Comment on "Biphoton double-slit experiment"

X. Oriols
Department d'Enginyeria Electrònica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain
(Received 1 December 2003; revised manuscript received 6 July 2004; published 31 January 2005)

In a recent paper [Phys. Rev. A **68**, 033803 (2003)] experimental results on a double-slit configuration with two entangled bosons are presented. The authors argue that their data contradicts the de Broglie–Bohm interpretation of quantum mechanics. In this Comment we show that this conclusion is incorrect.

DOI: 10.1103/PhysRevA.71.017801 PACS number(s): 42.50.Ar, 03.65.Ta

I. INTRODUCTION

The de Broglie–Bohm (dBB) formalism of quantum mechanics [1–3] was initially proposed by Louis de Broglie in 1926, and finally formulated, in mathematical terms, by David Bohm in 1952. Since then, all attempts to prove a measurable difference with the Copenhagen interpretation, named here standard quantum mechanics (SQM), have been unfruitful [1–3]. In a recent article [4], experimental data for a two-bosons system impinging upon a double slit are compared with dBB predictions obtained by two different theoretical groups [5–9]. The authors conclude that their experimental data reproduces the SQM predictions and contradicts the dBB theory. In particular, they wrote: "Thus, if the theoretical prediction is confirmed, our result will represent the first negative test of the de Broglie–Bohm theory." The goal of this Comment is to show that their experimental results do not provide any negative test of the dBB theory, but only verify a particular quantum-optical prediction. First, in Sec. II, we explain that the well-known *particle conservation law* of the dBB theory guarantees that Bohm trajectories exactly reproduce SQM observable results. Then, in order to provide an additional evidence, we show that Bohm trajectories reproduce SQM predictions for the particular two-entangled bosons system studied in Refs. [5–8]. Finally, in Sec. III, we discuss the differences between the wave function used to deduce the dBB predictions and the one used for obtaining SQM results.

II. PARTICLE CONSERVATION LAW

In this section we will show that the conclusion drawn in Ref. [4]: "The analysis of these data allows a test of standard quantum mechanics against the de Broglie–Bohm theory," is in clear contradiction with a general and well-known property of the dBB theory. Let us sketch the essential points of this property known as the *particle conservation law* for Bohm trajectories [3]. We define a two-particle system in a two-dimensional space by the wave function $\Psi(x_1,x_2,y_1,y_2,t)$, where x_i and y_i are the positions of particles $i=\{1,2\}$. The velocity vector of each particle can be defined, according to the dBB theory, as

$$v_j(x_1,x_2,y_1,y_2,t)=\frac{\hbar}{m}\mathrm{Im}\left\{\frac{\partial\Psi(x_1,x_2,y_1,y_2,t)/\partial_j}{\Psi(x_1,x_2,y_1,y_2,t)}\right\}, \quad (1)$$

where the index j represents each one of the four spatial variables $j=\{x_1,y_1,x_2$ or $y_2\}$. We rewrite the many particle wave function in the polar form: $\Psi(x_1,x_2,y_1,y_2,t)=R(x_1,x_2,y_1,y_2,t)\exp\{iS(x_1,x_2,y_1,y_2,t)/\hbar\}$ and introduce it into the Schrödinger equation. Then, a *particle conservation law* appears naturally within the dBB formalism [1–3] when the imaginary part of the Schrödinger equation is considered:

$$\frac{\partial R^2(x_1,x_2,y_1,y_2,t)}{\partial t}+\sum_j\frac{\partial}{\partial_j}[R^2(x_1,x_2,y_1,y_2,t)v_j(x_1,x_2,y_1,y_2,t)]=0. \quad (2)$$

This equation explicitly guarantees that the modulus of the SQM wave function is always reproduced by counting Bohm trajectories.

Let us point out the importance of Eq. (2). According to the dBB theory, the initial distribution of Bohm trajectories at time t has to be proportional to the probability presence $R^2(x_1,x_2,y_1,y_2,t)$ at each configuration point (for a large number $N\rightarrow\infty$ of Bohm particles). Then, at time $t+dt$, Eq. (2) guarantees that the number of particles at each configuration point is also proportional to $R^2(x_1,x_2,y_1,y_2,t+dt)$, when each particle moves the infinitesimal distance $v_j(x_1,x_2,y_1,y_2,t)dt$ [3,10,11]. Therefore, once the distribution of particles is correctly selected at time t, Eq. (2) guarantees that these Bohm trajectories will also reproduce the modulus of the wave function at any other time.

At this point let us discuss the origin of the misleading predictions provided by Refs. [5–8]. Let us notice that any arbitrary limitations for the selection of the initial positions of Bohm trajectories that is incompatible with the initial distribution of Bohm particles mentioned before [determined by $R^2(x_1,x_2,y_1,y_2,t)=|\Psi(x_1,x_2,y_1,y_2,t)|^2$] is totally inconsistent with the basics of the dBB theory [3]. If such "arbitrary" constrictions are imposed "by hand" on the initial positions, then only a subset of the possible trajectories is considered, and hence the predictions are not consistent with the dBB theory itself. This is exactly the mistake of Refs. [5–8]. For example, when the authors of Ref. [5] mention, just below their Eq. (18), that "if a $t=0$ the center of mass of the two particles is exactly on the x axis…," they are really assuming that the wave function that describes the two-particle systems is not described by their wave packet [Eq. (4) in Ref. [5]], but by a different wave function defined as an "eigenstate" of the center of mass (i.e., whose probability presence is only different from zero at two equidistant configurations points). This obvious explanation that invalidates the main conclusion of Refs. [5–8] (and also the conclusions of Ref. [4]) has already been pointed out by several different authors before [3,12,13]. However, recently the author of Ref. [8] has refuted this argument. He argues that at time $t+dt$ not all particles can be taken into account to try to reproduce $R^2(x_1,x_2,y_1,y_2,t+dt)$, but only the pairs of particles that accomplishes an additional constriction imposed on the positions of the entangled bosons [introduced as $\delta(y_1(t)+y_2(t))$ in Eq. (15) in Ref. [8]]. However, any restriction on Bohm trajectories can only come as a direct consequence of the wave function and, then, all trajectories will satisfy it by construction. In this case, there is no need to consider such constriction two times. On the contrary, if the additional constriction is not compatible with the dBB initial distribution of particles and with Eq. (1), then the constriction itself is incompatible with the dBB formulation and cannot be assumed.

In summary, the hypothetical discrepancies between the SQM and dBB theories presented in Refs. [5–8] are not plausible because they contradict the well-known *particle conservation law* of the dBB theory. As a consequence, the conclusion of Ref. [4] against the dBB theory is also incorrect.

A. On the possibility of detecting two Bohm particles at the same semiplane

Now, we will show, in detail, that the dBB theory applied to the particular system described in Refs. [5–8] exactly reproduce the SQM results. In fact, we are only providing a particular example of the previous general *particle conservation law* consubstantial to the dBB theory. The experimental setup presented in Ref. [4] consists on two undistinguishable photons send simultaneously upon a double-slit scenario (see Fig. 1). After passing through the double slit, the two entangled photons are detected simultaneously at the screen (at y_D and $y_{D'}$). In Fig. 1, we define two symmetric semiplanes (named semiplane A and semiplane B) separated by a line perpendicular to the screen and located at the middle between the slits. The authors of Ref. [4] (based on the theoretical work presented by two groups [5–9]) said that the dBB theory predicts the impossibility of detecting the two entangled bosons, simultaneously, at the same semiplane: "the coincidence signal (number of times that the particles are detected at y_D and $y_{D'}$) is predicted (by the dBB theory) to be strictly zero when the two detectors are in the same semiplane with respect to the double-slit symmetry axis." On the contrary, their experimental data and the SQM results effectively provide the coincidence of two bosons at the same semiplane. Here, we show that the "strictly zero" assumption of Ref. [4] is false because the dBB theory can effectively predict the coincidence of two bosons at the same semiplane, in perfect agreement with the SQM results.

First, let us get into those seminal works of the theoretical groups [5–8] used by the authors of Ref. [4] to deduce their conclusion. According to those authors, a two-boson wave function can be written as

$$\Psi(x_1,x_2,y_1,y_2,t) = \frac{1}{\sqrt{2}}\{\Psi_A(x_1,y_1,t)\Psi_B(x_2,y_2,t) + \Psi_A(x_2,y_2,t)\Psi_B(x_1,y_1,t)\} \tag{3}$$

that assures its symmetrical behavior when the particle positions are interchanged (let us notice that other definitions of the symmetrical many-particle wave function are possible). Here, $\Psi_{A/B}(x,y,t)$ is the wave function of a single-particle in a two-dimensional (x,y) space (the labels A and B describes the wave packet properties). Let us discuss the Gaussian wave packet configuration described in [5,6]. According to Eqs. (2)–(8) in Ref. [6], a single particle in a two-dimensional space with energy E can be defined, at any time, by the product of a plane wave in the x direction: $\Xi(x,t) = 1/\sqrt{2\pi}\exp(ik_x x - Et/\hbar)$ and a time-dependent Gaussian wave packet in the y direction:

$$\Phi_{A/B}(y,t) = \left(\frac{2a^2}{\pi}\right)^{1/4}\frac{e^{i\varphi}}{\left(a^4+\dfrac{4\hbar^2t^2}{m^2}\right)^{1/4}}e^{i(k_{A/B}(y-y_{A/B}))} \times\exp\left(-\frac{\left[y-y_{A/B}-\dfrac{\hbar k_{A/B}}{m}t\right]^2}{a^2+\dfrac{2i\hbar t}{m}}\right), \tag{4}$$

where a is the spatial dispersion of the wave packet, m the particle mass, k_x is the wave vector in the x direction, $k_{A/B}$ its wave vector in the y direction, and $\varphi=-\theta-\hbar k_A^2 t/(2m)$ with $\tan(2\theta)=2\hbar t/(ma^2)$ [14]. Each wave packet is centered at one of the slits. Thus $y_{A/B}$ are the symmetrical central position of the initial wave packets (see Fig. 1).

Now, let us provide a simple argumentation to show that Bohm trajectories can be detected at the same semiplane. According to the dBB formalism [3], the modulus of the wave function at the initial time $|\Psi(x_1=0,x_2$

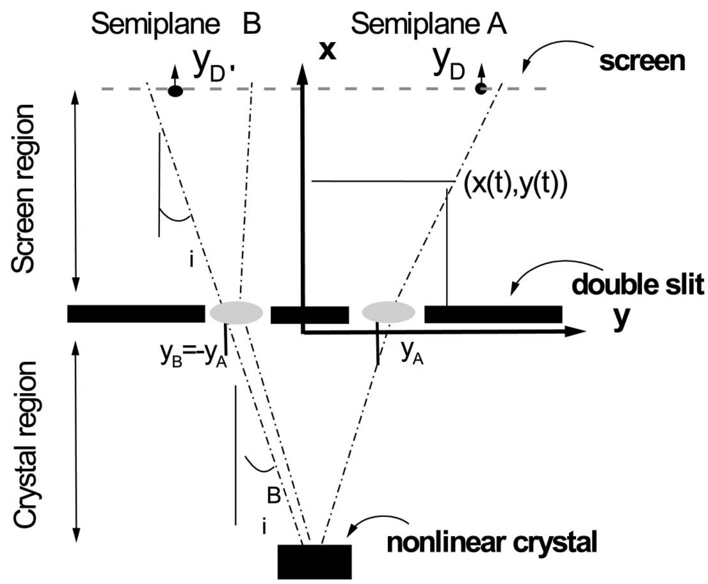

FIG. 1. Schematic representation of the x-y coordinate system used in this Comment. The location of the screen, the double slit, and the nonlinear crystal is depicted. The origin of the spatial coordinates ($x=0$, $y=0$) is located in the middle, between slits. The particle detectors are located at points y_D and $y_{D'}$ at the screen. The angles θ_i are defined in the *screen region* and θ_i^B in the *crystal region*.

$=0, y_1(0), y_2(0), t=0)|^2$, determines the initial positions of all pairs of bosons, $y_1(0)$ and $y_2(0)$. The probability of selecting two boson at semiplane B is not zero even if we choose a large distance between the central positions of the wave packets, $d \equiv y_A - y_B \to \infty$, or initial positions of the particles very far from the center of the double slit, $y_1(0) = y_2(0) \to -\infty$. Therefore, we can always select two initial positions at semiplane B far enough from the origin, $y=0$, so that the particle positions, $y_1(t)$ and $y_2(t)$ remain in the same semiplane B when they arrive at the screen. We admit that the probability of this selection can be, in general, very small, but not *strictly* zero. This small probability is enough to provide an irrefutable counterexample against the *impossibility* of detecting the two Bohm trajectories at the same semiplane. This argument can be generalized to any type of wave function whose modulus tends asymptotically to zero.

At this point, in order to provide an additional numerical evidence, we will use a quantum Monte Carlo method [15]. Here, we have adapted our approach [15] to the particular two-boson system described by Eqs. (3) and (4). For convenience, we have used the wave packet parameters typical of nanometric semiconductors with the effective mass equal to 0.067 times the free electron mass, $a=17$ nm, $k_A = 0.296 \times 10^9$ m^{-1}, $k_B = -0.296 \times 10^9$ m^{-1} and $k_X = 0.289 \times 10^9$ m^{-1}. The boson trajectories starts at the configuration point $(x_1(0)=0, y_1(0))$ and $(x_2(0)=0, y_2(0))$ at $t=0$ fs. The bosons positions $y_1(0)$ and $y_2(0)$ are selected according to the initial probability presence $|\Psi(x_1(0)=0, x_2(0)=0, y_1(0), y_2(0), t=0)|^2$. The trajectories are computed by numerically integrating Bohm velocities [Eq. (1)] for the many-particle wave function until they reach, simultaneously, the screen at $(x_1 = 14\ \text{nm}, y_1(t))$ and $(x_2 = 14\ \text{nm}, y_2(t))$ for the time $t=28$ fs [16]. In Fig. 2, we show the number of times that the two particles are detected at the screen (named *coincidences* in Ref. [4]); one particle within the interval $[y_D, y_D + \Delta y]$ and the other within $[y_{D'}, y_{D'} + \Delta y]$, where $\Delta y = 2$ nm. As in Ref. [4], we have plotted the number of coincidences for different configurations: one detector is located at $y_{D'} = -4$ nm and the position of the other, y_D, varies along the screen. The number of simulated pairs of bosons increases from 10 000 in Fig. 2(a) to 100 000 in Fig. 2(b), and finally to 1 000 000 pairs in Fig. 2(c). We clearly show in Fig. 2(c) that the probability of detecting two particles simultaneously at semiplane B is different from zero, in contradiction with the hypothesis of Ref. [4]. The agreement between the dBB and SQM results in Fig. 2 is excellent, when the number of simulated Bohm trajectories tends to infinity (see solid line in Fig. 2).

In summary, the sentence argued by the authors of Ref. [4] that "the coincidence signal is predicted (by the dBB theory) to be strictly zero when the two detectors are in the same semiplane with respect to the double-slit symmetry axis" is incorrect. As we have evidenced, the dBB theory effectively predicts the possibility of detecting two bosons at the same semiplane, in complete agreement with the SQM theory. Let us notice that the authors of Refs. [5–8] in their reply [7,8,17] to previously published arguments [12,13] against their predictions (similar to the ones presented here), admit the possibility of detecting two Bohm trajectories at the same semiplane (at least for short distance between the slits and wide spatial dispersion of the wave packets). Such possibility is not mentioned or discussed by the authors of Ref. [4] in their manuscript.

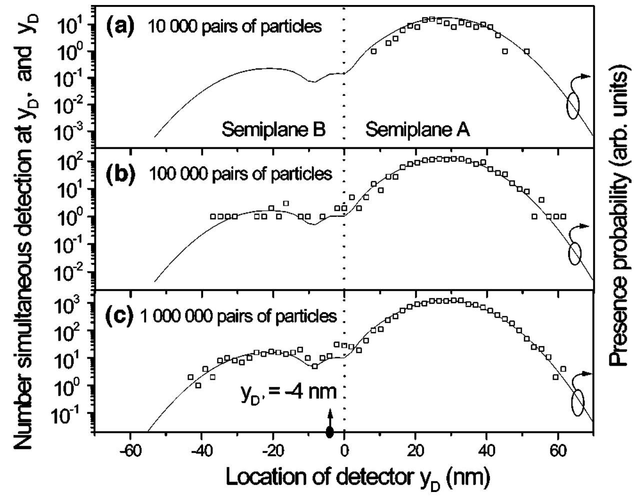

FIG. 2. In open squares, number of times that Bohm trajectories of two entangled bosons (using Gaussian wave packets) are detected simultaneously at the screen, as a function of the location of one of the detectors, y_D, when the other detector is fixed at semiplane B ($y_{D'} = -4$ nm) using a separation between slits $d \equiv y_A - y_B = 20$ nm. In the solid line, the SQM probability presence for $x_1 = x_2 = 14$ nm and $t = 28$ fs. The total number N of simulated pairs of bosons is $N = 10\ 000$ in (a), $N = 100\ 000$ in (b), and $N = 1\ 000\ 000$ in (c). There is an excellent agreement between SQM and dBB predictions for the coincident detection at the same semiplane.

III. DIFFERENCES BETWEEN THE "MASSIVE BOSON WAVE FUNCTION" AND THE "PHOTON WAVE FUNCTION" USED IN REF. [4]

The previous section is devoted to showing that the conclusion of Ref. [4] is incorrect because it is built on the wrong predictions of Refs. [5–8]. In this last section, we emphasize that, indeed, the following assumption of the authors of Ref. [4]: "our (experimental) scheme realizes the configuration recently suggested by two theoretical groups to test the dBB theory against SQM" is also incorrect because different wave functions are used for SQM and dBB predictions. According to dBB theory, Bohm trajectories are *intimately* connected to the wave function used to deduce its wavelike nature. Therefore, in order to compare the SQM and dBB predictions, the same wave function must be used in both cases. This obvious condition has not been respected in Ref. [4]. This inconsistency completely invalidates, *a priori*, any possible comparison between the SQM and dBB results done in Ref. [4].

We define the "photon wave function" as the expressions used in Ref. [4] to compute the SQM results. Such "photon wave function" is partially defined in the expressions (1), (2), and (3) in Ref. [4]. Due to the complexity in the quantum-optical parametric light conversion theory, only an analytical expression for the coincidence pattern is provided in expression (4) of Ref. [4]. At this point, let us notice that the experiments (and also the SQM predictions from the "photon wave function") are obtained for massless photons, where Maxwell equations obviously play an important role in determining the interference pattern [18]. However, the dBB predictions of Refs. [5–8] are obtained for massive nonrelativistic bosons where the interference pattern is described only by the Schrödinger equation (and not by Maxwell equations). Therefore, not only different wave functions, but different theoretical frameworks have been used for the SQM and dBB modeling of the experimental setup. In fact (see Chap. 12 in Ref. [3]) the attempt to extend the causal interpretation of the quantum theory to cover photons (with relativistic velocities) has not been extensively investigated yet.

We define the "massive boson wave function" as the analytical expression used by the authors of Refs. [5–8] to study the entangled bosons systems [rewritten in our Comment by expressions (3) and (4)]. Even under the assumption that the massive two-bosons framework can provide a reasonable approximation to the experimental setup described in Ref. [4] (that would obviously need some kind of justification by the authors of Ref. [4]), there is another important inconsistency between the "photon" and the "massive boson" wave functions. The "massive boson wave function" is independent of the distance between the nonlinear crystal and the double slit, because it is initially "defined" at the double slit, instead of at the nonlinear crystal (see Fig. 1). In principle, the "massive boson wave function" would have to be defined at the nonlinear crystal where the entangled two-particle system is created. Bosons would reflect back when impinging upon the double slit, and interference effects would appear in the crystal region (see Fig. 1). In fact, this is exactly the procedure that leads to the "photon wave function" described by expressions (1), (2), and (3) in Ref. [4]. The "photon wave function" is initially defined at the nonlinear crystal and photons are propagated by the "diffracted field" [expression (2) in Ref. [4]] through both slits until they reach the screen. Thus the "photon wave function" used to obtain SQM does not only depend on the diffraction angle observed by the detectors from the double slit (labeled as θ_i in the *screen region* of Fig. 1 and in Ref. [4]), but also on the incidence angle related with the nonlinear crystal position (labeled as θ_i^B in the *crystal region* in Fig. 1 and in Ref. [4]). Therefore, the distance between the nonlinear crystal and the double slit plays a role in determining the SQM coincidence, but it has no role in determining the dBB predictions. This is just more clear evidence that the wave function used to deduce dBB is different from the SQM one.

In summary, there are two fundamental differences between the "massive boson wave function" used to deduce the dBB predictions and the "photon wave function" used to obtain SQM results in Ref. [4]. First, the former is obtained for massive bosons instead of relativistic massless photons. Second, the "massive boson wave function" is arbitrarily defined at the double slit, and not at the nonlinear crystal as the "photon wave function." In conclusion, the obvious condition of dealing with a unique wave function whenever SQM and dBB predictions are compared is not respected in Ref. [4]. This fact implies that the results of Ref. [4] do no more than verify a particular quantum-optical prediction, with no possible import for dBB theory.

IV. CONCLUSIONS

In this Comment, we analyze the conclusions presented in Ref. [4] from a double-slit experiment with two entangled bosons. We provided two different arguments to show that the sentence affirmed by the authors, "The analysis of these data allows a test of standard quantum mechanics against the de Broglie–Bohm theory" [4], is incorrect.

The first argument is focused in analyzing the verisimilitude of expecting different predictions from SQM or dBB theories. To refute such possibility, in Sec. II, we have reminded that dBB results must be in complete agreement with SQM ones (as seen in Fig. 2), as a consequence of a *particle conservation law* consubstantial to the dBB interpretation of quantum mechanics. In particular, the authors of Ref. [4] affirm that "the coincidence signal (number of times that particles are detected at y_D and $y_{D'}$) is predicted (by the dBB theory) to be strictly zero when the two detectors are in the same semiplane with respect to the double-slit symmetry axis." The authors based their "strictly zero" dBB prediction on the work of two theoretical groups [5–8]. The type of prediction made by [5–8] is false because those authors impose, by hand, an "arbitrary" constriction on the initial positions of Bohm trajectories, and they only consider a subset of possible trajectories. Hence their predictions are not consistent with the dBB theory itself and, as a consequence, the hypothetical discrepancy between SQM and the dBB theories obtained in Ref. [4] is incorrect. We have explicitly shown that the coincidence of simultaneous particles at the same semiplane can be different from zero within the dBB theory for the wave functions described in [5–8], in complete agreement with the SQM results.

Second, we show that the sentence of the authors of Ref. [4] that "our (experimental) scheme realizes the configuration recently suggested by two theoretical groups to test the dBB theory against SQM" [4] is, in fact, incorrect because the "massive boson wave function" used in Refs. [5–8] does only provide a rude approximation to the real experimental system described in Ref. [4]. The "massive boson wave function" is arbitrarily defined at the double slit and avoids the consideration of the possible interference patterns at the "crystal region" (see Fig. 1). Moreover, what is even more meaningful to emphasize the differences is that the dBB predictions are obtained for massive bosons rather than for the massless relativistics photons used in the experiment and also in the SQM predictions. Therefore, not only different wave functions, but different theoretical frameworks have been used for the SQM and dBB modeling of the experimental setup.

In conclusion, the mentioned article provides an interesting experimental verification of a quantum-optical prediction for photons, but it does not import any consequence to the dBB theory. The conclusion affirmed by the authors of Ref. [4], "The analysis of these data allows a test of standard quantum mechanics against the de Broglie–Bohm theory," is incorrect. The seminal theoretical works of Refs. [5–8] are the main reason for their misleading conclusions.

ACKNOWLEDGMENT

This work has been partially supported by the Ministerio de Ciencia y Tecnología and FEDER through Project No. TIC 2003-08213-C02-02.

[1] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1997).
[2] F. Selleri, *Quantum Paradoxes and Physical Reality*, Fundamental Theories of Physics (Kluwer Academic, The Netherlands, 1990).
[3] P. R. Holland, *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, England, 1995).
[4] G. Brida, E. Cagliero, G. Falzetta, M. Genovese, M. Gramegna, and E. Predazzi, Phys. Rev. A **68**, 033803 (2003).
[5] M. Golshani and O. Akhavan, J. Phys. A **34**, 5259 (2001).
[6] M. Golshani and O. Akhavan, e-print quant-ph/0103100.
[7] Partha Ghose, e-print quant-ph/0103126.
[8] Partha Ghose, e-print quant-ph/0001024.
[9] P. Ghose, A. S. Majumdar, S. Guha, and J. Sau, Phys. Lett. A **290**, 205 (2001).
[10] X. Oriols, F. Martín, and J. Suñé, Phys. Rev. A **54**, 2594 (1996).
[11] J. Suñé and X. Oriols, Phys. Rev. Lett. **85**, 894 (2000).
[12] L. Marchildon, J. Mod. Opt. **50**, 873 (2003); e-print quant-ph/0007068; e-print quant-ph/0101132.
[13] W. Struyve, W. De Baere, J. De Neve, and S. De Weirdt, J. Phys. A **36**, 1525 (2003).
[14] Let us notice that the angle $\varphi=-\theta-\hbar k_A^2 t/(2m)$ with $\tan(2\theta)=2\hbar t/(ma^2)$ is omitted in Eqs. (5) and (6) in Ref. [6] [see the definition of a Gaussian wave packet in C. Cohen-Tanoudji, B. Diu, and F. Laloë, *Quantum Mechanics* (Wiley, New York, 1977)]. In any case, since this angle is independent of y, it does not affect Bohm velocities.
[15] The description of our quantum Monte Carlo simulator can be found in X. Oriols, F. Martin, and J. Suñé, Appl. Phys. Lett. **79**, 1703 (2001); **80**, 4048 (2002).
[16] Since Bohm velocity in the x direction is determined by a plane wave, $v_x=\hbar k_x/m$, both particles are located always at the same position in the x direction, $x_1(t)=x_2(t)$. It is assumed that both particles arrive simultaneously at the double slit at $t=0$ [i.e., $x_1(0)=x_2(0)=0$].
[17] O. Akhavan and M. Golshani, e-print quant-ph/0305020.
[18] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).