a size s such that $\frac{s^{m-1}}{s^{m-1}+(n-s)^{m-1}} > \frac{1}{n} = u_i^n$ because to announce s for the first player in the protocol dominates n. If the $s+1$ player does not announce $n-s$ payoff is even higher.

Finally, as in a first-price-auction, when $m \to \infty$ in the Nash equilibrium given a π the all coalition except the biggest one will use their entire endowments in effort. The biggest one will invest $t+\varepsilon$ where t is the size of the second biggest coalition. Let us assume that in the case of ties the contest is won by the first coalition formed. Therefore, in the Bloch stable coalition structure s_1 must be the biggest coalition. However the player s_1+1-th player in th protocol is indifferent among all the announcements in $\{1, n-s_1\}$ because all yield a payoff of zero. In order to select only the coarsest stable coalition structure the player s_1+1 will announce $n-s_1$. Knowing this player 1 in the protocol will announce

$$s_1 = \arg\max_{s\in\{1,\ldots,n\}} \frac{(2s-n)^\alpha}{s} = \min\{\frac{1}{2}\frac{n}{1-\alpha}, n\}.$$

and the payoff will be

$$u_i^{s_1} = \begin{cases} 2\alpha^\alpha(1-\alpha)^{1-\alpha}n^{\alpha-1} & \text{if } \alpha < \frac{1}{2} \\ n^{\alpha-1} & \text{otherwise} \end{cases}$$

■

Proof of Proposition 18. In equilibrium the FOC states.

$$m(k-1)((s-r^S)(1-\omega) - \theta(s-r^S)^2) = kr^S(s(1-\omega) - (s+1)\theta(s-r^S)).$$

From here, some calculation yields the equilibrium level of effort

$$r^S = \frac{n}{2k} + \frac{(m-n-mk)k(1-\omega) + \theta nm(k-1)}{2\theta\,(mk+k-m+n)\,k}$$
$$\frac{\sqrt{(m-n-mk)^2k^2(1-\omega)^2 + (m(k-n)(k-1) - n(n+k))2\theta nk(1-\omega) + \theta^2 n^2(n+k)^2}}{2\theta\,(mk+k-m+n)\,k}$$

so one can get a (complicated) expression for u_i^S. If $\theta > \frac{1-\omega}{n+1}$ then the solution to the production problem of the grand coalition is interior and the individual payoff under free access is

$$u_i^N = \frac{(1-\omega)(1+n\omega)}{(n+1)^2\,\theta}.$$

Computations yield that

$$u_i^S(r^S) > u_i^N \Leftrightarrow m < \frac{(\theta n-k)(n+1)+kn(1-\omega)}{k(k-1)}\frac{n(2n\omega+\omega-n)+k(1+n\omega)}{(1+n\omega)(2n\omega+\omega-n)} = \widetilde{m}(n,k,\omega,\theta).$$

Moreover, given that $\widetilde{m}(n,k,\omega,\theta)$ is increasing in θ it is clear that one can derive a treshold $\widetilde{\theta}$ such that $m=\widetilde{m}$. Hence, the existence of an interior solution of the free access problem guarantees as well the Pareto dominance of conflict if $\frac{1-\omega}{n+1} \geq \widetilde{\theta}$. ■

Proof of Lemma19. Let r' be the short hand notation of $\frac{\partial r}{\partial r^{-S}}$ and $r(r^{-S})$ the best response strategy of the member of S. Then indirect payoff function and its derivative with respect to r^{-S} are

$u_i^*(r^{-S}) = \frac{(sr(r^{-S}))^m}{(sr(r^{-S}))^m+r^{-S}}[\frac{1}{s}f(s-sr(r^{-S}))-\omega(1-r(r^{-S}))]$.

$$\begin{aligned}\frac{\partial u_i^*(r^{-S})}{\partial r^{-S}} &= \frac{\partial u_i^*}{\partial r}r' + \frac{\partial u_i^*}{\partial r^{-S}} \\ &= sp^S[r'\frac{s-1}{1-r} - \frac{1}{R}][\frac{1}{s}f - \omega(1-r)]\end{aligned}$$

where:

$$\frac{\partial r(r^{-S}))}{\partial r^{-S}} = \frac{\frac{m(sr)^{2m-1}}{R^2}[\frac{1}{s}f(s-sr)-\omega(1-r)]}{\frac{mr^{-S}(sr)^{m-1}}{R}[\frac{m+1}{r}-\frac{s-1}{1-r}][\frac{1}{s}f-\omega(1-r)]-f''-(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})} > 0.$$

Then $\frac{\partial u_i^*}{\partial r^{-S}}$ has no clear sign because $r'>0$. After making some computations, one can check that the sign of $\frac{\partial u_i^*}{\partial r^{-S}}$ is given by the sign of

$$\frac{-\frac{mr^{-S}(sr)^{m-1}}{R^2}[\frac{m+1}{r}-\frac{s-1}{1-r}\frac{1}{1-p^S}]][\frac{1}{s}f-\omega(1-r)]+f''+(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})}{\frac{mr^{-S}(sr)^{m-1}}{R}[\frac{m+1}{r}-\frac{s-1}{1-r}][\frac{1}{s}f-\omega(1-r)]-f''-(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})}$$

Unfortunately we should restrict to conditions that ensure the negative sign of this derivative. It is sufficient to show that

$\frac{m+1}{r} - \frac{s-1}{1-r}\frac{1}{1-p^S} > 0$

Finally, first order condition allows us to rewrite this condition in terms of production and conflict technology only as stated in the text. ■