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Citation: Applied Physics Letters **86**, 242503 (2005); doi: 10.1063/1.1947370
View online: http://dx.doi.org/10.1063/1.1947370
View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/86/24?ver=pdfcov
Published by the AIP Publishing

Flux-flow critical-state susceptibility of superconductors

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(Received 7 December 2005; accepted 5 May 2005; published online 8 June 2005)

The field-amplitude H_m and circular frequency ω dependent ac susceptibility, $\chi=\chi'-j\chi''$, of a hard superconducting cylinder with flux-flow type current-voltage characteristic is calculated. A remarkable feature of the resultant $\chi(H_m,\omega)$ is that both the maximum χ'', χ''_m, and $d\lg H_m(\chi''_m)/d\lg\omega$ increase with increasing ω. This behavior is observed in actual Bi-2223/Ag tapes and $YBa_2Cu_3O_{7-\delta}$-coated conductors. Our result provides a useful tool to study the intergranular critical state in high-temperature superconductors. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1947370]

The magnetic properties of hard superconductors were first explained by Bean, who proposed what was later called the critical-state (CS) model, i.e., the volume supercurrent induced in hard superconductors is limited by the critical-current density J_c.[1,2] For an infinite cylinder or slab in a longitudinal applied ac field, $H=H_m e^{j\omega t}$, the magnetization $M(H)$ curves and complex ac susceptibility, $\chi=\chi'-j\chi''$, were derived analytically from the CS model and given in Ref. 3. Considering strong demagnetizing effects, $M(H)$ and $\chi(H_m)$ curves were derived from the CS model for perpendicularly magnetized thin tapes and disks.[4,5] Accurate results of transverse $\chi(H_m)$ were obtained for rectangular bars with finite width and thickness.[6]

The common behaviour of the CS $\chi(H_m)$ for any sample geometry is that with increasing H_m, χ' increases from a negative value $-\chi_0$ corresponding to the complete shielding state up to zero, whereas χ'' increases from zero to a maximum χ''_m at $H_m=H_m(\chi''_m)$ and then decreasing to zero. Since $\chi_0=[1-N_m(\chi=-1)]^{-1}$, N_m being the magnetometric demagnetizing factor,[7] χ_0 increases with decreasing the aspect ratio along the field direction from 1 upwards. However, χ''_m/χ_0 and a properly normalized $H_m(\chi''_m)$ are not significantly influenced by demagnetizing effects. For example, for the transverse CS χ of rectangular bars and the axial CS χ of cylinders, χ''_m/χ_0 ranges from 0.21 to 0.24 for the aspect ratio being either much greater or much smaller than unity, whereas the corresponding $H_m(\chi''_m)/J_c$ equals 0.5 or 1 times the smallest dimension (diameter or thickness). Thus, one can conclude that after a proper normalization, the $\chi(H_m)$ calculated easily for infinitely long samples can be an acceptable approximation for the perpendicular $\chi(H_m)$ of thin samples, which are seriously subject to demagnetizing effects and more difficult to be calculated.[8]

It is well known that in hard type-II superconductors, the power dissipation accompanied by volume currents arises from the depinning of the Abrikosov vortices (AVs) driven by the currents. Considering such depinning to be thermally activated, the experimental temperature and observation-time dependence of J_c was explained using the flux creep model first proposed by Anderson.[9] The AV pinning is weak and a collective creep can take place in high-temperature superconductors (HTSs), owing to their small coherent length ξ so a small pinning volume per AV and their point-defects dominated pinning centers.[10] From the current-dependent activation energy of collective depinning, Brandt has recommended a power-law current-voltage dependence $E(J)=E_c|J/J_c|^n J/|J|$, J_c being the J when the electrical field $E=E_c=10^{-4}$ V/m and index $n\gg 1$, to approximately characterize the behavior of collective creep, and computed the perpendicular $\chi(H_m,\omega)$ of disks that follow this $E(J)$.[11,12] It turns out that in this case for a finite n, $\chi(H_m)$ at a fixed ω is similar to that of the CS model and it shifts to higher H_m with increasing ω in such a way that $\chi(H_m,\omega)$ depends only on $H_m/\omega^{1/(n-1)}$.[11,12]

When interpreting the χ measurements of HTSs by the CS model, the frequency dependence of χ has usually been ignored in most previous works. We have measured the χ of a $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+\delta}$ (Bi-2223) monofilamentary tape and a $YBa_2Cu_3O_{7-\delta}$ (YBCO) single crystal film at 77 K as a function of H_m and ω.[13,14] The results turn out to be consistent with Brandt's calculation of χ with power-law $E(J)$, i.e., both χ''_m and $d\lg H_m(\chi''_m)/d\lg\omega$ are roughly constant with increasing ω, having $n=19$ and 31 for the tape and film, respectively, so that the CS is basically controlled by the collective flux creep.

However, we find that the $\chi(H_m,\omega)$ of HTS samples is not always like this. For some coated YBCO conductors, the perpendicular $\chi(H_m,\omega)$ shows a behavior that with increasing ω both χ''_m and $d\lg H_m(\chi''_m)/d\lg\omega$ increase.[14] In biaxially textured YBCO-coated conductors J_c is limited by networks of low-angle grain boundaries (GBs). Different from the AVs in the grains with a core size of ξ, the vortices in the low-angle GBs are Abrikosov-Josephson vortices (AJVs) with a core size along the GB greater than ξ.[15] Owing to the smoothing off of pinning potentials related to the presence of more pinning centers over the larger core dimension along GB and to the increase of the elastic energy relevant to the AJV bending parallel to GB, the AJVs are less pinned by

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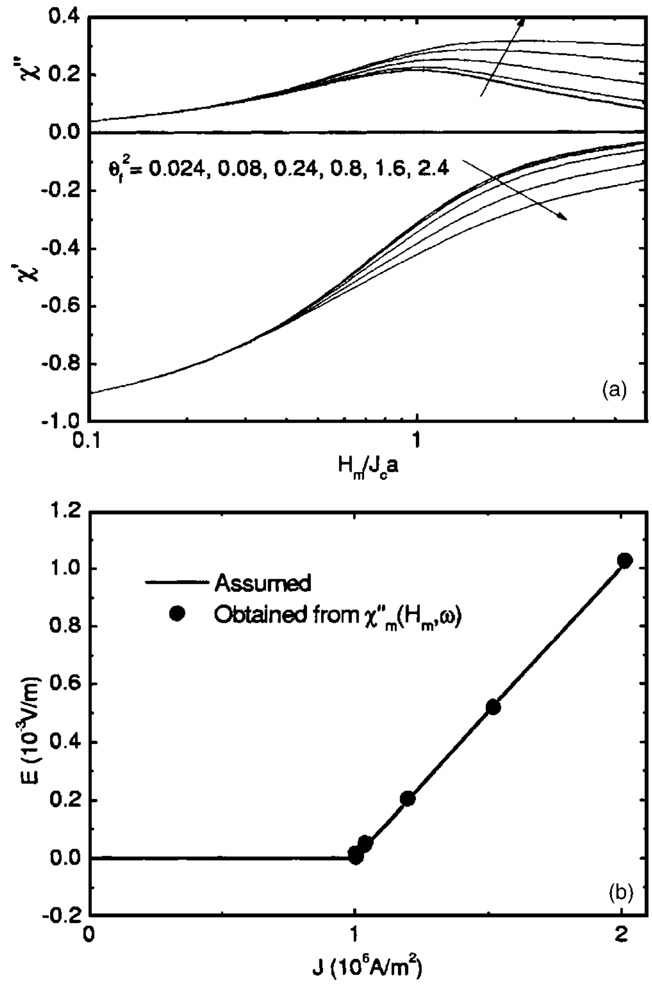

FIG. 1. (a) For a superconducting cylinder of radius a, critical-current density J_c, and flux-flow conductivity σ_f, the calculated ac χ as a function of the reduced field amplitude $H_m/(J_c a)$ for different values of dimensionless frequency $\theta_f^2 = a^2\mu_0\omega\sigma_f$. (b) The assumed flux-flow $E(J)$ relation compared with that calculated using Eqs. (4) and (5) from (a) when $a=1$ mm, $J_c=10^6$ Am^{-2}, and $\sigma_f=10^9$ m^{-1} Ω^{-1}. Arrows in (a) indicate the direction of increasing θ_f^2, and symbols in (b) represent data calculated from $\chi''_m(H_m,\omega)$ in (a).

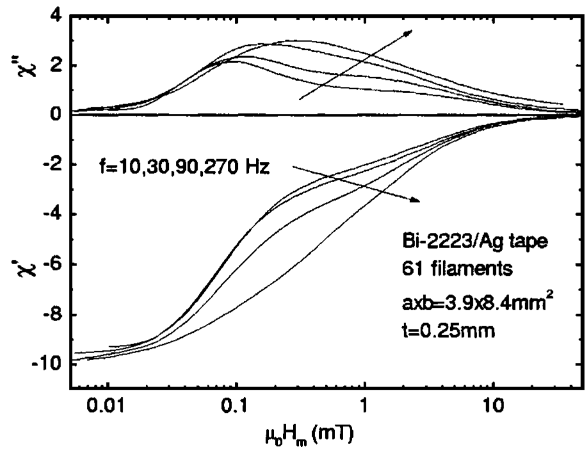

FIG. 2. The measured ac susceptibility of a Bi-2223 monofilamentary tape at 77 K and different values of frequency f as a function of $\mu_0 H_m$. Arrows indicate the direction of increasing f.

point defects along the GB than AVs are in the grains, but mainly pinned by dislocations aligned in the GB.[16] If pinning of dislocations is small and uniform enough, with increasing current, the dissipation will first occur from thermally activated AJV creep and quickly changing into vortex flow channeling along GBs, with a basically linear $E(J)$ curve above J_c. This behavior has been inferred from systematic experimental studies that have been carried out mainly on transport properties of bicrystalline films.[17–19]

Since $\chi(H_m,\omega)$ for flux-flow (limited) CS has not been studied, we calculate in this letter the flux-flow CS χ of a long cylinder of radius a using the method proposed by Brandt.[11,12] Based on observations of the current-voltage characteristic of YBCO bicrystals and some low-temperature superconductors,[17,20] we assume for the flux-flow CS that $E=0$ if $|J|\leqslant J_c$ and $[J-\mathrm{sgn}(J)J_c]/E=\sigma_f$ if $|J|>J_c$, where J_c is the critical current density and σ_f is the flux-flow conductivity, both being assumed to be magnetic field independent for simplicity. For the cylindrical geometry with magnetic field applied along the z axis, the induced electrical field **E** and the vector potential **A** in the London gauge only have a nonzero component (ϕ) being functions of r and t, $E(r,t)$ and $A(r,t)$. We divide the cylinder into $n=100$ cylindrical elements of equal radial dimension, borded at $r_i=ia/n, i=1,2,\dots,n$, with J in each element being uniform J_i and A at $r=r_i$ being A_i. Assuming $A(r=0)=0$, the Faraday law is written as

$$E[J_i(t)] = -\dot{A}_i^c(t) - \dot{A}_i^a(t) \quad (i=1,2,\dots,n), \tag{1}$$

where the $E(J)$ function has been defined above, the partial A for the applied field $A_i^a(t) = -\frac{1}{2}\mu_0 r_i H_m \sin\omega t$, and the partial A produced by currents $A_i^c(t)=\Sigma_j^n f_{ij}J_j(t)$, f_{ij} being the contribution of current in element j per unit J_j to A_i^c. Setting reasonable parameter values of $a=10^{-3}$ m, $J_c=10^6$ A/m, and $\sigma_f=10^{-9}$ m^{-1} Ω^{-1} and increasing stepwise t from 0, the equation system is solved numerically for each pair of given H_m and ω. In order to simulate accurately the physical process and to reach a stable symmetrical periodical state, three periods are calculated with 10^6 time steps in each period. With the stabilized J_i, the magnetic moment of the cylinder and finally $\chi(H_m,\omega)$ are computed in the usual way as described in Ref. 21.

The calculated $\chi(H_m,\omega)$ curves are plotted in Fig. 1(a), where H_m is normalized to $J_c a$ and a dimensionless frequency (following the tradition of eddy-current research)[22] $\theta_f^2=a^2\mu_0\sigma_f\omega$ is used so that the result becomes universal for any values of a, J_c, and σ_f. The curves for $\theta_f^2=0.024$ in Fig. 1(a) coincide very well with the CS χ for cylinder.[3] With increasing $\lg\theta_f^2$, we see both χ''_m and $d\lg[H_m(\chi''_m)/J_c a]/d\lg\theta_f^2$ to increase, which is qualitatively in agreement with the experimental results of the coated conductor reported in Ref. 14.

Such behavior has been found not only in coated conductors but also in some Bi-2223 tapes. In Fig. 2, we give $\chi(H_m,f)$ curves of a Bi-2223/Ag multifilamentary tape measured at 77 K. The critical current of the tape $I_c\approx 5$ A, which is rather low compared with the above mentioned monofilamentary tape with $I_c>30$ A. It was prepared without the last two thermomechanical treatments, each of which consisted of a cold rolling with small thickness reduction followed by 830 °C annealing, necessary for obtaining a good alignment and links of grains. Similar to the Bi-2223/Ag tape 61TH1 studied in Ref. 23, the intergranular J_c in this sample is dominated by weak links. We see from Fig. 2 that besides a two-stage feature in $\chi(H_m)$ curves, which indicates both the weak and strong-link contributions,[23,24] there is a remarkable similarity between the measured $\chi(H_m, f)$ and the curves calculated from flux-flow CS in Fig. 1(a).

This flux-flow feature found in χ of the multifilamentary Bi-2223/Ag tape, where J_c is dominated by weak links, is interesting. As studied in Refs. 15, 25, and 26, the GBs in this case may be regarded as extremely incoherent planar defects with very small depairing (Josephson) current density J_J, for which no AJVs can form like in coated conductors, where J_J of low-angle GBs acting as coherent planar defects can be close to the depairing current density of the bulk. As explained in Refs. 23 and 27, weak-link critical currents may arise from either Josephson-vortex pinning like in a type-II superconductor when the maximum Josephson currents, I_J's, between neighboring grains are very small, with a flux creep dissipation mechanism of the intergranular CS, or from the I_J's themselves when they are large. The Bi-2223/Ag tape has much greater I_J's than those of sintered YBCO, and the weak-link critical currents of Bi-2223/Ag tapes should result from I_J's of individual Josephson junctions themselves. In this case, the flux-flow behavior should be a consequence of the resistively shunted Josephson-junction network.

In such a network, the effect of GB between any pair of neighboring grains i and j is modeled by a parallel circuit of a Josephson junction and a resistance R together with a void of area A_v, where flux can pass through. After a field change, the gauge-invariant phase difference θ_{ij} of Josephson junctions in the induced intergranular current path may be chosen automatically to be near $(2n' \pm 1/2)\pi$, which corresponds to a CS with all Josephson junctions carrying a current close to I_J's. The flux penetration will involve a θ_{ij} jump $(n' \rightarrow n' \pm 1)$, which corresponds to a fluxon (Φ_0) to traverse through the Josephson junction, leading to an ac Josephson current to flow through R. Thus, the final result is very similar to the modeled flux-flow CS.

The following equations for J and E may be used for obtaining the approximate $E(J)$ relation from frequency dependent $H_m(\chi''_m)$:

$$J = H_m(\chi''_m)/a, \tag{2}$$

$$E = 2\mu_0 a\omega H_m(\chi''_m)/(3\pi). \tag{3}$$

The $E(J)$ function for the cylinder obtained using Eqs. (2) and (3) from the calculated $\chi(H_m, \omega)$ in Fig. 1(a) is plotted in Fig. 1(b) by symbols. We see that they agree quite well with the actually assumed $E(J)$ function (solid line) for $\chi(H_m, \omega)$ calculation. This agreement justifies the technique of determination of $E(J)$ curve from ac χ measurements, which has first been used in Ref. 14, where radius a is replaced by the thickness or sides of the film after physical consideration.

In conclusion, we have calculated ac χ of a superconducting cylinder with flux-flow $E(J)$ characteristic. The results may be useful for studying the intergranular CS of HTSs, such as coated YBCO conductors and multifilamentary Bi-2223/Ag tapes. Especially, we have demonstrated that the intergranular CS in HTSs is not resulting from Josephson vortex pinning as popularly believed but from the maximum Josephson currents through GBs themselves. We have also justified that the $E(J)$ characteristic of HTSs can be conveniently determined by field and frequency-dependent contactless χ measurements.

This work has been supported by MEC project FIS2004-02792 and Generalitat de Catalunya (SGR 2001-00189 and CeRMAE).

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