Macroeconomic Effects in Centralized and Decentralized Wage Setting Systems*

Valeri Sorolla[†]
Universitat Autònoma de Barcelona
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Abstract

We present a model of a monetary economy with two systems of wage setting: a decentralized system and a centralized system. In the decentralized system there is a union per firm that sets the firm's wage. In the centralized system there is a unique union that sets a common wage for all firms. We find that, when there is unemployment, the equilibrium wage set in the centralized wage setting system is lower than the one set in the decentralized wage setting and that both depend on the size of the unemployment benefit and on the degree of pro-workerism of the government. When the unemployment benefit is a given quantity, we find that monetary policy has real effects in both systems and implies nominal wage rigidity in the centralized system. On the contrary, when the unemployment benefit growths with the size of the economy, monetary policy has no real effects and implies real wage rigidity in both systems.

Key Words: Centralized and Decentralized Collective Bargaining, Trade Unions, Unemployment, Wages

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†Adress: Departament d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain. Tel: 3435812728, Fax: 3435812012, e-mail: valeri.sorolla@uab.es

Abstract

Presentamos un modelo de una economía monetaria con dos sistemas de fijación de salarios: un sistema descentralizado y uno centralizado. En el sistema descentralizado hay un sindicato en cada empresa que fija el salario de la misma. En el sistema centralizado existe un único sindicato que fija el mismo salario para todas las empresas. Encontramos que, cuando hay paro, el salario fijado en el sistema centralizado es menor que el fijado en el descentralizado y que los dos dependen del tamaño del subsidio de paro y de cuanto pro-trabajadores sea el gobierno. Cuando el subsidio de paro está dado, encontramos que la política monetaria tiene efectos reales en los dos sistemas y implica rigidez nominal en el sistema centralizado. Cuando el subsidio de paro crece con el tamaño de la economía, la política monetaria no tiene efectos reales y implica rigidez real en los dos sistemas.

1 Introduction

This paper analyzes the interaction between labor market and government institutions in a macroeconomic set up. We study the effect of left-wing governments and monetary policy, in decentralized and centralized wage setting systems, on wages, prices, employment and on the welfare of the agents. As Flanagan (1999) says in a recent essay: "The corporatist tradition rest on the assertion that interactions between labor market and government institutions influence macroeconomic performance, but the literature contains too little theory to provide a reliable guide to measurement". The model presented in this paper pretends to give an step in this direction.

More specifically we present a market economy with many identical firms where wages are set unilaterally by unions. We analyze two alternative systems of wage setting : a firm-level (decentralized) system and a national-level centralized) system. In the firm-level system, there is one union per firm and each union sets the firm wage. In the national level system, there is a national union which sets a common wage for all firms. The difference between the two systems is that the union in the national-level system takes into account that the gains from demanding higher wages are partly offset by higher prices, i.e., it takes into account the price externality (Calmfors, 1993). In the firm- level system trade unions take the price level as given, because the effect of their own wage demands on prices is negligible. In both systems the government taxes employed workers and owners and issues money in order to provide unemployment benefits to unemployed workers.

Sorolla (1995) analyzes the effect of a more pro-worker (left-wing) government on the price and unemployment levels in a centralized wage system. Sorolla (1993) studies if a centralized wage setting always implies wage restraint. Compared with these papers the model presented here endogenizes the determination of the unemployment benefit and introduces monetary policy. Calmfors and Driffill (1988) analyze how the wage set by unions changes as the wage setting system becomes more centralized. They argue that, under certain conditions, as the level of centralization increases, unions take into account that the wage set in one sector will affect the price of the product made in other sectors (take into account the price externality) and then the wage follows an inverse U form. They support their argument by a simulation. The main difference between our model and Calmfors and Driffill's model is that we only have two possible levels of wage setting: decentralized and centralized. On the other hand, our simpler general equilibrium model allows to given general results about the effect of the unemployment benefit, the degree of pro-workerism of the government and monetary policy and presents also welfare effects, questions not analyzed by Calmfors and Driffill. Danthine and Hunt (1994) extend Calmfors and Driffill's model to two countries. Layard et al. (1991, pp. 129-133) consider that negotiation al the centralized level internalizes the fiscal externality (Calmfors, 1993), i.e., that if unions increase the wage workers will pay more taxes. Holden and Raaum (1991) analyze the fiscal externality at the industry level.

Another characteristic of our model is that there is only one type of labor.

Calmfors (1993) surveys different models that study conditions under which there is wage restraint when different unions that represent different types of labor set the wage together. The literature on corporatism also investigates the relationship between wages and the degree of centralization of wage negotiations (see Flanagan (1999)). Using a differential game, De Andrés (1988) studies the effect on wages as the wage setting system becomes more centralized.

The paper is organized as follows. In section 2 we present the individual agents and we compute the price function. In section 3 we solve the program for the firm-level union and the national-level union. In section 4 we compare the reply functions, we complete the model by introducing the government and we analyze the existence of equilibria. In section 5 we compare the equilibria and we analyze the welfare effects. We conclude with a summary of the main results in section 6. All proofs are contained in the appendix.

2 The Individual Agents and the Price Function

The economy has three goods: a good y produced by labor l and money M that we use as numeraire. The individual agents are m identical firms and $n+1$ consumers. The jth firm produces output according to the production function $f(l_j) = Al_j^a$ $(0 < a < 1)$ where A represents the productivity level and l_j is the amount of labor employed by this firm. The first n consumers are workers: each of them is endowed with one unit of labor and zero units of money. The $(n+1)$-th consumer is the owner of all firms: he receives the profits of all firms and he owns neither labor nor money. We assume that all individual agents take prices and wages as given. Given p (the price of the produced good) and w_j (the wage at the jth firm) the jth firm chooses l_j in order to maximize $pAl_j^a - w_j l_j$. The solution of this program gives the labor demand function of the jth firm, $l^d(\frac{w_j}{p}) = (\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}}$, and the profit function, $\pi(w_j,p) = (1-a)Ap(\frac{1}{aA}\frac{w_j}{p})^{\frac{a}{a-1}}$.

We assume that all consumers have the same Cobb-Douglas utility function $U(y_i, \frac{M_i}{p}) = (y_i)^b(\frac{M_i}{p})^{1-b}$ [1], $(0 < b < 1)$ where y_i and M_i are the quantity of the produced good and money "consumed" by individual i. Labor is not argument in the utility function and we assume that all workers supply their unit of labor inellastically, that is, total labor supply is fixed at n. Consumer i's optimization program is: given p and his income I_i choose y_i and M_i in order to maximize $(y_i)^b(\frac{M_i}{p})^{1-b}$ subject to: $py_i + M_i = I_i$. The solution to this program gives the output demand function of the ith consumer $y^d(p, I_i) = \frac{bI_i}{p}$ and the demand of money function $M^d(p, I_i) = (1-b)I_i$. The indirect utility function $U(y^d(p, I_i), M^d(p, I_i))$ is denoted by $V(p, I_i)$ and can be computed as $b^b(1-b)^{1-b}\frac{I_i}{p}$. We define the indirect utility function of one unit of income as $\hat{V}(p) = V(p, 1)$ which can be computed as $b^b(1-b)^{1-b}\frac{1}{p}$. We write $V(p, I_i) =$

[1] In this paper the interpretation of M is money, in the three-commodity framework used by many authors, for example Hart (1982), M is interpreted as a non-produced good as "gold" which implies to use $(y_i)^b(M_i)^{1-b}$ instead of $(y_i)^b(\frac{M_i}{p})^{1-b}$ as the utility function. See also Hart (1982) for different interpretations of M.

$\hat{V}(p)I_i$ because U is an homothetic function.

We assume that if given w_1, ..., w_f and p there is unemployment then some workers work full time and the others remain unemployed. Each unemployed worker receives an unemployment benefit s.

There is a central bank that supplies $\bar{M}$ units of money, this amount is given to the government who, given the amount of unemployment, pays the unemployment benefit and taxes the income of the employed workers at the tax rate t_w and the income of the owner of the firms at the tax rate t_c. We assume that the government balances its budget. Thus the after tax income of a worker employed in the jth firm is $(1-t_w)w_j$ and the after tax income of the owner of the firms is $(1-t_c)\sum_{j=1}^{m}\pi(w_j,p)$.

We assume perfect competition in the output market, i.e., the price of the produced good balances demand and supply. Formally, given $(w_1,...,w_f,t_w,t_c,s)$ the price p balances demand and supply in the output market if the following equation holds:

$$\sum_{j=1}^{m} A(l^d(\frac{w_j}{p}))^a = \tag{1}$$

$$\sum_{j=1}^{m} l^d(\frac{w_j}{p})y^d(p,(1-t_w)w_j) + y^d(p,s)(n-\sum_{j=1}^{m} l^d(\frac{w_j}{p})) + y^d(p,(1-t_c)\sum_{j=1}^{m}\pi(w_j,p))$$

It is easy to prove (see appendix) that if the government has a balanced budget, then the price p clears the output market[2], when there is no excess demand of labor, if and only if:

$$p = \hat{p}(w_1,...,w_f) = \frac{1}{a^a A}(\frac{b}{1-b}\bar{M})^{1-a}(\sum_{j=1}^{m} w_j^{(\frac{a}{a-1})})^{a-1}. \tag{2}$$

Of course, given $(w_1,...,w_f)$ we need to check that there is no excess demand of labor, i.e., $\sum_{j=1}^{m} l^d(\frac{w_j}{\hat{p}(w_1,...,w_f)}) \leqq n$.

It is easy see, using equation (2) that if a particular wage w_j increases, then p increases.

As we said in the introduction, we consider two alternative systems of wage setting: a firm-level system and a national-level system. In the firm-level system, there is one union per firm and each union sets the firm's wage. Each union assumes that the wage that it sets does not affect the price of the produced good. This assumption is justified because, when m is large, we can check, using equation (2), that the influence of a particular w_j on p becomes negligible.

In the national-level system, there is a union which sets a common wage, denoted by w, for all firms. In this case a change in this common wage affects the price. The national union takes this effect into account. Formally, given (w,t_w,t_c,s), when there is no excess demand of labor the price p satisfies:

[2] If the government has a balanced budget constraint it is easy to prove, using Walras' Law, that if the output market is in equilibrium the money market is also in equilibrium.

$$m(l^d(\frac{w}{p}))^a = \tag{3}$$
$$ml^d(\frac{w}{p})y^d(p,(1-t_w)w) + y^d(p,s)(n - ml^d(\frac{w}{p})) + y^d(p,(1-t_c)m\pi(w,p)),$$

$$ml^d(\frac{w}{p}) \leqq n. \tag{4}$$

Proposition 1 *Given $w \geqq w_c$ equations (3) and (4) are satisfied if and only if $p = \tilde{p}(w) = \frac{1}{A}(\frac{b\bar{M}}{(1-b)m})^{1-a}(\frac{w}{a})^a$ where w_c is the competitive (market clearing) wage.*

We refer to $\tilde{p}(w)$ as the price function. Note that $\tilde{p}(w)$ does not depend on t_w, t_c and s. This result comes from assuming identical, homothetic individual utility functions and the government's balanced budget constraint. Together they imply that the price only depends on aggregate income which, in turn, only depends on the wage. Note that the price function is defined for wages greater than or equal to the competitive wage. The competitive wage is computed by solving the equation $ml^d(\frac{w}{\tilde{p}(w)}) = n$. If the wage is less that the competitive wage then there is an excess demand of labor but we do not consider this situation because it is easy to see that a national union would never find in its interest to set a wage lower than the competitive wage.

Now we define the eventual labor demand function of one firm, $\tilde{l}(w)$, as the level of employment determined by the wage when (3) and (4) are taken into account, i.e., $\tilde{l}(w) = l^d(\frac{w}{\tilde{p}(w)})$, the eventual profit function for one firm: $\tilde{\pi}(w) = \tilde{p}(w)A\tilde{l}(w)^a - w\ \tilde{l}(w)$, the aggregate eventual labor demand function: $\tilde{L}(w) = m\ \tilde{l}(w)$ and the aggregate eventual profit function $\tilde{\Pi}(w) = m\tilde{\pi}(w)$. The computations are given by the following proposition:

Proposition 2 $\tilde{l}(w) = \frac{b\bar{M}}{(1-b)m}\frac{a}{w}$, $\tilde{L}(w) = \frac{b\bar{M}}{(1-b)}\frac{a}{w}$, $\tilde{\pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)m}$, $\tilde{\Pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)}$.

Observe that the aggregate wage bill, $w\tilde{L}(w)$, and profits, $\tilde{\Pi}(w)$, are constant with respect to w. These are particular characteristics of the model and they are due to the assumption of Cobb-Douglas production and utility functions. Observe also that $\tilde{L}(w)$ and $\tilde{\Pi}(w)$ do not depend on t_w and t_c. This always true as long as everybody has the same homothetic utility function and means that the government can not affect the price level and the employment level via fiscal policy. Note also that an increase in $\bar{M}$ implies a higher price level and less unemployment. Finally, an increase in A implies a lower price but the same labor demand. This is because the decrease in the real wage offsets the increase in labor demand due to the increase in labor productivity.

3 Wage Setting in the Firm-Level System and in the National-Level System

In the firm-level system the program of the jth union is the following:

Program T_f: Given p, t_w and s choose w_j in order to maximize:

$T_f(w_j) = \frac{l^d\left(\frac{w_j}{p}\right)}{\frac{n}{m}}V((1-t_w)w_j, p) + \frac{\frac{n}{m} - l^d\left(\frac{w_j}{p}\right)}{\frac{n}{m}}V(s,p)$ subject to: $l^d\left(\frac{w_j}{p}\right) \leqq \frac{n}{m}$.

Program T_f is motivated by the assumption that workers are a priori assigned to unions and that a given union only cares about the welfare of workers affiliated to it. We assume that the n workers are symmetrically distributed between unions which implies that $\frac{n}{m}$ workers are affiliated to a given union.

Recall that, when the wage is set at the firm-level, the wage set in the jth firm does no affect the price level. The jth union sets the wage in order to maximize the expected utility of a worker affiliated to the jth union, that is, the probability of a worker affiliated to the jth union being employed in the jth firm times the utility of an employed worker plus the probability of a worker affiliated to the jth union not being employed in the jth firm times the utility of an unemployed worker. The jth union takes p, t_w and s as given. Program T_f generates the best reply function of the jth union.

Proposition 3 *For all $p > 0$, $s \geqq 0$ and $t_w < 1$ Program T_f has a unique solution denoted by $\hat{w}_f(p, t_w, s)$ which is a continuous function.*

We can check from the proof of Proposition 3 that $\hat{w}_f$ is increasing in t_w and s .

We assume that the national union chooses the wage in order to maximize the expected utility of a worker, that is, the probability of being employed in any firm times the utility of an employed worker plus the probability of being unemployed times the utility of an unemployed worker. Recall that, when the wage is set at the national level, the wage affects the price. The program of the national union is the following:

Program T_u: Given t_w and s choose w in order to maximize:

$T_u(w) = \frac{\tilde{L}(w)}{n}V((1-t_w)w, \tilde{p}(w)) + \frac{n-\tilde{L}(w)}{n}V(s, \tilde{p}(w))$.

The solution to Program T_u generates the best reply function of the national union.

Proposition 4 *For all $s \geqq 0$ and $t_w < 1$ Program T_u has a unique solution denoted by $\tilde{w}_u(t_w \mid s)$ which is a continuous function and satisfies $\tilde{w}_u(t_w \mid s) \geqq w_c$.*

We can check from the proof of Proposition 4 that $\tilde{w}_u$ is increasing in t_w and s.

4 Equilibrium

First note that at the firm level all unions set the same wage because they have an identical program. This wage is given by the function $\hat{w}_f(p, t_w, s)$ derived in Proposition 3. On the other hand, the price depends on this common wage according to the function $\tilde{p}(w)$. The following proposition shows that the equation $w = \hat{w}_f(\tilde{p}(w), t_w, s)$ has a unique solution:

Proposition 5 *For all $s \geqq 0$ and $t_w < 1$ a solution to the equation $w = \hat{w}_f(\tilde{p}(w), t_w, s)$ exists and is unique. This solution is denoted by $\tilde{w}_f(t_w \mid s)$ which is a continuous function and satisfies $\tilde{w}_f(t_w \mid s) \geqq w_c$.*

The function $\tilde{w}_f(t_w \mid s)$ indicates for each t_w and s the wage set by any union at the firm level and it also incorporates the equilibrium condition in the output market. The following proposition compares $\tilde{w}_f(t_w \mid s)$ and $\tilde{w}_u(t_w \mid s)$ for a given t_w and s .

Proposition 6 *If $s > (1 - t_w)a^2 \frac{b}{1-b} \frac{\bar{M}}{m}$, then $\tilde{w}_f(t_w \mid s) > \tilde{w}_u(t_w \mid s) > w_c$. If $s \leqq (1 - t_w)a^2 \frac{b}{1-b} \frac{\bar{M}}{m}$, then $\tilde{w}_f(t_w \mid s) = \tilde{w}_u(t_w \mid s) = w_c$.*

Proposition 6 says that for a given tax rate, if the unemployment benefit is large enough, then the wage set in the national-level system is lower that the wage set by in the firm-level system, i.e., there is wage moderation at the national level. In other words, internalizing the price externality implies wage moderation as long as there is unemployment. This result seems to confirm the intuition that the internalization of the price externality always implies wage moderation, but this intuition is not true. In Sorolla (1993), where the model follows the three-commodity framework, there are situations where the internalization of the price effect implies a wage at the national level higher that the one set at the firm level. The explanation is the following: First, the only difference between programs T_f and T_u is that the second one internalizes the price effect. This point becomes transparent rewriting the utility function of the jth union as: $\frac{ml^d(\frac{w_j}{p})}{n} V((1 - t_w)w_j, p) + \frac{n - ml^d(\frac{w_j}{p})}{n} V(s, p)$ and the utility function of the national union as: $\frac{ml^d(\frac{w}{\tilde{p}(w)})}{n} V((1-t_w)w, \tilde{p}(w)) + \frac{n-ml^d(\frac{w}{\tilde{p}(w)})}{n} V(s, \tilde{p}(w))$. Second the increase of the price via wage has two effects: a positive one and a negative one. The positive effect is due to the increase in the labor demand, which induces an increase of the welfare of the workers who change from unemployed to employed. The negative effect is due to the decrease of the welfare of employed and unemployed workers because of the increase of the price of the produced good. Finally, if the negative effect offsets the positive one the wage set at the national level must decrease. It is easy to see that this happens if either t_w or s are high because, on the one hand, the increase of the workers who become employed is small. On the other hand, a high s implies a high negative effect due to the decrease of the welfare of the unemployed workers. In the case studied in this paper it turns out that for a high enough s $(s > (1 - t_w)a^2 \frac{b}{1-b} \frac{\bar{M}}{m})$ we have wage moderation at the national level and if s is below this quantity then we have full employment in both wage setting systems.

The problem of comparing $\tilde{w}_f(t_w \mid s)$ and $\tilde{w}_u(t_w \mid s)$ for a given tax rate on employed workers and the unemployment benefit is that, if wages set by the firm and national unions are different, then the level of unemployment is different and the government may set different taxes and unemployment benefits. Thus we have to complete the model by introducing the government.

Given the unemployment benefit and the money supply of the central bank the government chooses the tax rates in order to maximize a Cobb-Douglas utility function with the expected utility of the worker to the β power and the utility of the owner of the firms to the $1-\beta$ power. The parameter β reflects the degree of pro-workerism of the government and we assume that $0 < \beta < 1$. Recall that the government must balance its budget. We also assume that the government takes w as given, this implies that the government has only one degree of freedom when deciding t_w and t_c. We choose t_w as the decision variable of the government because it appears directly in the utility function of the union. Clearly, once t_w is selected then t_c is determined according to the function $t_c = \tilde{t}_c(w, t_w, s) = \frac{s(n-\tilde{L}(w))-t_w w\tilde{L}(w)-\bar{M}}{\tilde{\Pi}(w)}$. The government solves the following program:

Program G: Given w, s, $\bar{M}$ and β choose t_w in order to maximize:

$G(t_w) = (T_u(w))^{\beta}(V((1-\tilde{t}_c(.))\tilde{\Pi}(w), \tilde{p}(w)))^{1-\beta}$ subject to $t_w \leqq 1$, $\tilde{t}_c(.) \leqq 1$, and $(1-t_w)w \geqq s$.

The restriction $(1-t_w)w \geqq s$ implies that the government, for a given wage, will never tax an employed worker at a tax rate that makes him worse off than an unemployed worker. Note that the program of the government is the same whether wages are set at the firm or national level. This is because, when the wage is set at the firm level we consider the case where the wage set by every union is the same which is the relevant one.

The solution to program G generates the best reply function of the government.

Proposition 7 *Program G has a solution for all* $w \geqq w_c$ *if and only if* $s \leqq \frac{\bar{M}}{(1-b)n}$. *The solution is unique, we denote it by* $\tilde{t}_w(w \mid s, \bar{M}, \beta)$ *which is a continuous function.*

If $s > \frac{\bar{M}}{(1-b)n}$ then there is no solution to the program of the government because the feasible set is empty. From the proof of Proposition 7 we also obtain that if $s < \beta\frac{\bar{M}}{(1-b)n}$ then $\frac{\partial \tilde{t}_w}{\partial w} > 0$, $\frac{\partial \tilde{t}_w}{\partial s} > 0$ and $\frac{\partial \tilde{t}_w}{\partial \beta} < 0$ and the tax rate on the owner does not depend on w and s. This last result means, in particular, that for any wage set by the union and, in consequence, for any level of unemployment, the owner is always taxed at the same tax rate.

Now we define an equilibrium with wage setting at the firm level as the pair $(w_f^*, t_{w,f}^*)$ satisfying $w_f^* = \tilde{w}_f(t_{w,f}^* \mid s)$, $t_{w,f}^* = \tilde{t}_w(w_f^* \mid s, \bar{M}, \beta)$ and an equilibrium with wage setting at the national level as the pair $(w_u^*, t_{w,u}^*)$ satisfying $w_u^* = \tilde{w}_u(t_{w,u}^* \mid s)$, $t_{w,u}^* = \tilde{t}_w(w_u^* \mid s, \bar{M}, \beta)$.

Theorem 8 *An equilibrium with wage setting at the firm level and an equilibrium with wage setting at the national level always exist and they are unique.*

Given s, $\bar{M}$ and β we write $(\hat{w}_f(s,\bar{M},\beta), \hat{t}_{w,f}(s,\bar{M},\beta))$ and $(\hat{w}_u(s,\bar{M},\beta), \hat{t}_{w,u}(s,\bar{M},\beta))$ for the unique firm-level and national-level equilibrium.

5 Comparative Statics

Now we compare under which conditions the wage at the firm-level equilibrium is greater than the wage at the national-level equilibrium or, in other words, when there is wage restraint at the national level.

Theorem 9 *If* $s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n}$ *then* $\hat{w}_f(s,\bar{M},\beta) = \hat{w}_u(s,\bar{M},\beta) = w_c$. *If* $a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ *then* $\hat{w}_f(s,\bar{M},\beta) > \hat{w}_u(s,\bar{M},\beta) > w_c$.

Theorem 9 says that given s, $\bar{M}$ and β if s is high enough with respect to $\bar{M}$ and β then there is wage restraint, otherwise there is full employment. As noted above this is a characteristic of this model which does not mean that in general the internalization of the price externality always implies wage moderation. Now we compare the welfare of the individual and collective agents under the two wage setting systems. The key observation is that, as it is easy to see from the expressions of $T_f(w_j)$ and $T_u(w)$, if the wage set at the firm level is the same for all firms, then the expected utility of a worker affiliated to the jth union is equal to the expected utility of a worker given this wage. Consequently we talk in general about the expected utility of a worker for a given wage.

Theorem 10 *For a given s, $\bar{M}$ and β the lower the nominal wage, the higher the expected utility of a worker and the utility of the government. It follows that if the national-level system has a lower equilibrium wage, then the expected utility of a worker and the utility of the government are higher in the national-level system than in the firm-level system.*

Theorem 10 says that it is not true that in general a centralized wage setting system, where the price externality is taken into account, implies a higher expected utility of a worker, this only happens if the centralized wage setting system implies wage restraint. The intuition is that it is the interaction between unions and governments what determines the equilibrium wage and taxes. If the equilibrium wage is lower, then the tax rate on an employed worker is also lower, the unemployment level is lower and, finally, the price is lower. All these effects taken together imply that the expected utility of a worker is higher. On the other hand, the owner of the firm is better off with a lower nominal wage. This is because, as we have seen, profits and the owner's tax rate are constant with respect to w but the price level is lower with a lower nominal wage. The unemployed workers are also better off with a lower nominal wage because the unemployment benefit remains constant but the price level is lower. Finally two effects affect the welfare of an employed worker: first, it is immediate to check that, the utility of an employed worker is increasing in w but, second, a higher nominal wage implies more unemployment and a higher tax rate on an employed worker which means that we can not say in general in which wage setting system an employed worker is better off.

Now we present the effect of a change in s, $\bar{M}$, β and A on the equilibrium of the economy when there is unemployment. The sign of the partial derivatives is given by the following table:

$$\begin{array}{lllll}
\frac{\partial \hat{w}_f}{\partial s} > 0 & \frac{\partial \tilde{p}(\hat{w}_f)}{\partial s} > 0 & \frac{\partial \tilde{L}(\hat{w}_f)}{\partial s} < 0 & \frac{\partial \hat{t}_{w,f}}{\partial s} > 0 & \frac{\partial \tilde{t}_c}{\partial s} = 0 \\
\frac{\partial \hat{w}_f}{\partial \beta} < 0 & \frac{\partial \tilde{p}(\hat{w}_f)}{\partial \beta} < 0 & \frac{\partial \tilde{L}(\hat{w}_f)}{\partial \beta} > 0 & \frac{\partial \hat{t}_{w,f}}{\partial \beta} < 0 & \frac{\partial \tilde{t}_c}{\partial \beta} > 0 \\
\frac{\partial \hat{w}_f}{\partial \bar{M}} < 0 & \frac{\partial \tilde{p}(\hat{w}_f)}{\partial \bar{M}} > 0 & \frac{\partial \tilde{L}(\hat{w}_f)}{\partial \bar{M}} > 0 & \frac{\partial \hat{t}_{w,f}}{\partial \bar{M}} < 0 & \frac{\partial \tilde{t}_c}{\partial \bar{M}} = 0 \\
\frac{\partial \hat{w}_f}{\partial A} = 0 & \frac{\partial \tilde{p}(\hat{w}_f)}{\partial A} < 0 & \frac{\partial \tilde{L}(\hat{w}_f)}{\partial A} = 0 & \frac{\partial \hat{t}_{w,f}}{\partial A} = 0 & \frac{\partial \tilde{t}_c}{\partial A} = 0
\end{array}$$

$$\begin{array}{lllll}
\frac{\partial \hat{w}_u}{\partial s} > 0 & \frac{\partial \tilde{p}(\hat{w}_u)}{\partial s} > 0 & \frac{\partial \tilde{L}(\hat{w}_u)}{\partial s} < 0 & \frac{\partial \hat{t}_{w,u}}{\partial s} > 0 & \frac{\partial \tilde{t}_c}{\partial s} = 0 \\
\frac{\partial \hat{w}_u}{\partial \beta} < 0 & \frac{\partial \tilde{p}(\hat{w}_u)}{\partial \beta} < 0 & \frac{\partial \tilde{L}(\hat{w}_u)}{\partial \beta} > 0 & \frac{\partial \hat{t}_{w,u}}{\partial \beta} < 0 & \frac{\partial \tilde{t}_c}{\partial \beta} < 0 \\
\frac{\partial \hat{w}_u}{\partial \bar{M}} = 0 & \frac{\partial \tilde{p}(\hat{w}_u)}{\partial \bar{M}} > 0 & \frac{\partial \tilde{L}(\hat{w}_u)}{\partial \bar{M}} > 0 & \frac{\partial \hat{t}_{w,u}}{\partial \bar{M}} < 0 & \frac{\partial \tilde{t}_c}{\partial \bar{M}} = 0 \\
\frac{\partial \hat{w}_u}{\partial A} = 0 & \frac{\partial \tilde{p}(\hat{w}_u)}{\partial A} < 0 & \frac{\partial \tilde{L}(\hat{w}_u)}{\partial A} = 0 & \frac{\partial \hat{t}_{w,u}}{\partial A} = 0 & \frac{\partial \tilde{t}_c}{\partial A} = 0
\end{array}$$

Table 1

Note that the effects of a change on s, β and A are the same in the firm and national wage setting systems the only difference being that in the national system the wage is lower and then we have lower prices and less unemployment. Note also that monetary policy has real effects in both the firm-level system and the national-level system but in the firm-level system an expansive monetary policy implies a decrease in the nominal wage but a nominal rigidity in the national-level system. The basic explanation for the real effect of money in this model is that an increase in $\bar{M}$ implies a decrease in the workers tax rate and then unions react. In the firm-level system unions decrease the nominal wage. In the national-level system it is known that the decrease in the nominal wage will imply a decrease in prices that is a decrease in labor demand. In this case the negative effect of taking into account the price effect is the decrease of welfare from people who move from employment to unemployment and the positive one the increase of welfare of both employed and unemployed due to the reduction of the price. It turns out that one effect equals the other and then the national union maintains the nominal wage, that is, there is nominal wage rigidity due to monetary policy. This means that this model derives, instead of assuming, nominal wage rigidity as the optimal behavior of the national union.

Note that in both wage setting systems the higher the degree of pro-workerism of the government or the lower the unemployment benefit the lower the wage. This explains why a country with a firm-level wage setting with a more pro-worker government or a lower unemployment benefit may present a wage lower than the wage of another country with a national-level wage setting but a less pro-worker government or a higher unemployment benefit. In other words, the degree of centralization is only one factor that may explain wage restraint, but the amount of the unemployment benefit and the degree of pro-workerism of the government are factors that also are needed to be taken into account.

Note finally that an increase in A, a positive productivity shock, do not affect the nominal wage and employment and the price level decreases, that is, the real wage moves procyclically and employment remains constant. This result contrast with the one obtained by McDonald and Solow (1981) where an increase in A maintains the real wage constant but employment increases. The difference between both results is due to the fact that in McDonald and Solow's model the union sets the real wage, whereas in our model it sets the nominal wage. Solving the program of the union they obtain that the real wage set by the union does not depend on the productivity shock. Consequently, because labor demand does depend on the productivity shock, a higher A implies the same real wage and a higher level of employment. In our model, we also obtain that the nominal wage set by the union does not depend on A, but now both labor demand and the price function do depend on A, in such a way that an increase in A leaves employment unaffected.

So far the unemployment benefit has been considered as an exogenous variable without specifying how it is determined as in Sorolla (1995). There is not a clear way of making the unemployment benefit endogenous and there is a variety of assumptions about the size of the unemployment benefit in the literature. In models of growth with unemployment Pissarides (1990) assumes $s = vw$ and Tabellini and Daveri (1997) $s = vF(.)$. Nevertheless in these models there is no money which means that s is determined in real terms. A reasonable assumption for our model is to make s a proportion v of the nominal gross national product, i.e., $s = v(\tilde{p}(w)A\tilde{l}(w)^a + \bar{M}) = v\frac{\bar{M}}{(1-b)}$ which means that the unemployment benefit growths with the size of the economy. Under this assumption the effect of a change in v, $\bar{M}$, β and A on the equilibrium of the economy when there is unemployment is given by the following table:

$\frac{\partial \hat{w}_f}{\partial v} > 0$	$\frac{\partial \tilde{p}(\hat{w}_f)}{\partial v} > 0$	$\frac{\partial \tilde{L}(\hat{w}_f)}{\partial v} < 0$	$\frac{\partial \hat{t}_{w,f}}{\partial v} > 0$	$\frac{\partial \tilde{t}_c}{\partial v} = 0$
$\frac{\partial \hat{w}_f}{\partial \beta} < 0$	$\frac{\partial \tilde{p}(\hat{w}_f)}{\partial \beta} < 0$	$\frac{\partial \tilde{L}(\hat{w}_f)}{\partial \beta} > 0$	$\frac{\partial \hat{t}_{w,f}}{\partial \beta} < 0$	$\frac{\partial \tilde{t}_c}{\partial \beta} > 0$
$\frac{\partial \hat{w}_f}{\partial \bar{M}} > 0$	$\frac{\partial \tilde{p}(\hat{w}_f)}{\partial \bar{M}} > 0$	$\frac{\partial \tilde{L}(\hat{w}_f)}{\partial \bar{M}} = 0$	$\frac{\partial \hat{t}_{w,f}}{\partial \bar{M}} = 0$	$\frac{\partial \tilde{t}_c}{\partial \bar{M}} = 0$
$\frac{\partial \hat{w}_f}{\partial A} = 0$	$\frac{\partial \tilde{p}(\hat{w}_f)}{\partial A} < 0$	$\frac{\partial \tilde{L}(\hat{w}_f)}{\partial A} = 0$	$\frac{\partial \hat{t}_{w,f}}{\partial A} = 0$	$\frac{\partial \tilde{t}_c}{\partial A} = 0$
$\frac{\partial \hat{w}_u}{\partial v} > 0$	$\frac{\partial \tilde{p}(\hat{w}_u)}{\partial v} > 0$	$\frac{\partial \tilde{L}(\hat{w}_u)}{\partial v} < 0$	$\frac{\partial \hat{t}_{w,u}}{\partial v} > 0$	$\frac{\partial \tilde{t}_c}{\partial v} = 0$
$\frac{\partial \hat{w}_u}{\partial \beta} < 0$	$\frac{\partial \tilde{p}(\hat{w}_u)}{\partial \beta} < 0$	$\frac{\partial \tilde{L}(\hat{w}_u)}{\partial \beta} > 0$	$\frac{\partial \hat{t}_{w,u}}{\partial \beta} < 0$	$\frac{\partial \tilde{t}_c}{\partial \beta} > 0$
$\frac{\partial \hat{w}_u}{\partial \bar{M}} > 0$	$\frac{\partial \tilde{p}(\hat{w}_u)}{\partial \bar{M}} > 0$	$\frac{\partial \tilde{L}(\hat{w}_u)}{\partial \bar{M}} = 0$	$\frac{\partial \hat{t}_{w,u}}{\partial \bar{M}} = 0$	$\frac{\partial \tilde{t}_c}{\partial \bar{M}} = 0$
$\frac{\partial \hat{w}_u}{\partial A} = 0$	$\frac{\partial \tilde{p}(\hat{w}_u)}{\partial A} < 0$	$\frac{\partial \tilde{L}(\hat{w}_u)}{\partial A} = 0$	$\frac{\partial \hat{t}_{w,u}}{\partial A} = 0$	$\frac{\partial \tilde{t}_c}{\partial A} = 0$

Table 2

The effects of changes in v , β and A are the same than the ones obtained with s given. The interesting result in this case is that monetary policy has no effects on employment. In this case an expansive monetary policy implies a decrease in the workers that rate but an increase in the unemployment benefit which produces a nominal wage increase in both systems in such a way that lives employment unaffected. This increase in the nominal wage leaves the real wage unaffected, that is there is real wage rigidity, and then labor demand do not change.

There is another way of endogenizing the unemployment benefit that is giving to the government the power of setting s in its program. In this case if the utility function of the government is the one used in Program G then there is an indeterminacy in the solution, because of the linearity of the expected utility of the worker between s and t_w . It turns out that the government sets t_c but is indifferent between any s and t_w that satisfies the government budget constraint. A reasonable way of eliminating this indeterminacy is changing the utility function of the government to $(\tilde{L}(w)V((1-t_w)w,\tilde{p}(w)))^{\alpha\beta}((n-\tilde{L}(w))V(s,\tilde{p}(w)))^{(1-\alpha)\beta}(V((1-\tilde{t}_c(.))\tilde{\Pi}(w),\tilde{p}(w)))^{(1-\beta)}$ where α is the weight that the government gives to the welfare of the employed workers, and $1-\alpha$ is the weight that the government gives to the welfare of the unemployed workers. In this case we also obtain that monetary policy has not real effects. This case is not presented in detail because the results are similar to the case when $s = v(\tilde{p}(w)\tilde{l}(w)^a + \bar{M})$ and we think that last case is more plausible.

6 Main Results

In this paper we present a model of a monetary economy with two systems of wage setting: a decentralized (firm-level) system and a centralized (national-level) system. In the firm-level system there is a union per firm that sets the firm's wage considering that the wage do not affect the price level. In the national-level system there is a unique union that sets a common wage for all firms and considers that changes in this common wage affect the price level. In both systems the government taxes owners and workers for covering the unemployment benefit which is also covered by monetary policy.

We find that, when there is unemployment, the equilibrium wage set in the national-level wage setting system is lower than the one set in the firm-level wage setting system, we also explain why this result may change using different models. We also find, when the unemployment benefit is a given quantity, that the equilibrium wage depends on the size of the unemployment benefit and on the degree of pro-workerism of the government which explains why countries with a firm-level wage setting system but with a more pro-worker government or a higher unemployment benefit may have lower wages than other countries with a national-level wage setting system and a less pro-worker government or a lower unemployment benefit.

When the unemployment benefit is a given quantity we find that monetary policy has real effects in both wage setting systems and implies nominal wage rigidity in the national-level system. This means that this model derives, instead of assuming, nominal wage rigidity as the optimal behavior of the national union.

An increase in productivity implies nominal wage rigidity in both systems and no effects on employment.

When the unemployment benefit growths with the size of the economy then the results are reversed, monetary policy has no real effects and implies real wage rigidity. This means that the way in which the size of the unemployment benefit is decided is really important for the real effects of government's monetary policy.

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8 Sketched Proofs

Proof. of Equation (2). Substituting $l^d(\frac{wj}{p}) = (\frac{1}{aA}\frac{wj}{p})^{\frac{1}{a-1}}$, $\pi(w_j, p) = (1-a)Ap(\frac{1}{a}\frac{w_i}{p})^{\frac{a}{a-1}}$ and $y^d(p, I_i) = \frac{bI_i}{p}$ in equation (1) and solving for p we obtain: $p = \frac{1}{a^a A}(\sum_{j=1}^m (w_j)^{\frac{a}{a-1}})^{a-1}(\frac{b\bar{M}}{(1-b)})^{1-a}$.

Proof. of Proposition 1. Substituting $w_j = w$ for all j in equation (1) we get: $p = \tilde{p}(w) = \frac{1}{A}(\frac{w}{a})^a(\frac{b\bar{M}}{(1-b)m})^{1-a}$. We obtain $ml^d(\frac{w}{\tilde{p}(w)}) \leqq n$ if and only if $w \geqq a\frac{b}{1-b}\frac{\bar{M}}{n}$. The competitive wage w_c satisfies $ml^d(\frac{w_c}{\tilde{p}(w_c)}) = n$ which yields $w_c = a\frac{b}{1-b}\frac{\bar{M}}{n}$. ■

Proof. of Proposition 2. Substituting equation (3) in $l^d(\frac{w}{\tilde{p}(w)})$ we get: $\tilde{l}(w) = l^d(\frac{w}{\tilde{p}(w)}) = \frac{b\bar{M}}{(1-b)m}\frac{a}{w}$ and then $\tilde{L}(w) = m\tilde{l}(w) = \frac{b\bar{M}}{(1-b)}\frac{a}{w}$. Substituting equation (3) in $\tilde{\pi}(w) = \tilde{p}(w)\tilde{l}(w)^a - w\tilde{l}(w)$ we get $\tilde{\pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)m}$ and then $\tilde{\Pi}(w) = w\tilde{\pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)}$. ■

Proof. of Proposition 3. Using $V(p, I_i) = \hat{V}(p)I_i$ it is easy to see that the solution to Program T_f is the same that the one obtained maximizing: $T_f^o = \{l^d\left(\frac{w_j}{p}\right)(1-t_w)w_j + s(\frac{n}{m} - l^d\left(\frac{w_j}{p}\right))\}$ subject to: $l^d\left(\frac{w_j}{p}\right) \leqq \frac{n}{m}$. Applying standard static optimization techniques we get: $\hat{w}_f(p, t_w, s) = \left\{\begin{matrix} \frac{s}{(1-t_w)a} \text{ if } s>(1-t_w)a^2p(\frac{n}{m})^{a-1} \\ ap(\frac{n}{m})^{a-1} \text{ if } s\leqq(1-t_w)a^2p(\frac{n}{m})^{a-1} \end{matrix}\right\}$.
■

Proof. of Proposition 4. Using $V(p, I_i) = \hat{V}(p)I_i = b^b(1-b)^{1-b}\frac{I_i}{p}$ and substituting $\tilde{p}(w)$ and $\tilde{L}(w)$ we obtain: $T_u(w) = \{(\frac{(1-t_w)ab\bar{M}}{1-b} + sn)w^{-a} - \frac{sab\bar{M}}{1-b}w^{-a-1}\}K$ where $K = \frac{Aa^a m^{1-a}(1-b)^{2-a-b}}{n\bar{M}^{1-a}b^{1-a-b}}$. Applying standard static optimization techniques we get:

$$\tilde{w}_u(t_w \mid s) = \left\{\begin{matrix} \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn} \text{ if } s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \\ w_c = a\frac{b}{1-b}\frac{\bar{M}}{m} \text{ if } s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \end{matrix}\right\}.$$

■

Proof. of Proposition 5. Substituting $p = \tilde{p}(w)$ in $\hat{w}_f(p, t_w, s)$ we get:

$$\tilde{w}_f(t_w \mid s) = \left\{\begin{matrix} \frac{s}{(1-t_w)a} \text{ if } s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \\ w_c = a\frac{b}{1-b}\frac{\bar{M}}{m} \text{ if } s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \end{matrix}\right\}.$$

■

Proof. of Proposition 6. Using $\tilde{w}_u(t_w \mid s)$ and $\tilde{w}_f(t_w \mid s)$ we see that if $s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$ then $\tilde{w}_u(t_w \mid s) = \tilde{w}_f(t_w \mid s) = w_c$. If $s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$ then $\tilde{w}_f(t_w \mid s) = \frac{s}{(1-t_w)a}$ and $\tilde{w}_u(t_w \mid s) = \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn}$ and it is easy to check that $\frac{s}{(1-t_w)a} > \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn}$ if and only if $s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$. ■

Proof. of Proposition 7. Using $V(p, I_i) = \hat{V}(p)I_i$ we can rewrite the utility function of the government as: $(1-t_w)w\tilde{L}(w) + s(n - \tilde{L}(w)))^\beta((1-t_c)\tilde{\Pi}(w))^{1-\beta}\hat{V}(\tilde{p}(w))\frac{1}{n^\beta}$. Taking into account the government budget constraint the utility function of the government becomes: $G(t_w) = (1-t_w)w\tilde{L}(w) + s(n-\tilde{L}(w)))^{\beta}(\tilde{\Pi}(w) + \bar{M} + t_w w\tilde{L}(w) - s(n-\tilde{L}(w)))^{1-\beta}\hat{V}(\tilde{p}(w))\frac{1}{n^{\beta}}$. The solution that maximizes $G(t_w)$ is the same that the one that maximizes: $(1-t_w)w\tilde{L}(w) + s(n-\tilde{L}(w)))^{\beta}(\tilde{\Pi}(w) + \bar{M} + t_w w\tilde{L}(w) - s(n-\tilde{L}(w)))^{1-\beta}$. Applying standard static optimization techniques we obtain:

$$\tilde{t}_w(w \mid s, \bar{M}, \beta) = \left\{ \begin{array}{c} \frac{(ab-\beta)\bar{M}+(1-b)sn}{ab\bar{M}} - \frac{s}{w} \; if \; s \leqq \beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ 1 - \frac{s}{w} \; if \; \beta\frac{1}{1-b}\frac{\bar{M}}{n} < s \leqq \frac{1}{1-b}\frac{\bar{M}}{n} \end{array} \right\} \text{ and}$$

$$\tilde{t}_c(w, \tilde{t}_w, s, \bar{M}, \beta) = \left\{ \begin{array}{c} 1 - \frac{(1-\beta)}{(1-a)b} \; if \; s \leqq \beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ \frac{-(ab+(1-\beta))\bar{M}+(1-b)sn}{(1-a)b\bar{M}} \; if \; \beta\frac{1}{1-b}\frac{\bar{M}}{n} < s \leqq \frac{1}{1-b}\frac{\bar{M}}{n} \end{array} \right\}.$$

■

Proof. of Theorem 8. Studying the intersection between $\tilde{w}_f(t_w \mid s)$ and $\tilde{t}_w(w \mid s, \bar{M}, \beta)$ we get:

$$(\hat{w}_f(s, \bar{M}, \beta), \hat{t}_{w,f}(s, \bar{M}, \beta)) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ (\frac{(1-a)b\bar{M}s}{\beta\bar{M}-(1-b)sn}, \frac{((1-a)ab-\beta)\bar{M}+(1-b)sn}{(1-a)ab\bar{M}}) \text{ if } a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \end{array} \right\}.$$

Studying the intersection between $\tilde{w}_u(t_w \mid s)$ and $\tilde{t}_w(w \mid s, \bar{M}, \beta)$ we get:

$$(\hat{w}_u(s, \bar{M}, \beta), \hat{t}_{w,u}(s, \bar{M}, \beta)) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ (\frac{bs}{\beta}, \frac{(ab-(1+a)\beta)\bar{M}+(1-b)sn}{ab\bar{M}}) \text{ if } a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \end{array} \right\}.$$

■

Proof. of Theorem 9. Comparing $\hat{w}_f(s, \bar{M}, \beta)$ and $\hat{w}_u(s, \bar{M}, \beta)$. ■

Proof. of Theorem 10. Using the government budget constraint we can rewrite the expected utility of a worker in the firm wage setting system, $\frac{1}{n}\{(1-\hat{t}_{w,f})\hat{w}_f\tilde{L}(\hat{w}_f) + s(n-\tilde{L}(\hat{w}_f))\}\hat{V}(\tilde{p}(\hat{w}_f))$, and the national wage setting system $\frac{1}{n}\{(1-\hat{t}_{w,u})\hat{w}_u\tilde{L}(\hat{w}_u) + s(n-\tilde{L}(\hat{w}_u))\}\hat{V}(\tilde{p}(\hat{w}_u))$ as $\{\hat{w}_f\tilde{L}(\hat{w}_f) + t_c\tilde{\Pi}(\hat{w}_f) + \bar{M}\}\hat{V}(\tilde{p}(\hat{w}_f))$ and $\{\hat{w}_u\tilde{L}(\hat{w}_u) + \tilde{t}_c\tilde{\Pi}(\hat{w}_u) + \bar{M}\}\hat{V}(\tilde{p}(\hat{w}_u))$ respectively. If $s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ then the last expressions become $\beta\frac{1}{1-b}\frac{\bar{M}}{n}\hat{V}(\tilde{p}(\hat{w}_f))$ and $\beta\frac{1}{1-b}\frac{\bar{M}}{n}\hat{V}(\tilde{p}(\hat{w}_u))$. This means that the expected utility of a worker is greater with the wage setting system that implies a lower equilibrium wage because $\hat{V}(\tilde{p}(w))$ is decreasing in w. The government is better off when w is lower because the expected utility of a worker is greater when w is lower and the owner is better off when w is lower. ■

Proof. of Table 1. All the partial derivatives are computed directly using $(\hat{w}_f(s, \bar{M}, \beta), \hat{t}_{w,f}(s, \bar{M}, \beta))$ and $(\hat{w}_u(s, \bar{M}, \beta), \hat{t}_{w,u}(s, \bar{M}, \beta)$. ■

Proof. of Table 2. Substituting $v\frac{\bar{M}}{(1-b)}$ in $(\hat{w}_f(s, \bar{M}, \beta), \hat{t}_{w,f}(s, \bar{M}, \beta))$ and

$(\hat{w}_u(s,\bar{M},\beta),\hat{t}_{w,u}(s,\bar{M},\beta)$ we get

$$(\hat{w}_f(v,\bar{M},\beta),\hat{t}_{w,f}(v,\bar{M},\beta)) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } v \leqq \frac{a\beta}{n} \\ (\frac{(1-a)bv\bar{M}}{(1-b)(\beta-vn)}, \frac{(1-a)ab-\beta+vn}{(1-a)ab}) \text{ if } \frac{a\beta}{n} < v < \frac{\beta}{n} \end{array} \right\} \text{ and}$$

$$(\hat{w}_u(v,\bar{M},\beta),\hat{t}_{w,u}(v,\bar{M},\beta) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } v \leqq \frac{a\beta}{n} \\ (\frac{bv\bar{M}}{(1-b)\beta}, \frac{ab-(1+a)\beta++vn}{ab}) \text{ if } \frac{a\beta}{n} < v < \frac{\beta}{n} \end{array} \right\},$$

and all the partial derivatives are computed directly using $(\hat{w}_f(v,\bar{M},\beta),\hat{t}_{w,f}(v,\bar{M},\beta))$ and $(\hat{w}_u(v,\bar{M},\beta),\hat{t}_{w,u}(v,\bar{M},\beta))$. ■ ■

9 Appendix II Detailed Proofs

(Not for publication)

Proof. of Equation (2). Substituting $l^d(\frac{wj}{p}) = (\frac{1}{aA}\frac{wj}{p})^{\frac{1}{a-1}}$, $\pi(w_j, p) = (1-a)Ap(\frac{1}{aA}\frac{w_i}{p})^{\frac{a}{a-1}}$ and $y^d(p, I_i) = \frac{bI_i}{p}$in equation (1) we obtain:

$$\sum_{j=1}^{m} A(\frac{1}{aA}\frac{w_j}{p})^{\frac{a}{a-1}} = \sum_{j=1}^{m}(\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}}\frac{b(1-t_w)w_j}{p} + \frac{bs}{p}(n - \sum_{j=1}^{m}(\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}}) + \frac{b(1-t_c)\sum_{j=1}^{m}(1-a)Ap(\frac{1}{aA}\frac{wj}{p})^{\frac{a}{a-1}}}{p} \quad (5)$$

Now if the government has a balanced budget constraint we have:

$$n - \sum_{j=1}^{m} A(\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}} = \sum_{j=1}^{m}(\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}}(1-t_w)w_j + (1-t_c)\sum_{j=1}^{m}(1-a)Ap(\frac{1}{aA}\frac{w_j}{p})^{\frac{a}{a-1}} + \bar{M}. \quad (6)$$

Substituting (6) in (??) we get:

$$\sum_{j=1}^{m} A(\frac{1}{aA}\frac{w_j}{p})^{\frac{a}{a-1}} = \frac{b}{p}(\sum_{j=1}^{m}(\frac{1}{aA}\frac{w_j}{p})^{\frac{1}{a-1}}w_j + \sum_{j=1}^{m}(1-a)Ap(\frac{1}{aA}\frac{w_j}{p})^{\frac{a}{a-1}} + \bar{M}),$$

and simplifying last equation we get:

$$p^{\frac{1}{1-a}}\sum_{j=1}^{m} A(\frac{w_j}{aA})^{\frac{a}{a-1}} = \frac{b\bar{M}}{(1-b)}.$$

Finally, solving for p last equation we get:

$$p = \frac{1}{a^a A}(\sum_{j=1}^{m}(w_j)^{\frac{a}{a-1}})^{a-1}(\frac{b\bar{M}}{(1-b)})^{1-a}.$$

■

Proof. of Proposition 1. Substituting $w_j = w$ for all j in equation (1) we get:

$$p = \tilde{p}(w) = \frac{1}{A}(\frac{w}{a})^a(\frac{b\bar{M}}{(1-b)m})^{1-a}.$$

Now we obtain $ml^d(\frac{w}{\tilde{p}(w)}) \leqq n$ if and only if $w \geqq a\frac{b}{1-b}\frac{\bar{M}}{n}$. The competitive wage w_c satisfies $ml^d(\frac{w_c}{\tilde{p}(w_c)}) = n$ which yields $w_c = a\frac{b}{1-b}\frac{\bar{M}}{n}$. ■

Proof. of Proposition 2. Substituting equation (3) in $l^d(\frac{w}{\tilde{p}(w)})$ we get: $\tilde{l}(w) = l^d(\frac{w}{\tilde{p}(w)}) = \frac{b\bar{M}}{(1-b)m}\frac{a}{w}$ and then $\tilde{L}(w) = m\tilde{l}(w) = \frac{b\bar{M}}{(1-b)}\frac{a}{w}$. Substituting equation (3) in $\tilde{\pi}(w) = \tilde{p}(w)A\tilde{l}(w)^a - w\tilde{l}(w)$ we get $\tilde{\pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)m}$ and then $\tilde{\Pi}(w) = w\tilde{\pi}(w) = (1-a)\frac{b\bar{M}}{(1-b)}$. ■

Proof. of Proposition 3. Using $V(p, I_i) = \hat{V}(p)I_i$ we rewrite Program T_f as given p, t_w and s choose w_j in order to maximize: $\{l^d\left(\frac{w_j}{p}\right)(1-t_w)w_j + s(\frac{n}{m} - l^d\left(\frac{w_j}{p}\right))\}\hat{V}(p)\frac{m}{n}$ subject to: $l^d\left(\frac{w_j}{p}\right) \leqq \frac{n}{m}$. First, in order to satisfy the restriction, any solution w_j must be such that $w_j \geqq w_{cp}$ where w_{cp} satisfies $l^d(\frac{w_{cp}}{p}) = (\frac{1}{a}\frac{w_{cp}}{p})^{\frac{1}{a-1}} = \frac{n}{m}$, that is $w_j \geqq w_{cp} = ap(\frac{n}{m})^{a-1}$. Now we solve Program T_f without the restriction. The program is: given p, t_w and s choose w_j in order to maximize: $\{l^d\left(\frac{w_j}{p}\right)(1-t_w)w_j + s(\frac{n}{m} - l^d\left(\frac{w_j}{p}\right))\}\hat{V}(p)\frac{m}{n}$.Note that $\hat{V}(p)\frac{m}{n}$ is a constant an then the solution of this program is the same that the one obtained maximizing: $T_f^o = \{l^d\left(\frac{w_j}{p}\right)(1-t_w)w_j + s(\frac{n}{m} - l^d\left(\frac{w_j}{p}\right))\}$. The first order condition for a maximum is: $(1-t_w) + \frac{(1-t_w)}{a-1} - \frac{s}{(a-1)w_j} = 0$ and solving for w_j we obtain: $w_j = \frac{s}{(1-t_w)a}$. One can check that the second derivative of T_f^o is $\frac{(a-1)s}{((a-1)w_j)^2}$which implies that if $w_j > 0$ then T_f^o is a concave function and then $w_j = \frac{s}{(1-t_w)a}$ is a global maximum. Finally we have $w_j > w_{cp}$ if and only if $s > (1-t_w)a^2p(\frac{n}{m})^{a-1}$. Summarizing the jth union best reply function is: $\hat{w}_f(p, t_w, s) = \left\{\begin{matrix} \frac{s}{(1-t_w)a} \text{ if } s>(1-t_w)a^2p(\frac{n}{m})^{a-1} \\ ap(\frac{n}{m})^{a-1} \text{ if } s\leqq(1-t_w)a^2p(\frac{n}{m})^{a-1} \end{matrix}\right\}$. ■

Proof. of Proposition 4. Using $V(p, I_i) = \hat{V}(p)I_i = b^b(1-b)^{1-b}\frac{I_i}{p}$ we can rewrite Program T_u as given t_w and s choose w in order to maximize: $T_u(w) = \frac{1}{\tilde{p}(w)}\{(1-t_w)w\tilde{L}(w) + s(n - \tilde{L}(w))\}\frac{b^b(1-b)^{1-b}}{n}$ where substituting $\tilde{p}(w)$ and $\tilde{L}(w)$ we obtain: $T_u(w) = \{(\frac{(1-t_w)ab\bar{M}}{1-b} + sn)w^{-a} - \frac{sab\bar{M}}{1-b}w^{-a-1}\}K$ where $K = \frac{Aa^a m^{1-a}(1-b)^{2-a-b}}{n\bar{M}^{1-a}b^{1-a-b}}$. The first order condition for a maximum is: $-a(\frac{(1-t_w)ab\bar{M}}{1-b} + sn)w^{-a-1} + (a+1)\frac{sab\bar{M}}{1-b}w^{-a-2} = 0$. Solving the equation for w we get: $w' = \frac{(1+a)sb\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn}$. It is not difficult to show that $\frac{dT_u(w)}{(dw)^2} < 0$ at w' which means that w' is a global maximum. Finally we have $w' > w_c = a\frac{b}{1-b}\frac{\bar{M}}{n}$ if and only if $s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$. Then the national union's best reply function is:

$$\tilde{w}_u(t_w \mid s) = \left\{\begin{matrix} \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn} \text{ if } s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \\ w_c = a\frac{b}{1-b}\frac{\bar{M}}{m} \text{ if } s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \end{matrix}\right\}. \tag{7}$$

■

Proof. of Proposition 5. By the proof of proposition 3 we have that the best reply function of the jth union is: $\hat{w}_f(p, t_w, s) = \left\{ \begin{smallmatrix} \frac{s}{(1-t_w)a} \text{ if } \frac{s}{(1-t_w)a} > w_{cp} \\ w_{cp} \text{ if } \frac{s}{(1-t_w)a} \leqq w_{cp} \end{smallmatrix} \right\}$ where w_{cp} satisfies $l^d(\frac{w_{cp}}{p}) = \frac{n}{m}$. Now substituting $p = \tilde{p}(w_{cp})$ in last equation and solving for w_{cp} we get $w_{cp} = a\frac{b}{1-b}\frac{\bar{M}}{n} = w_c$. Then the best reply function of the jth union becomes $\tilde{w}_f(t_w \mid s) = \left\{ \begin{smallmatrix} \frac{s}{(1-t_w)a} \text{ if } \frac{s}{(1-t_w)a} > w_c \\ w_{cp} \text{ if } \frac{s}{(1-t_w)a} \leqq w_c \end{smallmatrix} \right\}$ and substituting w_c we finally get:

$$\tilde{w}_f(t_w \mid s) = \left\{ \begin{matrix} \frac{s}{(1-t_w)a} \text{ if } s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \\ w_c = a\frac{b}{1-b}\frac{\bar{M}}{m} \text{ if } s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n} \end{matrix} \right\}. \quad (8)$$

■

Proof. of Proposition 6. Using $\tilde{w}_u(t_w \mid s)$ and $\tilde{w}_f(t_w \mid s)$ we see that if $s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$ then $\tilde{w}_u(t_w \mid s) = \tilde{w}_f(t_w \mid s) = w_c$. If $s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$ then $\tilde{w}_f(t_w \mid s) = \frac{s}{(1-t_w)a}$ and $\tilde{w}_u(t_w \mid s) = \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn}$ and it is easy to check that $\frac{s}{(1-t_w)a} > \frac{(1+a)bs\bar{M}}{(1-t_w)ab\bar{M}+(1-b)sn}$ if and only if $s > (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$. ■

Proof. of Proposition 7. Using $V(p, I_i) = \hat{V}(p)I_i$ we can rewrite the utility function of the government as:
$(1-t_w)w\tilde{L}(w) + s(n - \tilde{L}(w)))^\beta((1-t_c)\tilde{\Pi}(w))^{1-\beta}\hat{V}(\tilde{p}(w))\frac{1}{n^\beta}$. Taking into account the government budget constraint the utility function of the government becomes: $G(t_w) = (1-t_w)w\tilde{L}(w) + s(n-\tilde{L}(w)))^\beta(\tilde{\Pi}(w) + \bar{M} + t_w w\tilde{L}(w) - s(n - \tilde{L}(w)))^{1-\beta}\hat{V}(\tilde{p}(w))\frac{1}{n^\beta}$. The solution that maximizes $G(t_w)$ is the same that the one that maximizes: $(1-t_w)w\tilde{L}(w)+s(n-\tilde{L}(w)))^\beta(\tilde{\Pi}(w)+\bar{M}+t_w w\tilde{L}(w)-s(n-\tilde{L}(w)))^{1-\beta}$. The first order condition for a maximum implies: $\frac{-\beta w\tilde{L}}{(1-t_w)w\tilde{L}+s(n-\tilde{L})} + \frac{(1-\beta)w\tilde{L}}{\tilde{\Pi}+\bar{M}+t_w w\tilde{L}-s(n-\tilde{L})} = 0$. Solving for t_w we get $t_w = 1 - \beta - \beta\frac{\tilde{\Pi}+\bar{M}}{w\tilde{L}} + \frac{s(n-\tilde{L})}{w\tilde{L}} = \frac{(ab-\beta)\bar{M}+(1-b)sn}{ab\bar{M}} - \frac{s}{w}$ and t_w is a global maximum because $G(t_w)$ is an strictly concave function. Using the government budget constraint we get: $t_c = 1 - \frac{(1-\beta)}{(1-a)b}$. It is easy to check that $t_c < 1$. We have that $(1-t_w)w > s$ if and only if $s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$. If $s \geqq \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ the restriction $(1-t_w)w \geqq s$ is binding which means $t_w = 1 - \frac{s}{w}$. In this case $t_c = \frac{-(ab+(1-\beta))\bar{M}+(1-b)sn}{(1-a)b\bar{M}}$. Finally we have that $t_c \leqq 1$ if and only if $s \leqq \frac{1}{1-b}\frac{\bar{M}}{n}$. Summarizing the solution to the government's program is:

$$\tilde{t}_w(w \mid s, \bar{M}, \beta) = \left\{ \begin{matrix} \frac{(ab-\beta)\bar{M}+(1-b)sn}{ab\bar{M}} - \frac{s}{w} \; if \; s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ 1 - \frac{s}{w} \; if \; \beta\frac{1}{1-b}\frac{\bar{M}}{n} \leqq s \leqq \frac{1}{1-b}\frac{\bar{M}}{n} \end{matrix} \right\} \text{ and} \quad (9)$$

$$\tilde{t}_c(w, \tilde{t}_w, s, \bar{M}, \beta) = \left\{ \begin{matrix} 1 - \frac{(1-\beta)}{(1-a)b} \; if \; s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ \frac{-(ab+(1-\beta))\bar{M}+(1-b)sn}{(1-a)b\bar{M}} \; if \; \beta\frac{1}{1-b}\frac{\bar{M}}{n} \leqq s \leqq \frac{1}{1-b}\frac{\bar{M}}{n} \end{matrix} \right\}. \quad (10)$$

Proof. of Theorem 8. First we compute the firm-level equilibrium. We start with the case $s = 0$, then $s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ and $\tilde{t}_w = \frac{ab-\beta}{ab} < 1$. On the other hand $s \leqq (1-t_w)a^2\frac{b}{1-b}\frac{\bar{M}}{n}$ which means $\tilde{w}_f = w_c$. Now we consider the case $s > 0$. If $s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ then inverting $\tilde{t}_w$ we get: $w = \tilde{w}_o(t_w \mid s, \bar{M}, \beta) = \frac{abs\bar{M}}{(1-t_w)ab\bar{M}-\beta\bar{M}+(1-b)sn}$ and $\frac{\partial \tilde{w}_o}{\partial t_w} > 0$. Now we define the function $\tilde{w}_{fo}(t_w \mid s) = \frac{s}{(1-t_w)a}$. Computing the intersection between $\tilde{w}_o$ and $\tilde{w}_{fo}$ we get $(\underline{w}, \underline{t_w}) = (\frac{(1-a)b\bar{M}s}{\beta\bar{M}-(1-b)sn}, \frac{((1-a)ab-\beta)\bar{M}+(1-b)sn}{(1-a)ab\bar{M}})$. Thus if $\underline{w} > w_c$, that is if $a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s$ then $(\underline{w}, \underline{t_w})$ is an equilibrium. Finally it is easy to check that $\frac{\partial \tilde{w}_o}{\partial t_w} > \frac{\partial \tilde{w}_{fo}}{\partial t_w}$, which implies that if $t_w < \underline{t_w}$ (resp. $t_w > \underline{t_w}$) then $\tilde{w}_{fo}(t_w \mid s) > \tilde{w}_o(t_w \mid s, \bar{M}, \beta)$ (resp. $\tilde{w}_{fo}(t_w \mid s) < \tilde{w}_o(t_w \mid s, \bar{M}, \beta)$). This last property implies the segment of $\tilde{w}_f(t_w \mid s)$ where $\tilde{w}_f(t_w \mid s) = w_c$ never crosses the best reply function of the government and then $(\underline{w}, \underline{t_w})$ is the unique equilibrium. Figure 1 offers an illustration of this reasoning. If $\underline{w} \leqq w_c$, that is, if $s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n}$, then the segment of $\tilde{w}_f(t_w \mid s)$ where $\tilde{w}_f(t_w \mid s) = w_c$ now crosses the best reply function of the government and, thus, the equilibrium is $(w_c, \frac{ab-\beta}{ab})$. Figure 2 offers an illustration of this reasoning. If $\beta\frac{1}{1-b}\frac{\bar{M}}{n} \leqq s \leqq \frac{1}{1-b}\frac{\bar{M}}{n}$ then the best reply function of the government is $\tilde{t}_w(w \mid s, \bar{M}, \beta) = 1-\frac{s}{w}$. Inverting this function we get: $w = \tilde{w}_{oo}(t_w \mid s) = \frac{s}{(1-t_w)w}$ and in this case, it is easy to check that for all $t_w \leqq 1$ $\tilde{w}_{fo}(t_w \mid s) > \tilde{w}_{oo}(t_w \mid s)$. This last property implies that the equilibrium does not exist because $\tilde{w}_f(t_w \mid s)$ never crosses the best reply function of the government. Figure 3 offers an illustration of this reasoning. This concludes the proof for the existence of an equilibrium with wage setting at the firm level. The existence of an equilibrium with wage setting at the national level is proved in the same way analyzing the intersection between $\tilde{w}_u(t_w \mid s)$ and $\tilde{t}_w(w \mid s, \bar{M}, \beta)$. Summarizing the firm-level equilibrium and the national-level equilibrium are:

$$(\hat{w}_f(s,\bar{M},\beta), \hat{t}_{w,f}(s,\bar{M},\beta)) = \begin{Bmatrix} (w_c, \frac{ab-\beta}{ab}) \text{ if } s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ (\frac{(1-a)b\bar{M}s}{\beta\bar{M}-(1-b)sn}, \frac{((1-a)ab-\beta)\bar{M}+(1-b)sn}{(1-a)ab\bar{M}}) \text{ if } a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \end{Bmatrix}$$

$$(\hat{w}_u(s,\bar{M},\beta), \hat{t}_{w,u}(s,\bar{M},\beta)) = \begin{Bmatrix} (w_c, \frac{ab-\beta}{ab}) \text{ if } s \leqq a\beta\frac{1}{1-b}\frac{\bar{M}}{n} \\ (\frac{bs}{\beta}, \frac{(ab-(1+a)\beta)\bar{M}+(1-b)sn}{ab\bar{M}}) \text{ if } a\beta\frac{1}{1-b}\frac{\bar{M}}{n} < s < \beta\frac{1}{1-b}\frac{\bar{M}}{n} \end{Bmatrix}.$$

■

Proof. of Theorem 9. Comparing $\hat{w}_f(s,\bar{M},\beta)$ and $\hat{w}_u(s,\bar{M},\beta)$. ■

Proof. of Theorem 10. Using the government budget constraint we can rewrite the expected utility of a worker in the firm wage setting system, $\frac{1}{n}\{(1-\hat{t}_{w,f})\hat{w}_f\tilde{L}(\hat{w}_f) + s(n-\tilde{L}(\hat{w}_f))\}\hat{V}(\tilde{p}(\hat{w}_f))$, and the national wage setting system, $\frac{1}{n}\{(1-\hat{t}_{w,u})\hat{w}_u\tilde{L}(\hat{w}_u) + s(n-\tilde{L}(\hat{w}_u))\}\hat{V}(\tilde{p}(\hat{w}_u))$, as $\{\hat{w}_f\tilde{L}(\hat{w}_f) + t_c\tilde{\Pi}(\hat{w}_f) +$

$\bar{M}\}\hat{V}(\tilde{p}(\hat{w}_f))$ and $\{\hat{w}_u\tilde{L}(\hat{w}_u) + \tilde{t}_c\tilde{\Pi}(\hat{w}_u) + \bar{M}\}\hat{V}(\tilde{p}(\hat{w}_u))$ respectively. If $s < \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ then the last expressions become $\beta\frac{1}{1-b}\frac{\bar{M}}{n}\hat{V}(\tilde{p}(\hat{w}_f))$ and $\beta\frac{1}{1-b}\frac{\bar{M}}{n}\hat{V}(\tilde{p}(\hat{w}_u))$. This means that the expected utility of a worker is greater with the wage setting system that implies a lower equilibrium wage because $\hat{V}(\tilde{p}(w))$ is decreasing in w. The government is better off when w is lower because the expected utility of a worker is greater when w is lower and the owner is better off when w is lower. ∎

Proof. of Table 1. All the partial derivatives are computed directly using $(\hat{w}_f(s,\bar{M},\beta),\hat{t}_{w,f}(s,\bar{M},\beta))$ and $(\hat{w}_u(s,\bar{M},\beta),\hat{t}_{w,u}(s,\bar{M},\beta)$. The unique partial derivative that is difficult to compute is: $\frac{\partial\tilde{p}(\hat{w}_f)}{\partial\bar{M}}$. Using (3) we get that $\frac{\partial\tilde{p}(\hat{w}_f)}{\partial\bar{M}} = (\frac{1-a}{\bar{M}} + \frac{a}{\hat{w}_f})\tilde{p}(\hat{w}_f)$. Substituting $\hat{w}_f$ in this expression we get that $\frac{\partial\tilde{p}(\hat{w}_f)}{\partial\bar{M}} > 0$ if and only if $s < \frac{a\beta\bar{M}}{a(1-b)n-(1-a)^2b}$ and it is easy to check that $\frac{a\beta\bar{M}}{a(1-b)n-(1-a)^2b} > \beta\frac{1}{1-b}\frac{\bar{M}}{n}$ which implies that for an equilibrium with unemployment $s < \frac{a\beta\bar{M}}{a(1-b)n-(1-a)^2b}$ and then $\frac{\partial\tilde{p}(\hat{w}_f)}{\partial\bar{M}} > 0$. ∎

Proof. of Table 2. Substituting $v\frac{\bar{M}}{(1-b)}$ in $(\hat{w}_f(s,\bar{M},\beta),\hat{t}_{w,f}(s,\bar{M},\beta))$ and $(\hat{w}_u(s,\bar{M},\beta),\hat{t}_{w,u}(s,\bar{M},\beta)$ we get

$$(\hat{w}_f(v,\bar{M},\beta),\hat{t}_{w,f}(v,\bar{M},\beta)) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } v \leqq \frac{a\beta}{n} \\ (\frac{(1-a)bv\bar{M}}{(1-b)(\beta-vn)}, \frac{(1-a)ab-\beta+vn}{(1-a)ab}) \text{ if } \frac{a\beta}{n} < v < \frac{\beta}{n} \end{array} \right\}$$

$$(\hat{w}_u(v,\bar{M},\beta),\hat{t}_{w,u}(v,\bar{M},\beta) = \left\{ \begin{array}{c} (w_c, \frac{ab-\beta}{ab}) \text{ if } v \leqq \frac{a\beta}{n} \\ (\frac{bv\bar{M}}{(1-b)\beta}, \frac{ab-(1+a)\beta++vn}{ab}) \text{ if } \frac{a\beta}{n} < v < \frac{\beta}{n} \end{array} \right\},$$

and all the partial derivatives are computed directly using $(\hat{w}_f(v,\bar{M},\beta),\hat{t}_{w,f}(v,\bar{M},\beta))$ and $(\hat{w}_u(v,\bar{M},\beta),\hat{t}_{w,u}(v,\bar{M},\beta))$. ∎ ∎

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