

$\Delta S = 0$  BARYONIC CURRENT COUPLED TO THE MUONIC CURRENT

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(Received 29 April 1966)

Recently it has been suggested,<sup>1</sup> in an attempt to reformulate the discrete symmetry operations, that the electronic current interacts with the strangeness-conserving baryonic current  $\cos\theta(V_\mu^{(0)} + A_\mu^{(0)})$ , but that the muonic current interacts with  $\cos\theta(V_\mu^{(0)} - A_\mu^{(0)})$ . It is claimed that the experimental results are consistent with this hypothesis provided  $g_A = 1.2g_V$  and  $g_P = 24g_A$ . The purpose of this Letter is to show that we have enough experimental evidence to rule out this hypothesis.

We are going to consider the two following systems of coupling constants: (i)  $g_A = -1.18g_V$ ,  $g_P = 7g_A$  and (ii)  $g_A = +1.18g_V$ ,  $g_P = \lambda g_A$ . Furthermore, we assume  $g_V = 1.1152 \times 10^{-11}$  MeV<sup>-2</sup>,  $g_M = 3.7g_V$ , and that the second-class currents are absent.

(1) Muon capture in hyperfine states of H<sup>1</sup>.—The capture rate in the two hyperfine states is<sup>2</sup>

$$\Lambda_- = 22.55 \{G_V - 3G_A + G_P\}^2,$$

$$\Lambda_+ = 22.55 \{(G_V + G_A - G_P)^2 + (8/9)G_P^2\},$$

where  $G_V$ ,  $G_A$ , and  $G_P$  are the usual combinations of coupling constants. We have used the value 98.6 MeV/c for the neutrino momentum. Using the experimental results of Culligan et al.<sup>3</sup> on <sup>9</sup>F<sup>19</sup> and Primakoff theory,<sup>4</sup> we get for the hydrogen

$$\frac{\Delta\Lambda}{\Lambda} \equiv \frac{\Lambda_- - \Lambda_+}{\frac{1}{4}[3\Lambda_+ + \Lambda_-]} = 3.65 \pm 0.66,$$

where the error quoted is due uniquely to the error attached to the measure on <sup>9</sup>F<sup>19</sup>. Using the set (i) we get  $\Delta\Lambda/\bar{\Lambda} = 3.73$  in very good agreement with the experiment.

If the second set of constants is used we obtain  $\Delta\Lambda/\bar{\Lambda} < 0$  independently of the value of  $\lambda$ .

(2) Muon capture by liquid hydrogen.—The capture rate is

$$\Lambda(pp\mu^-) = 2\gamma \left\{ \frac{3}{4}\Lambda_- + \frac{1}{4}\Lambda_+ \right\}.$$

An accurate evaluation of  $2\gamma$  has been made by Halpern<sup>5</sup> and Wessel and Phillipson<sup>6</sup> with the result  $2\gamma = 1.01 \pm 0.05$ . Rothberg<sup>7</sup> has measured this quantity using counters and his val-

ue is  $476 \pm 40$  sec<sup>-1</sup>. (For a discussion on lower results obtained with bubble-chamber experiments, see Ref. 2.)

With the set (i) we get  $\Lambda = 475$  sec<sup>-1</sup>. To obtain the same value with the second set, it is necessary to take  $\lambda = 88$  or  $\lambda = -30$ . If  $\lambda = -24$  we get  $\Lambda = 388$  sec<sup>-1</sup>.

(3) Capture by He<sup>3</sup>.—The experimental value of the capture rate is  $1468 \pm 40$  sec<sup>-1</sup> (average of five experiments<sup>8</sup>). Using the value 103.085 MeV/c for the neutrino momentum, we get<sup>8</sup> for the transition rate

$$\Lambda = 200(g_V^2 + 3\Gamma^2),$$

where  $\Gamma^2 = g_A^2 + \frac{1}{3}(g_P^2 - 2g_A g_P)$ , and the average value obtained using Gaussian and Irving functions has been considered. The theoretical error in this expression is less than 5%. Using the set (i) we get  $\Lambda = 1462$  sec<sup>-1</sup>. To obtain the experimental result with the second set of constants, it is necessary to take  $\lambda = +46$  or  $\lambda = -8$ . If  $\lambda = -24$  we get  $\lambda = 2647$  sec<sup>-1</sup>.

(4) Capture by C<sup>12</sup>.—The experimental value<sup>9</sup> is  $\Lambda = (6.75^{+0.30}_{-0.75}) \times 10^3$  sec<sup>-1</sup>. The theoretical value due to Foldy and Walecka<sup>10</sup> is

$$\Lambda = 546[5.73G_A^2 + 1.65[G_P^2 - 2G_P G_A - 0.53(G_A - G_P)G_A] - 0.12G_A G_V].$$

The theoretical error in this formula is around 8%. Using the set (i) we get  $\Lambda = 6.18 \times 10^3$  sec<sup>-1</sup> in good agreement with the experimental results. If (ii) is used the values of  $\lambda$  necessary to obtain  $\Lambda = 6700$  sec<sup>-1</sup> are  $\lambda = +56$  or  $\lambda = -19$ . For  $\lambda = -24$  we get  $\Lambda = 8043$  sec<sup>-1</sup>.

(5) Primakoff's formula.—The total  $\mu^-$ -capture rates by complex nuclei are in good agreement<sup>11,12</sup> with Primakoff's formula<sup>4</sup> (see Bell and Løvseth<sup>13</sup> for a discussion on the theoretical validity of this formula):

$$\Lambda(Z, A) = Z_{\text{eff}}^4 \gamma \left( \frac{\nu}{m_\mu} \right)^2 \left\{ 1 - \frac{A - Z}{2A} \delta \right\},$$

with

$$\gamma = 272 \frac{G_V^2 + 3\Gamma^2}{(\beta/2)^2 + 3g_A^2} \text{ sec}^{-1}.$$

Assuming an average value of  $\nu = 85$  MeV/c, then the best fit of the experimental rates is obtained with  $\delta = 3.13$  and  $\gamma(\nu/m_\mu)^2 = 183$  sec $^{-1}$ . With the coupling constants (i) we get  $\gamma(\nu/m_\mu)^2 = 184$  sec $^{-1}$ . With the constants (ii) we fit the experimental result with  $\lambda = 54$  or  $\lambda = -8$ . For  $\lambda = -24$  we get the value 298 sec $^{-1}$  which is too high.

(6) Angular distribution of emitted neutrons.—The angular distribution of the neutrons emitted after the capture of polarized  $\mu^-$  is described by a parameter  $\alpha$  introduced by Primakoff<sup>4</sup> which is defined by

$$\alpha = \frac{G_V^2 - G_A^2 + G_P^2 - 2G_A G_P}{G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P}.$$

From measurement in Ca<sup>40</sup> and S<sup>32</sup>, it is known that  $\alpha = -1 \pm 0.15$ .<sup>14</sup> The calculated value using (i) is  $\alpha = -0.36$  in bad agreement with the experiment. More experiments of this kind are needed to clear up this discrepancy completely. With the coupling (ii) we get always  $\alpha > 0$ , in particular for  $\lambda = -24$  we get  $\alpha = +0.60$  in

worse agreement with the experiment.

From all this it is clear that there is strong experimental evidence in support of the constants (i) and the same evidence rules out completely the second set of constants.

One of us (R.P.) would like to thank the Junta de la Energía Nuclear for its financial support.

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<sup>5</sup>A. Halpern, Phys. Rev. Letters 13, 660 (1964).

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<sup>7</sup>J. E. Rothberg *et al.*, Phys. Rev. 132, 2664 (1963).

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<sup>9</sup>E. J. Maier *et al.*, Phys. Rev. 133, B663 (1964).

<sup>10</sup>L. L. Foldy and J. D. Walecka, to be published.

<sup>11</sup>J. C. Sens, Phys. Rev. 113, 679 (1959).

<sup>12</sup>V. L. Telegdi, Phys. Rev. Letters 8, 327 (1962).

<sup>13</sup>J. S. Bell and J. Løvseth, Nuovo Cimento 32, 433 (1964).

<sup>14</sup>V. S. Evseev *et al.*, Phys. Letters 6, 193 (1963); Acta Phys. Polon. 21, 313 (1962).

## EQUIVALENT REPRESENTATIONS IN SYMMETRIZED TENSORS

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(Received 4 May 1966)

It has been noted that for many detailed properties of group representations, such as generator matrix elements and certain Clebsch-Gordan coefficients, tensor methods have not been used.<sup>1</sup> The reason is because the basis of decomposed tensors has usually been only implicit.<sup>2</sup> The cause of this trouble is in a portion of a theorem given by Weyl<sup>3</sup> which states that if  $C$  is a Young symmetrizer, then the tensors  $CF$  form an irreducibly invariant subspace. Except for one-dimensional representations, this is incorrect whether it is interpreted to mean a single tableau and some set of tensors  $F$  or whether it is interpreted to mean some set of tableaux (of a given Young pattern) and a single tensor  $F$ . This has caused considerable confusion in the physics literature. Most authors use the first interpretation,<sup>4-6</sup> but the second one has also appeared.<sup>7</sup>

We now show how to construct the irreducibly invariant subspaces. Denoting tableaux

column and row permutations by  $q$  and  $p$ , respectively, it is well known that for any given tableau a minimal left ideal is generated by the Young symmetrizer

$$PQ = \sum_{pq} pq \delta_q, \quad (1)$$

where the sums are over all column and row permutations. The left ideals obtained from the standard tableaux are linearly independent and span the whole ring so that a Peirce resolution of the unit element ( $e$ ) of the ring can be written

$$e = \sum_{i, \mu} (N^\mu / G)(PQ)_i^\mu, \quad (2)$$

where  $N^\mu$  is the dimension of the representation ( $\mu$ ) and  $G$  is the order of the permutation group  $S_r$ . The sum is over all standard tableaux of all patterns of  $S_r$ . If a pair of tableaux  $\tau_i^\mu$