Phase-only filter with improved discrimination

E. Ahouzi, J. Campos, and M. J. Yzuel

Department of Physics, Autonomous University of Barcelona, E-08193 Bellaterra, Spain

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The optimization of a phase-only filter (POF) in terms of discrimination capability is presented. The notion of a phase-difference histogram and its modification are proposed for selecting the support function of the POF. Some numerical results obtained with the conventional POF and the optimized POF are given. The discrimination capability is increased significantly.

In the field of pattern recognition by optical correlation methods, the use of phase-only filters (POF's) has been well investigated.[1–3] In the literature, several types of optimization of POF's in terms of some parameters that evaluate the correlation have been proposed. In Ref. 2 Kumar and Bahri introduced the notion of optimal support for a POF in the sense of maximizing the signal-to-noise ratio, and they showed that it is necessary to block certain spatial frequencies. The support function indicates which pixels in the filter have magnitudes of one and which pixels have zero magnitudes. The support function of a POF for maximizing the peak-to-correlation energy (PCE) criterion has been proposed by Kumar *et al.*[3] Other support functions to optimize different criteria or multicriteria are studied in Ref. 4. The discrimination capability (DC) is one of the most important parameters for evaluating the performance of a filter for pattern recognition. Optical correlation in real time demands filter designs that maximize this parameter. In Refs. 5 and 6 some optimization in terms of correlation discrimination of a POF and a binary POF are proposed. In Ref. 5 Awwal *et al.* proposed an amplitude-modulated POF for achieving improved correlation discrimination, in which the POF is associated with the inverse amplitude of the Fourier transform of the target. In Ref. 6 the ternary-phase-amplitude filter is proposed, in which a zero-modulation state is added to the binary POF. The zero-modulation state of ternary filters is used to block the energy in those regions of spatial frequency that have a high ratio of nontarget-to-target energy. In all these designs the support functions are calculated when only the amplitude information is taken into account. Nevertheless, the phase distribution plays a crucial role in the DC. Recently Zhou *et al.*[7] proposed a technique based on phase-difference prewhitening, and they obtained an improvement in interclass multiobject discrimination. In this Letter we introduce the notion of a phase-difference histogram, and we propose a method that optimizes the DC for a POF with a support function based on the phase information. We show a formal procedure based on the modification of a phase-difference histogram by blocking some frequencies. We investigate the DC by computer simulation, and we show the improvement in the DC that is obtained with this method.

Let us assume that $t(x,y)$ and $d(x,y)$ represent the pattern to be recognized and the patterns to be rejected, respectively, when the target $[t(x,y)]$ is centered at the origin and the nontarget $[d(x,y)]$ is placed in the scene at coordinates such that the maximum of the cross correlation appears at the origin. Let

$$T(u,v) = |T(u,v)|\exp[i\Phi_t(u,v)],$$
$$D(u,v) = |D(u,v)|\exp[i\Phi_d(u,v)] \tag{1}$$

be the Fourier transforms of the target $[t(x,y)]$ and the nontarget $[d(x,y)]$, respectively, where Φ_t and Φ_d are their phase distributions. When the POF is used, the correlation functions in the Fourier domain for the autocorrelation and the cross correlation are given by

$$C_t(u,v) = |T(u,v)|,$$
$$C_d(u,v) = |D(u,v)|\exp[i\Delta\Phi(u,v)], \tag{2}$$

respectively, where $\Delta\Phi(u,v) = \Phi_d(u,v) - \Phi_t(u,v)$ is the phase difference. The DC is a parameter that measures the ability of the correlator to discriminate between two similar input objects. This parameter may be defined[7] as the ratio of the difference between the autocorrelation and the cross correlation to the autocorrelation; let us say that

$$\mathrm{DC} = 1 - \frac{|\iint |D(u,v)|\exp[i\Delta\Phi(u,v)]\mathrm{d}u\mathrm{d}v|^2}{|\iint |T(u,v)|\mathrm{d}u\mathrm{d}v|^2}. \tag{3}$$

Let us define the set $(S_j)_{1\leq j\leq N}$ as the set of pixels in the Fourier domain that have a difference of phases in the interval $(j-1)\delta\phi < \Delta\Phi(u,v) \leq j\delta\phi$ as

$$S_j = \{(u,v) \in \mathrm{FP}/(j-1)\delta\phi < \Delta\Phi(u,v) \leq j\delta\phi\}, \tag{4}$$

where $\delta\phi$ is the phase-quantization step, defined as $\delta\phi = 2\pi/N$, N is the number of phase-quantization levels, and FP is the Fourier plane. Then the phase of the pixels in sector S_j are approximated by $\Delta\Phi_j =$

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$j\delta\phi$. The parameter DC will be approximated by the following equation:

$$\mathrm{DC}_N = 1 - \frac{\left|\sum_j^N \exp(i\Delta\Phi_j) \iint_{s_j} |D(u,v)| \mathrm{d}u\mathrm{d}v\right|^2}{\left|\sum_j^N \iint_{s_j} |T(u,v)| \mathrm{d}u\mathrm{d}v\right|^2}. \quad (5)$$

Let us define the weighted phase-difference histogram P_j as the sum of the amplitude values of the Fourier transform of a nontarget over sector S_j. Then P_j is given by

$$P_j = \iint_{S_j} |D(u,v)| \mathrm{d}u\mathrm{d}v. \quad (6)$$

The method discussed here enhances the DC by modifying the weighted phase-difference histogram of a given target and a given nontarget. The enhancement that we obtain depends on the number of quantization levels N. By substitution of Eq. (6) into Eq. (5) and taking into account that if two sectors are in phase opposition they fulfill $\Delta\Phi_{N/2+j} = \Delta\Phi_j + \pi$, the DC becomes[5]

$$\mathrm{DC}_N = 1 - \frac{\left|\sum_j^{N/2} (P_j - P_{N/2+j}) \exp(i\Delta\Phi_j)\right|^2}{\left|\sum_j^N \iint_{S_j} |T(u,v)| \mathrm{d}u\mathrm{d}v\right|^2}. \quad (7)$$

We improve the DC by maximizing DC_N, which we obtain by minimizing the difference $|P_j - P_{N/2+j}|$ for any $j = 1 \ldots N/2$. This can be done when some frequencies are blocked. By using this formulation we propose to add zero modulation to the POF by selecting the support function that minimizes the difference $|P_j - P_{N/2+j}|$. The resulting modified histogram will satisfy the following condition:

$$|P_j - P_{N/2+j}| \leq \varepsilon. \quad (8)$$

The discrimination improvement of the POF, as well as the other performance measurements that evaluate the behavior of the filter, such as the PCE, light efficiency[8] (η), and the signal-to-noise ratio, will depend on the value of ε. DC_N is bounded by a polynomial function of ε, as shown in the following relation:

$$\mathrm{DC}_N(\varepsilon) \geq 1 - \frac{N}{2E_{\mathrm{nbt}}} \varepsilon^2, \quad (9)$$

where E_{nbt} is the total nonblocked energy of the target. The boundary established in relation (9) for DC_N shows that the DC is improved for small values of ε. The dimension of ε^2 corresponds to energy.

To maximize DC_N according to Eq. (7) and to take into account the condition given in relation (8), we order, according to the modulus of the Fourier transform, the samples of nontarget in each sector S_j and the samples that correspond to the sector in phase opposition to S_j. The order is established from the highest to the lowest values. The proposed algorithm consists of choosing, for a given ε, a minimum number of pixels for which the amplitude will become zero. If, for instance, $P_j \geq P_{N/2+j}$, we make the amplitude equal to zero in S_j starting with the first pixels of the ordered series. In each step, P_j decreases until the condition given in relation (8) is satisfied.

We investigate the comparative performance in terms of the DC between similar objects for the conventional POF and the optimized POF by the phase-difference histogram method. To illustrate the improvements of the discrimination with the proposed method, we perform numerical experiments by using 26 × 26 alphanumeric characters placed in a 128 × 128 array. The scene that we study is shown in Fig. 1. We have selected two similar letters: **E** and **F** (**F** is contained in **E**). The target is **F**. Figure 2 shows the correlation planes for the POF [Fig. 2(a)] and the optimized POF [Fig. 2(b)] for $N = 100$ and $\varepsilon = 10$. The DC improvement is clearly shown by a comparison of Fig. 2(b) with Fig. 2(a). The normalized cross-correlation maximum for **E** decreases from 0.88 in Fig. 2(a) to 0.04 in Fig. 2(b). For the classical matched filter, the maxima of the autocorrelation and the cross correlation (**E** and **F**) are equal. The price that we pay for an improvement in the discrimination is a decrease in efficiency. This happens normally in

Fig. 1. Input scene.

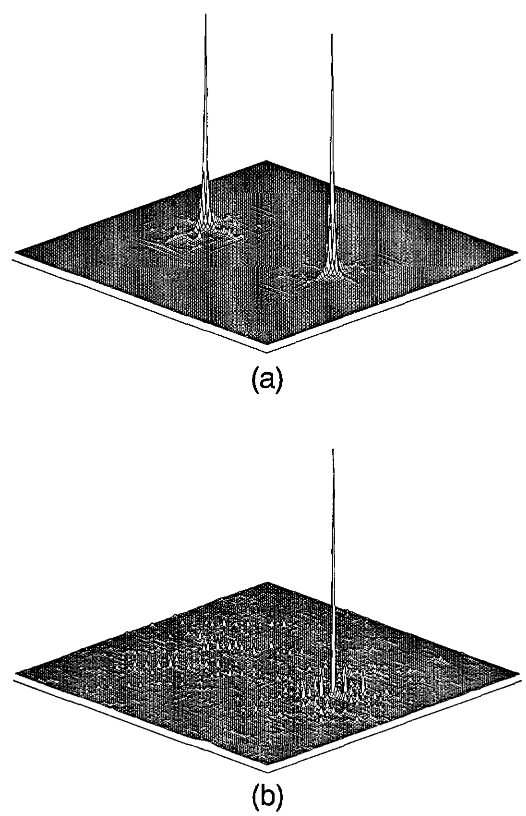

Fig. 2. Correlation plane with (a) a conventional POF, (b) an optimized POF with $N = 100$, $\varepsilon = 10$.

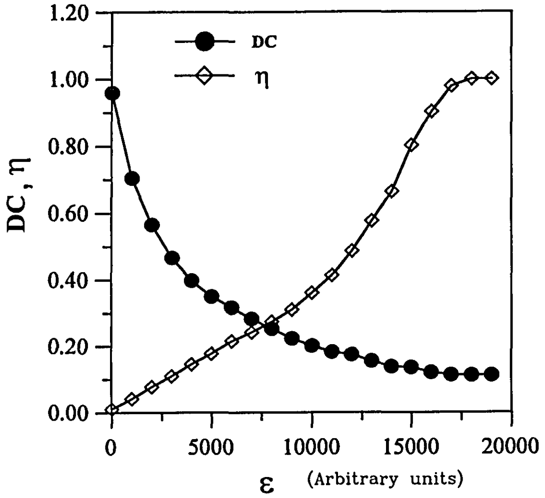

Fig. 3. Variation of the efficiency (η) and the DC as a function of ε for $N = 100$. η is evaluated for the autocorrelation.

the methods that increase discrimination. Figure 3 shows the efficiency (η) and the DC as functions of ε. We can see that, when the discrimination capability is improved, the efficiency decreases. The relation between the improvement of the discrimination and the deterioration of the efficiency depends on the scene used in the experiment. Because of the great similarity between **E** and **F** we need a rather small value of ε to get good discrimination (Fig. 3). This fact leads to a high decrease in the efficiency. In a future study we will investigate the selection of the pixels to be blocked in each sector to improve other performance criteria, such as the signal-to-noise ratio, the PCE, and η. We will also investigate the generalization of the method to take into account several objects to be rejected.

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