Grand-Unification Effects in the Soft Supersymmetry Breaking Terms

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In minimal supergravity theories the soft supersymmetry breaking (SSB) parameters are universal near the Planck scale. Nevertheless, one often assumes universality at the grand-unification scale $M_G \approx 10^{16}$ GeV, and corrections to the SSB parameters arising from their evolution between the Planck and grand-unification scales are neglected. We study these corrections in minimal SU(5) and show that large splittings between the scalar mass parameters can be induced at M_G. These effects are model dependent and lead to significant uncertainties in the predictions of supersymmetric models.

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The minimal supersymmetric extension of the standard model (MSSM) is a well-motivated candidate to describe the physics beyond the standard model [1]. The unknown origin of supersymmetry breaking is parametrized by soft supersymmetry breaking (SSB) terms in the Lagrangian, i.e.,

$$-\mathcal{L}_{\rm soft} = m_i^2|\Phi_i|^2 + B_{ij}\Phi_i\Phi_j + A_{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M_\alpha\lambda_\alpha^2 + \text{H.c.}, \tag{1}$$

where Φ_i (λ_α) are the scalar (gaugino) fields. Equation (1) introduces a large number of new arbitrary parameters and is impractical for phenomenological studies. A better situation appears if the supersymmetry is a local symmetry, i.e., supergravity. In the minimal supergravity model, the effective Lagrangian below $M_P \equiv M_{\rm Planck}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV [1] consists of a global supersymmetric theory with SSB terms as in Eq. (1) but with universal values [1], i.e.,

$$m_i^2 \equiv m_0^2, \qquad B_{ij} \equiv B_0\mu_{ij}, \qquad A_{ijk} \equiv A_0 Y_{ijk}, \qquad M_\alpha \equiv M_{1/2}, \tag{2}$$

where μ_{ij} and Y_{ijk} are, respectively, the bilinear and trilinear couplings in the superpotential. The deviations from the universal boundary condition (2) at lower scales are calculated using renormalization group (RG) methods, and given only four soft parameters one can predict the superpartner mass spectrum.

If the MSSM is embedded in a grand-unified theory (GUT) at the scale $M_G \approx 10^{16}$ GeV suggested by coupling constant unification [2], then the evolution of the parameters between M_P and M_G depends on the GUT and is strongly model dependent. Nevertheless, it is often assumed that applying (2) at M_G rather than at M_P is a good approximation, because M_G is close to M_P. One then uses the generic MSSM RG equations (RGEs) between M_G and the weak scale [3,4].

In this Letter we will examine the corrections to the SSB parameters arising from their evolution between the Planck and the GUT scales. We will show that these corrections induce large deviations in the SSB parameters from their universal values. The corrections are typically proportional to $(\alpha/\pi)m^2\ln(M_P/M_G)$, where α and m^2 are a generic coupling and soft mass parameter, respectively. Although these corrections are not enhanced by large logarithms, they can be significant due to the following.

(1) The number of particles above M_G, N, is large as a result of the large symmetry group, and one roughly has $\alpha/\pi \to N\alpha/\pi$. (See also Ref. [5].)

(2) Large Yukawa couplings that are typically present in GUTs and that grow with the energy. In addition to the large top Yukawa coupling, one has to introduce extra large couplings to avoid too large of a proton decay rate.

The above corrections depend on the details of the GUT model and represent uncertainties in the low-energy predictions. Gravitational and other effects could also affect the boundary condition (2) and would only add to the uncertainty.

For definiteness and simplicity we consider the minimal SU(5) model. The Higgs sector of the model consists of three supermultiplets, $\Sigma(\mathbf{24})$ in the adjoint representation, $\mathcal{H}_1(\bar{\mathbf{5}})$ and $\mathcal{H}_2(\mathbf{5})$, each containing a SU(2) doublet H_i and a color triplet H_{C_i}. The matter superfields are in the $\bar{\mathbf{5}} + \mathbf{10}$ representations $\phi(\bar{\mathbf{5}})$ and $\psi(\mathbf{10})$. The superpotential is given by

$$W = \mu_\Sigma \operatorname{tr}\Sigma^2 + \frac{1}{6}\lambda'\operatorname{tr}\Sigma^3 + \mu_H\mathcal{H}_1\mathcal{H}_2 + \lambda\mathcal{H}_1\Sigma\mathcal{H}_2 + \frac{1}{4}h_t\epsilon_{ijklm}\psi^{ij}\psi^{kl}\mathcal{H}_2^m + \sqrt{2}h_b\psi^{ij}\phi_i\mathcal{H}_{1_j}. \tag{3}$$

[We define $\Sigma = \sqrt{2}T_a w_a$ where T_a are the SU(5) generators with $\operatorname{tr}\{T_aT_b\} = \delta_{ab}/2$, and we only consider Yukawa couplings for the third generation.] Dimension-five operators induced by the color triplet give large contributions $\propto 1/M_{H_C}^2$ to the proton decay rate [6]. To suppress such operators, the mass of the color triplets has to be large, $M_{H_C} \equiv (\lambda/g_G)M_V \gtrsim M_V$, where M_V is the heavy gauge boson mass, implying $\lambda \gtrsim g_G \approx 0.7$. Thus, one-loop corrections proportional to λ produce important effects. Below M_P the effective Lagrangian also contains the SSB terms

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$$-\mathcal{L}_{\rm soft} = m_{\mathcal{H}_1}^2|\mathcal{H}_1|^2 + m_{\mathcal{H}_2}^2|\mathcal{H}_2|^2 + m_\Sigma^2 {\rm tr}\{\Sigma^\dagger\Sigma\} + m_5^2|\phi|^2 + m_{10}^2{\rm tr}\{\psi^\dagger\psi\}$$
$$+ \Big[B_\Sigma\mu_\Sigma {\rm tr}\,\Sigma^2 + \frac{1}{6}A_{\lambda'}\lambda'{\rm tr}\,\Sigma^3 + B_H\mu_H\mathcal{H}_1\mathcal{H}_2 + A_\lambda\lambda\mathcal{H}_1\Sigma\mathcal{H}_2$$
$$+ \frac{1}{4}A_t h_t\epsilon_{ijklm}\psi^{ij}\psi^{kl}\mathcal{H}_2^m + \sqrt{2}A_b h_b\psi^{ij}\phi_i\mathcal{H}_{1j} + \frac{1}{2}M_5\lambda_\alpha\lambda_\alpha + {\rm H.c.}\Big]. \qquad (4)$$

From M_P to M_G the SSB terms evolve according to the RGEs of the SU(5) model with Eq. (2) as a boundary condition. Thus, we expect a breakdown of universality at M_G for SSB parameters of fields that are in different SU(5) representations. The SU(5) RGEs for the SSB parameters and Yukawa couplings are given by

$$\frac{dm_{10}^2}{dt} = \frac{1}{8\pi^2}\Big[3h_t^2(m_{\mathcal{H}_2}^2 + 2m_{10}^2 + A_t^2) + 2h_b^2(m_{\mathcal{H}_1}^2 + m_{10}^2 + m_5^2 + A_b^2) - \frac{72}{5}g_G^2M_5^2\Big],$$
$$\frac{dm_5^2}{dt} = \frac{1}{8\pi^2}\Big[4h_b^2(m_{\mathcal{H}_1}^2 + m_{10}^2 + m_5^2 + A_b^2) - \frac{48}{5}g_G^2M_5^2\Big],$$
$$\frac{dm_{\mathcal{H}_1}^2}{dt} = \frac{1}{8\pi^2}\Big[4h_b^2(m_{\mathcal{H}_1}^2 + m_{10}^2 + m_5^2 + A_b^2) + \frac{24}{5}\lambda^2(m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_\Sigma^2 + A_\lambda^2) - \frac{48}{5}g_G^2M_5^2\Big],$$
$$\frac{dm_{\mathcal{H}_2}^2}{dt} = \frac{1}{8\pi^2}\Big[3h_t^2(m_{\mathcal{H}_2}^2 + 2m_{10}^2 + A_t^2) + \frac{24}{5}\lambda^2(m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_\Sigma^2 + A_\lambda^2) - \frac{48}{5}g_G^2M_5^2\Big],$$
$$\frac{dm_\Sigma^2}{dt} = \frac{1}{8\pi^2}\Big[\frac{21}{20}\lambda'^2(3m_\Sigma^2 + A_{\lambda'}^2) + \lambda^2(m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_\Sigma^2 + A_\lambda^2) - 20g_G^2M_5^2\Big],$$
$$\frac{d\lambda'}{dt} = \frac{\lambda'}{16\pi^2}\Big[\frac{63}{20}\lambda'^2 + 3\lambda^2 - 30g_G^2\Big], \quad \frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2}\Big[\frac{21}{20}\lambda'^2 + 3h_t^2 + 4h_b^2 + \frac{53}{5}\lambda^2 - \frac{98}{5}g_G^2\Big],$$
$$\frac{dh_t}{dt} = \frac{h_t}{16\pi^2}\Big[9h_t^2 + 4h_b^2 + \frac{24}{5}\lambda^2 - \frac{96}{5}g_G^2\Big], \quad \frac{dh_b}{dt} = \frac{h_b}{16\pi^2}\Big[10h_b^2 + 3h_t^2 + \frac{24}{5}\lambda^2 - \frac{84}{5}g_G^2\Big], \qquad (5)$$

where $t = \ln Q$. The RGE for the gauge coupling in $d\alpha_G/dt = -3\alpha_G^2/2\pi$, and similarly $dM_5/dt = -3\alpha_G M_5/2\pi$. The RGEs for the trilinear SSB parameter A_i can be obtained from the RGEs of the corresponding Yukawa coupling Y_i by

$$\frac{dY_i}{dt} = \frac{Y_i}{16\pi^2}[a_{ij}Y_j^2 - bg_G^2] \to \frac{dA_i}{dt} = \frac{1}{8\pi^2}[a_{ij}Y_j^2A_j - bg_G^2M_5]. \qquad (6)$$

We can omit the RGEs for μ_Σ, μ_H, B_Σ, and B_H, which are arbitrary parameters that decouple from the rest of the RGEs.

The evolution of the SSB parameters from M_P to M_G is dictated by a competition between the positive Yukawa terms (i.e., scalar contributions) and the negative gauge terms (i.e., gaugino contributions) in the RGEs. We can distinguish two scenarios: (A) For moderate values of $M_{1/2} \equiv M_5(M_P)$ the contribution from the gauge sector is small. In this case, the RGEs of $m_{\mathcal{H}_1}^2$ and $m_{\mathcal{H}_2}^2$ have a large contribution proportional to λ^2 and both masses are diminished as the energy scale decreases. For $h_t \gg h_b, m_{\mathcal{H}_2}^2$ decreases faster than $m_{\mathcal{H}_1}^2$, but also m_{10}^2 (for the third family) is diminished in that case. (B) For large values of $M_{1/2}$ the RGEs are dominated by the negative gaugino contribution so that all the SSB parameters increase as the energy scale decreases. The scalar masses are enhanced by an additive factor

$$\Delta m_i^2 = -\frac{c_i}{3}\Big[1 - \frac{1}{[1 + (3\alpha_G/2\pi)\ln(M_G/M_P)]^2}\Big]M_{1/2}^2, \qquad (7)$$

where $c_i = \frac{72}{10}$ $(\frac{24}{5})$ for i in the **10** (**5**) representation. One has $\Delta m_i^2 \approx 0.5(0.3)M_{1/2}^2$.

Examples of scenarios (A) and (B) are given in Figs. 1(a) and 1(b), respectively. We see that the violation of the universality of the SSB parameters at M_G can be substantial. In particular, the soft masses of the Higgs fields are typically split from the matter field masses. For $M_{H_C} = 1.4M_V$ (i.e., $\lambda \approx 1$ at M_G) the splitting can be as large as 100%.

In order to analyze the implications of these soft mass splittings in the supersymmetric spectrum and phenomenology, we have to run the SSB parameters from M_G down to m_Z [7]. Below M_G the effective theory corresponds to the MSSM:

$$W = \mu H_1H_2 + h_tQH_2U + h_bQH_1D + h_\tau LH_1E, \qquad (8)$$

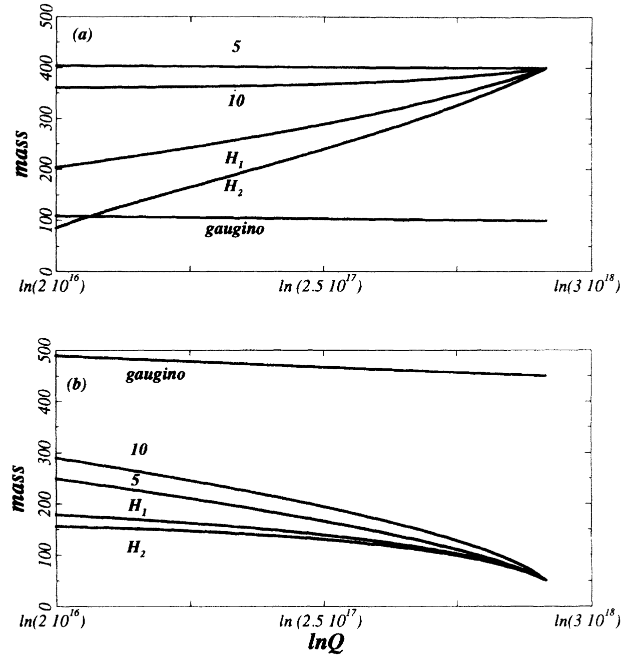

FIG. 1. The evolution of the soft mass parameters of the third family, $\phi(\bar{\mathbf{5}})$ and $\psi(\mathbf{10})$, the Higgs $\mathcal{H}_i$ and the gaugino between the Planck and grand-unification scales for (a) scenario (A) with $m_0 = A_0 = 400$ GeV and $M_{1/2} = 100$ GeV; and (b) scenario (B) with $m_0 = A_0 = 50$ GeV and $M_{1/2} = 450$ GeV. In both cases $m_t = 160$ GeV; $\tan\beta = 1.25$; and the boundary conditions $\lambda = 1$ (i.e., $M_{H_C} = 1.4 M_V$) and $\lambda' = 0.1$ at M_G are assumed. All masses are in GeV.

where $Q(L)$ and $U, D(E)$ are, respectively, the quark (lepton) SU(2) doublet and singlet superfields. The tree-level matching conditions of the SSB parameters between the SU(5) model and the MSSM are $m^2_{\mathcal{H}_i}(M_G) = m^2_{H_i}(M_G)$, $m^2_{10}(M_G) = m^2_{Q,U,E}(M_G)$, and $m^2_{\bar{5}}(M_G) = m^2_{D,L}(M_G)$. In this Letter we present only a qualitative analysis of the GUT effects in the low energy quantities. A comprehensive numerical study, together with the details of the numerical procedures, will be given elsewhere.

In scenario (A) the parameters $m^2_{\mathcal{H}_i}$ have substantial shifts and the parameters $m^2_{H_i}(m_Z)$ are modified. The latter enter the minimization conditions of the weak-scale Higgs potential. Therefore, corrections to $m^2_{H_i}$ modify the region of the MSSM parameter space that is consistent with electroweak symmetry breaking (EWSB). The μ parameter is also affected by the GUT corrections to $m^2_{H_i}$ since it is extracted from the minimization conditions of the weak-scale Higgs potential [1,3,4]. This leads to corrections to observables that depend on μ, such as the Higgsino mass, the Higgsino-gaugino mixing, and the left-right scalar quark mixing. Also, consistency with constraints from color and charge breaking [4] can be affected. *A priori,* one would expect that the Higgs boson masses are also affected. Notice, however, that the latter depend only on the sum $m^2_{H_i} + |\mu|^2$, which is not modified significantly (the shift in $m^2_{H_i}$ is approximately compensated by the shift in $|\mu|^2$). The masses of the lightest chargino and neutralinos are typically proportional to $M_{1/2}$ and are only slightly affected. There are large deviations for the third generation squark masses. Not only can m^2_{10} be shifted for large h_t [see Fig. 1(a)] but, in addition, the evolution of m^2_Q and m^2_U from M_G down to m_Z depends on the value of $m^2_{H_2}$ that is sensitive to the GUT physics.

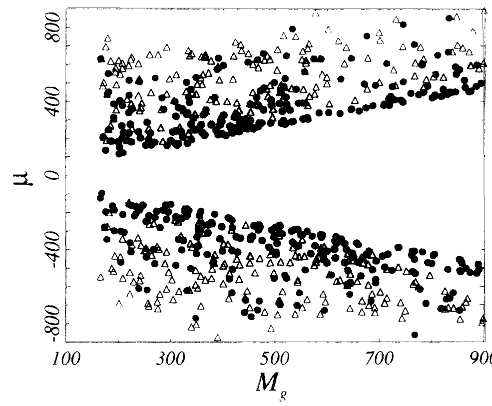

FIG. 2. Scatter plot of the μ parameter vs the gluino mass within the allowed parameter space (see above) for $m_t = 180$ GeV and $\tan\beta = 42$. Triangles (filled circles) correspond to universality [Eq. (2)] at the Planck (grand-unification) scale. λ and λ' are as in Fig. 1. All masses are in GeV.

In scenario (B), all the scalar masses are shifted [Eq. (7)] from their universal value at M_P. The squark squared masses, however, receive large corrections in the running from M_G down to m_Z ($\approx 6M^2_{\frac{1}{2}}$) so that (7) represents only a few percent of their values. This is not the case for the sleptons, where such corrections are much smaller [$\approx 0.5M^2_{\frac{1}{2}}$ ($0.1M^2_{\frac{1}{2}}$) for $\tilde{e}_L$ ($\tilde{e}_R$)] and the increment (7) can even double their masses.

In Figs. 2 and 3 we present examples of the GUT effects in the low-energy predictions. In Fig. 2 we take a large $\tan\beta \equiv \langle H_2\rangle/\langle H_1\rangle$ and show the allowed values (i.e., consistent with EWSB and experimental bounds on the particle masses—see [4]) of μ vs the gluino mass $M_{\tilde{g}}$ when the evolution from M_P to M_G is considered (triangles) or neglected (filled circles). The correlation between μ and $M_{\tilde{g}}$, which exists when neglecting the M_P to M_G evolution [3,4], is "smeared" when that evolution is included and μ is larger in the latter case. We also find that $m^2_{H_i}(m_Z)$ are often both negative when the GUT effects are considered [$m^2_{\mathcal{H}_i}(m_G)$ are diminshed together because $h_t \approx h_b$], which is inconsistent with EWSB. The allowed parameter space is then significantly reduced. In Fig. 3 we show the light t-scalar mass $m_{\tilde{t}_1}$ vs $M_{\tilde{g}}$ for $\tan\beta \approx 1$ ($h_t \approx 1$). μ is now large (~ 1 TeV) and is less sensitive to the GUT effects. Corrections to $m_{\tilde{t}_1}$ are mainly via the diminished $h_t^2 m^2_{H_2}$ term in the respective RGEs below M_G. $\tilde{t}_1$ is therefore heavier and some points which correspond to a tachyonic t scalar and are excluded when the M_P to M_G evolution is neglected can be allowed. Note also that the correlation between $m_{\tilde{t}_1}$ and $M_{\tilde{g}}$ is weakened by the GUT corrections. We find that correlations between predictions are generically modified due to the model-dependent smearing from the M_P to M_G evolution.

Even if the universal boundary condition (2) for the SSB parameters is taken at M_G, there is some arbitrariness in the value of M_G due to mass splittings between the particles at the GUT scale, i.e., threshold effects. The largest threshold corrections to the SSB parameters arise from the SU(2) triplet and singlet components of the Σ superfield, Σ_3 and Σ_1, and are given by

$$m_{H_i}^2(M_G) = m_{\mathcal{H}_i}^2(M_G) + \frac{\lambda^2}{4\pi^2}(m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_\Sigma^2 + A_\lambda^2)\left[\frac{3}{4}\ln\frac{M_{\Sigma_3}}{M_G} + \frac{3}{20}\ln\frac{M_{\Sigma_1}}{M_G}\right], \quad (9a)$$

$$\Delta A_{t,b}(M_G) = \frac{\lambda^2}{4\pi^2}A_\lambda\left[\frac{3}{4}\ln\frac{M_{\Sigma_3}}{M_G} + \frac{3}{20}\ln\frac{M_{\Sigma_1}}{M_G}\right]. \quad (9b)$$

(We identify $M_G = \max\{M_V, M_{H_C}\}$. Details will be given elsewhere.) Since the masses M_{Σ_3} and $M_{\Sigma_1} \equiv 0.2M_{\Sigma_3}$ can be much smaller than M_G, these corrections can be substantial. For $M_{\Sigma_3} \approx 10^{-2}M_G$ and $\lambda \approx 1$, we have $m_{H_i}^2(M_G) \approx 0.6m_0^2$.

In extended supersymmetric GUTs the corrections could be larger. In extended SU(5) large representations are introduced and the positive scalar contribution to the RGEs is larger. Hence, the SSB parameters decrease faster with the scale. However, one has to be aware of a possible breakdown of perturbation theory. An interesting scenario occurs in models in which $\mathcal{H}_1$ and $\mathcal{H}_2$ couple with different strength to the other Higgs supermultiplets. For example, in the missing partner SU(5) model $W = \lambda_1\mathcal{H}_1\Sigma(\mathbf{75})\Phi(\mathbf{50}) + \lambda_2\mathcal{H}_2\Sigma(\mathbf{75})\Phi(\overline{\mathbf{50}}) + \cdots$, and if $\lambda_2 > \lambda_1$, the evolution from M_P to M_G splits the two Higgs scalar masses. That splitting can now affect the low-energy Higgs boson masses and reduce the degree of fine-tuning that is typically required to achieve EWSB in scenarios with large $\tan\beta$ (in which the Higgs scalar masses are not split by Yukawa interactions). In models where the rank of the group is larger than the rank of the SM group, e.g., SO(10), one has an additional contribution to the scalar masses that arises from the D terms [8]. However, because the light fields can be embedded in fewer representatives the $M_P - M_G$ evolution may split fewer SSB parameters.

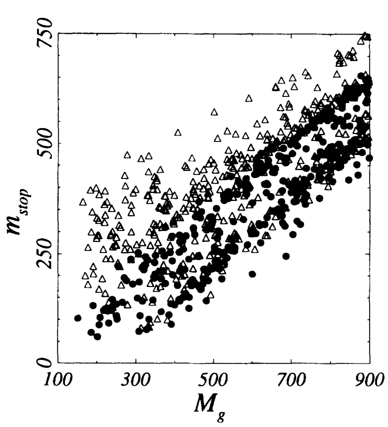

FIG. 3. Same as in Fig. 2 except the light-t-scalar mass vs the gluino mass and m_t = 160 GeV and $\tan\beta$ = 1.25.

To summarize, we have shown that large deviations from universality at M_G can be generated when considering (i) the model-dependent evolution from M_P to M_G and (ii) threshold corrections at M_G. We have also shown that the above leads to a modification of the allowed parameter space, smears predicted correlations, and affects certain low-energy predictions such as the μ parameter and the t-scalar mass. These corrections have to be considered as uncertainties when analyzing possible future evidence for supersymmetry. On the other hand, such corrections could provide a probe of the high scale.

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