Lasing without inversion with frequency up-conversion in a Doppler-broadened V-type three-level system

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We study lasing without inversion in a Doppler-broadened V-type three-level system in the frequency up-conversion regime. We show that the ratio \( R = \omega / \omega_0 \) of generated laser frequency to driving laser frequency that can be achieved using a vapor cell is modest \((R \approx 2)\). Our analysis demonstrates that using an atomic beam the frequency of the generated field can be substantially larger than that of the driving field \( R \sim 10 \). Two different configurations are considered: (a) copropagating driving and laser beams orthogonal to the atomic beam, and (b) counterpropagating driving and atomic beams and laser beam at a determined angle with the driving beam. [S1050-2947(99)05807-2]

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I. INTRODUCTION

In the past much work has been devoted to studying atomic coherence effects in quantum optics and laser spectroscopy [1]. One of these effects that has attracted more attention is lasing without inversion (LWI), because of its potential for short-wavelength lasing. For conventional lasers based on population inversion, continuous wave (cw) lasing in this spectral domain is unpractical due to the \( \omega^3 \) scaling of the Einstein \( A \) coefficient. In LWI an external driving field acting on one transition generates the atomic coherence that relaxes the population inversion condition for laser oscillation in an adjacent transition. Obviously, LWI is more interesting if the generated laser field can have a frequency higher than the external coherent driving field. This is the so-called frequency up-conversion regime.

Theoretically, it has been shown that LWI can be achieved in many different schemes [2], although to the best of our knowledge, only a few papers have focused on the frequency up-conversion problem [3–7]. Several experiments provided clear-cut evidence, first, of amplification without inversion (AWI) in the transient regime [8], then in the steady-state regime [9], and, finally, of LWI [10–12]. Although the last experiments demonstrated the feasibility of LWI, any of them operated in the frequency up-conversion regime. Up to now, the largest laser to driving frequency ratio \( R = \omega / \omega_d \approx 1.35 \) was achieved in a lasing with population inversion below threshold experiment [13].

Here we address theoretically the frequency up-conversion issue by considering one of the simplest schemes for LWI, namely, the V scheme (Fig. 1). One of the main features of our treatment is the explicit inclusion of Doppler broadening, which is known to play, \( a \) priori, a negative role in LWI when the driving and lasing frequencies are appreciably different [3]. In fact, the first experiments demonstrating continuous wave LWI [10,11] used an “almost Doppler-free” configuration. Other authors have studied previously Doppler-broadened V scheme AWI and LWI [6,14]. Karawajczyk and Zakrzewski [14] fixed the laser to driving frequency ratio to \( R = 1.5 \) and considered a population relaxation rate larger for the driven transition than for the laser transition. Boon et al. have studied theoretically and experimentally electromagnetically induced transparency (EIT) [15] and inversionless amplification [6] on a blue probe in a four- and a three-level Doppler-broadened mismatched V-type system, respectively. The mismatch in the EIT experiment and in the numerical simulation for the inversionless amplification case was \( R = 1.85 \) and \( R = 2 \), respectively. In both cases, the population relaxation rate for the driven transition was larger than the corresponding one for the probed transition, i.e., \( \gamma_{23} > \gamma_{13} \) (Fig. 1). Although this is the most general real situation even in the frequency up-conversion regime, one can also find real three-level systems for which the \( \omega^3 \) dependence of the Einstein \( A \) coefficient dominates over its dependence on the square of the dipole matrix element, thus resulting in \( \gamma_{23} < \gamma_{13} \). We will see below that with \( \gamma_{23} < \gamma_{13} \), LWI is possible in a narrower range of detunings and is more affected by the Doppler effect than in the opposite case. These conclusions are drawn in the uniform field approximation, i.e., when propagation effects for drive and laser fields are neglected. However, when these effects are included, it results [3,4] that in the frequency up-conversion regime the maximum possible inversionless gain, limited by drive field absorption during its propagation, is larger for \( \gamma_{23} < \gamma_{13} \) than for \( \gamma_{23} > \gamma_{13} \), at least in the ab-

\[ \Delta_{\text{h}} \]

\[ \Delta_{\alpha} \]

\[ 11 > \]

\[ 12 > \]

\[ 13 > \]

\[ \beta \]

\[ \gamma_{23}, \gamma_{13} \]

\[ \Delta \]

\[ \alpha \]

\[ \Delta_{\beta} \]

\[ \Delta_{\alpha} \]

FIG. 1. Closed V scheme under investigation. \( 2 \alpha \) (2\( \beta \)) is the Rabi frequency of the probe (driving) field, \( \Delta_{\alpha} \) (\( \Delta_{\beta} \)) is the probe (driving) field detuning, \( \Delta \) is an incoherent pump rate, and \( \gamma_{ij} \) is the spontaneous population decay rate from state \( i \) to state \( j \).
sence of Doppler broadening [3,4]. Here we analyze both cases, i.e., $\gamma_{23} > \gamma_{13}$ and $\gamma_{23} < \gamma_{13}$.

II. THEORY AND RESULTS

A. Homogeneous broadening

Consider the closed V-type three-level system shown in Fig. 1. A coherent driving field from an external source couples to transition $|2\rangle \rightarrow |3\rangle$ with Rabi frequency $2\beta$ and detuning $\Delta_{g}$ from atomic resonance and prepares the atoms to generate a laser field inside a unidirectional ring laser cavity (see Fig. 3). The coupling of this laser field to transition $|1\rangle \rightarrow |3\rangle$ is characterized by a Rabi frequency $2\alpha$ and detuning $\Delta_{r}$ from atomic resonance. In addition, an incoherent pumping process between levels $|3\rangle$ and $|1\rangle$ with a population transfer rate $\Lambda$ brings some population into the upper level of the lasing transition. Using standard semiclassical methods, we find the following set of Maxwell-Schrödinger equations for this system in the rotating wave, uniform field and slowly varying envelope approximations:

$$\dot{\rho}_{11} = -\gamma_{13}\rho_{11} + \Lambda(\rho_{33} - \rho_{11}) - 2\alpha \text{Im} \rho_{31},$$

$$\dot{\rho}_{22} = -\gamma_{23}\rho_{22} - 2\beta \text{Im} \rho_{32},$$

$$\dot{\rho}_{33} = \gamma_{13}\rho_{11} + \gamma_{23}\rho_{22} + \Lambda(\rho_{11} - \rho_{33}) + 2\alpha \text{Im} \rho_{31} + 2\beta \text{Im} \rho_{32},$$

$$\dot{\rho}_{32} = -(\Gamma_{32} - i\Delta_{\rho})\rho_{32} + i\alpha\rho_{12} - i\beta(\rho_{33} - \rho_{22}),$$

$$\dot{\rho}_{31} = -(\Gamma_{31} - i\Delta_{\alpha})\rho_{31} + i\beta\rho_{21} - i\alpha(\rho_{33} - \rho_{11}),$$

$$\dot{\rho}_{12} = -[\Gamma_{12} - i(\Delta_{\rho} - \Delta_{\beta})]\rho_{12} + i\alpha\rho_{32} - i\beta\rho_{13},$$

$$\dot{\alpha} = -\kappa\alpha + g \text{Im} \rho_{31},$$

$$\Delta_{r} = \Delta_{r}^{c} + g(\text{Re} \rho_{31}/\alpha).$$

Dissipative processes are described by means of the population relaxation rates $\gamma_{13}$ and $\gamma_{23}$ of lasing and driven transitions, respectively, and coherence relaxation rates given, in the so-called radiative limit where there are no dephasing collisions, by $\Gamma_{31} = (\gamma_{13} + 2\Lambda)/2$, $\Gamma_{32} = (\gamma_{23} + \Lambda)/2$, and $\Gamma_{12} = (\gamma_{13} + \gamma_{23} + \Lambda)/2$. $\kappa$ is the damping rate of the laser field due to cavity losses and $g = \omega_{13}N\mu_{13}^{2}/2\hbar\epsilon_{0}$ the gain parameter of the lasing transition where $\omega_{13}$ is the corresponding transition frequency, $\mu_{13}$ the electric dipole matrix element, $N$ the density of atoms, $\hbar$ Planck’s constant, and $\epsilon_{0}$ the dielectric permittivity. Finally, $\Delta_{r}^{c}$ is the cavity detuning from the atomic resonance $\omega_{13}$.

These equations consider homogeneous broadening and uniform driving and laser fields along the resonator axis, a case that has been studied extensively in the past. Let us recall some relevant results. Zhu [16] showed that this system can exhibit LWI either with both fields tuned to resonance ($\Delta_{r} = \Delta_{\beta} = 0$) or with both fields detuned from resonance but close to the two-photon resonance ($\Delta_{r} = \Delta_{\beta} \neq 0$). In the first case lasing can be achieved without population inversion either at the lasing transition or at the two-photon Raman transition (i.e., with $\rho_{33} > \rho_{11} > \rho_{22}$), while in the second case the domain of detunings for which there is no Raman inversion is usually small. It is interesting to remark that the gain that can be achieved in the detuned case is substantially larger than the corresponding one in the resonant case. Necessary conditions for LWI in the fully resonant case are [16]

$$\gamma_{23} > \gamma_{13},$$

$$\Lambda > \gamma_{13}^{2}(\gamma_{23} - \gamma_{13}).$$

Recently, it has been shown that inversionless gain in the V scheme can also be achieved if either the probe [4,17] or the driving field [4] is tuned away from resonance and the population on the transition coupled by the driving field is inverted. In [4,5] it was shown that condition (2a) is no longer necessary for LWI in the cases where one or two fields are detuned, respectively. This is a relevant result for the frequency up-conversion regime when propagation effects are included [3,4], since then the main limitation on achievable AWI gain is due to spontaneous emission along the driven transition and the associated absorption of the driving field. In the following we will consider both $\gamma_{23} < \gamma_{13}$, which excludes LWI with $\Delta_{r} = \Delta_{\beta} = 0$, but is more immune to propagation effects, and $\gamma_{23} > \gamma_{13}$, which corresponds to the most general real situation.

It is easy to see that in the closed three-level system of Fig. 1 the steady-state population can be inverted neither at the transition $|2\rangle \rightarrow |3\rangle$ nor at the transition $|1\rangle \rightarrow |3\rangle$. This therefore excludes the frequency up-conversion regime investigated by Yelin and co-workers [4], and we are left only with the cases $\Delta_{r} = \Delta_{\beta} 
eq 0$ and $\Delta_{r} = \Delta_{\beta} = 0$ on which we focus in the following.

Let us obtain now the general threshold condition for continuous wave lasing in our system. Taking all time derivatives in Eqs. (1) equal to zero and solving Eqs. (1a)–(1d) to zeroth order and Eqs. (1e) and (1f) to first order in the laser field, we obtain a solution denoted by $\rho_{1j}^{(1)}$. According to Eq. (1g) the threshold condition for cw lasing is $\kappa < g \text{Im} \rho_{1j}^{(1)}/\alpha$, which simply states that the unsaturated probe gain per unit time must overcome cavity losses. This condition can be expressed as

$$d = \kappa(\Delta_{r}^{2} + \Gamma_{13}^{2})(\Delta_{r} - \Delta_{a}^{2} + \Gamma_{12}^{2})$$

$$+ \beta^{2}[2\Delta_{a}(\Delta_{r} - \Delta_{a}) + 2\Gamma_{12}\Gamma_{13} + \beta^{2}])$$

$$+ g[n_{31}(\Gamma_{13}(\Delta_{r} - \Delta_{a})^{2} + \Gamma_{12}^{2}(\Gamma_{12}^{2} + \beta^{2}))$$

$$+ \beta x_{23}[\Gamma_{13}(\Delta_{r} - \Delta_{a}) - \Gamma_{12}\Delta_{a}])$$

$$- \beta y_{23}[\Delta_{a}(\Delta_{r} - \Delta_{a}) + \Gamma_{12}\Gamma_{13} + \beta^{2}]) < 0,$$

where $n_{31} = \rho_{33} - \rho_{11}$ and $\rho_{23} = x_{23} + iy_{23}$ correspond to the solution to zeroth order in the laser field. Equation (3) shows that in the noninversion case, i.e., with $n_{31} > 0$, the driving field $\beta$ and the atomic coherence $\rho_{23}$ play an essential role in order to generate the laser field. In addition, it is easy to see that it is convenient to take $\Delta_{r} = \Delta_{a}$ since the positive contribution of $n_{31}$ to the left-hand side of Eq. (3) decreases substantially. A detailed discussion of Eq. (3) can be found in [5].
FIG. 2. The area inside the curves corresponds to the LWI domain in the plane ($\Delta_a$, $\Delta_b$) for (a) $\gamma_{23}=2\gamma_{13}$, and $\Lambda=0.3\gamma_{13}$, $\beta=5\gamma_{13}$, for (i) or $\Lambda=2\gamma_{13}$, $\beta=10\gamma_{13}$ for (j). For (b) $\gamma_{23}=0.5\gamma_{13}$, and $\Lambda=0.3\gamma_{13}$, $\beta=5\gamma_{13}$ for (k) or $\Lambda=\gamma_{13}$, $\beta=15\gamma_{13}$ for (l). In all the cases $\kappa=0.5\gamma_{13}$, and $g=1000\gamma_{13}$. Outside the region between the dotted lines with abscissa $\Delta^a_b (a=i,j,k,l)$ laser oscillation occurs with Raman inversion.

Figure 2 shows curves $d=0$ as a function of $\Delta_a$ and $\Delta_b$ for the following parameters: In Fig. 2(a) $\gamma_{23}=2\gamma_{13}$, and $\Lambda=0.3\gamma_{13}$, $\beta=5\gamma_{13}$, for (i) or $\Lambda=2\gamma_{13}$, $\beta=10\gamma_{13}$ for (j). In Fig. 2(b) $\gamma_{23}=0.5\gamma_{13}$, and $\Lambda=0.3\gamma_{13}$, $\beta=5\gamma_{13}$ for (k) or $\Lambda=\gamma_{13}$, $\beta=15\gamma_{13}$ for (l). In all these cases $\kappa=0.5\gamma_{13}$, and $g=1000\gamma_{13}$. Notice that we assume driving field intensities, $\beta/\gamma_{23}\leq 30$, relatively easy to attain with visible cw laser powers. Since the realization of an efficient incoherent pump represents one of the main difficulties in short-wavelength LWI experiments, we consider moderate incoherent pump rates $\Lambda/\gamma_{13}\leq 2$. There is positive net gain ($d<0$) inside the corresponding curve. Let us emphasize that $\Delta_a$ is not a parameter but the detuning of the generated field. Therefore, one must select the cavity detuning that yields a laser field with a detuning in the corresponding region of LWI in Fig. 2. This point was discussed in [5], where it was noted that there is a strong pushing of the laser frequency away from atomic resonance, at variance with what happens in conventional two-level lasers where it is pulled toward atomic resonance, and it will be taken into consideration in the results presented below. As is clearly seen in Fig. 2, it is possible to achieve LWI only near the diagonal $\Delta_a=\Delta_b$, i.e., near the two-photon resonance condition. Since the width of the two-photon resonance depends on $\Gamma_{12}=\gamma_{13}+\gamma_{23}+\Lambda$, one can see in Fig. 2, by comparing the four cases shown, that the smaller $\Gamma_{12}$ the narrower the domain of lasing. In cases (i) and (j) [Fig. 2(a)] the decay rates satisfy condition (2a), but condition (2b) is only fulfilled in case (j). In cases (k) and (l) [Fig. 2(b)] condition (2a) is not verified. Therefore on-resonance LWI is possible only in case (j). In cases (i), (k), and (l) LWI is possible for sufficiently detuned fields close to the two-photon resonance condition. In fact, the lasing domain appears in these cases for $\Delta_b>\Delta_a$, with $\Delta^a_a=\beta^2(\Lambda+\gamma_{23})(\gamma_{13}+\Lambda(\gamma_{13}-\gamma_{23})/\gamma_{23}\Lambda(\gamma_{13}+\gamma_{23}+\gamma_{13}))$, and a cavity detuning such that the two-photon resonance condition $\Delta_a=\Delta_b$ is approximately fulfilled [5]. Outside of the region delimited by the corresponding vertical dotted lines with abscissa $\Delta^a_a (a=i,j,k,l)$, laser oscillation occurs with Raman inversion ($\rho_{11}>\rho_{22}$). Inside of this region one has $\rho_{11}<\rho_{33}$ and $\rho_{11}<\rho_{22}$, i.e., one- and two-photon noninversion. Physical interpretations for the origin of this gain were given by Gryenberg, Pinard, and Mandel [18] in terms of interference between Feynman diagrams in the dressed basis, and by Mompart and Corbalán [17] in terms of dressed-state quantum interference and quantum-jump analyses. By increasing the incoherent pump power $\Lambda$ the region of lasing without one- and two-photon inversion becomes narrower. Pure LWI is still observable up to $\Lambda=6.1\gamma_{13}$ for cases (i) and (j) of Fig. 2(a); and $\Lambda=3.9\gamma_{13}$ for case (k), and $\Lambda=7.5\gamma_{13}$ for case (l) of Fig. 2(b). Above the corresponding $\Lambda$ value gain originates exclusively with Raman inversion.

B. Doppler broadening

The above discussion ignores Doppler broadening. In the following we will consider the three configurations shown schematically in Fig. 3 where atomic motion has been taken into account. In the first two cases [see Fig. 3(a)], driving and laser beams are copropagating and the atoms are either in an orthogonal atomic beam or in a vapor cell. In the third case [Fig. 3(b)], driving and atomic beams are counterpropagating and the laser beam makes an angle $\phi$ with the driving beam. Now, atoms with velocity $v$ toward the driving beam “see” both fields with velocity dependent detunings $\Delta_a(v)$ and $\Delta_b(v)$, given by

$$\Delta_a(v) = \Delta_a + \omega_a c^{-1} s,$$

$$\Delta_b(v) = \Delta_b + \omega_b c^{-1} s,$$  

where $\Delta_a$ and $\Delta_b$ are the nominal detunings, $\omega_a$ and $\omega_b$ the corresponding field frequencies, $c$ the velocity of light, and $s$ a parameter that depends on the case under consideration: $s = 1$ for the first two cases and $s = \cos \phi$ for the third case. According to Eqs. (1a)–(1f) the response of the atoms to the fields is now velocity dependent and one must perform in the laser amplitude and frequency determining Eqs. (1g) and (1h), respectively, the following integrations over the velocity distribution $N(v)$ to obtain the contribution of all the atoms:

$$\dot{a} = -\kappa a + \int g(v) \text{Im} \rho_{31}(v) dv,$$

$$\Delta_a = \Delta_a^0 + \int \big| g(v) \big| (\text{Re} \rho_{31}(v)/a) dv,$$  

where $g(v) = \omega_{13} N(v) \mu_{13}^2 2\hbar \varepsilon_0$ and $N(v) dv$ is the density of atoms with velocity $v$. For the first two cases, to describe the Doppler broadening in a vapor and in an orthogonal atomic beam not perfectly collimated, we will take the standard Maxwellian distribution:
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FIG. 3. Configurations considered in this paper: (a) driving and probe fields copropagating through a vapor cell and through an orthogonal atomic beam; (b) driving and atomic beam counterpropagating and the laser field forming an angle $\phi$ with the driving beam.

\[
N(v) = \frac{N_0}{\sigma \sqrt{\pi}} \exp(-v^2/\sigma^2), \quad (6)
\]

where $\sigma$ is the most probable velocity. In the third case, the longitudinal velocity distribution in an atomic beam is given by [19]

\[
N(v) = \frac{N_0}{\sigma^3} v^3 \exp(-v^2/\sigma^2), \quad (7)
\]

where now the most probable velocity is $\sqrt{2} \sigma$. To characterize the velocity distribution we will use in the three cases the well-known expression of the full width at half maximum of the absorption profile of a purely Doppler-broadened vapor, given for the lasing transition by

\[
\Delta \omega_{D,a} = 2 \sqrt{2} \ln 2 \frac{\sigma}{c} \omega_a. \quad (8)
\]

To obtain the numerical results we have integrated Eqs. (1a)–(1f) and Eqs. (5a) and (5b) by using a Runge-Kutta-Felhberg routine. We have split the velocity distribution into discrete groups or velocity classes with all the atoms belonging to a given group taken as moving with the same velocity. In all cases shown below we have considered a velocity distribution with more than 99.7% of the atoms and a large enough number (>100) of velocity classes to assure the convergence of the results.

I. Vapor cell

We now consider parameters as for case (k) in Fig. 2(b), with $\Delta_\beta=\Delta'_\alpha=0$. Taking into account Eq. (4), the points representing different velocity classes in the inset of Fig. 4 lie on a straight line ($\overline{TT'}$) passing through the origin with slope $R = \omega_a/\omega_\beta$. For this line we plot a length $\Delta \gamma_{13}/\gamma_{13}$ as in Fig. 2(b) [as in Refs. 6, 7], the domain of detunings corresponding to three standard deviations in the velocity distribution. Since one must consider the contributions of all the atoms simultaneously, one sees in this inset that lasing is only possible with $R \sim 1$ and if the Doppler broadening is large enough that the contribution of the atoms in the gain regions ($\overline{QT}$ and $\overline{QS'}$) overcomes that of the atoms in the absorption region near zero-detuning ($\overline{QQ'}$). This is confirmed by the numerical results shown in Fig. 4, which one sees that the domain of lasing as well as the amplitude of the generated field rapidly decreases when $R$ increases. This configuration is therefore not interesting for frequency up-conversion LWI. In this configuration LWI occurs without Raman inversion, i.e., with $\rho_{11}<\rho_{22}$, only at the left side of the dotted vertical line shown in Fig. 4. However, one can notice that even in this case there is a significant contribution to gain of velocity classes with Raman inversion (velocity classes $\overline{ST}$ and $\overline{S'T'}$).

From a comparison between Figs. 2(a) and 2(b) it is clear that with $\gamma_{23}>\gamma_{13}$ [as in Fig. 2(a)], the domain of lasing begins either at $\Delta_\alpha=0$ or closer to $\Delta_\alpha=0$ than for cases in Fig. 2(b). This means that more low velocity classes, which contain most of the atoms, contribute now to gain. Therefore, LWI in the case $\gamma_{23}>\gamma_{13}$, already studied in Refs. 6, 14, is less affected by the Doppler effect and is possible for appreciably larger values of $R$ than in the opposite case. Thus, for parameters as for case (j) in Fig. 2, at $R=2$ the domain of lasing covers the range $\Delta \omega_{D,a}/\gamma_{13} = 0.99$, with a peak value for the generated field, $\alpha R = 3.2$, and lasing occurring in the presence of Raman inversion for $\Delta \omega_{D,a}/\gamma_{13} > 56.6$. 

FIG. 4. Amplitude of the laser field, $\alpha$, as a function of the Doppler broadening in a vapor cell for different values of the up-conversion ratio ($R$). In all the cases $\gamma_{23}=0.5 \gamma_{13}$, $\Lambda=0.3 \gamma_{13}$, $\beta=5 \gamma_{13}$, $\kappa=0.5 \gamma_{13}$, $g=1000 \gamma_{13}$, and $\Delta_\beta=\Delta'_\alpha=0$. At the right side of the vertical dotted line, laser oscillation occurs with Raman inversion. These results have been obtained by integration over all the velocity classes along lines ($\overline{TT'}$) of slope $R$ as the one represented in the inset for $R=1.05$ and $\Delta \omega_{D,a}=28.4 \gamma_{13}$. The absorbent velocity classes are located on the segment $\overline{QQ'}$ and the velocity groups contributing to gain correspond to the segments $\overline{QT}$ and $\overline{QS'}$. In some of them ($\overline{ST}$ and $\overline{S'T'}$) there is Raman inversion.
FIG. 5. Amplitude of the generated field, $\alpha$, in an orthogonal atomic beam as a function of the residual Doppler broadening for (continuous line) $\gamma_2 = 0.5 \gamma_{13}$, $\Lambda = \gamma_{13}$, $\beta = 15 \gamma_{13}$, $\Delta_0 = 23 \gamma_{13}$, and $\Delta'_a = 20 \gamma_{13}$; or (dashed line) $\gamma_2 = 2 \gamma_{13}$, $\Lambda = 2 \gamma_{13}$, $\beta = 10 \gamma_{13}$, and $\Delta_0 = \Delta'_a = 8 \gamma_{13}$. In both cases $\kappa = 0.5 \gamma_{13}$ and $g = 1000 \gamma_{13}$. The Doppler integration over all the velocity classes is represented in the inset of the figure by the lines $SS'$ (for $\Delta \omega_{D\alpha} = 8.4 \gamma_{13}$) or $UU'$ (for $\Delta \omega_{D\alpha} = 7.9 \gamma_{13}$) with slope $R = 10$. The integration starts at point $P$ or $P'$ and the velocity classes that contribute to laser oscillation are situated along the segments ($QQ'$ or $TT'$). There is no Raman inversion in any of the two lasing domains.

2. Orthogonal atomic beam

Let us investigate (see Fig. 5) the influence of the residual Doppler broadening in the case of the atomic beam configuration displayed in Fig. 3(a). We consider parameters as for cases (j) and (l) in Fig. 2, with $\Delta_0 = \Delta'_a = 8 \gamma_{13}$ in the first case and $\Delta_0 = 23 \gamma_{13}$ and $\Delta'_a = 20 \gamma_{13}$ in the second one. For these parameters, in the absence of Doppler broadening, a numerical integration of Eqs. (1) shows that after a transient the system reaches a steady lasing state without inversion either at the transition $|1\rangle \leftrightarrow |3\rangle$ or at the transition $|1\rangle \leftrightarrow |2\rangle$, with $\alpha = 7.1 \gamma_{13}$ and $\Delta_a = 11.6 \gamma_{13}$ in case (j), or $\alpha = 3.2 \gamma_{13}$ and $\Delta_a = 27.2 \gamma_{13}$ in case (l). These states are represented by points $P'$ and $P$, respectively, in the inset of Fig. 5. In the presence of Doppler broadening, the points representing the detunings of different velocity classes [see Eqs. (4)] lie on a straight line passing through either $P$ or $P'$ with a slope $R$ [20]. Increasing $\Delta \omega_{D\alpha}$, i.e., the length of the line $SS'$ (or $UU'$), and/or $R$ one should expect a decrease of the generated field $\alpha$ and an eventual disappearance of lasing when the amplitifying atoms ($QQ'$ or $TT'$) cannot compensate the action of the absorbing atoms ($QS$ and $Q'S'$ or $TU$ and $T'U'$). This is confirmed by the numerical results shown in Fig. 5 [continuous and dashed lines represent cases (l) and (j), respectively] for $R = 10$, where it appears that the domain of lasing extends to values of $\Delta \omega_{D\alpha}$ more than an order of magnitude larger than the homogeneous width of the lasing transition. This means that the atomic beam does not need to be extremely well collimated. We have checked that in the entire lasing domain, in both cases, the total population in level $|1\rangle$ is the smallest among the level populations, as one would expect in view of the inset to Fig. 5.

3. Collinear driving and atomic beams making an angle with the laser field

We now consider the configuration shown in Fig. 3(b) in which the driving and atomic beams are counterpropagating and the laser beam makes an angle $\phi$ with the driving beam. Whatever the frequency up-conversion ratio $R$ is, we can
maintain the two-photon resonance for all the velocity classes by tuning the angle \( \phi \) in such a way that \( R \cos \phi = 1 \).

We again consider cases (j) and (l) in Fig. 2. With \( \Delta_\beta = \Delta_\alpha = 0 \) and \( R \cos \phi = 1 \), the line representing different velocity classes coincides with the diagonal of the first quadrant in Fig. 2. Therefore, all the atoms contribute to lasing in case (j). However, for the above drive and cavity tunings, the slowest atoms do not contribute to lasing in case (l), for which it is more convenient to take the nominal field detunings inside the amplifying region. We have studied case (l) with two different field detunings: \( \Delta_\beta = \Delta_\alpha > 0 \) (blue detuned fields from atomic resonance) and \( \Delta_\beta = \Delta_\alpha < 0 \) (red detuned fields). In both of them the slope \( R \cos \phi \) of the integration line has been kept as 1. Continuous lines in Fig. 6(a) show results for case (l) with nominal detunings \( \Delta_\beta = \Delta_\alpha = 21 \gamma_{13} \) which correspond to point \( P \) in the inset of Fig. 6(a). Dashed lines in this figure correspond to case (j) with detunings \( \Delta_\beta = \Delta_\alpha = 0 \), which are represented by point \( P' \) in the inset. We have considered two values for the up-conversion ratio, \( R = 5 \) and \( R = 10 \). At the right side of the corresponding vertical arrows (the longest ones correspond to \( R = 10 \)) laser oscillation occurs with Raman inversion. In both cases (l) and (j), all the velocity classes contribute to gain (segments \( PQ \) or \( P'Q' \) in the inset) although for large enough Doppler broadening some of them have Raman inversion (\( QR \) or \( Q'R' \)).

Figure 6(b) shows the amplitude of the laser field, \( \alpha \), as a function of its angular deviation from the driving field, with other parameters as for case (l) in Fig. 6(a). Although the angular domain of lasing decreases when the up-conversion ratio \( R \) increases, for \( R = 10 \) it is still large enough to allow a easy experimental control.

In Fig. 7 we have chosen as nominal field detunings \( \Delta_\beta = \Delta_\alpha = -50 \gamma_{13} \), for case (l) and \( \Delta_\beta = \Delta_\alpha = -25 \gamma_{13} \), for case (j). Such states are represented by points \( P \) and \( P' \), respectively, in the inset of this figure. With these detunings, the LWI region \( (\rho_{11} > \rho_{11} < \rho_{22}) \) for \( R = 10 \) and \( \cos \phi = 0.1 \) is larger than the corresponding one in Fig. 6(a) and it is located at the right side of the vertical line, in case (l), or between the two vertical lines in case (j). Note, however, that most of the gain in the LWI regions is produced by velocity classes with Raman inversion (segments \( PQ \) or \( P'Q' \) and \( U'T' \)).

III. CONCLUSIONS

We have analyzed frequency up-conversion LWI in a Doppler-broadened \( V \) scheme. We have considered cases with a spontaneous population decay rate for the laser transition either larger or smaller than for the driven transition. We have considered modest incoherent pump rates \( (\Lambda \approx 2 \gamma_{13}) \) and driving field intensities \( (\beta \approx 15 \gamma_{13}) \). We have shown that the frequency up-conversion ratio that can be achieved using a vapor cell is modest \( (R \approx 2) \). Two alternative arrangements have been discussed involving an atomic beam where the driving and laser frequencies can be substantially different \( (R \approx 10) \). Broad domains of lasing without inversion at the one- and at the two-photon Raman transitions are found for both arrangements. Moreover, by using a sample of laser cooled atoms one could obtain the results shown in Figs. 5, 6(a), and 7 for \( \Delta \omega_{D_0} = 0 \). The above results show that the Doppler-broadened \( V \) scheme has more potential for frequency up-conversion LWI than previously expected.

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For each value of $\Delta \omega_{D}^{a}$, the points $P$ and $P'$ may be slightly displaced vertically because dispersive effects, and therefore the unknown generated laser frequencies, do not need to be equal in the homogeneous and in the Doppler-broadened cases.