Compensation and Span of Control in Hierarchical Organizations

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This article presents evidence on the relationship between compensation ratios and spans of control within hierarchical organizations. We find that compensation ratios are lower than span of control at any position within the hierarchy, which is consistent with an elasticity of compensation to a number of subordinates lower than one. Managers' human capital endowments determine a significant part of the salary differences throughout hierarchical levels, as predicted by models of talent allocation in hierarchies. Differences in the size of firms should be attributed more to differences in their number of hierarchical levels than to variations in the span of control.

I. Introduction

Organizational hierarchies are characterized by two main features: the span of control in each managerial position and the evolution of salaries throughout hierarchical levels. A significant amount of work has been done on the distribution of salaries in multilevel organizations, but research into the span of control, and especially the relationship between salaries and span of control in intermediate hierarchical levels, is much

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more scarce.[1] This article contributes to the understanding of hierarchical organizations by providing empirical evidence on the evolution of salaries and span of control throughout hierarchical organizations as the result of allocating managerial talent in multilevel job positions in a sample of Spanish firms. We also provide evidence that the evolution may be the result of a rational process of managerial talent allocation, where talent is approximated by managers' human capital.

Earlier theoretical papers on the internal organization of firms (Simon 1957; Williamson 1967) model hierarchies by assuming that salaries and span of control are exogenous to the model. Later, Beckmann (1977) assumes that firms choose the span of control to mitigate the "loss of control" that occurs in hierarchies, but salaries are still exogenous. Geanakoplos and Milgrom (1991) analyze the optimal allocation of managerial time in order to coordinate the decisions of imperfectly informed employees, deriving implications for salaries and span of control. In all this literature, the hierarchy is viewed as a coordinating device. Incentive issues are introduced in Calvo and Wellisz (1978, 1979) and Qian (1994), but only Qian (1994) models the two features of the hierarchy, salaries and span of control, though assuming homogeneous managers and workers. Another line of research (Rosen 1982; Waldman 1984; Gibbons and Waldman 1999) investigates the allocation of different talented managers to previously established job positions, with implications for wage formation. However, this ignores the issue of the span of control at the different hierarchical levels.

In order to guide the empirical research, we extend the classical hierarchical models of coordination with loss of control, assuming that the profit-maximizing firm can chose managers with different human capital endowments to be placed in each hierarchical position. Managers who are more talented lower the control loss and improve overall productivity, but they earn higher salaries and increase production costs. By choosing the profit-maximizing level of human capital in each position, firms are endogenously determining the span of control and the salaries earned by the manager.[2]

[1] Some relevant papers on the distribution of salaries in hierarchical organizations are Lazear (1992); Baker, Gibbs, Holmstrom (1993, 1994*a*, 1994*b*); Lambert, Larcker, and Weigelt (1993); Main, O'Reilly, and Wade (1993); and Eriksson (1999). There is also an extensive literature on the relationship between top managers' compensation and firms' size; see Rosen (1992) or Murphy (1985, 1999) for a synthesis. Extensions of these analyses to intermediate hierarchical levels are scarce (Gerhart and Milkovich 1990; Leonard 1990), and the only evidence on the evolution of the span of control and compensation at middle hierarchical levels that we are aware of is Baker et al. (1993).

[2] The static nature of the model drives us to ignore the problems of asymmetric information on managers' intrinsic ability and the consequences of learning about this over time (as analyzed by Gibbons and Waldman 1999).

The empirical framework predicts that, under convex cost functions of human capital production, firms choose a span of control greater than one in each hierarchical level, so the usual inverted tree structure emerges. Firms also want managers who are more talented to be allocated into higher hierarchical positions, so we should observe higher salaries being paid in these positions. Moreover, in the profit-maximizing solution, the ratio between compensations of managers and of their direct subordinates is lower than managers' span of control. This result has implications for the value of the elasticity of substitution between managers and direct subordinates in a given hierarchical level and for the elasticity of managers' compensation with respect to the size of the firm.

The empirical evidence is presented in three ways. First, we estimate the average ratio of compensation between managers and their direct subordinates, and compare it with the managers' span of control at each hierarchical level, using data from a large sample of Spanish firms and managers. The comparison will provide evidence about elasticity of substitution between managers and their direct subordinates, greater than one.

Second, the empirical analysis is extended to evaluate the proportion of compensation variance throughout hierarchical levels in the same sample, which is explained by differences in the endowment of human capital in each level. This is relevant because theories such as efficiency wages (Calvo and Wellisz 1978, 1979; Qian 1994, e.g.) and tournament competition (Lazear and Rosen 1981; Rosen 1986, e.g.) predict that salaries will increase throughout hierarchical levels even when the manager population is homogeneous in talent, due to incentive reasons. However, our framework and in general the optimal human capital allocation models (Rosen 1982; Waldman 1984, e.g) explain compensation differences as a function of heterogeneity in ability and talent.

Third, we decompose the elasticity of compensation to the size of resources under control for the managers in the sample, in intra- and interlevel effects. Since the latter appear to be more important than the former, we conclude that in the firms of the sample, differences in their size are explained more by differences in their number of hierarchical levels than by differences in their span of control within a level. This result helps to explain variations in the estimated elasticity of managerial compensation to the size of the firm. Rosen (1992) reports that many studies find an elasticity of around 0.25 in samples of top managers, but Leonard (1990) and Gerhart and Milkovich (1990) obtain an elasticity of around 0.005 for top and middle managers.

This article is organized as follows. Section II presents a review of the literature on hierarchical organizations and outlines the main empirical implications to be tested with the available data. Section III presents a description of the sample data and the results of the empirical estimations. The conclusions summarize the main results.

II. Literature Review and Hypothesis

Hierarchical structures can be understood as a collection of labor contracts that link workers of a given level with the subordinates that report directly to them (Holmström and Tirole 1989). These contracts induce a level of output Q as a function of the number of direct workers, L_n, and the inputs of their hierarchical superiors, which implies an average productivity for each direct worker of b_n. Therefore, $Q = b_n L_n$, where n is the lowest level, direct workers.

Let $a_j = b_{j+1}/b_j \leq 1$ be the decrease in average productivity of direct workers that takes place when the hierarchy changes from j levels to $j+1$ levels, and let $t_j = L_{j+1}/L_j$ be the number of subordinates at level $j+1$ that report to one manager of level j, that is, the span of control. Thus, the output of the hierarchy may be written as

$$Q = b_n L_n = b_n \prod_{j=0}^{n-1} t_j = b \prod_{j=0}^{n-1} t_j a_j,$$

where b is the productivity of top management, $j = 0$. Notice that a_j can also be interpreted in terms of loss of control at level j (Williamson 1967).

We assume that a_j is not fixed, but depends on the span of control t_j and on the differences of human capital endowment accumulated from the top of the hierarchy z_0 to the direct subordinates of the manager in level j, z_{j+1}:

$$a_j = t_j\left(\prod_{i=0}^{j} \frac{z_{j+1}}{z_i}\right) - 1 = t_j(z_{j-1}/z_0) - 1.$$

$0 \leq z_{j+1} \leq z_0$ and $t_j \geq 1$ in order[3] that $a_j \leq 1$. Therefore, a_j decreases with the span of control t_j and increases with the human capital endowment of the direct subordinates, z_{j+1}. Normalizing the talent of top managers to one and substituting in the production function above, we obtain

$$\begin{aligned} Q &= b_n L_n = b_n \prod_{j=0}^{n-1} t_j = b \prod_{j=0}^{n-1} t_j a_j = b\{t_0[t_1 \ldots (t_{n-1})^{(z_n/z_{n-1})} \ldots]^{(z_2/z_1)}\}^{(z_1/z_0)} \\ &= b \prod_{j=0}^{n-1} t_j^{z_{j+1}} = bL_n^{z_n} \prod_{j=1}^{n-1} L_j^{z_j - z_{j+1}}. \end{aligned}$$

[3] Actually, this is equivalent to assuming that when $t_j \leq 1$ and/or $z_0 \leq z_j$, supervision is perfect, $a_j = 1$. We omit the analysis of those cases in which $t_j < 1$ and/or $z_0 < z_j$, because if there are positive profits, they will be dominated by $t_j = 1$ and/or $z_0 = z_j$.

We can therefore define $\sigma_{j,j+1}$ as the elasticity of substitution between managers of level j and their subordinates in level $j+1$, for a given level of output,

$$\sigma_{j,j+1} = \frac{\partial \ln Q / \partial \ln L_{j+1}}{\partial \ln Q / \partial \ln L_j} = \frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}} = \frac{\Delta z_{j+1}}{\Delta z_j}.$$

Beckmann (1977) and Rosen (1982) assume that the elasticity of substitution above is a fixed parameter exogenous to the decisions of the firm. We argue that firms can choose the elasticity of substitution by modifying the differences in human capital endowments between consecutive levels.

The profit-maximization problem of the firm will be written as

$$\begin{aligned} \max P_{L_1,\ldots L_n,\; z_1\ldots,z_n} &= pb \prod_{j=0}^{n-1} t_j^{z_j+1} - \sum_{j=1}^{n} w_j L_j \\ &= pbL_n^{z_n} \prod_{j=1}^{n-1} L_j^{z_j - z_{j+1}} - \sum_{j=1}^{n} e^{f(z_j)} L_j, \\ &\text{subject to } 1 \geq z_j \geq 0, \text{ and } t_j \geq 1, \end{aligned}$$

where p is the price of output and w_j is the salary of workers at level j, which depends on workers' human capital endowment, $w_j = e^{f(z_j)}$. We assume that $f(z_j)$ is increasing and is a convex function of z_j, that is, there are decreasing returns in the production of managerial talent.[4]

Qian (1994) and Calvo and Wellisz (1978) use efficiency wage arguments to assume that managerial effort e_j is endogenous. Their assumptions can easily be incorporated into our model by identifying loss of control with level of effort, $e_j = a_j = t_j^{z_{j+1}-1}$. The effective productivity of managers in level $j+1$, compared with the effort of supervisors in level j, decreases with the span of control (supervision) and increases with compensation. The variable z_j is now interpreted as the response of effort to changes in salary, $z_j = f^{-1}(\ln w_j)$.

Appendix A shows that, at the profit-maximizing solution, the value of z_j increases as j approaches the top of the hierarchy, but at a decreasing rate, $0 < z_j - z_{j+1} = \Delta z_j < \Delta z_{j+1} = z_{j+1} - z_{j+2}$. The first result implies that, in the optimal solution, compensation increases as one moves to the top of the hierarchy, $w_j > w_{j+1}$. The second result is explained by the convexity of the cost function $f(z_j)$.

The pattern of human capital allocation implies that, in the profit-

[4] For empirical evidence of such functional forms, see Mincer (1974) or Blaug (1992).

maximizing solution, the elasticity of substitution between managers and direct subordinates will be higher than one,

$$\sigma_{j,j+1} = \frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}} = \frac{\Delta z_{j+1}}{\Delta z_j} > 1.$$

Moreover, we also know that in the optimal solution, the ratio of total input costs will be equal to the elasticity of substitution between inputs,

$$\frac{w_{j+1} L_{j+1}}{w_j L_j} = \sigma_{j,j+1}.$$

Therefore,

$$\frac{L_{j+1}}{L_j} = \frac{w_j}{w_{j+1}} \sigma_{j,j+1}. \quad (1)$$

Equation (1) has two clear implications. First, the span of control $t_j = L_{j+1}/L_j$ will be greater than one, and consequently the inverted tree structure of the hierarchy emerges; this result comes from $w_j > w_{j+1}$ and $\sigma_{j,j+1} > 1$. Second, if we compare span of control in j, t_j, with the compensation ratio also in j, $\beta_j = w_j/w_{j+1}$, we should observe $t_j/\beta_j = \sigma_{j,j+1} > 1$, that is, the span of control will be larger than the compensation ratio.

Finally, notice that a simple extension of our model has implications for the relation between compensation and number of total subordinates reporting to managers placed at different hierarchical levels and firms. For this purpose, we assume that firms differ in the number of hierarchical levels, n_i, so the compensation of the manager allocated at hierarchical level j in the firm i will be

$$\ln w_{j,i} = \ln\left(w_{n_i,i} \prod_{r=j}^{n_i-1} \frac{w_{r,i}}{w_{r+1,i}}\right) = \ln\left(w_{n,i} \prod_{r=j}^{n_i-1} \beta_{r,i}\right).$$

In order to simplify, we assume that direct worker compensation is the same for all the firms, $w_n^i = w_d$, that the compensation ratios and the span of control are also constant for all the firms and hierarchical levels, $\beta_{j,i} = \beta$, $t_{j,i} = t$, and, consequently, $\sigma_{j,j+1,i} = \sigma$. Under these assumptions, the number of direct workers commanded by a manager at the hierarchical level j and firm i will be $A_{j,i} = t^{n_i - j}$, and the managers' compensation can be written as

$$\ln w_{j,i} = \ln(w_d \beta^{n_j - j}) = \ln w_d + (n_i - j) \ln \beta = \ln w_d + \left(\frac{\ln \beta}{\ln t}\right) \ln A_{j,i}. \quad (2)$$

Taking into account that the elasticity of substitution among managers and their direct subordinates is greater than one, $t/\beta = \sigma > 1$, the elasticity of managers' compensation with respect to number of direct workers commanded by the manager, will be less than one, $(\ln \beta / \ln t) < 1$.

Equation (2) generalizes a result of Rosen (1982) to multilevel hierarchies and to the case of endogenously determined elasticity of substitution. Rosen shows that, in a two-level hierarchy, a sufficient condition for an elasticity of manager compensation to the number of direct workers lower than one is an elasticity of substitution among managers and direct workers higher than one; however, the value of this elasticity is not derived from the model.

In the argumentation above, the differences in the number of direct workers among firms only derive from extensions of the hierarchical levels. Obviously, differences in the span of control can also exist between firms and among positions inside a given firm. How important the number of hierarchical levels are, compared with heterogeneity within firms and among firms in span of control, when determining the observed differences in firm size is a relevant empirical question.

Extensions

The predictions that salaries increase with hierarchical level and that the span of control in each level is greater than one are also obtained from models that assume homogeneous workers and different levels of effort, as in Calvo and Wellisz (1979) and Qian (1994). Of course, we could have situations where managers differ in both human capital and effort. Calvo and Wellisz (1979) allow for differences in the quality of supervision in a model with a fixed number of hierarchical levels. They find that managers who are more able should be placed in higher hierarchical positions, earn higher salaries, and command larger spans of control. The third prediction, that the optimal span of control increases with the hierarchical level, does not follow from our model, which tells us nothing about the evolution of the span of control, of the increase in salaries, or of the elasticity of substitution among managers and subordinates throughout hierarchical levels.

The information-processing model of the managerial function developed by Geanakoplos and Milgrom (1991) shows that the ablest managers will be allocated to the top of the hierarchy only when their decisions affect the subsequent information processing of their subordinates. In these cases, the authors point out that certain substitutions can exist between managerial abilities and span of control, something that is also captured in our framework.

Another issue, ignored in the model, is the possible use of the tournament type of incentives to extract greater managerial effort. By attaching higher salaries to higher hierarchical positions and filling vacancies with the worker who exerts more effort at the level below, a strong incentive system is created, which will stimulate effort with low direct supervision. These are the foundations of the tournament theory, put forward by Lazear and Rosen (1981). Salaries are again attached to jobs, but no prediction is made with respect to the implications for decision on the span of control in the hierarchy. Rosen (1986) goes on to develop a model with an elimination tournament structure, and predicts that the shape of the compensation function throughout hierarchical levels should present a "convex shape," that is, compensation increases should be higher at the upper levels of the hierarchy.

Empirical Predictions

The presentation above makes clear predictions about the evolution of span of control and compensation within hierarchies. We now summarize these predictions before proceeding to the empirical analysis.

i) The span of control is greater than one and may be higher in upper hierarchical positions than in lower ones (as in the Calvo and Wellisz [1979] model).

ii) Compensation increases with the hierarchical level, and the rate of increase may be higher in top hierarchical positions if tournament effects are present.

iii) In any hierarchical position, managers' span of control will be higher than the ratio between managers' compensation and that of their direct subordinates. This result is implied by an elasticity of substitution between managers and their direct subordinates, higher than one.

iv) Differences in human capital endowment should explain most of the variance of compensation throughout hierarchical levels, if firms use managerial talent to increase productivity. Under the pure incentive effects of hierarchy, the variance of compensation throughout hierarchical levels should be independent of human capital endowments.

The same arguments apply to the influence of human capital endowments on the level of responsibility (number of subordinates) assigned to each manager.

v) The elasticity of managerial compensation to the size of resources commanded by the manager is lower than one in all managerial positions. However, the importance of intralevel versus interlevel variability in determining this elasticity is an empirical question. The answer will tell us about the sources of difference in the size of firms, difference in the number of hierarchical levels, and difference in span of control within levels.

III. The Empirical Evidence

Most of the empirical predictions in the previous section can be tested using data from a single firm. In fact, Baker et al. (1993) present descriptive statistics about compensation and span of control of a single firm over time that are consistent with our predictions i–iii. However, our database concerns managers from different firms, and at the same time, the number of managers from the same firm is very low; firm-by-firm analysis is therefore not viable. Account will be taken of interfirm heterogeneity and the estimated values of the span of control and salary ratios will be averages across firms after controlling for firm-specific effects. However, we have information about human capital variables for each manager, and therefore it is possible to investigate how important these variables are in explaining the pattern of managerial talent allocation and compensation (prediction iv). Finally, we can relate our results with previous empirical evidence on the elasticity of compensation to the size of resources controlled by a manager and decompose it into interlevel and intralevel effects (prediction v).

The origin of our database is similar to that used by Leonard (1990) and Gerhart and Milkovich (1990), in the sense that has been provided by a consulting firm on human resources, Ingenieros Consultores Sociedad Anónima. We have information about 9,694 managers distributed throughout 669 Spanish firms and six hierarchical levels, starting from the position of the general manager. Although average firm size in the sample is much smaller than the average size in American samples (569 employees and $138.5 million in sales compared with 30,000 employees and $5 billion in sales), the sample is still biased toward relatively large firms.[5] The data are pooled for the years 1990, 1991, and 1992, which implies that some firms may be repeated. However, reasons of confidentiality do not allow us to know the identity of each firm.[6]

We have some information of a personal character (education, age, job tenure) for each manager, as well as a description of his or her job (compensation, hierarchical level, and the functional area inside the firm—production, marketing, finance, and personnel). Table 1 presents the descriptive statistics of the variables considered in the analysis. Notice that compensation includes only base salary plus bonuses, although other forms of compensation such as shares in the firm, stock options, and so on, were very rare in Spain in the early 1990s and were basically nonexistent for middle-level managers.

One important variable in our analysis is the level of the manager in the organizational hierarchy. The questionnaire sent to the firms asks for

[5] Only 7.6% of the population of Spanish firms have more than 50 employees. In our sample, 81% of the firms have more than 50 employees.

[6] The conclusions do not change if we estimate the model for each separate year.

Table 1
Descriptive Statistics

	Cases	Compensation w	Subordinates A	University Degree: Graduate S_1 (%)	University Degree: Undergraduate S_2 (%)	Age h	Job Tenure an	Firms	Sales
General managers ($j = 0$)	669	13,096	486	71.75	20.63	48.18	8.88	669	14,121
Division managers ($j = 1$)	185	9,599	164	70.81	18.92	45.48	6.76	76	16,018
Production:									
Level 1 ($j = 2$)	933	6,861	67	48.12	36.01	44.52	7.68	464	12,733
Level 2 ($j = 3$)	1,576	4,948	46	29.19	41.50	43.07	7.81	452	16,720
Level 3 ($j = 4$)	652	4,287	48	25.46	32.21	43.30	8.12	217	16,660
Level 4 ($j = 5$)	168	4,026	79	16.67	46.63	43.67	7.14	54	34,160
Marketing:									
Level 1 ($j = 2$)	841	7,703	32	46.13	37.34	43.45	6.53	505	15,730
Level 2 ($j = 3$)	1,226	5,810	17	30.75	45.76	41.71	6.00	399	19,326
Level 3 ($j = 4$)	396	5,145	35	31.57	35.35	40.06	5.07	138	27,421
Level 4 ($j = 5$)	100	5,117	73	16.00	46.63	43.67	4.64	26	32,577
Finance:									
Level 1 ($j = 2$)	813	7,200	23	58.92	28.17	42.03	6.95	560	13,877
Level 2 ($j = 3$)	1,049	5,189	9	39.94	37.08	40.41	7.04	461	19,233
Level 3 ($j = 4$)	312	4,969	9	32.05	35.90	40.93	6.71	138	36,070
Level 4 ($j = 5$)	98	4,326	6	50.00	30.43	39.02	5.88	36	62,689
Personnel:									
Level 1 ($j = 2$)	255	6,954	15	53.73	36.47	43.98	7.35	250	26,636
Level 2 ($j = 3$)	295	5,044	19	33.56	42.37	42.76	7.37	227	22,278
Level 3 ($j = 4$)	103	4,819	11	40.78	35.92	40.03	6.51	64	58,921
Level 4 ($j = 5$)	23	3,563	4	60.87	30.43	34.96	3.43	15	82,187
All sample	9,694	6,341	67	40.81	36.69	42.83	7.12	669	14,121

NOTE.—Average compensation is expressed in thousands of pesetas, sales in millions of pesetas (1990 value) for the managers ("Cases") and firms ("Firms") represented in each category. Average age and job tenure are expressed in years. The category omitted in education is that of managers without a university degree.

the manager whose compensation data are requested, to be placed in one of six hierarchical levels—starting from the general manager, followed by division manager, functional area manager, and three consecutive hierarchical levels below the functional area manager in each area. The hierarchical levels do not explicitly refer to compensation levels, but there is no way of knowing to what extent the levels respond to the actual steps followed by the managers in being promoted within the firm, as in Baker et al. (1993, 1994*a*). However, it is reasonable to assume that we should be closer to the description of the hierarchy in Baker et al., for a single firm, than to the hierarchy in Main et al. (1993) and Eriksson (1999), where only managers in very top positions are considered, that is, vice presidents who compete for the job of CEO. We should also bear in mind that we have a large number of small firms in our sample, and therefore, in some job positions only a few firms will register observations. As we will show below, this is taken into account in the measurement of compensation ratios and spans of control in each hierarchical position.

A more general issue is the use of single-firm or multifirm data to test propositions about internal labor markets and firm's organization. Having complete data on job positions and compensation for a single firm over time, as in Baker et al. (1994*a*, 1994*b*), avoids interfirm heterogeneity and captures time dynamics. Static data from managers belonging to many firms and jobs allow us to make interfirm comparisons and to introduce small firms into the analysis. The theoretical propositions are all static and concern common points of the hierarchical design of any organization; in this case, therefore, it is important to test the propositions for a range of different organizations. Previous empirical evidence shows that organizational hierarchies tend to be stable over time (Baker et al. 1994*a*), and therefore the static nature of our analysis should not be an obstacle to generalizing the results.

Nevertheless, firms in our database have different hierarchical structures, and for some job positions, we only have information for a limited sample of managers. This means that heterogeneity across firms deserves special attention and our article develops a specific methodology to deal with this. The recognition of heterogeneity is also important in correctly interpreting certain empirical evidence obtained from aggregated firm-level data, such as the elasticity of general mangers' compensation to the size of the firm.

Compensation, Span of Control, and Elasticities of Substitution

We first concentrate on the evolution of salaries and span of control throughout the hierarchy (empirical predictions i–iii). In accordance with the theoretical model, general managers are assigned to level 0, and the salary of the lowest hierarchical level is exogenous. Define $w_{5,i}$ as the total compensation of managers in level 5 at firm i, the lower level for which information is available. The wage of manager k at the hierarchical level j of the firm i, can be expressed, in logs, as

$$\ln w_k = \ln\left(w_{5,i}\prod_{r=j}^{4}\beta_r e^{\varepsilon_k}\right),$$

where β_r is the ratio between managers' and direct subordinates' compensation, $\beta_r = w_r/w_{r+1}$, and ε_k captures the compensation variation within hierarchical levels inside the same firm or differences in salary increases across the different firms in the sample.

The equation above can be written as

$$\begin{aligned}\ln w_k &= \ln\left(w_{5,i}\prod_{r=j}^{4}\beta_r e^{\varepsilon_k}\right)\\ &= \ln w_{5,i} + \sum_{r=j}^{4}\ln\beta_r + \varepsilon_k \qquad (3)\\ &= \sum_i \tau_i E_{i,k} + \sum_{j=0}^{4}\alpha_j N_{j,k} + \varepsilon_k,\end{aligned}$$

where E_i is a dummy variable capturing firm-specific effects and is equal to one for managers of firm i, and is otherwise zero. The coefficient $\tau_i = \ln w_{5,i}$ provides an estimation of manager compensation in the lowest level of the hierarchy in firm i. The dummy variable N_j takes the value of one for managers at level j, and is otherwise zero. The coefficient $\alpha_j = \sum_{r=j}^{4}\ln\beta_r$ measures the average increase in compensation from the lowest level of the hierarchy to level j.

The model takes into account the data limitation due to missing information on managers in certain positions and due to differences in firm size. Consider the case of missing information in level 4 of firm i. In this case, firm i will be included to estimate compensation increases between levels 5 and 3, but not in computing compensation increases between hierarchical levels 5 and 4 or 4 and 3. However, if a firm is small and only has managers at level 4, this firm will not be considered when computing salary increases between hierarchical levels 4 and 5. Therefore $\ln w_{5,i}$ is estimated by taking into account the fact that compensation at level 4 for this firm i will be $\ln w_{5,i} + \ln\beta_4$. It is not therefore necessary to have full information on all the hierarchical levels for each firm in order to estimate the model. In this way, we can work with firms of different sizes and with missing values, as is the case for the great majority of our firms, according to columns of "firms" and "cases" in table 1.

Once α_j has been estimated, the rate of increase in compensation between levels j and $j+1$, $\ln \beta_j = \ln w_j/w_{j+1}$, is computed as

$$\ln \hat{\beta}_j = \hat{\alpha}_j - \hat{\alpha}_{j+1}, \quad \alpha_5 = 0, \quad j = 0, 1, \ldots, 4.$$

Similarly, we can estimate the equations

$$\begin{aligned} \ln A_k &= \ln\left(A_{5,i} \prod_{r=j+1}^{5} t_r e^{\nu_k}\right) = \ln A_{5,i} + \sum_{r=j+1}^{5} \ln t_r + \nu_k \\ &= \sum\nolimits_i \lambda_i E_{i,k} + \sum_{j=0}^{4} \mu_j N_{j,k} + \nu_k; \end{aligned} \tag{4}$$

$$\begin{aligned} \ln(A_k/w_k) &= \ln\left(\sigma_{n_i,5} \prod_{r=j+1}^{5} \sigma_{r-1,r} e^{\nu_k} e^{-\varepsilon_k}\right) \\ &= \sum\nolimits_i (\lambda_i - \tau_i) E_{i,k} + \sum_{j=0}^{4} (\mu_j - \alpha_j) N_{j,k} + \nu_k - \varepsilon_k, \end{aligned} \tag{5}$$

and obtain the logs for span of control

$$\ln \hat{t}_j = \hat{\mu}_j - \hat{\mu}_{+1j}, \hat{\mu}_5 = 0, j = 0, 1, \ldots, 4,$$

and elasticity of substitution

$$\ln \hat{\sigma}_{j,j+1} = (\hat{\mu}_j - \hat{\alpha}_j) - (\hat{\mu}_{j+1} - \hat{\alpha}_{j+1}), j = 0, 1, \ldots, 4.$$

The percentage of the variance in compensation explained by the dummy variables of firms and level is reasonably high, at around 80%. The estimation of average span of control (in logs) and average elasticity of substitution (also in logs), across firms and levels, also shows reasonably high R^2, except in the area of production.

The empirical results are consistent with theoretical predictions. Compensation increases as we move to the top of the hierarchy and the span of control and elasticity of substitution between managers and their direct subordinates are both greater than one.

Notice also that, with the exception of division managers with respect to functional area managers, the rate of increase in salaries is higher in the top positions of the hierarchy, as the tournament theory predicts.[7] However, the magnitude of these increases is much lower than in other tournament studies. For example, Main et al. (1993) estimate an increase of 150% in compensation for the CEO with respect to the first vice president, and we obtain increases of less than 50% for the general man-

[7] Equation (3) was estimated with the null hypothesis of $\ln \beta_j$ constant and equal for all j. This hypothesis was rejected at the 1% level of statistical significance.

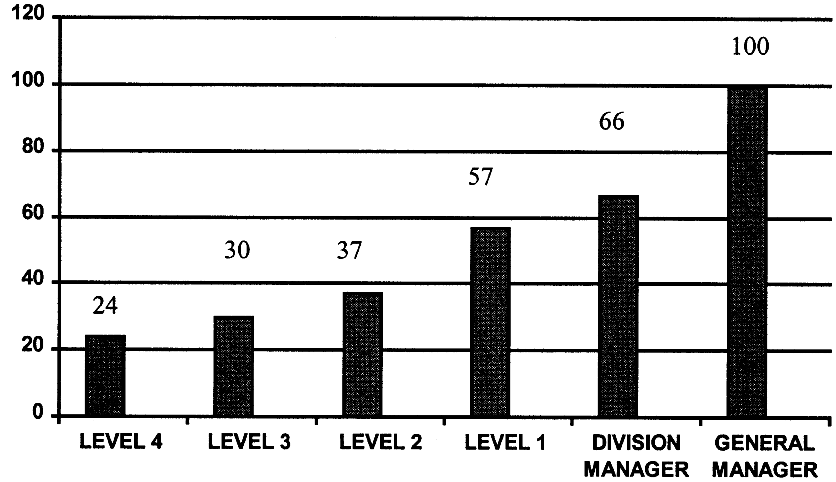

FIG. 1.—Average estimated compensation by hierarchical level. General managers' compensation is set at 100. Average compensation across functional areas is obtained by weighting estimated values by number of observations in each functional area.

ager with respect to the division manager.[8] Figure 1 shows the evolution of estimated average compensations throughout hierarchical levels, consistent with the "convexity" predicted by the tournament theory.

According to table 2, ln t_j tends to be higher for the top hierarchical positions,[9] which would be consistent with the supervision and talent allocation model of Calvo and Wellisz (1978). Finally, a value of ln $\sigma_{j,j+1}$ significantly different from zero confirms that average compensation ratios β_j are lower than the average span of control t_j as the model predicts.

The estimated value for elasticity of substitution $\sigma_{j,j+1}$ is higher at the position of the general manager.[10] According to the model, this means that, at the top of the hierarchy, the same increase of managerial talent requires greater compensation increments than at lower levels (the cost function of producing talent is more convex).

The Explanatory Power of Human Capital Variables

If human capital variables were unimportant in explaining the allocation of managers to hierarchical positions, with the resulting implications for

[8] Differences in average sizes for firms across samples and in their institutional environments (ownership and governance) may explain these differences.

[9] Equation (4) was estimated with the null hypothesis of $\ln t_j$ constant and equal for all j. The hypothesis was rejected at the 1% level of statistical significance.

[10] Equation (5) was estimated with the null hypothesis of $\ln \sigma_{j,j+1}$ constant and equal for all j. The hypothesis was rejected at the 1% level of statistical significance.

Table 2
Salaries, Number of Subordinates, and Elasticities of Substitution

Manager	Production			Marketing			Finance			Personnel		
	$\ln\hat{\beta}_j$	$\ln\hat{t}_j$	$\ln\hat{\sigma}_{j,j+1}$	$\ln\hat{\beta}_j$	$\ln\hat{t}_j$	$\ln\hat{\sigma}_{j,j+1}$	$\ln\hat{\beta}_j$	$\ln\hat{t}_j$	$\ln\hat{\sigma}_{j,j+1}$	$\ln\hat{\beta}_j$	$\ln\hat{t}_j$	$\ln\hat{\sigma}_{j,j+1}$
General	.4602**	1.3979**	.9377**	.2838**	1.2520**	.8558**	.4727**	1.4965**	1.0239**	.4122**	1.5642**	1.1519**
Division	.1468**	.1917	.0449	.1192**	.9648**	.8456**	.1268**	.9778**	.8510**	.2873**	1.5554**	1.2681**
Level 1	.4402**	.9961**	.5559**	.3981**	1.0856**	.6874**	.4629**	1.0671**	.6042**	.4212**	.6269**	.2056+
Level 2	.2509**	.3276**	.0766	.2188**	.5302**	.3113**	.1950**	.4638**	.2687**	.1995**	1.0251**	.8255**
Level 3	.2481**	.5276**	.2795*	.2027**	.0149	−.1878	.1992**	.6730**	.4739**	.2570**	.7808**	.5238+
f	26.3183	6.8658	4.5159	17.2408	10.8293	7.5324	18.5732	11.4524	7.1026	14.0873	8.6738	5.7551
R^2	.8155	.5356	.4313	.7880	.7001	.6189	.8303	.7510	.6517	.8620	.7937	.7185
Number of firms		578			577			640			357	
Observations		4,048			3,277			3,090			1,176	

NOTE.—The statistics f and R^2 correspond to eqq. (3)–(5). To obtain the level of significance for each one of the parameters presented, we estimate eqq. (3)–(5) after changing the hierarchical level omitted; a total of 60 estimations, five estimations for each equation and functional area.

+ Significantly different from zero at 10%.
* Significantly different from zero at 5%.
** Significantly different from zero at 1%.

compensation and responsibility (resources under their control), then we would find evidence in favor of the hypothesis that hierarchies respond to the objective of providing incentives for homogeneous workers.

Methodologically, we want to test what part of the explanatory power of hierarchical levels is due to differences in human capital endowment throughout hierarchical levels, when it comes to explaining differences in compensation and the number of total subordinates of a given manager (our measure of resources controlled). To do so, we add our information on the human capital of each manager (formal education categorized in university graduates, S_1, university undergraduates, S_2, and others; age, h; and job tenure, an) to the equations of compensation and number of subordinates postulated above. Next, we decompose the total variance explained by these equations into that part explained by hierarchical levels ($N \equiv N_j$), firm-specific effects ($E \equiv E_i$), human capital variables, $H \equiv \{S_1, S_2, h, an\}$, and all possible combinations among the three. Table 3 shows the results of the exercise.

The first three rows indicate the contribution to the R^2 of the regression, when each variable is added to the model and the other two are already included. As is the case in other studies (see Leonard 1990), human capital variables add little explanatory power when we control for levels and firms (between 2% and 3.5% in the case of compensation and almost zero in the case of number of subordinates). Nevertheless, this accounts only for variability within hierarchical levels. To evaluate the full contribution of human capital variables, we also have to take into account the explanatory power between hierarchical levels, which is part of the interaction effects.

Row 5 provides this information. In the case of compensation, the interaction between levels and human capital explains around 50% of the variance explained by hierarchical levels ($0.2736/[0.3131 + 0.2736 - 0.0045 - 0.0196] = 0.49$ in the area of production). This means that half of the total explanatory power attributed to levels (half of the 57% in the functional area of production) is due to differences in human capital endowment throughout hierarchical levels. The proportion is quite similar across functional areas and when we consider the number of subordinates.

The explanatory power of human capital variables, variance in compensation and number of subordinates is more than 10 times higher when it comes from differences in human capital endowments throughout hierarchical levels, than when it comes from differences in human capital within a hierarchical level. The hypothesis of human capital as determinant of managerial allocation in hierarchies is difficult to reject. Although the fact that the remaining half of the variance explained by hierarchical levels is due to other (unknown) factors leaves room for incentive effects, such as those predicted by the efficiency wage and tournament literature. However, it should be noted that the information on human capital is limited

Table 3
Explanatory Power of Levels, Firms, and Human Capital Variables

	Production	Marketing	Finance	Personnel
Compensation:				
1. Levels	.3131	.2937	.3158	.3091
2. Firms	.1802	.2321	.2156	.1689
3. Human capital	.0294	.0174	.0223	.0207
4. Levels and firms	−.0045	.0013	−.0198	.0517
5. Levels and human capital	.2736	.2288	.3130	.3466
6. Firms and human capital	.0727	.0375	.0566	.0456
7. Firms and human capital and levels	−.0196	−.0054	−.0509	−.0598
8. Total	.8449	.8054	.8526	.8828
No. of subordinates:				
1. Levels	.1718	.2928	.3576	.3726
2. Firms	.3005	.2879	.2235	.2426
3. Human capital	.0041	.0040	.0025	.0021
4. Levels and firms	−.0500	−.0134	−.0199	.0040
5. Levels and human capital	.0940	.1414	.2064	.2164
6. Firms and human capital	.0322	.0117	.0188	.0125
7. Firms and human capital and levels	−.0129	−.0202	−.0354	−.0544
8. Total	.5397	.7041	.7535	.7958

NOTE.—The values presented in each row are computed as follows: Row 1 $\equiv R^2_{H,N,E} - R^2_{H,E}$; Row 2 $\equiv R^2_{H,N,E} - R^2_{H,N}$; Row 3 $\equiv R^2_{H,N,E} - R^2_{N,E}$; Row 4 $= R^2_N + R^2_E - R^2_{N,E} -$ Row 7; Row 5 $= R^2_H + R^2_N - R^2_{H,N} -$ Row 7; Row 6 $= R^2_H + R^2_E - R^2_{H,E} -$ Row 7; Row 7 $= R^2_{H,N,E} - (R^2_{H,E} + R^2_{H,N} + R^2_{N,E}) + (R^2_H + R^2_E + R^2_N)$; Row 8 $\equiv R^2_{H,N,E}$. For example, in the case of compensation, $R^2_{H,N,E}$, is the percentage of the compensation variance explained by the equation $\ln w_k = c_0 + d_1 S_{1,k} + d_2 S_{2,k} + eh_k + fh^2_k + gan_k + \sum_i \lambda_i E_{i,k} + \sum_{j=0}^{4} \mu_j N_{j,k} + u_k$. The expression R^2_H is the percentage of the compensation variance explained by the equation $\ln w_k = c_0 + d_1 S_{1,k} + d_2 S_{2,k} + eh_k + fh^2_k + gan_k + u_k$. The expression R^2_N is the percentage of the compensation variance explained by the equation $\ln w_k = c_0 + \sum_{j=0}^{4} \mu_j N_{j,k} + u_k$. Finally, $R^2_{H,N}$ is the percentage of the compensation variance explained by the equation $\ln w_k = c_0 + d_1 S_{1,k} + d_2 S_{2,k} + eh_k + fh^2_k + gan_k + \sum_{j=0}^{4} \mu_j N_{j,k} + u_k$.

(we do not have details on investment in training different from university degrees) and a part of these unknown factors could also be related to differences in human capital endowments.

The Elasticity of Compensation to Size

Many papers have documented the fact that CEOs' compensation increases with the size of the firm they manage.[11] Moreover, the elasticity of compensation (salary and bonus) to size (sales, assets, and employees) clusters around the estimated value of 25% and is always lower than one (see Rosen 1992). The human capital perspective of wage formation and managerial talent allocation presented immediately above explains this

[11] Murphy (1985) shows that this result is quite robust to the inclusion of performance variables and firm-specific effects to the regression model.

evidence. There are, however, a few additional issues worth considering. Is elasticity of compensation to size lower than one for managers in each level of the hierarchy? What are the main determinants of this elasticity, interlevel or intralevel variability of compensation and span of control?

To answer these questions, we consider a hypothetical sample of managers in which manager k receives compensation w_k and has a total number of direct and indirect subordinates equal to A_k. The elasticity of compensation to number of subordinates would be estimated by ordinary least squares regression of the log-linear model,

$$\ln w_k = c_0 + c_1 \ln A_k + e_k, \tag{6}$$

where c_0 and c_1 are the parameters to be estimated and e_k is the error term. If managers belong to firms that have observations in all hierarchical positions; if firms have constant and equal span of control in all points of the hierarchy; if the relation between salaries in two consecutive hierarchical positions is constant; and if there is no intralevel variation of salaries and number of subordinates; then, from equation (2), the elasticity of compensation to number of subordinates, $\hat{c}_1$, will be equal to

$$\hat{c}_1 = \frac{\ln\beta}{\ln t}.$$

The theory predicts, and is supported by the empirical evidence, that $\ln t > \ln\beta$ and, therefore, $\hat{c}_1 < 1$. The actual estimated value of $\hat{c}_1$ is consistent with many combinations of β and t. Simon (1957) considers that reasonable values for β and t are 1.5 and 3, respectively, which implies a value of $\hat{c}_1 = 0.37$, higher than the empirical regularity of 0.25 observed in most studies with U.S. data.

The homogeneity conditions assumed above will, in general, not be satisfied and managers will most probably come from heterogeneous hierarchies in terms of number of levels, observations in each position, compensation, and span of control. Appendix B shows how such heterogeneity determines the value of compensation elasticity to number of subordinates, $\hat{c}_1$. This value depends on the interlevel average values of salary increases $\ln\beta_j$ and the span of control log $\ln t_j$ throughout the observed levels $\hat{c}_{INTER}$; on the intralevel heterogeneity of manager and subordinate compensation $\hat{c}_{INTRA}$; on the elasticity for unobserved hierarchical levels $\hat{c}_{UNOBS}$; and the consequences of missing managers or levels $\hat{c}_{MISSING}$.

An alternative but complementary way of taking into account the heterogeneity in the sample used to estimate $\hat{c}_1$ is to obtain separate estimates of $\hat{c}_{1,j}$ in each hierarchical level, $j = 0, \ldots, 5$, together with their interaction effect $\hat{c}_{AV}$. The latter is in fact the elasticity obtained in the hypothetical case that all executives earn average compensation and supervise an average

Table 4
Determinants of the Elasticity of Compensation to Number of Subordinates

	Production		Marketing		Finance		Personnel	
	E	W	E	W	E	W	E	W
General managers $\hat{c}_{1,0}$	.032	.111	.023	.138	.026	.159	.015	.189
Division managers $\hat{c}_{1,1}$	.067	.027	.076	.028	.050	.045	.072	.082
Level 1 functional area $\hat{c}_{1,2}$	.105	.200	.108	.145	.148	.121	.165	.064
Level 2 functional area $\hat{c}_{1,3}$	.084	.297	.116	.189	.151	.121	.104	.098
Level 3 functional area $\hat{c}_{1,4}$	.092	.119	.066	.077	.087	.037	.143	.025
Level 4 functional area $\hat{c}_{1,5}$	.032	.044	.095	.023	.043	.009	.307	.004
Interlevel averages $\hat{c}_{AV}$	.460	.202	.288	.400	.295	.508	.281	.537
Parameter $\hat{c}_1$	.156		.165		.196		.185	
Interobserved levels $\hat{c}_{INTER}$	.437	.336	.293	.573	.305	.690	.278	.700
Intraobserved levels $\hat{c}_{INTRA}$	.044	.464	.055	.300	.046	.249	.046	.206
Unobserved levels $\hat{c}_{UNOBS}$	.146	.393	.131	.350	.167	.279	.108	.287
Missing values $\hat{c}_{MISSING}$	.354	−.193	.293	−.223	.331	−.218	.261	−.190

NOTE—E = elasticity; W = weight. For a complete explanation of the table, see app. B.

number of subordinates, computed for those managers placed at the same hierarchical level and functional area. Appendix B also shows how to aggregate the estimated parameters $\hat{c}_{1,j}$ and $\hat{c}_{AV}$ in order to obtain the elasticity of compensation to number of subordinates, $\hat{c}_1$.

Table 4 shows the results of estimating compensation elasticity to the number of subordinates $\hat{c}_1$, and of applying the two decomposition procedures just described, with our data on Spanish managers.[12] The elasticity of compensation to number of subordinates obtained for all the managers in the sample $\hat{c}_1$ is higher than the elasticity of compensation to size (sales or employees) obtained with general managers only. In fact, $\hat{c}_1$ is higher than any $\hat{c}_{1,j}$, which implies that interlevel effects $\hat{c}_{AV}$ are relevant to estimating the elasticity of compensation to size. Notice also that the interlevel elasticity obtained with the observed levels $\hat{c}_{INTER}$, and the elasticity obtained for the average values at each hierarchical level of compensation and number of subordinates $\hat{c}_{AV}$ are very similar. Moreover, their values

[12] We have estimated general managers' compensation elasticity with respect to the volume of sales of the firm, equal to 0.075, much lower than the 0.25 obtained from U.S. samples, although higher than the 0.03 value for the number of subordinates shown in table 4.

are very close to the elasticity obtained by averaging the ratios in each of the functional areas. Results appear displayed in table 2.

The main conclusion from this analysis is that the intralevel elasticity of compensation to size, although significant, is very low compared with the interlevel elasticity of compensation. Leonard (1990) and Gerhart and Milkovich (1990) obtain even lower intralevel elasticity, with values at around 0.005, 10 times lower than ours. This result suggests that differences in firm size are basically due to differences in the extension of hierarchical levels, and much less to differences in the span of control at the various hierarchical levels. This fact explains that the elasticity of compensation to size, computed with data on compensation and span of control in homogeneous hierarchies, is much higher than the elasticity obtained by pooling heterogeneous managerial data, as is the case with equation (6). Actually, with our data, the implicit elasticity in the estimated values of $\ln \beta_j$ and $\ln t_j$ obtained for each hierarchical position is around 0.30 in the hierarchy ranging through the functional areas outside production (marketing, finance, personnel) and is 0.45 in the hierarchy ranging from general management through production. These results imply differences in substitution elasticity across functional areas, a fact also reflected in table 2. According to the model, the same increase of managerial talent in the functional area of production requires lower compensation increments than in the rest of functional areas (the cost function of producing talent is less convex in the functional area of production).

IV. Conclusion

Although hierarchical organizations are characterized by the distribution of salaries and spans of control in job positions, most of the previous research has focused only on the distribution of salaries. The span of control, and especially its relation with compensation, has been neglected. Our article provides estimations of interlevel compensation ratios and span of control from a sample of Spanish managers with job positions from general managers down to level 4 in a given functional area. The interpretation and implications of the estimated values are derived from extensions of models that view the hierarchy as a coordination device, allowing the hiring of more talented, and therefore more expensive, managers to reduce control losses or increase information-processing capabilities.

In our empirical framework, compensation and span of control in each point of the hierarchy will be jointly determined as the profit-maximizing solution to the managerial allocation problem, where the exogenous variable is the cost of contracting a manager of a given talent. Firms prefer to allocate managers who are more talented to higher hierarchical positions, but talent increases at a decreasing rate. This also implies that, in the optimal solution, the span of control in each position and the elasticity of substitution between managers of two consecutive hierarchical levels are both greater than one. The empirical implication of these results is that the positive association observed between managers' compensation and number of subordinates under their control (size of the firm) obeys the fact that the ablest managers earn higher salaries and occupy higher hierarchical positions.

The empirical evidence from a sample of Spanish firms and managers is consistent with these predictions. Average increases in compensation between consecutive hierarchical levels are all positive and significantly different from zero, and the average estimates of span of control and implicit substitution elasticity between subordinates and their direct managers are greater than one. The article also finds that the explanatory power of human capital variables in empirical models of the determinants of compensation and resources controlled by managers (around 30% of dependent variable variance) can be attributed almost entirely (90%) to differences in human capital endowment throughout hierarchical levels. The rest would be due to differences in human capital endowments within levels. This also means that half of the explanatory power of hierarchical levels, when the dependent variables are compensation and the number of subordinates, can be attributed to the fact that the average endowment of human capital increases as one gets closer to the top of the hierarchy.

Therefore, theories that consider human capital variables as determinants of managers' differences in compensation and level of responsibility within organizations cannot be rejected with our data. Of course, there is still room for other theories that explain differences in compensation throughout hierarchical levels in terms of the need to provide incentives for managers' efforts.

The paper also looks at the empirical evidence for the relationship between CEO compensation and the size of the firms. Rosen (1982) modeled this relation in a two-level hierarchy where the elasticity of substitution between managerial talent and direct workers was exogenous. In our framework, this elasticity is endogenously determined by the decision of the firm as regards the amount of human capital (talent) allocated in each hierarchical position. Under convex cost functions of producing human capital, in the optimal solution the elasticity of substitution will be greater than one. For this reason, the ratio between the manager compensation and that of their direct subordinates will be lower than the managers' span of control. Consequently, in a pooled sample of managers, the elasticity of compensation to the size of the resources they manage will be lower than one. The article decomposes the estimated elasticity from pooled samples in different ways, which show that the interlevel effects are more important than those of the intralevel in determining the observed elasticity of compensation to size. This means that differences in the size of firms are due to differences in hierarchical levels, rather than to differences in span of control within levels.

One important limitation of our data is that the number of observations from the same firm is fairly small and that we cannot observe the evolution of the managers over time. For this reason, we cannot replicate studies such as those of Baker et al. (1994*a*, 1994*b*) at the firm level, nor can we provide estimates of wage increases and span of control within each firm in the sample. However, we are able to obtain average estimates of the relevant variables across firms and job positions, and our empirical models explain a considerable part of the variance in compensation and subordinates within the sample. Moreover, we can provide evidence on the importance of properly accounting for intrafirm heterogeneity when performing measurements using firm-level data.

Appendix A

Characterization of the Solution to the Allocation Problem

The profit-maximization problem is written as

$$\max P_{L_{1,\ldots}L_{n},z_{1\ldots},z_{n}} = pb\prod_{j=0}^{n-1} t_j^{z_j+1} - \sum_{j=1}^{n} w_j L_j = pbL_n^{z_n}\prod_{j=1}^{n-1} L_j^{z_j - z_{j+1}} - \sum_{j=1}^{n} e^{f(z_j)} L_j,$$

subject to $1 \geq z_j \geq 0, t_j \geq 1$.

The theoretical results in the main text of the article are obtained from the interior solution to the problem. Before deriving these, we show the conditions that allow us to exclude corner solutions.

i) For any intermediate level k, $t_{k-1} = 1$ cannot be an optimal solution since we can save $w_k L_k$ excluding level k and still produce the same level of output. For example, if the hierarchy has only one level, if $t_0 = L_1 = 1$, then profits would be $P = pb - w_1$, lower than pb, the output of the top managers when producing alone.

ii) $z_k = 0$ is not optimal for any k; $z_k = 1$ is not optimal for any k such that $2 \leq k \leq n$. The derivative of P with respect to L_j is

$$\frac{\partial P}{\partial L_j} = (z_j - z_{j+1})\frac{pQ}{L_j} - w_j.$$

If $z_j \leq z_{j+1}$, this derivative is always negative. When $z_k = 1$, we have $z_{k-1} - z_k \leq 0$, and when $z_k = 0$, we have $z_{k-1} - z_k \leq 0$. Therefore, if $z_k = 1$, we choose $t_{k-2} = 1$, and when $z_k = 0$, we choose $t_{k-1} = 1$ as optimal spans of control. But from condition i we know that a firm without levels $k - 1$ or k dominates these solutions.

iii) Placing a manager with the highest talent in level 1, $z_1 = 1$, the profit function can be written as

$$P = \left[\left(pb\prod_{j=1}^{n-1} t_j^{z_{j+1}} - \sum_{j=2}^{n} w_j \frac{L_j}{L_1}\right) - e^{f(1)}\right]L_1.$$

For example, in the one-level case, profits will be $P = [pb - e^{f(1)}]L_1$. If the term in brackets were positive, firms could earn infinite profits by choosing $L_1 = \infty$. This would be equivalent to replicating infinite times firms with $n - 1$ hierarchical levels by managers of talent equal to one, $z_1 = 1$. We exclude this solution by assuming that competition in product and/or labor markets will adjust prices p such that, in equilibrium, $P^* = e^{f(1)}$; that is, managers of talent $z = 1$ will earn a competitive profit (or salary equal to their opportunity cost).

Characterization of the Interior Solution

First-order conditions:

$$\frac{\partial P}{\partial L_j} = (z_j - z_{j+1})\frac{pQ}{L_j} - w_j = 0 \qquad \text{for } j = 1, 2, \ldots, n-1. \tag{A1}$$

$$\frac{\partial P}{\partial L_j} = z_n\frac{pQ}{L_j} - w_n = 0 \qquad \text{for } j = n.$$

$$\frac{\partial P}{\partial z_{j+1}} = \ln(t_j)pQ - f'(z_{j+1})w_{j+1}L_{j+1} = 0 \qquad \text{for } j = 0, \ldots, n-1. \tag{A2}$$

Second-order conditions: The Hessian matrix must be negative semi-definite. Among the conditions to be satisfied, we have

$$\frac{\partial^2 P}{\partial L_j \partial L_j} = (z_j - z_{j+1})(z_j - z_{j+1} - 1)\frac{pQ}{L_j^2} < 0;$$

$$\frac{\partial^2 P}{\partial z_j \partial z_j} = [\ln(t_j)]^2 pQ - \{f''(z_{j+1}) + [f'(z_{j+1})]^2\}w_{j+1}L_{j+1} < 0. \tag{A3}$$

If there is an interior solution to the maximization problem, this solution will be characterized by the following conditions:

1. If the firms obtain profits, $\sum_{j=1}^{n} w_j L_j/pQ < 1$. From equation (A1) we obtain that $\sum_{j=1}^{n} w_j L_j/pQ = \sum_{j=1}^{n} z_j - z_{j+1} = z_1 < 1$. The talent of the general managers ($z_0 = 1$) will therefore be greater than the talent of their direct subordinates ($z_1 < 1 = z_0$).

From (A1) we also obtain that $(z_j - z_{j+1}) = w_j L_j/pQ > 0$. Workers will be distributed throughout the hierarchical levels as a function of their talent z_j and, since w_j is increasing in z_j, those located at levels closer to the top of the hierarchy will receive greater compensation.

2. Combining (A1) and (A2), we obtain

$$\ln(t_j) = f'(z_{j+1})(z_{j+1} - z_{j+2}) > 0, \tag{A4}$$

the number of managers assigned to any hierarchical level increases as we move up the hierarchy (a span of control greater than one).

3. *Proposition.* The differences of z_j between contiguous hierarchical levels will decrease as we go up the hierarchy $(z_{j+1} - z_{j+2}) > (z_j - z_{j+1})$.

Proof. Comparing equation (A1) referring to hierarchical level $j + 1$ with that referring to hierarchical level j, we obtain, $w_{j+1}L_{j+1}/w_jL_j = (z_{j+1} - z_{j+2})/(z_j - z_{j+1})$, and, in logarithmic terms:

$$\begin{aligned}\ln(t_j) &= \ln\frac{L_{j+1}}{L_j} = \ln\frac{w_j}{w_{j+1}} + \ln\frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}} \\ &= [f(z_j) - f(z_{j+1})] + \ln\frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}}.\end{aligned} \tag{A5}$$

a) For any pair of values, z_j and z_{j+2}, between zero and one, $1 > z_j > z_{j+2} > 0$, the value of $z_{j+1} = (z_j + z_{j+2})/2$, does not fulfill the necessary optimality conditions. Replacing (A5) in (A4) we obtain

$$\ln\frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}} = f'(z_{j+1})(z_{j+1} - z_{j+2}) - [f(z_j) - f(z_{j+1})].$$

Since for $z_{j+1} = (z_j + z_{j+2})/2$, $\ln(z_{j+1} - z_{j+2})/(z_j - z_{j+1}) = 0$, the above equation will only be zero when $f'' = 0$.

This shows that at the optimum solution, if f'' is greater than zero, the differences in z_j between hierarchical levels will never be equal.

b) At the point analyzed, $z_{j+1} = (z_j + z_{j+2})/2$, if $f'' > 0$ is fulfilled, then the first-order condition will not be satisfied, because using the above relationships,

$$\frac{\partial P}{\partial z_{j+1}} = \ln\frac{z_{j+1} - z_{j+2}}{z_j - z_{j+1}} + [f(z_j) - f(z_{j+1})] - f'(z_{j+1})(z_{j+1} - z_{j+2}) > 0.$$

For the existence of a single interior maximum, the second-order equation,

$$\frac{\partial^2 P}{\partial z_{j+1}\partial z_{j+1}} = -\{f''(z_{j+1}) + [1 - (z_{j+1} - z_{j+2})][f'(z_{j+1})]^2\}w_{j+1}L_{j+1} < 0,$$

must be satisfied for any value of z_{j+1} between z_j and z_{j+2}, implying that the optimum value z^*_{j+1} will be greater than the average point, $z^*_{j+1} > (z_j + z_{j+2})/2$. In other words, at the optimum, $2z^*_{j+1} > (z_j + z_{j+2})$ and consequently $(z^*_{j+1} - z_{j+2}) > (z_j - z^*_{j+1})$. Q.E.D.

Table A1
Interior Solutions

Variables	Number of Hierarchical Levels $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
z_1	.466	.660	.771	.837	.859	.850
z_2	. . .	.353	.543	.659	.721	.751
z_3	. . .	. . .	.290	.465	.571	.638
z_4	. . .	. . .	. . .	.249	.405	.510
z_5	. . .	. . .	. . .	. . .	.219	.365
z_6	. . .	. . .	. . .	. . .	. . .	.198
t_0	96.596	28.925	14.164	8.499	6.122	5.060
t_1	. . .	25.068	13.448	8.423	5.854	4.650
t_2	. . .	. . .	12.676	8.328	5.656	4.313
t_3	. . .	. . .	. . .	8.305	5.493	4.001
t_4	. . .	. . .	. . .	. . .	5.351	3.672
t_5	. . .	. . .	. . .	. . .	. . .	3.313
Profits (P)	.251 w_0	.548 w_0	.842 w_0	1 w_0	.935 w_0	.741 w_0

Example

Consider the case where manager compensation with talent z_j, relative to top manager compensation, $z_0 = 1$, is given by

$$\frac{w_j}{w_0} = \frac{w(z_j)}{w(z_0)} = \exp(f(z_j) - f(z_0)) = \exp(-10 + 7z_j + 3z_j^2).$$

Additionally, if the top manager $z_0 = 1$ does not hire any workers, the relative compensation obtained by thus working alone is $pb/w_0 = e^{-2.8836}$.

The interior optimal solutions, z_j and t_j for different values of n, hierarchical levels, have been computed (see table A1) solving numerically the first-order conditions rewritten as

$$\frac{\Delta z_{n-1}}{e^{f(z_{n-1})-f(z_n)}} = \frac{z_n}{e^{f'(z_n)z_n}} \text{ and } \frac{\Delta z_j}{e^{f(z_j)-f(z_{j+1})}} = \frac{\Delta z_{j+1}}{e^{f'(z_{j+1})\Delta z_{j+1}}} \text{ for } j = 1, \ldots, n-2;$$

$$t_j = e^{f'(z_{j+1})(z_{j+1}-z_{j+2})} \text{ for } j = 1, \ldots, n-1;$$

$$t_{n-1} = e^{f'(z_n)(z_n)} \text{ and } z_n = \frac{w_n L_n}{pQ}.$$

The illustration confirms that in the interior solutions $\Delta z_{j+1} > \Delta z_j$ and $t_j > 1$. Profits increase with n until $n = 4$, where they are maximized and reach a level where they just compensate the opportunity cost of the general manager; that is, this would also be the competitive equilibrium solution.

Appendix B

The Elasticity of Compensation to Number of Subordinates

Estimation of the Elasticity of Compensation to Subordinates with Data on Increases in Compensation and Span of Control

Given equations (3) and (4) we can state

$$\ln w_k = \ln A_k + (\tau_i - \lambda_i)E_{i,k} + \sum_{j=0}^{4}(\alpha_j - \mu_j)N_{j,k} + \varepsilon_k - \nu_k.$$

If we estimate equation (6)

$$\ln w_k = c_0 + c_1 \ln A_k + e_k, \tag{B1}$$

the estimated value of c_1 will be equal to

$$\begin{aligned}\hat{c}_1 = \frac{\operatorname{Cov}(w_k, A_k)}{\operatorname{Var}(\ln A_k)} &= \frac{\operatorname{Cov}(\alpha_j N_{j,k} + \tau_i E_{i,k}, \mu_j N_{j,k} + \lambda_i E_{i,k})}{\operatorname{Var}(\ln A_k)} + \frac{\operatorname{Cov}(\varepsilon_k, \nu_k)}{\operatorname{Var}(\ln A_k)} \\ &= \hat{c}_{INTER}\frac{\operatorname{Var}(\mu_j N_{j,k})}{\operatorname{Var}(\ln A_k)} + \hat{c}_{INTRA}\frac{\operatorname{Var}(\nu_k)}{\operatorname{Var}(\ln A_k)} + \hat{c}_{UNOBS}\frac{\operatorname{Var}(\lambda_i E_{i,k})}{\operatorname{Var}(\ln A_k)} \\ &\quad + \hat{c}_{MISSING}\frac{2\operatorname{Cov}(\lambda_i E_{i,k}, \mu_j N_{j,k})}{\operatorname{Var}(\ln A_k)},\end{aligned}$$

where $\hat{c}_{INTER} = \operatorname{Cov}(\alpha_j N_{j,k}, \mu_j N_{j,k})/\operatorname{Var}(\mu_j N_{j,k})$ is the interlevel elasticity in the observed levels; the intralevel elasticity is defined as $\hat{c}_{INTRA} = \operatorname{Cov}(\varepsilon_k, \nu_k)/\operatorname{Var}(\nu_k)$; the elasticity in the unobserved levels is $\hat{c}_{UNOBS} = \operatorname{Cov}(\tau_i E_{i,k}, \lambda_i E_{i,k})/\operatorname{Var}(\lambda_i E_{i,k})$, and the consequences of a missing manager or levels is measured by

$$\hat{c}_{MISSING} = \frac{\operatorname{Cov}(\tau_i E_{i,k}, \mu_j N_{j,k}) + \operatorname{Cov}(\lambda_i E_{i,k}, \alpha_j N_{j,k})}{2\operatorname{Cov}(\lambda_i E_{i,k}, \mu_j N_{j,k})}.$$

Given the following assumptions:

i) Errors are noncorrelated; $\operatorname{Cov}(\varepsilon_k, \nu_k)/\operatorname{Var}(\nu_k) = 0$.

ii) We have information for all the hierarchical levels and for all the executives that occupy these in each of the firms; $(\operatorname{Cov}(\tau_i E_{i,k}, \mu_j N_{j,k}) + \operatorname{Cov}(\lambda_i E_{i,k}, \alpha_j N_{j,k}))/2\operatorname{Cov}(\lambda_i E_{i,k}, \mu_j N_{j,k}) = 0$. The parameter $\hat{c}_1$ will be equal to

$$\hat{c}_1 = \frac{\operatorname{Var}(\mu_j N_{j,k})}{\operatorname{Var}(\ln A_k)}\frac{\operatorname{Cov}(\alpha_j N_{j,k}, \mu_j N_{j,k})}{\operatorname{Var}(\mu_j N_{j,k})} + \frac{\operatorname{Var}(\lambda_i E_{i,k})}{\operatorname{Var}(\ln A_k)}\frac{\operatorname{Cov}(\tau_i E_{i,k}, \lambda_i E_{i,k})}{\operatorname{Var}(\lambda_i E_{i,k})}.$$

We define $\alpha_j = \sum_{r=j}^{4}\ln\beta_r$, $\mu_j = \sum_{r=j}^{4}\ln t_r$, $\tau_i = \ln w_{n_i} + \sum_{r=5}^{n_i-1}\ln\beta_r$, and $\lambda_i = \sum_{r=5}^{n_i-1}\ln t_r$, where n_i is the number of hierarchical levels that make up the firm i and w_{n_i} is the compensation of their direct workers. Considering the above equations and the relation $\ln t_r = \ln \beta_r + \ln \sigma_{r,r+1}$, the value of the parameter $\hat{c}_1$ will be

$$\hat{c}_1 = \frac{\mathrm{Cov}\left(\sum_{r=j}^{4} \ln (t_r/\sigma_{r,r+1}) N_{j,k},\ \sum_{r=j}^{4} x_r N_{j,k}\right)}{\mathrm{Var}(\ln A_k)}$$

$$+ \frac{\mathrm{Cov}\left(\left(\ln w_{n_i} + \sum_{r=5}^{n_i-1} \ln (t_r/\sigma_{r,r+1})\right) E_{i,k},\ \sum_{r=5}^{n_i-1} (\ln t_r) E_{i,k}\right)}{\mathrm{Var}(\ln A_k)}$$

$$= \frac{\mathrm{Cov}\left(\ln A_k - \left(\sum_{r=j}^{n_i-1} \ln \sigma_{r,r+1}\right) \ln (A_k)\right)}{\mathrm{Var}(\ln A_k)}$$

$$= 1 - \frac{\mathrm{Cov}\left(\sum_{r=j}^{n_i-1} \ln \sigma_{r,r+1}, \ln (A_k)\right)}{\mathrm{Var}(\ln A_k)}.$$

iii) If we further assume that increases in salaries and number of subordinates are constant throughout the hierarchy $\ln t_j = \ln t$, $\ln \beta_j = \ln \beta$, and $\ln \sigma_{j,j+1} = \ln \sigma$, then the value of the estimated parameter will be

$$\hat{c}_1 = 1 - \frac{\mathrm{Cov}\left(\sum_{r=j}^{n_i-1} \ln \sigma,\ \ln (A_k)\right)}{\mathrm{Var}(\ln A_k)}$$

$$= \frac{\mathrm{Cov}((n_i - j)\ln \sigma, (n_i - j)\ln t)}{\mathrm{Var}((n_i - j)\ln t)} = 1 - \frac{\ln \sigma}{\ln t} = \frac{\ln \beta}{\ln t}.$$

Decomposition by Hierarchical Levels

We can also decompose the elasticity of compensation to subordinates by hierarchical levels.

$$\hat{c}_1 = \frac{\mathrm{Cov}(\ln (w_k),\ \ln (A_k))}{\mathrm{Var}(\ln A_k)} = \sum_{j=0}^{5} \hat{c}_{1,j} \frac{\mathrm{Var}(\ln (A_k) N_{j,k})}{\mathrm{Var}(\ln A_k)}$$

$$+ \hat{c}_{AV} \frac{\mathrm{Var}\left(\overline{(\ln A_K) N_{J,k}}\right)}{\mathrm{Var}(\ln A_k)}.$$

Where $\hat{c}_{1,j}$ is the elasticity of compensation to number of subordinates computed only with those executives allocated at hierarchical level j, $\hat{c}_{1,j} = \mathrm{Cov}(\ln w_k N_{j,k},\ \ln A_k N_{j,k}) / \mathrm{Var}(\ln A_k N_{j,k})$.

The parameter $\hat{c}_{AV}$ is the elasticity of compensation to number of subordinates in the case of all executives earning the average salary and directing the average number of subordinates within the hierarchical level that they occupy, $\overline{\ln(w_k)N_{j,k}}$ and $\overline{\ln(A_k)N_{j,k}}$, respectively; $\hat{c}_{AV} = \mathrm{Cov}(\overline{\ln(w_k)N_{j,k}}, \overline{\ln(A_k)N_{j,k}})/\mathrm{Var}(\overline{\ln A_k N_{j,k}})$.

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