Comment on "Quantum Wave Packet Dynamics with Trajectories"

In a recent Letter [1], Lopreore and Wyatt (LW) concluded that the usual view of particle tunneling within the Bohm interpretation (the quantum potential lowers the barrier so that the Bohm particles can travel classically over it [2,3]) is "misleading and incorrect for smooth barrier penetration." They claim that the Bohm particles feel only a significant quantum force during a short *boost phase* just after the launching of the wave packet (far from the barrier), and only those particles that acquire enough kinetic energy (KE) during this boost phase can pass over the barrier and contribute to the *tunneling* transmission. Consequently, they propose a *decoupling approximation* that consists of ignoring the quantum force after a short initial decoupling time. In this Comment, we argue that the results of LW correspond to a particular case in which tunneling is not significant. Thus, their conclusions about the tunneling mechanism are not valid.

For comparison with LW results, we consider the scattering of Gaussian wave packets, $|\Psi(t)\rangle$, by an Eckart potential $V(x) = V_0 \,\mathrm{sech}^2[a(x - x_b)]$. If the transmission coefficient of the eigenstate $|\Psi_k\rangle$ is $T(k)$, the total transmission probability T can be decomposed into pure tunneling (T_{tun}) and over-the-barrier (T_{OB}) components:

$$T = T_{\mathrm{tun}} + T_{\mathrm{OB}} = \int_0^{k_B} |a(k)|^2 T(k)\, dk + \int_{k_B}^{\infty} |a(k)|^2 T(k)\, dk\,, \qquad (1)$$

where $a(k) \equiv \langle \Psi_k \mid \Psi(t)\rangle$, and $k_B = \sqrt{\frac{2mV_0}{\hbar^2}}$ is the wave vector associated with the top of the barrier.

Figure 1 shows T, T_{tun}, and T_{OB} as a function of the barrier width. For wide barriers, T_{tun} tends to be negligible and T converges to T_{OB} and to the result given by the LW decoupling approximation. The case analyzed by LW corresponds to this limit, i.e., $T_{\mathrm{OB}} \gg T_{\mathrm{tun}}$. However, for thin enough barriers, tunneling tends to dominate the total transmission, and the decoupling approximation fails (it underestimates T). The increase of transmission due to tunneling can only be accounted for by particles that have KE lower than the barrier height V_0 after the boost phase. Since Bohm trajectories are *classical,* we must conclude that the quantum potential lowers the classical barrier, as is usually claimed [2,3] and has been clearly shown for Hamiltonian eigenstates [4,5]. The transmission coefficient calculated under the decoupling approximation depends on the barrier height but not on the barrier width (see Fig. 1). The decoupling approximation is reasonable for thick barriers but underestimates the transmission by orders of magnitude when tunneling dominates (inset of Fig. 1).

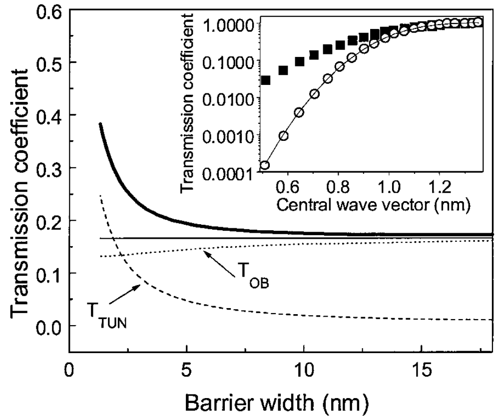

FIG. 1. Transmission coefficient of a Gaussian wave packet (central energy of 0.03 eV, spatial dispersion of 5 nm) impinging upon an Eckart barrier as a function of the barrier width at half maximum (BWHM) for a fixed barrier height, $V_0 = 0.04$ eV. The bold line corresponds to the total transmission coefficient (T), the dashed line to the tunneling component (T_{tun}), the dotted line to the over-the-barrier component (T_{OB}), and the continuous line to LW decoupling approximation. Inset: transmission coefficient versus central energy (wave vector) for two barrier widths: BWHM = 1.3 nm (squares) and BWHM = 18.8 nm (circles). The continuous line corresponds to the decoupling approximation.

In conclusion, the interpretation of tunneling given by LW is misleading and incorrect. Tunneling requires that the quantum potential lowers the classical barrier as previously claimed [2,3]. The decoupling approximation is valid only when quantum effects (tunneling and over-the-barrier resonances) are unimportant.

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