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Sequential Formation of Coalitions through Bilateral Agreements in a Cournot Setting*

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Abstract

We study a sequential protocol of endogenous coalition formation based on a process of bilateral agreements among the players. We apply the game to a Cournot environment with linear demand and constant average costs. We show that the final outcome of any Subgame Perfect Equilibrium of the game is the grand coalition, provided the initial number of firms is high enough and they are sufficiently patient.

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1 Introduction

The incentives that firms have to merge have recently been studied in non-cooperative games of endogenous coalition formation. The usual way of analyzing these games is by assuming that the forming of a coalition or the negotiation of a merger has no cost for the participants, in particular, many players may consider simultaneously whether to form a coalition or not. However, the decision to join in a single coalition involves negotiations or, for instance, the need to co-ordinate the productive technologies of the different firms. There are, therefore, transaction costs. Moreover, the transaction costs seem much higher the more firms that are involved in a single merger.[1]

For this reason, we consider relevant to study the opposite case: what happens when the merger process is carried out always bilaterally? With this we mean that, at any point in time, only two of the existing coalitions may decide to merge. Although we do not explicitly model transaction costs, this restriction could be understood as the outcome of situations in which there is a very important scale-effect in transaction costs (i.e., a merger that involves three or more coalitions would imply bearing so many transaction costs that it becomes unfeasible). The restriction we impose does not mean, however, that only small coalitions may be formed. By sequentially meeting over time, coalitions may grow in size. In other words, once some coalitions are formed, they may decide to continue with the process and form even larger entities.

The sector of firms that provide professional services (accounting, consulting, etc.) offers a relevant set of examples of such a sequential process of bilateral mergers. Some of the major firms in this sector, (i.e., Ernst & Young, KPGM and PricewaterhouseCoopers) are the outcome of a sequential process of mergers with a small number of parties involved. In particular, since Arthur Young opened an accounting firm in Chicago (1894), and the brothers Alvin and Theodore Ernst settled their firm in Cleveland (1903), at least four bilateral mergers have taken place before the present structure of Ernst and Young was arrived at.

[1] The cost of integrating more than two organizations is very large. This is the reason why most mergers involve only two firms. For example, Houston, James, and Ryngaert (2001) construct a sample of the largest bank mergers between 1985 and 1996. The 64 mergers in the sample are bilateral mergers.

The banking sector provides other examples. In Spain, the bank that is now known as SCH is the outcome of a merger between the Banco de Santander and the Banco Central Hispano which, in turn, was the result of the merger between the banks Central and Hispano. Similarly, the banks of Bilbao and Vizcaya first merged to form the BBV and then the new firm merged again with the Banco Argentaria to form the BBVA.

We model the formation of coalitions as a sequential process in which, at each moment in time, only two existing coalitions can decide to merge. We study the subgame perfect equilibria of such a game. The sequential process of coalition formation that we propose can be useful in analyzing sequential formation of bilateral agreements in several economic environments where groups of agents interact, including mergers, environmental cartels, and networks.

In this paper, we consider a market in which identical firms with constant returns facing linear demand compete à la Cournot. At each period, the firms make decisions on quantity. To focus our analysis on the incentives to form coalitions, we assume that production is a short-term decision. Also, at each period, two randomly chosen coalitions can merge in the existing partition. A merger means forming a cartel in which the partners decide on production jointly. The decision on the merger is made by taking the long-term profits into account.

As Salant, Switzer, and Reynolds (1983) point out, two firms (or coalitions) will not be interested in merging if they only consider the present period profits and if there are already at least three firms (coalitions) in the industry. Their result extends easily to our model: If the firms' discount rate is low enough, they will not merge at any period in the unique subgame perfect equilibrium of the game. Hence, the outcome is that all of the firms remain singletons.

The situation when firms are forward-looking is more interesting. In such a case, the firms may want to merge even if they lose profits in the short run. In fact, we show that when firms are patient enough, and there are enough firms in the industry, the final outcome of any subgame perfect equilibria is "the grand coalition". The firms form coalitions sequentially, growing gradually, so that finally they all end up together. We characterize the sequences of mergers that the firms will undertake in equilibrium. In those sequences, firms will accept some of the mergers and will reject others.

Moreover, the characteristics of our game allow us to analyze all of the subgame perfect equilibria, without restricting our attention to stationary strategies as it is usual in the literature. All the results remain true if we concentrate on the stationary subgame perfect equilibria of our coalition formation game.

The fact that, in a linear Cournot model, "the grand coalition" can result as the equilibrium of a game of coalition formation, is in contrast with other results on mergers presented in the literature. Several authors have addressed the question of the coalition structures that would prevail in this set up by analyzing the stability of the coalition structures.[2] This literature suggests that there would eventually be one large coalition and a few players as singletons. Our game never has these intermediate results: If there is a small number of players, or if the discount rate is low, all of the players remain as singletons, while "the grand coalition" is the only final outcome when both the set of players and the discount rate are large enough. In fact, "all singletons" and "the grand coalition" are the only two possible subgame perfect equilibrium outcomes of our game.

The difficulty to reach efficient outcomes through non-cooperative games of coalition formation has been discussed in different games. Kamien and Zang (1990) show that a merger can not involve many firms when the number of players is large using a model of coalition formation via acquisitions. This is also the case in Bloch (1996) and Ray and Vohra (1999) for a sequential game of coalition formation.[3] Ray and Vohra (1997) find an analogous result for a larger class of games using the notion of equilibrium binding agreements based on farsightedness. Diamantoudi and Xue (2002) prove that this negative result may still arise when arbitrary coalitional deviations are allowed. However, for

[2] In simultaneous games, we can refer to four stability concepts (Aumann (1967) and Hart and Kurtz (1983)). A coalition structure is α-stable if no group of firms can guarantee an improvement, independently of what the others do. A partition is β-stable if no group of firms has, for any possible reaction of the external players, a strategy that can improve its situation. A coalition structure is γ-stable (respectively, δ-stable) if no set of players has incentives to deviate when the players of their original coalitions split up (respectively, they still form a coalition). In the linear Cournot game, α-stable, β-stable, and γ-stable outcomes always have the form $\{s, 1, ..., 1\}$ with s being higher or equal to 80% of the market. The set of δ-stable outcomes, on the other hand, is empty.

[3] Section 5 provides a more detailed comparison of the papers by Bloch (1996) and Ray and Vohra (1999) with our analysis. It also provides a discussion on the implications of the differences between their approach and ours.

symmetric games like ours, the grand coalition is an stable outcome of their game.

Some authors have considered the sequential formation of mergers by studying how these decisions are inter-connected over time. Pesendorfer (2004) studies a model of merger formation with entry in the line of Kamien and Zang (1990), where certain firms acquire others by submitting bids and asking prices. In his model, "the grand coalition" cannot be formed in a single period, because all the firms are not present in the market from the beginning of the game. He concludes that even if frequent mergers are not profitable when the number of firms in the industry is small, they can become profitable as the number of firms increases. Gowrisankaran and Holmes (2000) analyze the steady states of an endogenous merger game, in which a dominant firm takes merger decisions regarding a competitive fringe. They show that monopoly and perfect competition always belong to the set of steady states in the game. In this paper, we have identified a new strategic effect of mergers: *new merges may foster future mergers, and thus may create a merger wave.*

Our work is also in line with Gul (1989), who analyzes a transferable utility economy in which random bilateral meetings occur. At each meeting, one of the agents makes a proposal to the other which he can either accept or reject. If the proposal is accepted, the resources of both agents are in the hands of the proposer from this moment on, otherwise, both players stay in the game. Gul (1989) shows that, under some conditions, all the players will eventually end up together and the expected payoff of each player in an efficient sequential perfect equilibrium is his Shapley value.[4]

In the following section we present the coalition-formation game. In Section 3, we analyze the outcomes of the game when firms are myopic, while in Section 4 we do the analysis when firms are forward looking. In Section 5, we show the extent to which our results are robust to several variations of our game.

[4] Seidmann and Winter (1998) also analyze gradual coalition-formation in games without externalities, although the agreements are not bilateral.

2 The Coalition-Formation Game

We study the sequential formation of coalitions between firms competing à la Cournot in a framework in which only *bilateral agreements* are allowed. We assume that, at each moment in time only two of the existing coalitions can decide to merge.

At the beginning of the game, there are n identical firms, with $n \geq 2$. We denote the set of firms by $N = \{1, ..., n\}$. Firms can form coalitions following a certain protocol that will be described later. Hence, at any point in time, these n firms form a partition of N, i.e., they constitute a *coalition structure.*

Let $\prod$ denote the set of coalition structures over N. Denote $\pi \in \prod$ an element of this set, that is, $\pi = \{S_1, ..., S_r\}$, with $S_a \subset N$ for all $a = 1, ..., r$, $\cup_{a=1}^{r} S_a = N$, and $S_a \cap S_b = \emptyset$ for all $S_a, S_b \in \pi$, with $S_a \neq S_b$. We denote by s_a the size of coalition S_a. Among the set of partitions, a particular coalition structure is the one in which all the agents are alone, i.e., all the coalitions are singletons. We denote such a partition by π^n and "the grand coalition" by $\pi^1 \equiv N$, i.e., the coalition structure with just one element. We denote by $(\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$, the coalition structure that results when we replace two elements of π, namely S_a and S_b, by their union. Therefore, if π is formed by r coalitions, $(\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ consists of $(r - 1)$ coalitions.

Firms make decisions at any time $t = 0, 1, 2, ...$ At time t, the present profits of a firm depend on the coalition structure at that time. We assume, for the sake of simplicity, that firms face a linear demand function and bear equal constant average costs. That is, the inverse demand function at time t is:

$$P(\sum_{j=1}^{n} q_j) = \alpha - \beta \sum_{j=1}^{n} q_j.$$

The production costs of firm i are given by:

$$C_i(q_i) = cq_i.$$

When firms merge, they form a cartel. That is, merging allows the firms to co-ordinate their quantity decisions. We calculate the firms' profits at any point in time, given a cartel structure (i.e., a coalition structure) $\pi = \{S_1, ..., S_r\}$. We assume that production is a short-term decision, being taken by short-term managers.[5] Given that there are r cartels in this structure and that marginal costs are equal for all firms in a cartel, cartel S_a chooses the total level of production q_a of its firms by solving the following maximization program:

$$\max_{q_a} \left\{ \left(\alpha - \beta \sum_{b=1}^{r} q_b \right) q_a - c q_a \right\}. \tag{1}$$

From this program we find that the equilibrium quantities are equal for all of the cartels and that they are equal to: $q^r = \frac{\alpha - c}{\beta(r+1)}$. Hence, the Cournot profits per-cartel V^r in a coalition structure with r cartels are:

$$V^r = \frac{(\alpha - c)^2}{\beta(r+1)^2}.$$

We normalize $\frac{(\alpha-c)^2}{\beta} = 1$, so:

$$V^r = \frac{1}{(r+1)^2}.$$

It can be easily verified that the efficient outcome, from the industry's point of view, is arrived at when all the firms merge, and "the grand coalition" is formed.

We assume that the sharing of profits among the firms that form the cartel is exogenously fixed and egalitarian. Therefore, the individual profits $V_i(\pi)$ of any firm i belonging to the cartel $S_a \in \pi$, with a size s_a, when there are r cartels in the coalition structure π, are:

$$V_i(\pi) = \frac{1}{(r+1)^2 s_a}. \tag{2}$$

All firms value future payoffs with a discount factor $\delta \in [0, 1)$. Therefore, if π_t is the coalition structure existing at time t, for $t \geq t^\circ$, the discounted payoff of firm i at time t° is $\sum_{t=t^\circ}^{\infty} \delta^{(t-t^\circ)} V_i(\pi_t)$.[6]

[5] It is well known that, in an infinite game like ours, there are strategies by which firms may reach implicit collusion in production if the discount rate is high enough (notice, however, that the set of equilibrium outcomes is usually very large). Our objective in this paper is the analysis of the incentives for coalition formation, so we will abstract from the possibility of collusion by assuming that production is a short-term decision. An equivalent assumption is that firms use Markov, or stationary, strategies when they decide their production level. In Section 5, we analyze a simpler game in which this assumption is not necessary because production takes place only once. In this game, all our results still hold.

[6] When $\delta = 0$, the discounted payoff of player i at time t^o is $V_i(\pi_{t^o})$.

We study the outcome of a *process of sequential coalition formation*. This infinite-horizon process is undertaken according to the following protocol. At each period t, there is first the decision to merge (stages $t.1$ and $t.2$) and secondly, (stage $t.3$), there is the decision on production. We have already described the result of the production stage, summarized by the profit function $V_i(\pi_t)$. More precisely:

At $t = 0$:

0.1 Two different firms i and j are randomly selected. All the firms have the same probability of being selected.

0.2 Firms i and j sequentially decide whether to merge or not. The merger occurs if both players agree.

The coalition structure at time $t = 0$ is then either $\pi_0 = (\pi^n \backslash \{\{i\}, \{j\}\}) \cup \{i, j\}$ if firms i and j have merged or $\pi_0 = \pi^n$ if they have not.

0.3 Each firm $k \in N$ obtains, at $t = 0$, profits $V_k(\pi_0)$.

Let us now consider any time $t \geq 1$. The coalition structure existing at $t-1$ was π_{t-1}. If $\pi_{t-1} = N$, then $\pi_t = N$. Otherwise:

$t.1$ Two coalitions S_a and S_b in π_{t-1} are randomly selected. All of the coalitions in π_{t-1} have the same probability of being selected.

$t.2$ Firms in coalitions S_a and S_b sequentially decide whether to merge. The merger is carried-out if all of the firms in coalitions S_a and S_b agree to it.[7]

The coalition structure at time t is either $\pi_t = (\pi_{t-1} \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if coalitions S_a and S_b have merged or $\pi_t = \pi_{t-1}$ if they have not.

$t.3$ Each player $k \in N$ obtains profits $V_k(\pi_t)$ at time t.

[7] The firms are the players of our game. When they decide on the merger, the members in S_a and S_b do not face a co-ordination problem because they chose sequentially. Therefore, if it is optimal for all of them, they will sequentially choose to merge. If the merger is not profitable for the firms in one of the coalitions the merger will not happen, because one of the firms will not accept it.

The solution concept that we consider is *Subgame Perfect Equilibrium* and we concentrate on *pure strategies.* We denote the set of Subgame Perfect Equilibria in pure strategies by SPE.

We must point out that the proposed process for the formation of coalitions is irreversible, in the sense that the players cannot dissolve a merger once it has been formed. Allowing for mergers to split up enlarges the set of possible SPE considerably.

Given the irreversibility of the coalition-formation process, the game will arrive at a situation in which the existing coalition structure at that specific period will remain forever, with probability one. We will refer to such a coalition structure as a *final coalition structure* or a *final outcome.* If there are SPE strategies that lead to a particular final outcome, then we say that it is an *SPE final outcome.*

3 Myopic Firms

The objective of this paper is to look at the SPE final outcomes of the game of sequential formation of coalitions. The easiest analysis is done in the simple benchmark where players have a completely *myopic behavior.* This is equivalent to assuming that $\delta = 0$, the case in which we have the static version of our game.

If the players are myopic, the firms in two coalitions S_a and S_b in partition π will decide to merge (if they are chosen by the protocol) at any period, if and only if:[8]

$$V_i(\pi) < V_i\left((\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}\right) \text{ for all } i \in S_a \cup S_b.$$

Let us suppose that the coalition structure π is formed by $r \geq 2$ coalitions. Then, the firms of S_a and S_b will want to merge and move to a structure with $r-1$ coalitions if:

$$\max\left\{\frac{1}{(r+1)^2 s_a}, \frac{1}{(r+1)^2 s_b}\right\} < \frac{1}{r^2(s_a+s_b)}.$$

Let us assume, without loss of generality, that $s_a \leq s_b$. The condition then becomes:

$$\frac{1}{(r+1)^2 s_a} < \frac{1}{r^2(s_a+s_b)},$$

[8]For convention, we make the implicit assumption that a player will only be willing to join a coalition if he makes a strictly positive gain by doing so.

or equivalently:

$$s_a > \frac{r^2}{2r+1} s_b.$$

Note that the previous equation implies $s_a > s_b$ as long as $r \geq 3$, which would be in contradiction with our hypothesis that $s_a \leq s_b$. Therefore, two coalitions of firms will never be interested in merging if they only care about present profits and if there are at least three existing coalitions in the industry. This is a well-known result in static games that goes back to Salant *et al.* (1983). In addition to this, and for the case $r = 2$, the previous inequality shows that two coalitions will merge to monopoly if and only if their sizes are not very different. More precisely, the required condition is that $s_a > (4/5)s_b$, for the case $s_a \leq s_b$.

The previous observation implies that *if there are at least three firms* in the market, *the only myopic final outcome of the game of coalition formation is "all singletons"*. That is, when $\delta = 0$ no merger will occur.

For low enough discount rates, a firm is not interested in compensating short-term loses with long-term gains. Therefore, the myopic final outcome will also be the SPE final outcome when the discount parameter δ is low enough. We state this result formally in the following proposition:

Proposition 1 *If $n \geq 3$ and the discount rate δ is low enough, then the only SPE final outcome of the process of sequential coalition-formation in the linear Cournot setting is that all firms remain singletons.*

Proof. Immediate, after the discussion for the case $\delta = 0$. ■

4 Forward-Looking Firms

When firms are forward-looking, they may have an interest in merging even in cases in which they would lose profits in the short run. The reason would be that by doing so they would obtain higher profits in the future. A (non-profitable) merger by two firms or two coalitions may further other mergers. Hence, although the initial merging firms (or coalitions) lose profits because of the first merger, they may improve their situation later on if other mergers are carried-out.

The following proposition restricts the set of potential SPE final outcomes of the sequential game for any discount rate. It shows that, in equilibrium, firms will surely not start merging to end up in a coalition structure with more than one coalition.

Proposition 2 *The SPE final outcome of the coalition formation game in a Cournot competition model can only be either a monopoly or "all singletons".*

Proof. We do the proof by contradiction. Let us suppose that the final outcome is a coalition structure π formed by r coalitions, with $2 \leq r \leq n-1$. Denote by S_a and S_b the last two coalitions that merged, say at period t^0, with $s_a \leq s_b$. In Section 3, we saw that, for a firm $i \in S_a$, if π includes at least 3 coalitions, then $V_i(\pi) > V_i((\pi \backslash \{S_a \cup S_b\}) \cup \{S_a, S_b\})$. In addition, firms in S_a would even get strictly higher profits if, at any period after t^0, other mergers not involving S_a take place. Therefore, for firms in S_a, the strategy of merging with S_b at t^0 (leading to the final outcome $\pi \neq \pi^1$) is strictly dominated by the strategy of not accepting any merger from t^0 on. Therefore, the firms in S_a have a profitable deviation. Hence, no SPE strategy profile can lead to a final outcome with r coalitions, for $2 \leq r \leq n-1$. ■

Proposition 2 shows that the process of coalition-formation in a linear Cournot model will only begin if it leads to full integration (monopoly). Otherwise, all of the firms will remain singletons. The reason for this result is that no pair of coalitions wants to be the last to merge (unless the merger leads to a monopoly). In equilibrium, therefore, a merger can only happen if the firms involved anticipate that it will be followed by another, and yet another, until "the grand coalition" is formed.

We are now interested in finding out when the SPE final outcome of the game of coalition-formation is a monopoly. We know that a necessary condition for a monopoly to emerge is that the discount rate should be high enough, since no merger takes place in equilibrium when the discount rate δ is low enough, as was shown in Proposition 1.

Given Proposition 2, we also know that two coalitions will never merge if there is not a sequence of unions leading up to full integration. Another necessary condition for the mergers to arise therefore, is that for every value of r, for $2 \leq r \leq n$, there must exist a coalition structure with r coalitions, such that at least two of them obtain smaller profits in this sort of structure than they would in a monopoly.

The profits of the members of a coalition of size s in a coalition structure with r cartels are strictly smaller than their profits in a monopoly if:

$$\frac{1}{(r+1)^2 s} < \frac{1}{4n}, \text{ i.e., } s > \frac{4n}{(r+1)^2}.$$

Given that s is a natural number, the condition can be re-written as:[9]

$$s \geq \underline{s}^r \equiv int\left\{\frac{4n}{(r+1)^2}\right\} + 1.$$

Hence, in a partition with r coalitions, a necessary condition for two coalitions to merge is that the size of each one be at least $\underline{s}^r$. This necessary condition has to be verified for every $r \geq 2$.

To formally state the conditions under which a monopoly might be the SPE outcome, let us denote by $\mathcal{M} \equiv \mathcal{M}^n$ the set of sequences of coalition structures $M = \{\pi^r\}_{r=1}^n$ such that π^n is "all singletons" and, for all $r = 2, ..., n$, $\pi^{r-1} = (\pi^r \backslash \{S_a^r, S_b^r\}) \cup \{S_a^r \cup S_b^r\}$, for some S_a^r and S_b^r in π^r satisfying $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$.

Similarly, for any $r^\circ = 1, ..., n$, we denote by $\mathcal{M}^{r^\circ}$ the set of sequences of coalition structures $M^{r^\circ} = \{\pi^r\}_{r=1}^{r^\circ}$ such that π^{r° is any partition of N with r° coalitions and, for all $r = 2, ..., r^\circ$, $\pi^{r-1} = (\pi^r \backslash \{S_a^r, S_b^r\}) \cup \{S_a^r \cup S_b^r\}$, for some S_a^r and S_b^r in π^r satisfying $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$.

According to the previous definition, $\mathcal{M}^1 = \{N\}$. Also, if the sequence $\{\pi^r\}_{r=1}^{r^\circ} \in M^{r^\circ}$, then $\{\pi^r\}_{r=1}^{r'} \in M^{r'}$, for any $r^\circ = 1, ..., n$ and $r' \leq r^\circ$.

Proposition 3 *For any n, there exists a $\bar{\delta} < 1$, such that for all $\delta \geq \bar{\delta}$, the SPE strategy profiles of the process of sequential coalition-formation satisfy the following properties. Consider a subgame in which the existing partition π^r contains r coalitions:*

(a) If coalitions S_a and S_b are chosen by the mechanism, the merger will not be accepted if $\min\{s_a, s_b\} < \underline{s}^r$ *or if* $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ *does not belong to any sequence of coalitions in $\mathcal{M}^{r-1}$.*

(b) If π^r belongs to some sequence of coalitions in $\mathcal{M}^r$, there are two coalitions S_a and S_b in π^r, such that the firms in S_a and S_b accept the merger if they are selected by the mechanism.

[9] We use $int\{m\}$ to denote the integer part of $m \in \mathbb{R}$.

(c) The final outcome will be a monopoly if and only if π^r belongs to some sequence of coalitions in $\mathcal{M}^r$. Otherwise, the final outcome will be π^r.

Proof. We prove the proposition by induction over r.
($r = 2$) Take any subgame where only two coalitions S_a and S_b are left, i.e., $\pi^2 = \{S_a, S_b\}$. In such a case, the merger of the two coalitions is N, hence it is always in $\mathcal{M}^1$.

($2.a$) If $\min\{s_a, s_b\} < \underline{s}^2$, any firm in the smallest coalition prefers to stay as a duopoly rather than become part of a monopoly. Therefore, every SPE involves rejection of the merger.

($2.b$) If π^2 belongs to some sequence of coalitions in $\mathcal{M}^2$, then $\min\{s_a, s_b\} \geq \underline{s}^2$. All the firms in S_a and S_b obtain higher profits by merging. As a consequence, accepting this merger is the only SPE strategy in this subgame.

($2.c$) Immediate, after ($2.a$) and ($2.b$).

Hence, the properties (a), (b) and (c) hold for all $\delta \in [0, 1]$.

We now make the induction hypothesis that there exists $\overline{\delta}^{r-1} < 1$, such that for all $\delta \geq \overline{\delta}^{r-1}$, properties ($a$), ($b$), and ($c$) hold for any $r' < r$ and for any $\pi^{r'}$. We prove that this induction hypothesis is also satisfied for r, where $r = 3, ..., n$.
(r) Let π^r be the existing partition.

($r.a$) Suppose that coalitions S_a and S_b in π^r have been chosen by the mechanism and the firms in these coalitions must decide whether to merge or not. Suppose that $\min\{s_a, s_b\} < \underline{s}^r$. According to property (c) of the induction hypothesis, the final outcome will be either a monopoly or $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if S_a and S_b merge and $\delta \geq \overline{\delta}^{r-1}$. Moreover, to reach monopoly it is necessarily the case that $\pi^{r-1} \in \mathcal{M}^{r-1}$, hence, the profits of the firms in $S_a \cup S_b$ will be lower than monopoly profits along all the path to monopoly. In both cases, the firms of the smallest coalition will obtain lower profits than in π^r. Hence, merging is a strategy that is strictly dominated (for the firms in the smallest coalition) by the strategy of never merging from this moment on.
Similarly, let us suppose that the partition $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ does not belong to any $M^{r-1} \in \mathcal{M}^{r-1}$. Then, according to property (c) of the induction hypothesis, the final outcome will be $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if S_a and S_b merge and $\delta \geq \overline{\delta}^{r-1}$. On the other hand, if the firms in one of the coalitions choose never to merge (not necessarily the optimal

strategy, but one possibility), they obtain, from this moment on, at least the benefits that they have under the structure π^r. Given that $r > 2$, $V_i(\pi^r) > V_i((\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\})$ for either every firm in S_a or every firm in S_b. Therefore, here also, merging is not the optimal strategy for any of the firms, either in S_a or in S_b.

We have shown, therefore, that property (a) of the induction hypothesis holds for r as long as $\delta \geq \bar{\delta}^{r-1}$.

$(r.b)$ We now prove that there exists a $\bar{\delta}^r < 1$ such that for $\delta \geq \bar{\delta}^r$, if π^r belongs to some $M^r \in M^r$ then the strategies of the members of (at least) two coalitions S_a and S_b in π^r will be to accept the merger if they are selected by the mechanism. We do the proof by contradiction. Suppose that π^r belongs to some $M^r \in M^r$ but no two coalitions in π^r ever accept the merger when they are selected at t. If this is the case, then the final outcome is π^r. Take a pair of coalitions S_a and S_b in π^r, such that $\min\{s_a, s_b\} \geq \underline{s}^r$ and $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ belongs to some $M^{r-1} \in M^{r-1}$ (the existence of such a pair of coalitions is guaranteed by the definition of M^r). The members of S_a and S_b obtain strictly higher profits in a monopoly than staying in π^r, since $\min\{s_a, s_b\} \geq \underline{s}^r$. Also, a monopoly is the final outcome if $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ is reached, given that it belongs to some $M^{r-1} \in M^{r-1}$ and property (c) of the induction hypothesis holds. Finally, the probability that one of the paths leading to monopoly is chosen by the mechanism is one, since every pair of coalitions has a positive probability of being chosen. Therefore, there is a $\bar{\delta}^r(S_a, S_b, \pi^r) < 1$ large enough such that firms in $S_a \cup S_b$ strictly prefer to arrive at a monopoly (at the rhythm according to the SPE strategies) than to stay in S_a and S_b forever. They will therefore, have incentives to change their strategy and accept the merger if $\delta \geq \bar{\delta}^r(S_a, S_b, \pi^r) < 1$. To do the argument for any (S_a, S_b, π^r), we take $\bar{\delta}^r$ as the maximum between $\bar{\delta}^{r-1}$ and all the (finite number of) $\bar{\delta}^r(S_a, S_b, \pi^r)$ for all possible pairs of coalitions (S_a, S_b) in any possible coalition structure π^r that belongs to some $M^r \in M^r$.

$(r.c)$ is a direct consequence of $(r.a)$, $(r.b)$, and property (c) of the induction hypothesis.

■

Proposition 3 gives a lot of information about SPE when the discount factor δ is high. It provides the two main characteristics of the SPE outcome. First, in a SPE strategy profile, the members of two randomly chosen coalitions will only decide to merge if the resulting coalition structure belongs to some sequence $M^r \in \mathcal{M}^r$. Secondly, when it is possible to keep the sequence of coalitions in $\mathcal{M}^r$, then at least one pair of coalitions will decide to merge. The two properties together imply that, if we start from a partition in some $\mathcal{M}^r$, the firms will form coalitions and end up all together.

Hence, from Proposition 3 we can conclude that if the "all singletons" coalition π^n belongs to some sequence $M \in \mathcal{M}$, then a monopoly is the final outcome. Moreover, a monopoly can only be reached through sequences in $\mathcal{M}$.

The next natural question is whether the set $\mathcal{M}$ that we have identified exists or not. To see that it is sometimes empty, it is sufficient to verify that it is empty for $n = 3$ or $n = 4$. The following lemma provides a sufficient condition for $\mathcal{M}$ to contain some sequence of coalition structures.

Lemma 1 *If n is large enough, then $\mathcal{M}$ is non-empty.*

Proof. We construct a sequence $M = \{\pi^r\}_{r=1}^n$ by starting from "the grand coalition", $\pi^1 = N$, and splitting up one coalition each time. We divide $S^1 = N$ into S_a^2 and S_b^2 with $s_a^2 = n - \underline{s}^2$ and $s_b^2 = \underline{s}^2$. For $r = 3$, S_a^3 and S_b^3 are obtained by dividing S_a^2 in such a way that $s_a^3 = s_a^2 - \underline{s}^3$ and $s_b^3 = \underline{s}^3$. For $r \geq 4$, we split the largest coalition in π^{r-1}, which corresponds to the largest coalition of those with the smallest index in π^{r-1}. In every step, the coalition that is divided ($\hat{S}$) is split in such a way that $s_a^r = \left|\hat{S}\right| - \underline{s}^r$ and $s_b^r = \underline{s}^r$ (see Figure 1).

[Insert Figure 1.]

The proof that the sequence M belongs to the set $\mathcal{M}$ when n is large enough is relegated to the Appendix. ■

We denote the set of natural numbers for which the set $\mathcal{M}$ is non-empty by $\mathcal{N}$. According to Lemma 1, for a sequence to exist in $\mathcal{M}$, the number of initial players is crucial. In fact it can be shown that the set $\mathcal{N}$ contains all the numbers higher than or equal to 37.[10] Let us explain why starting with a large number of firms facilitates arriving at a monopoly. Two coalitions must not be very different in size to be willing to merge, but this is a requirement to be fulfilled throughout the entire sequence of mergers. If, at any stage all of the coalitions are too similar, when two of them merge they create a great

[10] One can also check that $\mathcal{N}$ also includes 15, 22, 23, 26, 29 to 31, and 33 to 35.

coalition compared to the others, and the small ones may stop the process by free-riding on the big one. With many players, there is a way of having coalitions whose sizes are balanced enough at every stage.

To highlight the previous argument, consider the case of three firms. In order to reach "the grand coalition", a firm of size 2 has to merge with the a firm of size 1. This process will, however, not be completed because the duopoly is very asymmetric. The firm alone receives higher profits in the duopoly than the third it would obtain from the monopoly profits. Consider now, the case $n = 39$. For the same reason as stated before, a sequence of mergers that leads to a duopoly with a firm of size 26 and another of size 13 will never arrive at "the grand coalition". However, a path yielding a duopoly with firms of sizes 21 and 18 will eventually end up as a monopoly. In the previous step (a triopoly), two firms of sizes 10 and 11, for instance, are not too small and so they prefer to reach "the grand coalition" than to stay in the triopoly.

The next proposition states the main result of this paper by combining Proposition 3 and Lemma 1. It shows that if $n \in \mathcal{N}$ and δ is large enough, then the firms will enter into a sequential process of forming coalitions that will end up in the creation of a monopoly.

Proposition 4 *If $n \in \mathcal{N}$, there exists a $\bar{\delta} < 1$, such that $\forall \delta \geq \bar{\delta}$, the final outcome of any SPE of the process of sequential coalition-formation is "the grand coalition".*

Proof. The proof is straightforward, following Proposition 3 and Lemma 1 ■

In our coalition-formation game, only the extreme coalition structures, "all singletons" or "the grand coalition", can be equilibrium outcomes. Proposition 4 shows that, when the number of initial players is high enough and these players are patient enough, the efficient outcome is the only equilibrium outcome. That is, under these two conditions, the possibility of establishing bilateral agreements sequentially makes the firms merge in such a way that they end up being a monopoly.[11] This result is in contradiction with previous results of merger games. Indeed, as we have discussed in the Introduction, "the grand coalition" is often not an equilibrium (or stable) outcome and, when it is, it is not the only one.

[11] At this point it is worth recalling that, in this model, there is no entry of new firms. If entry was allowed, this would crucially alter the incentives of the agents to merge. For an analysis of mergers under entry see Pesendorfer (2004).

For the proof of the result, in particular for the proof of the non-emptiness of the set of sequences of coalition structures $\mathcal{M}$, we explicitly construct an algorithm that provides a particular sequence of coalition structures leading to monopoly when the number of firms is high enough. The construction is made easy by the property that, in Cournot oligopolies with linear demand, the profits of a firm only depend on the number of coalitions in the structure and the size of the coalition the firm belongs to. In addition to this, we can ensure that the SPE path will follow a sequence in $\mathcal{M}$ due to the property that players will only think about making bilateral agreements if they expect to end up all together. That is, at the coalition structures different from the grand coalition (and "all singleton"), at least one player in a non-singleton coalition is worse-off than at the beginning of the process. Although the intuition behind the result is strong, extending the algorithm to accommodate more general profit functions seems a difficult task.

Contrary to Cournot oligopoly games, there are other economic situations where agents have short-term incentives to merge. This is the case, for instance, when agents may agree on the level of public good provision or may reach trade agreements. In all these cases, the coalition formation process is fostered which can result in the formation of the grand coalition, even for small number of initial agents. To illustrate the previous argument, take the public good game of Ray and Vohra (2001). In this game, each player can be interpreted as a region that invests in pollution abatement that benefits all the regions. With a quadratic cost function of production of the public good, in the partition $(S_1, ..., S_r)$, player i in coalition S_i has a payoff given by:

$$V_i(\pi) = \frac{1}{2}s_i^2 + \sum_{S_j \in \pi \setminus S_i} s_j^2.$$

This is a situation with positive externalities, as the Cournot model we are considering in this paper. However, in contrast with our model, players (as well as coalitions of similar sizes) have an incentive to merge even if they do not expect further mergers.[12] In this game, it can be checked that the grand coalition is the only SPE outcome when the initial number of players is small, and players are sufficiently patient. Figure 2 illustrates this game for $n = 5$.

[Insert Figure 2]

[12] Hence, all singletons is never an SPE final outcome.

Each box in Figure 2 corresponds to each possible step of the game. It includes the vector of sizes of the coalitions in the partition that exists at this step, as well as the payoff of each player in each coalition (for instance in the second box, $v_2(\pi) = 5$ indicates that each player's payoff in a coalition of size 2 when the other players are singletons is 5). The arrows indicate the possible SPE paths.

It is easy to check that the only possible SPE outcome apart from monopoly, is a duopoly of the form $(4, 1)$ but, can such a duopoly be a SPE outcome? To reach this outcome, the only possible path would be one of the form $(a) \Longrightarrow (b) \Longrightarrow (d) \Longrightarrow (f)$.[13] However, we claim that this cannot be an SPE path because if the firms in the second two-player coalition in box (d) anticipate that they will end-up in a four-player coalition (in box (f)), they will never merge. Indeed, any of these two firms and the first two-player coalition in box (b) have incentives to form a three-player coalition and move to box (c), since they know that from this point on, the final outcome will certainly be the grand coalition.

5 Comments and Extensions

In this paper, we have shown that when the initial number of firms is large enough and they are forward-looking, a sequential process of bilateral agreements will lead to the creation of a monopoly ("the grand coalition"). In this section, we discuss the main ingredients of our model by proposing several other processes of gradual agreements and by comparing our framework with that of Bloch (1996) and Ray and Vohra (1999). We introduce modifications that affect the timing of the coalition formation and the production stages, the graduality of the process, the bilateral nature of the agreements, the protocol that chooses the candidates for mergers and the exogenous sharing rule.

Possibly, the closest papers to ours are Bloch (1996) and Ray and Vohra (1999), who also analyze an infinite-horizon sequential game. In their model, payoffs are only realized after the coalitions have been formed. In the coalition formation game previous to the production, the first agent, according to a rule of order, makes an offer to other agents

[13] It is clear that the alternative path that involves going from box (c) to box (f) cannot be part of an equilibrium.

to join him in a coalition. If all members accept the offer, the partnership is formed and the partners in the coalition leave the game. The first agent in the set of remaining players then makes a partnership proposal, and the game continues following the same rule until all of the players have left the game. If someone rejects, he will then have to make the next proposal. This model applies to general games. For the linear Cournot game, Bloch (1996) proves that, when players are ex-ante symmetric and the discount rate is high enough, the coalition structures that result from the stationary symmetric perfect equilibria in pure strategies contain a coalition whose size is about 80% of the market, while the other firms remain isolated. Hence, "the grand coalition" is not formed.

There are three main differences between the game proposed by Bloch (1996), and by Ray and Vohra (1999) and our proposal. First, in their analyses, if the offer is accepted, the coalition leaves the game, while in our approach the coalitions do not leave the game once they are formed. This is a reasonable assumption and it is essential for our analysis, as it allows coalitions to be formed gradually over time. In fact, the bilateral process of coalition formation would make no sense if the pairs formed in one stage could not, later on, join with others.

The second difference is that, in the previous papers, production takes place only after the coalitions have been formed. And thirdly, a player may make an offer to any set of partners. We now discuss the implications of the last two differences, as well as those of other potential modifications of the analysis performed.

5.1 Timing of the Production Stage

Our results continue to hold if we consider a game similar to the one described in Section 2 but where production takes place and profits are realized *only* after the whole process of coalition formation has ended. This is the framework that most models in the literature have considered.[14]

To adapt our model, consider the same protocol for coalition-formation as in Section 2 with the following difference: At $t.3$ (for any $t \geq 0$) all firms are asked sequentially whether to continue with the coalition-formation process (Y), or to move to the production stage

[14] Not only Bloch (1996) and Ray and Vohra (1999) but also, for instance, Montero (1999).

(N); if all the firms say Y then they go to $(t+1).1$ (they do not obtain profits at t); otherwise, each player $k \in N$ obtains profits $V_k(\pi_t)$ for any period $\tau \geq t$. That is, the formation of coalitions continues only if all the firms agree to it, any firm can decide to end the coalition formation stage and move to the production stage if it wishes. We refer to this game as "the bilateral coalition formation game with profits at the end".

Proposition 5 *If $n \in \mathcal{N}$, there exists a $\bar{\delta} < 1$, such that $\forall \delta \geq \bar{\delta}$, the final outcome of any SPE of the bilateral coalition formation game with profits at the end in a linear Cournot competition model is "the grand coalition".*

Proof. The proof goes along the same lines as the proof of Proposition 3. In this case, it is possible to make a more precise statement in part (b) of that Proposition: Coalitions S_a and S_b in π^r accept the merger if they are selected by the mechanism and if such decision minimizes the expected losses from discounting. This is the case since all the firms share the same objective when they decide whether to merge or not. Finally, the players' strategies when the existing coalition structure is π^r, specify that they will decide to move to the production stage if and only if π^r does not belong to any sequence of coalitions in $\mathcal{M}^r$. ∎

5.2 Protocol

The results in this paper are also robust to other protocols for the choice of the coalitions that, at each period, decide whether to merge or not. First, it is clear that, if a deterministic protocol selects the identity of the two coalitions that can merge, the results still hold, provided that the protocol is exhaustive in the set of possible couples for each coalition structure (i.e., all the possible pairs of coalitions in any coalition structure are called by the protocol at some moment).

The analysis can equally be developed in scenarios where the protocol selects one of the coalitions, which then has the possibility to offer a merger to any other coalition. To be more precise, consider the following protocol: At each period $t \geq 0$ where $\pi_{t-1} \neq N$ (with the obvious small differences when $t = 0$) :

- $t.1$ A coalition S_a in π_{t-1} is randomly selected. All of the coalitions in π_{t-1} have the same probability of being selected.

$t.2$ A (randomly chosen) firm in coalition S_a selects a coalition $S_b \in \pi_{t-1} \backslash S_a$.

$t.3$ Firms in coalitions S_a and S_b sequentially decide whether to merge. The merger is carried-out if all of the firms in S_a and S_b agree to it.

The coalition structure at time t is either $\pi_t = (\pi_{t-1} \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if S_a and S_b have merged or $\pi_t = \pi_{t-1}$ if they have not.

$t.4$ Each player $k \in N$ obtains profits $V_k(\pi_t)$ at time t.

We will refer to coalition S_a as a "leader".

Proposition 6 *If $n \in \mathcal{N}$, there exists a $\bar{\delta} < 1$, such that $\forall \delta \geq \bar{\delta}$, the final outcome of any SPE of the bilateral coalition formation game with with a leader in a linear Cournot competition model is "the grand coalition".*

Proof. It is very similar to the proof for propositions 3 and 4. The main difference is that, under the present protocol, the statements (a) and (b) in Proposition 3 are: (a) If the coalition S_a is chosen by the mechanism, no proposed merger will be accepted if $\min\{s_a, s_b\} < \underline{s}^r$ or if $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ does not belong to any sequence of coalitions in $\mathcal{M}^{r-1}$ for all $S_b \in \pi_{t-1} \backslash S_a$. (b) If π^r belongs to some sequence of coalitions in $\mathcal{M}^r$, there are two coalitions S_a and S_b in π^r, such that, if S_a is selected by the mechanism, the chosen firm in S_a will select the coalition S_b, and the firms in S_a and S_b will accept the merger. ■

5.3 Multilateral Agreements

The bilateral nature of the agreements is a key feature of our analysis. The study of the outcome of a coalition formation game where players have the possibility of forming coalitions of any size in a single round is not the subject of this paper. However, we suspect that the results obtained in this paper do not extend to the multilateral case, where forming coalitions of any size is costless. In particular, suppose that a protocol selects (as in Subsection 5.2) one of the coalitions which then has the possibility to offer a merger to any set of coalitions. Contrary to the case with bilateral mergers, a sufficient condition for the emergence of monopoly as an equilibrium outcome is that the discount factor is low enough (impatient players).[15] The reason is that, in this case, the players' strategic capacity to induce a more profitable coalition structure through unions that trigger other unions is lost, since the players are essentially short-term maximizers. Similarly, it is easy to check that a monopoly is the unique equilibrium outcome in the linear Cournot setting when the number of firms is small. Indeed, when there are less than five firms, at equilibrium, any proposer at the first round will always propose the formation of the grand coalition, and the other firms will accept the proposal.

5.4 Endogenous Sharing Rule

We have chosen to study the outcomes of a coalition formation procedure when the payoffs of the players, at any moment, depend exclusively on the coalition structure prevailing at that moment. Indeed, we have assumed an exogenous equal-sharing rule that is independent of the history. We could also study the outcomes of a similar procedure allowing for endogenous sharing rules that would depend on the bargaining power of the coalitions at the moment when they have to decide whether to merge or not. Although it may seem at first sight that allowing for endogenous sharing rules should help the formation of coalitions, since it allows for compensating players in any way, this possibility makes forming coalitions more difficult. The reason for this is that merging at an early stage lowers the bargaining power of the players in the continuation of the game. Hence, although the final mergers are easier to implement, the players have no incentive to start the process. The bilateral and sequential nature of the coalition formation process, avoids benefiting from the greater flexibility that, at least in principle, offers an endogenous sharing rule. This is in contrast with the multilateral process analyzed by Ray and Vohra (1999). They show that, in symmetric games, endogeneizing the sharing rule gives the same results as the exogeneous sharing rule model studied by Bloch (1996).

To be more precise, consider the following variation of our coalition formation game: Of the two coalitions that have to decide whether to merge or not, one of them is chosen randomly and must make a proposal to the other concerning the sharing of the surplus.

[15] A general model of coalition formation with multilateral agreements and endogenous sharing rule (contrary to our assumption of fixed sharing rule) is provided by Gomes (2003), where he proves that there is always immediate merger to monopoly in stationary subgame perfect equilibrium.

We refer to this variant as "the bilateral coalition formation game with endogeneous sharing rule".

Proposition 7 *The SPE final outcome of the bilateral coalition formation game with endogeneous sharing rule in a linear Cournot competition model is that all the players remain as singletons.*

Proof. By an argument similar to the one leading to Proposition 2, it is easy to check that the process of coalition-formation will only begin if it leads to monopoly. We prove, by contradiction, that the SPE final outcome cannot lead to monopoly. First, if a duopoly is formed, the two coalitions will have incentives to merge, each eventually obtaining an expected payoff of 1/8, since they share the benefits of the monopoly, i.e., 1/4 (in expected terms, the possible surplus will be shared equally between the two coalitions). Consider, now, the moment where (all the players have been merging continually and) the structure in the market is of three coalitions. The sum of the payoffs of the firms in each of the coalitions is 1/16. But this implies that no two coalitions, say S_1 and S_2, would have any incentive to merge: Firms in S_1 (as well as the firms in S_2) would obtain profits of 1/18 in the duopoly and end up with the same profits as in their initial situation, that is 1/16.[16] Therefore, the monopoly can not be reached and the only possible SPE final outcome is that all players remain as singletons. ■

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6 Appendix

Proof of Lemma 1. We prove that the sequence M constructed in the proof of Lemma 1 belongs to $\mathcal{M}$ when n is large enough. We do the proof by induction over r. For each r, we provide conditions over n under which the two "candidate" coalitions S_a^r and S_b^r satisfy $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$. Note that, since the minimum size of a coalition is 1, when $\underline{s}^r = 1$, the previous condition imposes no restriction on the size of the coalitions. This is the case if

$$int\left\{\frac{4n}{(r+1)^2}\right\} = 0, \text{ i.e., } r > r_{\max}(n) \equiv \sqrt{4n} - 1.$$

Therefore, we concentrate on $r \in [2, r_{\max}(n)]$.

$(r = 2)$ $\min\{s_a^2, s_b^2\} \geq \underline{s}^2$ holds if and only if $s_a^2 \geq \underline{s}^2$, that is $s^1 = n \geq 2\underline{s}^2$, i.e.,

$$n \geq 2\left(int\left\{\frac{4n}{9}\right\} + 1\right).$$

A sufficient condition for the above inequality to hold is $n \geq 18$.

$(r = 3)$ Since $s_b^3 = \underline{s}^3$, to check if the condition $\min\{s_a^3, s_b^3\} \geq \underline{s}^3$ is satisfied it suffices to check that $s_a^3 = s_a^2 - \underline{s}^3 \geq \underline{s}^3$, or equivalently that:

$$s_a^2 \geq 2\underline{s}^3, \text{ i.e., } n - \left(int\left\{\frac{4n}{9}\right\} + 1\right) \geq 2\left(int\left\{\frac{n}{4}\right\} + 1\right).$$

It can be shown that the above inequality always holds if $n \geq 37$.

For any $r \geq 4$, the sizes of the coalitions S_a^r and S_b^r sum up to the size of the largest coalition in π^{r-1} (denote it $\hat{S}^{r-1}$). Since $s_b^r = \underline{s}^r$, to check condition $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$, it suffices to verify that $s_a^r = \left|\hat{S}^{r-1}\right| - \underline{s}^r \geq \underline{s}^r$. If we take into account that the size of the biggest coalition in π^{r-1} has to be at least $\frac{n}{r-1}$, we have that:

$$\min\{s_a^r, s_b^r\} \geq \underline{s}^r \text{ if } \frac{n}{r-1} \geq 2\left(int\left\{\frac{4n}{(r+1)^2}\right\} + 1\right).$$

This inequality holds if:

$$f(r) \equiv \frac{2(r+1)^2(r-1)}{(r-3)^2} \leq n.$$

Since the function $f(r)$ is first decreasing (from $r = 4$ on) and then increasing, the previous inequality holds for all relevant r if it is satisfied at the extreme values $r = 4$ and $r = r_{\max}(n)$. It can be shown that this happens as long as n is large enough. In particular, this sufficient condition holds for every $n \geq 150$.

We now provide an algorithm to construct a sequence in $\mathcal{M}$. This algorithm allows to check that $\mathcal{M} \neq \emptyset$ for $n \in \{15, 22, 23, 29, 30, 31, 33, 34, 35\}$, and for every $n \geq 37$. We also include an example of the working of the algorithm when $n = 15$.

Algorithm:

The algorithm creates a sequence of mergers. If the algorithm does not stop in any iteration $t' \leq r_{\max} = 2\sqrt{n} - 1$, then the set $\mathcal{M}$ is not empty and the sequence created with the algorithm can be continued until it reaches π^n such that $M \in \mathcal{M}$.

For any initial number of firms n, let v^r be the vector that represents the sizes of the coalitions that will be formed at the step r of the algorithm. The vector v^r will have r components that sum up to n. We start from $v^1 = (n)$ and define $v_a^1 = n$.

For any $r = 1, 2, ..., n$, let $\underline{s}^r \equiv int\left\{\frac{4n}{(r+1)^2}\right\} + 1$.

At any iteration $t \geq 1$:

1. From the vector v^t, take the element v_a^t and split it into two numbers v_h^{t+1} and v_k^{t+1} such that:

$$\begin{aligned} v_h^{t+1} &= v_a^t - \underline{s}^{t+1} \\ v_k^{t+1} &= \underline{s}^{t+1}. \end{aligned}$$

2. Construct v^{t+1} as a vector with $t+1$ components: All the components v_i^t for $i \neq a$, v_h^{t+1}, and v_k^{t+1}.

3. Compute $v_a^{t+1} = \max\limits_{i=1,\ldots,r} v_i^{t+1}$ and $v_b^{t+1} = \min\limits_{i=1,\ldots,r} v_i^{t+1}$.

4. If $v_b^{t+1} < \underline{s}^{t+1}$, then stop and the algorithm is unable to produce a sequence in the set $\mathcal{M}$. If $v_b^{t+1} \geq \underline{s}^{t+1}$ and $t + 2 < r_{\max}$ then move to iteration $t + 1$. Finally, if

$v_b^{t+1} \geq \underline{s}^{t+1}$ and $t+2 \geq r_{\max}$ then the algorithm is over (since $\underline{s}^{t+2} = 1$) and the set $\mathcal{M}$ is non-empty.

Example: $n = 15$. $r_{\max} = 6.745$, hence the algorithm has, at most 5 rounds. $\underline{s}^2 = 7$, $\underline{s}^3 = 4$, $\underline{s}^4 = 3$, $\underline{s}^5 = 2$, $\underline{s}^6 = 2$, $\underline{s}^t = 1$, for all $t \geq 7$.

$t = 1$: From $v^1 = (15)$, define $v_h^2 = 15 - 7 = 8$ and $v_l^2 = 7$.

Then $v^2 = (8, 7)$, and $v_a^2 = 8$, $v_b^2 = 7 = \underline{s}^2$ and we move to iteration 2.

$t = 2$: From $v^2 = (8, 7)$, define $v_h^3 = 8 - 4 = 4$ and $v_l^3 = 4$.

Then $v^3 = (4, 4, 7)$, and $v_a^3 = 7$, $v_b^3 = 4 = \underline{s}^3$ and we move to iteration 3.

$t = 3$: From $v^3 = (4, 4, 7)$, define $v_h^4 = 7 - 3 = 4$ and $v_l^4 = 3$.

Then $v^4 = (4, 4, 4, 3)$, and $v_a^4 = 4$, $v_b^4 = 3 = \underline{s}^4$ and we move to iteration 4.

$t = 4$: From $v^4 = (4, 4, 4, 3)$, define $v_h^5 = 4 - 2 = 2$ and $v_l^5 = 2$.

Then $v^5 = (2, 2, 4, 4, 3)$, and $v_a^5 = 4$, $v_b^5 = 2 = \underline{s}^5$ and we move to iteration 5.

$t = 5$: From $v^5 = (2, 2, 4, 4, 3)$, define $v_h^6 = 4 - 2 = 2$ and $v_l^6 = 2$.

Then $v^6 = (2, 2, 2, 2, 4, 3)$, and $v_a^6 = 4$, $v_b^6 = 2 = \underline{s}^6$ and we stop the algorithm.

The outcome of the algorithm indicates how to arrive to the grand coalition when $n = 15$. Any path that leads to a partition with 6 coalitions of sizes $(2, 2, 2, 2, 4, 3)$ can be part of a SPE. From this partition on, the coalitions can follow the inverse path of mergers that the algorithm proposes.

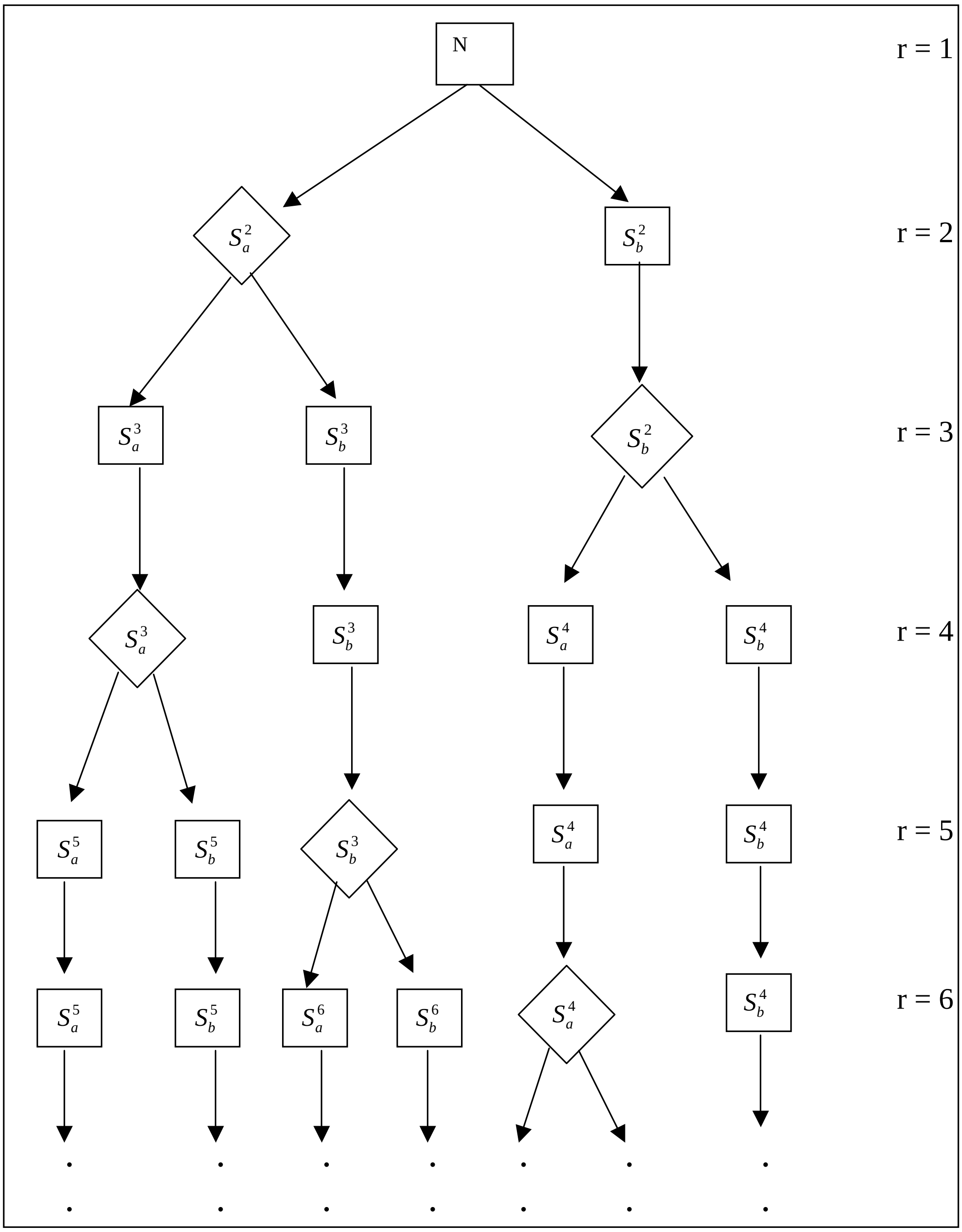

Figure 1: Outline of the Sequence of Moves

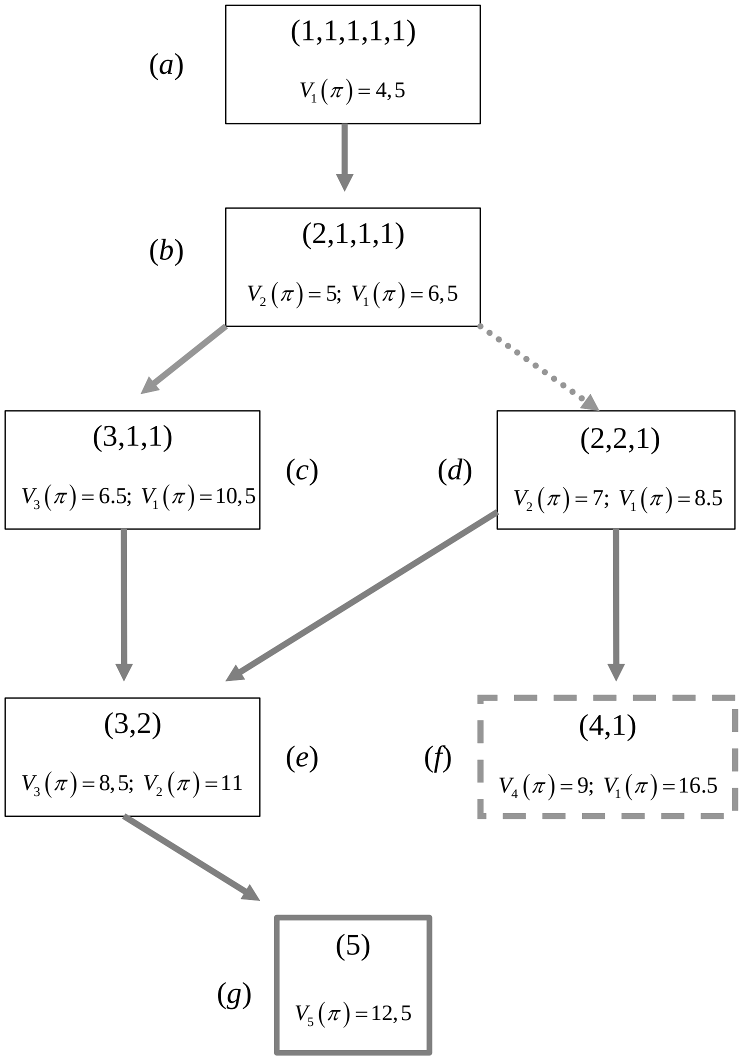

Figure 1: Figure 2: Public Good game with $n = 5$.