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Citation: Applied Physics Letters **78**, 2802 (2001); doi: 10.1063/1.1369619
View online: http://dx.doi.org/10.1063/1.1369619
View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/78/18?ver=pdfcov
Published by the AIP Publishing

Simple model to study soliton wave propagation in periodic-loaded nonlinear transmission lines

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(Received 20 June 2000; accepted for publication 15 March 2001)

A model to study soliton propagation characteristics in nonlinear transmission lines (NLTLs) periodically loaded with voltage dependent capacitances is presented. This is based on the LC ladder equivalent network of the NLTL, and can be applied to structures where the Korteweg–de Vries approach does not hold. Specifically, the model allows to numerically obtain soliton waveforms under arbitrary capacitance non linearity. To demonstrate its validity, it has been applied to structures loaded with symmetric capacitances, similar to those exhibited by actual heterostructure barrier varactors. The model can be of help to understand harmonic generation in monolithic NLTL-based frequency multipliers. © *2001 American Institute of Physics.*
[DOI: 10.1063/1.1369619]

Interest in monolithic nonlinear transmission lines (NLTLs) has grown during the last few years due to their applications in the fabrication of broadband high efficiency frequency multipliers operating above 100 GHz.[1–3] Essentially, a NLTL is a ladder network consisting of a high impedance propagating medium periodically loaded with nonlinear capacitive devices, such as Schottky diodes or heterostructure barrier varactors (HBVs) [much effort has been recently dedicated to the optimization of HBVs[4] since their symmetric capacitance–voltage (C–V) characteristic is of interest for the suppression of even order harmonics]. In these structures, nonlinearity and dispersion (which is introduced by periodicity), both combined can balance their effects and give rise to the propagation of electrical pulses with permanent profile, i.e., solitons.[5] The multiplicative process in NLTLs has been interpreted as a decomposition of the input signal waveform in a set of solitons of different amplitudes and velocities.[6] A waveform of multiple solitons per cycle is thus progressively produced and harmonic multiplication achieved. Since monolithic NLTLs support solitary pulses of picosecond duration, output signals with high frequency harmonic content (hundreds of GHz and THz) can potentially be produced with NLTL-based multipliers. Based on the lumped element equivalent circuit of the NLTL, in this work we present a numerical model to study soliton behavior in NLTLs. The main advantage of this model over previous proposals[7] is that it allows to obtain soliton waveforms regardless of the considered nonlinear capacitance. The model can be used to predict soliton propagation in fabricated NLTLs and is especially useful in studying the influence of nonlinear device on soliton characteristics, since C–V curves experimentally obtained on nonlinear loading devices can be introduced in the formulation. Therefore, the model can be of interest to understand harmonic generation in NLTLs and as a guide in multiplier design.

Figure 1 shows the schematic of a NLTL together with the lumped equivalent circuit model, which has been previously used to study harmonic multiplication[2,6] and pulse sharpening.[8] L and C_0 are the per-section inductance and capacitance, respectively, of the line, while $C_D(V)$ represents the nonlinear device capacitance. The study of soliton propagation in nonlinear networks of the type of Fig. 1(b) was already carried out by Hirota and Suzuki[7] almost three decades ago. They considered a nonlinear shunt capacitance of the form $C(V) \propto 1/(V - V_0)$ (where V_0 is a constant). In this particular case, the system is equivalent to the Toda lattice,[9] for which soliton solutions are well known, and can be described by the Korteweg–de Vries (KdV) equation in the long wave limit (i.e., nonlocalized solitons). Systems based on the KdV equation have been well studied and the response of such systems to initial disturbances that breakup into solitons has been described by the inverse scattering method.[10] However, for strong lumped solitons,[11] or to study soliton propagation in nonlinear LC networks with arbitrary $C(V)$ nonlinearity, a KdV approach cannot generally be applied. Therefore, a new procedure to obtain soliton solutions in NLTLs described by its lumped element equivalent circuit will be presented, and applied to structures with HBV-like nonlinear devices.

Regarding the validity of the lumped network to describe the distributed NLTL of Fig. 1(a), it has been commonly accepted to study the propagation of signals with wave-

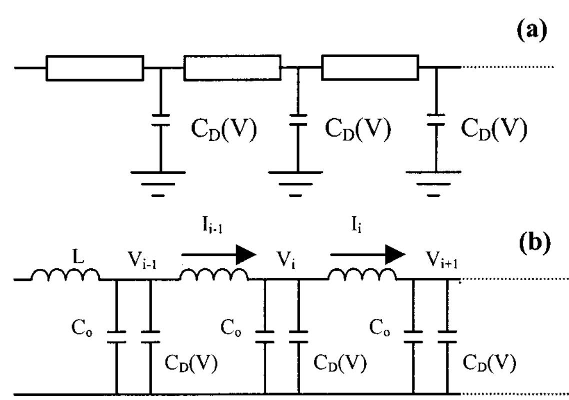

FIG. 1. Nonlinear transmission line circuit schematic (a) and lumped element equivalent circuit (b). Each transmission line section between nonlinear capacitances is modeled by a series inductor, L, and shunt capacitor, C_0.

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lengths exceeding the spacing between voltage-variable capacitors (a distributed model of the NLTL has been recently proposed to deal with wide-bandwidth signals[12]). On the other hand, although device and transmission line losses can degrade the performance of NLTLs acting as frequency multipliers, and cannot be discarded in realistic circuit simulations, this work is focused on the propagation of pulse-like solitons, which is only possible in NLTLs with negligible losses (diffusive solitons are possible in nonlinear dissipative media,[13] but these kink-shaped solitons are not interesting for harmonic generation). For this reason losses are not taken into account in the model.

Returning to Fig. 1(b), by imposing the condition that a signal pulse $V(t)$ propagates along the line without distortion, the following equation must be satisfied:

$$L\left[C[V(t)]\frac{d^2V(t)}{dt^2}+\left(\frac{dV(t)}{dt}\right)^2\frac{dC(V)}{dV}\right] = V(t+T)-2V(t)+V(t-T), \tag{1}$$

where T is the per-section propagation delay (i.e., the time needed by the signal to propagate between successive sections) and $C(V)=C_0+C_D(V)$. This is a second-order nonlinear differential equation that has an infinite number of solutions. We are only interested in obtaining those physical solutions that correspond to solitary waves, i.e., symmetric voltage pulses that satisfy the boundary condition $V(t\rightarrow\pm\infty)=0$. To this end, we have proposed an approximate numerical method that allows solutions of the type indicated above to be obtained with little computation.

First of all, to avoid the indeterminacy caused by the time translation invariance of Eq. (1), we will consider that pulse maxima always occur at $t=0$ s, where all odd order derivatives of $V(t)$ are zero as a consequence of pulse symmetry. This is an additional boundary condition for the solution of Eq. (1). The fact that the right-hand side (rhs) of Eq. (1) contains two terms exhibiting time displacement severely complicates the solution of this equation. We have obtained an approximate equation by expanding in Taylor series up to the fourth order the first and third term of the rhs of Eq. (1). Due to the structure of the rhs, the zero-order term and odd derivatives cancel and Eq. (1) can be simplified as follows:

$$L\frac{d}{dt}\left(C(V)\frac{dV(t)}{dt}\right)=T^2\frac{d^2V(t)}{dt^2}+\frac{T^4}{12}\frac{d^4V(t)}{dt^4}. \tag{2}$$

It will be later shown that solutions of Eq. (2) satisfy Eq. (1) to a good approximation.

If Eq. (2) is integrated we obtain:

$$\left(C(V)-\frac{T^2}{L}\right)\frac{dV(t)}{dt}=\frac{T^4}{12L}\frac{d^3V(t)}{dt^3}, \tag{3}$$

where the independent term is null, provided odd-order derivatives vanish at pulse maximum ($t=0$ s). Equation (3) can be integrated again to give

$$F[V(t)]-\frac{T^2}{L}V(t)=\frac{T^4}{12L}\frac{d^2V(t)}{dt^2}, \tag{4}$$

where $F(V)$ is a function responsible for nonlinearity given by

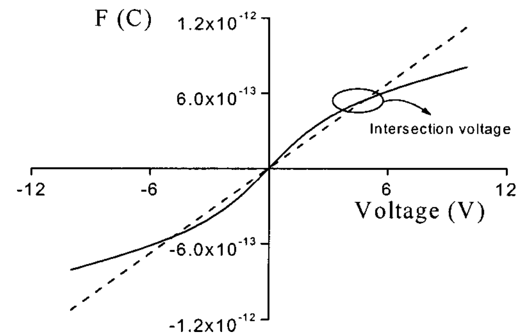


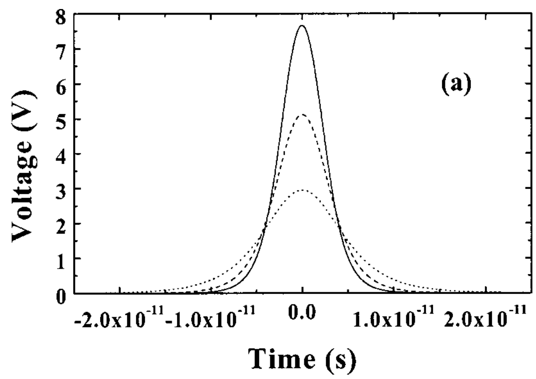
FIG. 2. Representation of $F(V)$ obtained from Eq. (5) with $C_0=4.35\times10^{-14}$ F and $C_D(V)=1.2\times10^{-13}$ sech(0.5 V) F (solid line). The second term on the lhs of Eq. (4) with $T=3.5$ ps and $L=1.087\times10^{-10}$ H is also depicted (dashed line).

$$F(V)=\int_0^V C(V')dV'. \tag{5}$$

Figure 2 depicts $F(V)$, which was obtained from the nonlinear capacitance given in the caption. The slope of this function asymptotically approaches C_0, the per-section capacitance of the line, while at zero bias the derivative is given by $C_0+C_D(0)$. An analysis of Eq. (4) for $t\rightarrow\pm\infty$ indicates that the integration constant is zero since all terms vanish at this limit.

Inspection of Eq. (4) reveals that solutions of the form indicated above (i.e., solitary waves) are possible. To demonstrate this, in Fig. 2 we have also depicted the second term of the left-hand side (lhs) of Eq. (4) (for the conditions indicated in the caption). Since the pulse maximum occurs at $t=0$ s, the lhs of Eq. (4) must be negative at this point (positive polarity pulses are considered). This is satisfied if the pulse amplitude is above the intersection voltage of Fig. 2. However, this is not enough to obtain solutions of the form indicated above. Namely, as voltage decreases from maximum, the curvature decreases, changes polarity at the crossing point and, finally, an oscillatory behavior is found, unless the pulse amplitude takes the precise value that makes the solution vanish for $t\rightarrow\pm\infty$. In view of the previous argument and Fig. 2, it is clear that solutions corresponding to solitary waves are only possible if an intersection point exists between both terms of the lhs of Eq. (4) above the origin. This limits the slope of the second term of the lhs to the range given by the asymptotic value of $F(V)$ and the derivative of $F(V)$ at the origin. This means that only solitons with per-section propagation delay T within the interval given by $\{(LC_0)^{1/2}-[LC(0)]^{1/2}\}$ can propagate in the line. For a given T in this interval, a single solution exists, and soliton amplitude increases as T decreases. This behavior is in agreement with previous work[7] and can be deduced from Fig. 2, where it is clearly seen that the intersection voltage increases as T approaches the lower limit of the above cited interval.

To numerically obtain solutions corresponding to solitons, Eq. (4) has been discretized, where the selected time interval is much lower than the per-section propagation delay, T. Then, an algorithm based on the shooting method has been applied, i.e., from the condition of pulse maximum at $t=0$ s, we have repeatedly solved Eq. (4) until a waveform that vanishes at the extremes has been obtained. To achieve convergence, a successive approximation scheme has been applied, that gives different pulse amplitudes at the start of



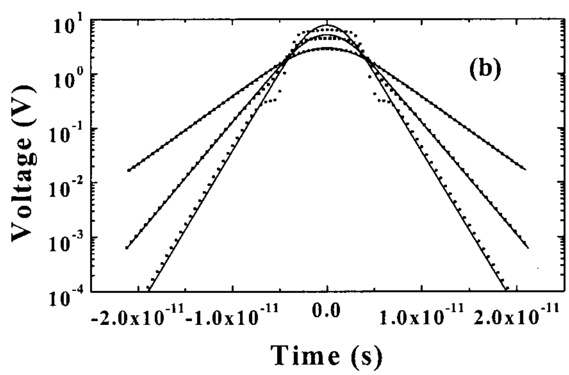


FIG. 3. Soliton waveforms obtained using the approximate model presented in this work (a) and estimated error (b). Each curve corresponds to a different per-section propagation delay: $T=3.5$ ps (solid line), $T=3.75$ ps (dashed line) and $T=4$ ps (dotted line). Model parameters indicated in Fig. 2, which are reasonable for actual NLTLs, have been used.

each iteration. In such a way, soliton shape can be obtained for each T within the cited limits. The main advantage of our method is versatility, which arises from the fact that soliton behavior can be studied whatever the C–V characteristic of the nonlinear capacitance is assumed (including the possibility to consider experimental C–V curves to analyze soliton propagation in actual structures). In Fig. 3, soliton waveforms with different propagation delays that correspond to the network of Fig. 1(b) demonstrate that higher amplitude solitons are faster. The nonlinear capacitance has been assumed to be given by $C_D(V)=1.2\times10^{-13}\,\mathrm{sech}(0.5\ V)$ F, which is a reasonable approximation to the capacitance dependence on bias for a typical HBV. In order to demonstrate the validity of the presented approximate model, we have proceeded as follows: we have isolated $V(t)$ on the rhs of Eq. (1); the solutions depicted in Fig. 3(a) have been introduced in the resulting equation and both parts compared. The results, represented in Fig. 3(b) as a logarithmic plot, show a small deviation over several decades. This demonstrates that our simple approximate approach provides solutions in very close agreement to the exact solutions of Eq. (1). Therefore our model can be used to study soliton propagation in NLTLs that cannot be described by the KdV or Toda lattice equations. Figure 4 depicts the interaction of two solitons obtained by numerical simulation of the network of Fig. 1(b). This figure demonstrates that solitons are stable and they preserve their identity after collision. This is an additional condition for considering our solutions as actual classical solitons (in contrast to envelope or hole solitons, which are well known wave entities in optical communications[14]).

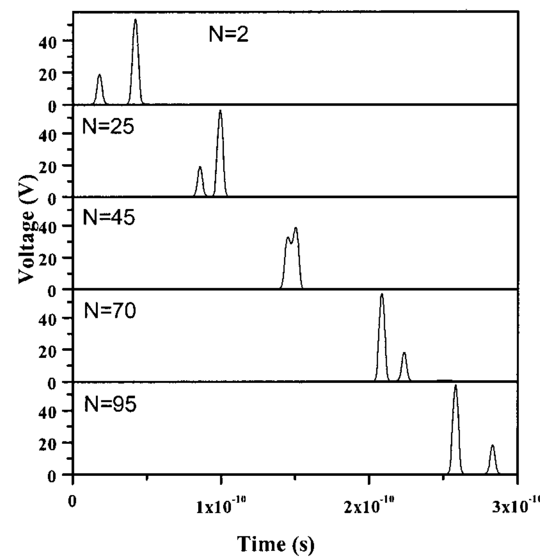


FIG. 4. Interaction of two solitons propagating in the same direction with different time delay ($T=2.5$ ps for the high amplitude soliton and $T=3$ ps for the smaller amplitude soliton). The same NLTL parameters as in Fig. 2 have been used. N indicates the section number of the line.

In conclusion, a numerical model has been presented in order to study soliton wave propagation in periodically loaded NLTLs that cannot be described by the KdV equation. By considering model parameters of typical HBV-like NLTL structures, soliton waveforms for different propagation delays have been numerically obtained. It has been found that the resulting voltage pulses are stable and maintain shape and speed after interaction. These results validate our model for use in studying soliton behavior in NLTLs. The model can be of help to understand soliton-like harmonic generation in NLTL-based frequency multipliers.

The authors would like to thank the *Quantum Devices and Terahertz Electronics Group* of IEMN (Lille University) for helpful discussions, and are also in debt to A. Calsina for assistance in mathematical aspects. This work has been supported by the DGES under Project No. PB97-0182.

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