Full polarization chaos in a pump-polarization modulated isotropic cavity laser

A. Kul'minskii and R. Vilaseca

Department de Física i Enginyeria Nuclear, Universitat Politécnica de Catalunya, Colom 11, E-08222 Terrassa, Spain

R. Corbalán

Department de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain

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We study the dynamic behavior of an optically pumped $J = 0 \rightarrow J = 1 \rightarrow J = 0$ laser operating with an isotropic ring cavity and a linearly polarized pump field whose direction of polarization is modulated by the sinusoidal law $\theta(t) = m \sin \Omega t$. Modulation frequencies Ω of the same order of magnitude as the transverse relaxation rate of the laser transition are considered here. At large enough modulation amplitudes, and for a detuned cavity, we obtain fully developed polarization chaos, which affects both the ellipticity and the orientation of the polarization ellipse as well as the laser intensity.

Until recently studies of laser dynamics have focused on laser systems in which the vector character of the light field does not play a significant role.[1] Recently, however, much attention has been devoted to laser systems in which the vector field orientation can also be involved in the global laser dynamics.[2–4]

In Ref. 2 we considered the case of an optically pumped $J = 0 \rightarrow J = 1 \rightarrow J = 0$ laser operating with an isotropic ring cavity and an axial magnetic field. Although the isotropic cavity allows the vector field to evolve freely, the presence of a pump field with fixed linear polarization breaks the spatial isotropy of the system with respect to rotations around the cavity axis and leads to gain anisotropy. As a result, we found that, although the laser field was linearly polarized in all the cases (assuming resonance conditions for pump and laser fields), it could show rich dynamics that affect simultaneously the modulus and the orientation of the laser field vector.

Here we study the dynamics of the same laser as in Ref. 2 but now without a magnetic field and with the linear polarization of the resonant pump beam modulated according to the sinusoidal law $\theta(t) = m \sin \Omega t$, where $\theta(t)$ represents the instantaneous angle between the linear polarization of the pump beam and the y axis (the pump and laser fields propagate along the axis of the ring cavity in the z direction) and m and Ω are the modulation amplitude and frequency, respectively. In practice this modulation can be accomplished by means of a longitudinal magnetic field acting on the pump laser or passing the pump beam through an electro-optical crystal possessing the transverse Pockels effect.[5] Because of modulation the laser system becomes nonautonomous. Moreover, for frequencies Ω of the order of the transverse relaxation rate $\gamma_\perp$ of the lasing transition and large amplitudes m, one can expect that the gain anisotropy associated with the pump polarization will be largely averaged, opening the possibility of observing polarization dynamics richer than those described in Ref. 2. In fact, as is shown below, for these conditions of modulation two main features distinguish the modulated laser from the unmodulated one: First, the modulated laser is more unstable in the sense that chaotic dynamics appears at pump intensities reduced by a factor of ~ 50. Second, the polarization of the generated laser is not always linear; for a detuned cavity it can be elliptical, with the ellipticity and the orientation of the polarization ellipse changing chaotically.

The equations describing our system are given in Ref. 6, and one easily sees that $2\theta(t)$ represents the phase difference between the left and right circular harmonics of the pump beam. It is worth mentioning that the modulation that we are considering acts only on the polarization state of a pump light beam. By contrast, modulation of a laser system is usually accomplished by action on the pump strength (or gain), on cavity parameters such as losses, detuning, and anisotropy, or on an applied magnetic field.[1,3,7] The generated field will be expressed as the superposition of a left (σ_+) and a right (σ_-) circularly polarized field of slowly varying Rabi frequency $\alpha_+(t)$ and $\alpha_-(t)$ and phase $\phi_+(t)$ and $\phi_-(t)$, respectively, which are coupled to the transitions $|J = 1, M = +1\rangle \rightarrow |J = 0, M = 0\rangle$ and $|J = 1, M = -1\rangle \rightarrow |J = 0, M = 0\rangle$, respectively. We use the amplitude m and the frequency Ω of modulation, as well as the cavity detuning, as the main control parameters. We fix the pump beam Rabi frequency at $\beta = 0.2\gamma_\perp$ and the relaxation parameters at the values given in Ref. 6 (with $\Gamma = 0.5\gamma_\perp$), which correspond to the 81.5-μm emission from an ammonia laser pumped at 10.8 μm by a N_2O laser.[8] Note that the adopted pump intensity (β^2) is approximately seven times weaker than the one corresponding to the instability threshold of the unmodulated laser.[2,6] Without modulation ($m = 0$) for $\beta = 0.2\gamma_\perp$ the laser emits in a stable regime a field with a linear polarization parallel to that of the pump beam.[2]

The results obtained after a scanning of the parameter space (m, Ω, Δ, β) in our system will be described

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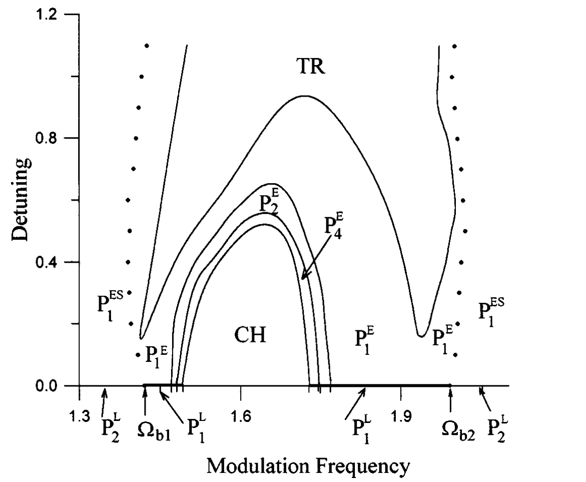

Fig. 1. Phase diagram on the modulation frequency $(\Omega/\gamma_\perp)$ versus the cavity detuning $(\Delta/\gamma_\perp)$ plane for a modulation amplitude $m = 5$ rad: P_n, periodic oscillation of the laser intensity; TR, torus attractor; CH, chaos. See text for other details.

elsewhere in detail. Here we concentrate on the most interesting situation, which occurs for $\Omega \sim \gamma_\perp$. At fixed Ω we find two qualitatively different dynamics, depending on whether the modulation amplitude is small or large. To characterize these two regimes we describe the results for $m = 2$ rad (small) and $m = 5$ rad (large).

For $m = 2$ rad and $\Delta = 0$, the generated field polarization is always linear and the dynamics are periodic or quasi-periodic, depending on the value of Ω that affects both the amplitude and the orientation of the laser field vector. Periodic dynamics appear to be associated with limit cycles of the first or second kind. In the first case all the field and material variables are periodic, whereas in the second case all are periodic except the phases ϕ_+ and ϕ_-. There is anholonomy of the phases, which leads to a rotation of the polarization plane of the generated field in spite of the fact that the amplitude of the oscillation that we impose on the pump field is less than π. Superimposed upon this rotation there is an oscillation at frequency Ω. The relations $\alpha_+(t) = \alpha_-(t)$ and $\phi_+(t) = -\phi_-(t)$ between the σ_+ and σ_- field components are verified.

Figure 1 is a phase diagram on the parameter plane (Ω, Δ) obtained by numerical integration of the laser equations in the case $m = 5$ rad. We analyze the results for $\Delta = 0$ first, and then those for $\Delta \neq 0$.

Tuned cavity ($\Delta = 0$). In this case, for all Ω, the relationship just mentioned between the σ_+ and σ_- field components holds, so that the generated field polarization remains linear. We find here periodic dynamics in laser intensity at frequencies 2Ω (domain P_2^L in Fig. 1) and Ω (P_1^L).[9] At Ω_{b1} and Ω_{b2} there is a change of behavior from P_2^L to P_1^L (or vice versa). When the modulation frequency is increased (decreased) from Ω_{b1} (Ω_{b2}) the P_1 attractor experiences period-doubling bifurcations and eventually enters the chaotic domain.

Detuned cavity ($\Delta \neq 0$). When the cavity is detuned, the generated field becomes elliptically polarized with $\alpha_+(t) \neq \alpha_-(t)$ and $\phi_+(t) \neq \phi_-(t)$. It is therefore convenient to introduce the following parameters: total laser intensity $I = (\alpha_+^2 + \alpha_-^2)/2$, ellipticity $\xi = (\alpha_+^2 - \alpha_-^2)/(\alpha_+^2 - \alpha_-^2)$, azimuth of the main axes of the polarization ellipse $\Phi = (\phi_+ - \phi_-)/2$, and mean field phase $\Phi_S = (\phi_+ + \phi_-)/2$. In the domains denoted P_1^{ES} and P_1^{E} in Fig. 1 the laser intensity oscillates periodically at frequency Ω, but in the P_1^{ES} domain $\alpha_+(t + \pi/\Omega) = \alpha_-(t)$ (i.e., the fields show the same time evolution except for a temporal shift π/Ω); in the P_1^{E} domain this equality no longer holds. The border between these two domains is represented by points in Fig. 1. In the P_1^{ES} domain the ellipticity shows symmetrical oscillations around zero whose amplitude increases with Δ and can reach the extreme values $\xi = +1$ and $\xi = -1$. This means that the laser field polarization evolves periodically according to this sequence: linear → elliptical right → linear → elliptical left → linear ..., and so on. In the domain P_1^{E} these ellipticity oscillations are shifted with respect to zero and are more irregular. The azimuth oscillates and rotates, as in resonance. For $\Delta \neq 0$ two independent solutions coexist that have the σ_+ and σ_- waves interchanged. They arise as a result of a symmetry breaking transformation that occurs when Δ is varied from zero, which affects only the variables ξ and Φ.

At the border with the torus domain in Fig. 1 the system enters a domain of quasi-periodic behavior. In this domain a new low frequency (envelope frequency) appears in the system's evolution, which is not commensurate with the modulation frequency Ω (fast oscillations). This quasi-periodic dynamics entails a symmetry-restoring process because, as can be seen from Fig. 2(a), the projection of the attractor on the (α_-, α_+) plane becomes symmetric. The trajectory in phase space connects the two coexisting periodic orbits that are found in the

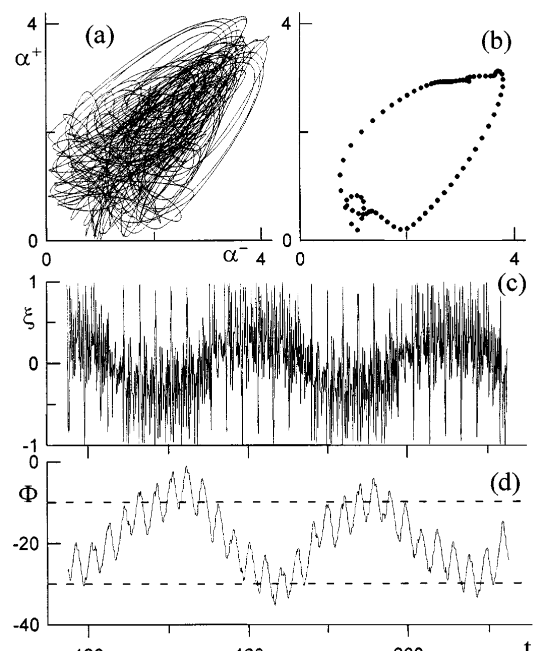

Fig. 2. Quasi-periodic dynamics for $m = 5$ rad, $\Omega = 1.6\gamma_\perp$, and $\Delta = \gamma_\perp$. (a) Attractor projection and (b) Poincaré stroboscopic map on the (α_-, α_+) plane. Time evolution of (c) the ellipticity ξ and (d) the azimuth Φ.

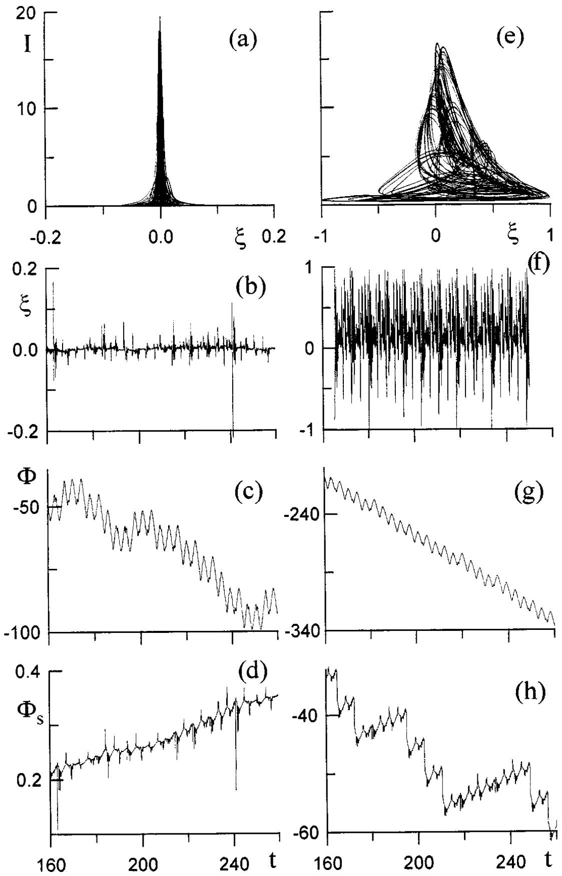

Fig. 3. Chaotic laser dynamics for $m = 5$ rad and $\Omega = 1.65\gamma_\perp$. For (a)–(d) $\Delta = 0.01\gamma_\perp$. For (e)–(h) $\Delta = 0.51\gamma_\perp$. (a), (e) Attractor projections on the (ξ, I) plane. Time evolution of (b), (f) the ellipticity ξ; (c), (g) the azimuth Φ; and (d), (h) the mean field phase Φ_S.

P_1^E domain, the alternation between them taking place at the new low frequency. Figures 2(b)–2(d) represent the corresponding Poincaré stroboscopic map and time evolution of the ellipticity ξ and the azimuth Φ, respectively. The ellipticity changes between right and left circular polarization. Notice that superimposed upon the oscillations and the irregular rotation of the azimuth there is a constant rotation at a small rate [there is a small global negative slope in the trace of Fig. 2(d)], which gives a measure of the asymmetry in the mean frequency pulling affecting the σ_+ and σ_- fields in this case with $\Delta \neq 0$.

Figure 3 shows the results of the laser dynamics in the chaotic domain of Fig. 1 for $\Omega = 1.65\gamma_\perp$. For Figs. 3(a)–3(d) (i.e., figures at the left in Fig. 3) the detuning is small, $\Delta = 0.01\gamma_\perp$; for Figs. 3(e)–3(h) (figures at the right) the detuning is larger, $\Delta = 0.51\gamma_\perp$. In both cases we observe a fully developed chaos that affects not only the light intensity but also the ellipticity ξ, the azimuth Φ, and the mean field phase Φ_S. For small detuning [Fig. 3(b)] the ellipticity oscillates with small amplitude around zero. Moreover, as can be seen from Fig. 3(a), the laser intensity reaches its maximum values when the polarization is linear, and the largest spikes in ellipticity occur when the laser intensity is small. Figures 3(e) and 3(f) show that an increase of cavity detuning Δ gives rise to increased ellipticity changes during the time evolution in such a way that the polarization of the laser beam changes from linear to almost circular with relatively large intensity. A comparison of Figs. 3(a) and 3(e) indicates that the symmetry-breaking phenomenon is manifest more clearly at large detuning: The attractor in Fig. 3(e) is clearly asymmetric and shows a preference for left-handed polarization ($\xi > 0$). The average slope in Figs. 3(c) and 3(g) shows that the global rotation rate increases as Δ is increased.

In conclusion, we have presented the results of calculations of the dynamics of a cavity isotropic laser optically pumped by a linearly polarized laser beam whose plane of polarization is modulated at a frequency Ω of the order of the transverse relaxation rate of the lasing transition. We have found rich polarization dynamics, including phenomena such as symmetry breaking affecting the ellipticity of the generated field; quasi-periodic behavior; and, for the first time to our knowledge in lasers, chaotic dynamics affecting simultaneously the amplitude, the ellipticity, and the azimuth of the generated laser field vector. All these phenomena appear for a detuned cavity at low pump intensities, i.e., in conditions that can easily be reached in an experiment.

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