SCI. MAR., 61 (Supl. 1): 57-62 SCIENTIA MARINA 1997

LECTURES ON PLANKTON AND TURBULENCE, *C. MARRASÉ, E. SAIZ and J.M. REDONDO (eds.)*

Intermittent turbulence: a short introduction*

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SUMMARY: This paper provides a short introduction to the consequences of intermittency on the statistical properties of fully developed turbulence, mainly on (a) scaling laws for the different moments of velocity, (b) energy distribution and (c) diffusion behaviour. The description of intermittency is carried out in the fractal β model and in a more general multifractal perspective.

Key words: Intermittency, turbulence, multifractals.

INTRODUCTION

Intermittent effects are one of the main topics of present research on turbulence. In this paper, we provide a brief introduction to the description of intermittent effects, with special emphasis on their role in energy distribution amongst eddies of different size, in scaling laws for the several moments of the velocity, and in diffusion processes. We also outline the main departures from the standard Kolmogorov model. From a biological point of view, energy distribution at different scales and scaling laws are important because planktonic microorganisms will be most affected by the properties of the flow at a scale comparable to their own size.

We will also very briefly review the 1941 Kolmogorov's theory, for the sake of completeness, and we introduce some phenomenological facts about intermittency and mention how they were taken into consideration in the 1962 version of the Kolmogorov theory. Later, we introduce the so-called β model, which takes into account the fractal characteristics of intermittent turbulence. And finally we present some recent efforts to take into account multifractal properties of this phenomenon. We have restricted ourselves to a presentation of the main ideas, rather than trying to be exhaustive. In the references mentioned at the end of the paper the reader will find a wide bibliography to pursue his or her own understanding.

*Received December 12, 1995. Accepted April 30, 1996.

THE 1941 KOLMOGOROV THEORY

In 1922, L.F. Richardson proposed fully developed turbulence as a hierarchy of eddies of different sizes. Such eddies could represent, for instance, the Fourier components of the velocity field. There is an energy supply to the biggest eddies, of a scale L_o. This energy flows to smaller and smaller eddies, and finally it is dissipated in small eddies of scale L_η where viscosity plays a dominant role. The mean rate of energy transfer per unit mass, denoted by ε, plays a central role in this scheme. One of the main problems of turbulence is to relate in a quantitative and rigorous way this scheme to the Navier-Stokes equation, which describes the evolution of the velocity field v of the fluid and which has the form

$$\underset{(a)}{\frac{\partial v}{\partial t} + v.\nabla v} = -\frac{1}{\rho}\nabla p + \underset{(b)}{\frac{\eta}{\rho}\nabla^2 v} + \underset{(c)}{f} \quad (1).$$

Here, ρ is the mass density, p the pressure and η the dynamic viscosity. The term (c) represents the external forcing acting on the fluid; the nonlinear term (a) implies a breaking of bigger eddies into smaller eddies; the viscous term (b) represents a dissipation of kinetic energy as internal energy of the fluid. However, though the main ideas of energy supply, energy transfer and energy dissipation are common to both schemes, it is very difficult to produce a definitive quantitative relation between both descriptions.

Some of the main questions concerning the energy cascade from large to small eddies are: a) how is the kinetic energy distributed amongst the eddies of different size in the steady state? b) which are the statistical properties of the hierarchy? c) how these statistical features affect the transport properties? We will try to illustrate how the consideration of intermittency influences the answer to these questions.

In 1941, Kolmogorov studied these problems from an especially illuminating perspective, which is reviewed in any standard textbook on turbulence or fluid mechanics (see, for instance, Landau and Lifshitz 1971, Mc Comb 1990 or any other monograph). He assumed: a) the properties of the eddy hierarchy only depend on ε, L and v; and b) there exists an intermediate "inertial range" of eddies $L_o >> L >> L_\eta$ where the viscosity does not play any role. In this range, all the properties should be determined by ε and L.

First of all, we ask how the characteristic speed $v(L)$ for eddies of size L depends on L. Since the dimensions of ε are velocity^2/time, the only dimensional combination of ε and L yielding a quantity with dimensions of speed is

$$v(L) \approx (\varepsilon L)^{1/3} \quad (2).$$

The fractional energy distribution per unit mass will be given by

$$E(L)(\mathrm{d}L/L) \approx v(L)^2(\mathrm{d}L/L) \approx (\varepsilon L)^{2/3}(\mathrm{d}L/L) \approx \varepsilon^{2/3}L^{-1/3}\mathrm{d}L \quad (3).$$

It is more common to write the energy distribution in terms of the wavevector k ($\approx 2\pi/L$). Taking into account that $\mathrm{d}k \approx \mathrm{d}L/L^2$, (3) yields

$$E(k)\mathrm{d}k \approx \varepsilon^{2/3}k^{-5/3}\mathrm{d}k \quad (4).$$

This is the famous Kolmogorov -5/3 law, which has been dealt with in detail by other authors in this colloquium. This law is only valid in the inertial range.

A much more difficult topic of research is, for instance, how the energy distribution behaves near L_η, the length scale where the viscous effects begin to be important, and which by simple dimensional arguments may be estimated as $L_\eta = (\nu^3/\varepsilon)^{1/4}$. For instance, some primitive models of the spectral dynamics of equilibrium turbulence proposed by Heisenberg do not agree with experiments for small L, and they have been superseded by more recent analyses which suggest exponential spectra in the high-wavenumber range. However, this is not the main topic of this presentation. The reader interested in it is referred to McComb (1990).

Now, we consider the influence of turbulence of diffusion. Assume that $v(L) \approx L^{\zeta_1}$, with ζ_1 a scaling exponent. Paladin and Vulpiani (1987, page 172) have shown that the moments of the relative distance R between a pair of particles scale with time as $<R^{2q}> \approx t^{2q/(1-\zeta_1)}$ in the limit of long times. As a consequence, the mean value of the square of the displacement R as a function of time in the long-time limit is given by

$$<R^2> \approx t^{2/(1-\zeta_1)} \quad (5).$$

This result is surprising, as one could intuitively expect that $<R^2>$ should be related to the second moments of the velocity fluctuations, i.e. to ζ_2. However, it may be qualitatively understood by considering $\mathrm{d}<R^2>/\mathrm{d}t \approx <R\mathrm{d}R/\mathrm{d}t> \approx <RV> \approx <R^{1+\zeta_1}>$, from where it follows (5) by integration.

In the Kolmogorov model (3), $\zeta_1 = 1/3$, and therefore it follows that

$$<R^2> \approx t^3 \quad (6)$$

in contrast with the behaviour of classical diffusion, where $<R^2> \approx t$. Note that this implies that turbulence enhances diffusion effects. Expression (6) is known as Richardson's law and has been reviewed by other lecturers in this course.

INTERMITTENCY

Kolmogorov implicitly assumed that ε changes smoothly with space and time, or that it is a constant. However, the experimental results show strong intermittent bursts instead of a steady behaviour of the fluctuations, indicating that ε strongly fluctuates. This may be understood from a geometrical point of view if the active turbulent regions do not fill the whole volume, but only a subvolume, in a very irregular way.

From a mathematical point of view, the fact that ε is not a constant but that it changes in an irregular way over time and space implies several important consequences on the statistics of the velocity and of the velocity gradient. Here, we will focus our attention on the scaling laws for the different moments of the velocity fluctuations of eddies of size L , which are given by

$$<v(L)^p> \approx L^{p/3} <\varepsilon(L)^{p/3}> \approx L^{\zeta p} \quad (7)$$

with ζ_p scaling exponents, which in Kolmogorov theory are simply given by $\zeta_p = p/3$, as it follows from (2). In the presence of intermittency this is not so, because the average value of ε will be different at different points in space. The values of the exponents ζ_p have been measured experimentally. In Table 1 we list the values of some of these exponents as compared with the Kolmogorov's values and with the approximate values predicted by the fractal and multifractal models. These values have been obtained by analysis of the turbulent velocity data obtained in a boundary layer on a smooth flat plate in a wind tunnel (Stolovitzky *et al*. 1993).

We will ask about which are the consequences of (7) on the energy distribution and on the diffusion scaling law. We will do that in the framework of a fractal and a multifractal model, but first of all we will shortly mention the 1962 version of Kolmogorov theory, which was formulated both by Kolmogorov himself and by Obukhov (see, for instance, Monin and Yaglom 1975 for a comprehensive summary). In this version, it is assumed that $\varepsilon(x)$ fluctuates according to a log-normal probability distribution function as

$$\Pr\left[\ln \varepsilon(x)\right] \approx \exp\left[-\frac{(\ln \varepsilon(x) - <\ln \varepsilon(x)>)^2}{2\mu}\right] \quad (8)$$

where μ describes the intensity of the fluctuations in $\ln\varepsilon$. It may also be shown that μ is related to the spatial correlation of the fluctuations in ε as

$$<\delta\varepsilon(r)\delta\varepsilon(r+L)> \approx <\varepsilon^2>(L/L_0)^{-\mu} \quad (9).$$

Relation (9) is often used to measure μ, which turns out to have values between 0.3 and 0.5. It follows that the scaling laws for the moments of the velocity are

$$< v(L) > \approx L^{p/3} L^{\mu p(3-p)/18} \quad (10).$$

This modifies the energy distribution function as

$$E(k) \approx \varepsilon^{2/3} k^{-5/3-\mu/9} \quad (11)$$

and the diffusion behaviour as

$$<R^2(t)> \approx t^{51/(18-\mu)} \quad (12).$$

This model shows that the scaling laws are modified when intermittent effects are taken into account. The model stimulated much experimental research during two decades. But, it has some shortcomings: a) all regions of the fluid are active, though not in a regular way; b) it lacks a simple geometrical interpretation; c) its scaling exponents are given by

$$\varsigma_p = \frac{p}{3} + \frac{\mu}{18} p(3-p) \quad (13).$$

This relation yields ζ_p negative for high p, in contrast to what is experimentally observed. The interest in fractal geometry has led in the last decade to a different way to describe intermittency.

TABLE 1. – Values of the scaling exponents of several moments of the velocity fluctuations

	ζ_2	ζ_4	ζ_6	ζ_8	ζ_{10}	ζ_{12}
Experiment	0.70	1.20	1.62	2.00	2.36	2.68
Kolmogorov 41	0.67	1.33	2.00	2.67	3.33	4.00
β model	0.72	1.27	1.92	2.36	2.91	3.46
random β model	0.71	1.22	1.72	2.16	2.42	2.75

THE Β MODEL: THE FRACTAL DIMENSION OF TURBULENCE

We have seen that not all the regions of the fluid are active from the point of view of turbulence. Such regions are so strongly convoluted that they are not completely space filling, a feature which is more and more accentuated as the scale size decreases. Thus, the active regions fill different fractions of the whole volume according to the size of the eddies. In the simplest assumption, one could imagine that in the process of breaking, the ensuing eddies fill less and less volume in an self-similar way.

Selfsimilar geometrical objects may be characterized by the so-called Hausdorff dimension or fractal dimension (Mandelbrot, 1982). The fractal dimension D_F of an object embedded in a space of a given dimension is defined through the relation $N(l) \approx l^{-D_F}$, where $N(l)$ is the number of hypercubes of edge l, for $l \to 0$, necessary to cover the object. The reader interested in the details of fractal objects is referred to Mandelbrot (1982).

The simplest selfsimilar model for turbulence is the so-called β model, which was proposed in 1964 by Novikov and Stewart, by Mandelbrot in 1976 and by Kraichnan in 1979 but which was considerably developed and popularized by Frisch, Sulem and Nelkin since 1978 (see also Mc Comb, 1990, pages 104-109, and Sreenivasan, 1991).

To achieve a maximum of simplicity, we will assume that each eddy breaks into eddies which have half the previous scale length. Therefore, the characteristic scale of the eddies of the *n*-generation, i.e. after *n* breaking processes, is

$$L(n) = L_0 2^{-n} \quad (14).$$

Here, the factor 2 is taken only for convenience, but the results for the scale laws would remain unchanged if one takes instead a different arbitrary factor.

Furthermore, we will assume that the number of eddies at the *n* generation is given by

$$N(n) = N_0 2^{Dn} \quad (15).$$

Therefore, the volume filled by the eddies of the *n* generation is

$$V(n) = N(n)L(n)^3 \approx N_0 L_0^3 2^{-(3-D)n} \quad (16).$$

It is seen that if D is different from 3, the eddies do not fill the whole space, but that smaller and smaller eddies occupy smaller and smaller regions of the volume. D may be fractionary and it is called the fractal dimension of intermittent turbulence. One usually writes as $\beta(n)$ the volume ratio of the eddies of *n*-generation to the initial volume, i.e.

$$\beta(n) = V(n)/V_0 = 2^{-(3-D)n} = [L(n)/L_0]^{3-D} \quad (17).$$

The energy per unit mass in eddies of the *n*-th generation is given by $E(n) \approx \beta(n)v(n)^2$. The turnover time $\tau(n)$ corresponding to eddies of the *n*-th generation is $\tau(n) \approx L(n)/v(n)$. It then follows that the energy transfer rate is

$$\varepsilon \approx \beta(n)v^3(n)\tau^{-1}(n) \quad (18).$$

Therefore, one may express $v(n)$ as

$$v(n) \approx [\varepsilon L(n)\beta^{-1}(n)]^{1/3} \quad (19)$$

and

$$E(n) \approx \beta(n)\varepsilon^{2/3}[L(n)/\beta(n)]^{2/3} \quad (20).$$

It follows from here that

$$E(k)\mathrm{d}k \approx \varepsilon^{2/3}k^{-5/3}[kL_0]^{-(3-D)/3}\mathrm{d}k \quad (21).$$

This expression clearly shows the deviation with respect to the Kolmogorov -5/3 law due to intermittent effects.

To obtain the value of D one needs further theoretical hypotheses or a comparison with experimental data. From the theoretical point of view, we underline the proposal of Fujisaka and Mori (1979), who evaluated $\mu = 3 - D$ from a maximum-entropy approach. Their result was $\mu = 1/3$, i.e. $D \approx 2.66$. Experimental measurements give for the fractal dimension of intermittent turbulence $D \approx 2.83$. Thus, according to this latter value, the energy spectrum scales as $k^{-1.72}$ instead of the Kolmogorov prediction $k^{-1.67}$.

The consequences for higher moments of the velocity are more relevant. It may be shown that the scaling laws for higher moments of the velocity are of the form

$$\langle v(L)^p \rangle \approx \varepsilon^{p/3} L^{\zeta_p} \quad (22)$$

with the scaling exponents ζ_p given by

$$\zeta_p = \frac{p}{3} + (3-p)\frac{(3-D)}{3} \quad (23).$$

In Table 1 it is seen that the values predicted by (23) are closer to the experimental data than those given by the Kolmogorov assumption (2). However, for $p > 8$ the disagreement with the experimental data is more and more acute in both models.

Concerning the behaviour of diffusion, it follows, taking into account that $\zeta_1 = (7 - 2D)/3$ and the relation (5) and assuming that $D \approx 2.83$

$$<R^2> \approx t^{3.28} \quad (24).$$

The exponent of t is higher than the value 3 obtained in the Richardson law without intermittency. Thus, intermittency further enhances diffusion.

Finally, one could also ask for the modification of the scaling laws for the moments near the viscous range. One may obtain (Stolovitzky *et al.*, 1993)

$$< v(L)^p > = \frac{A_p L_\eta^p (L/L_\eta)^p}{\left[1 + B_p (L/L_\eta)^2\right]^{\frac{p}{2} - \varsigma_p}} \quad (25)$$

with A_p and B_p constants which depend only on p. Note that when $L>>L_h$ one recovers the scaling law (7).

MULTIFRACTAL FEATURES OF INTERMITTENT TURBULENCE

The scaling exponents for the moments of the velocity found in (24) are not satisfactory at high p, where there is a reduction of the observed ζ_p with respect to the values predicted in (23), as it may be seen in Table 1. This cannot be explained in the framework of a simple fractal model as the β model, and one needs to take into account multifractal properties of intermittency. The essential idea of a fractal is the selfsimilarity at all scales. However, if instead of having the same rule to generate every breaking process, the breaking rules are drawn at random at each step in length scale, one has inhomogeneous multifractal objects instead of usual fractal objects. One single fractal dimension cannot completely characterize the multifractal, because it does not give enough information on the probability distribution of the random breaking process. Multifractal aspects were introduced in the analysis of intermittent turbulence by Frisch and Parisi in 1985 and have been explored in much detail since then (for a review, see Paladin and Vulpiani, 1987, or Sreenivasan, 1991).

To present this idea in the context we are considering, it is assumed that in the breaking process the ratio β_n, defined as the ratio of the volume occupied by the eddies of the $n + 1$ generation to that occupied by eddies of the n generation is not given *a priori*, as in the β model presented in the previous section, but that it is itself a statistical property of intermittency, characterized by a probability distribution function $P(\beta_n)$where the β_n are independent, identically distributed random variables. This model is known as random β model.

In this way, one would have

$$v_n \approx L_n^{1/3} \left(\prod_{i=1}^{n} \beta_i \right)^{-1/3} \quad (26)$$

and the moments of the speed would be given by

$$< v_n^p > = \iiint d\beta_1 d\beta_2 ... P(\beta_1, \beta_2, ...) v_n^p \quad (27).$$

Paladin and Vulpiani have taken for $P(\beta)$ a simple expression, allowing only for two different kinds of breaking processes: a breaking into vortex sheets ($\beta = 0.5$) and a standard breaking without volume reduction ($\beta = 1$). The bimodal probability distribution function they assume has the form

$$P(\beta) = x\delta(\beta - 0{,}5) + (1 - x)\delta(\beta - 1) \quad (28)$$

with δ the Dirac delta and x a parameter to be fitted experimentally. The limit $x = 0$ corresponds to the standard Kolmogorov model whereas $x = 1$ fits the β model. The value of x which yields a best fit of the scaling exponents with the experimental data turns out to be $x \approx 0.125$. With this value for x, one obtains a very good fit for the scaling exponents ζ_p as seen in Table 1. It is worth mentioning that multifractal characteristics do not have any effect on the diffusion properties, which only depend on the scaling law for the first moment of the velocity, which is virtually unchanged by multifractality.

SUMMARY

We have emphasized here the following ideas:

(1) The energy transfer between eddies of different scales strongly fluctuates from place to place, yielding intermittent bursts of turbulence.

(2) The active regions occupied by the eddies do not fill the whole volume, but only a (very irregular) subregion of it, which in the simplest model may be characterized by a fractal dimension $D \approx 2.87$. Since

$D < 3$, it is clear that the eddies do not fill the whole space, but that smaller eddies occupy less volume. Therefore, organisms of different size will also feel different intensity of the turbulence.

(3) The intermittency modifies the energy spectrum, by attributing less energy to small scales in comparison with that predicted by the standard Kolmogorov distribution; and it enhances diffusion with respect to the usual Richardson law of non-intermittent turbulence.

(4) More detailed analyses of intermittency must focus on the exponents of the scaling laws for the different moments of the velocity. We have seen that a simple fractal model (the β model) captures many essential features of intermittent turbulence, but that it fails to describe the nonlinear dependence of the scaling exponent ζ_p of the p-th order moment with p. To describe this nonlinearity a multifractal model is needed. A useful multifractal model is the so-called random β model presented.

We could have mentioned many other aspects of intermittency, namely, some theoretical estimations of the fractal dimension of turbulence, the scaling law behaviour for different moments of temperature fluctuations, the relations between the fluctuations of the energy transfer rate and the velocity fluctuations, the effects of compressibility on intermittency, the comparison of the effects of intermittency in the inertial range with those near the dissipation range, and so on, but our aim has been only to call the attention of the reader to this topic in a very simple introduction.

ACKNOWLEDGEMENTS.

This work has been partially financed by the Dirección General de Investigación Científica y Técnica of the Spanish Ministry of Education and Science under grant PB94-0718.

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