Nonlinearities or outliers in real exchange rates? ☆

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Abstract

Long-lasting misalignments in the real exchange rates are sometimes explained by the presence of a nonlinear adjustment process towards the long-run equilibrium. However, while it is possible that evidence of nonlinearity exists for some real exchange rates, outliers and nonlinearity may easily be confused. This paper uses robust methods to examine and compare the behaviour of Smooth Transition Autoregressive [STAR] models for the real exchange rates of 14 countries. The results show that the evidence for nonlinearity is reduced when considering outliers. Nonlinearity is also more common among developing economies.

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1. Introduction

The purchasing power parity (PPP) is a simple concept of long-run or equilibrium exchange rate in the literature that implies a constant equilibrium exchange rate.[1] In other words, the PPP maintains that there exists a deterministic steady state level of real exchange rate (the equilibrium real exchange rate) towards which the current value converges in the long run. Yet, even though its popularity, many studies that test PPP during the recent float cannot reject the random walk hypothesis for the real exchange rates of the currencies of all the major industrialized countries against one another, suggesting that deviations from PPP are permanent (for example, Engle, 2000).

Therefore, failure to provide evidence for the PPP has motivated several justifications. For instance, recently, a number of studies (Michael et al., 1997; Sarantis, 1999; Taylor et al., 2001) have explained the persistent misalignments in the real exchange rates during the post-Bretton Woods period by the presence of a nonlinear adjustment process

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[1] Other concepts of equilibrium exchange rates, as the Fundamental Equilibrium Exchange Rate (FEER) by Williamson (1985), the Behavioral Equilibrium Exchange Rate (BEER) by Clark and MacDonald (1998) and the Natural Real Exchange Rate (NATREX), introduced by Stein (1994), have also been suggested in the literature.

towards the PPP.[2] While the most common causes for nonlinearity are the existence of transactions costs, mainly due to the cost of transportation (Dumas, 1992), other arguments, such as the heterogeneity of opinions in the foreign exchange market concerning the equilibrium level of the nominal exchange rate (see Taylor and Allen, 1992; Kilian and Taylor, 2003), the speculative attacks on currencies (Flood and Marion, 1998), the presence of target zones (Krugman, 1991), noise traders (De Long et al., 1990) or the heterogeneity of the interventions of central banks (Dominguez, 1998) have also been advanced.[3]

In general terms, the idea of nonlinearities in the adjustment process implies that there exists a band for the real exchange rate (RER) within which the marginal cost of arbitrage exceeds the marginal benefit. The thresholds then not only reflect shipping costs and trade barriers *per* se, but also they are the result of the sunk cost of international arbitrage and the resulting tendency for traders to wait for sufficiently large arbitrage opportunities to open up before entering the market. That is, the profits from commodity arbitrage, which is generally thought to be the ultimate force behind maintaining the PPP, do not make up for the cost involved in the necessary transactions for small deviations from the equilibrium real exchange rate. This implies the existence of a band around the equilibrium rate in which there is no tendency of the real exchange rate to revert to its equilibrium value. Outside this band, commodity arbitrage becomes profitable, forcing the real exchange rate towards the band.

To capture the phase-dependent properties, most of the empirical studies on real exchange rates specify the switch between regimes as a function of past values of the real exchange rate by means of smooth transition autoregressive (STAR) models, as suggested by Terasvirta and Anderson (1992), Granger and Terasvirta (1993) and Terasvirta (1994). Based on this approaches, these studies conclude that the RER in industrialized countries behave more like a unit root process the closer they are to long-run equilibrium and, conversely, become more mean reverting the further they are from it.[4]

However, even if STAR characterizations may be useful to explain deviations from the Purchasing Power Parity (PPP) in some cases, it is possible that this apparent nonlinearity is due to outlier observations in the series.[5] Indeed, many linear economic time series are contaminated by occasional outliers. In particular, given the changes in exchange rate regimes, financial or political crisis and other sharp disturbances, real exchange rates are subject to substantial variations. These turbulent histories appear as structural shifts or as outliers in the real exchange rates. In the presence of some aberrant observations, nonlinearity tests may incorrectly point towards nonlinear structures.

Yet, it is also possible that the nonlinear properties of the series are reflected in a few observations. One may be tempted to view these nonlinear data points as aberrant observations and remove them dramatically, thus destroying intrinsic nonlinearity (van Dijk, 1999). Indeed, it may be the case that the presence of these outliers allows one to detect the way and speed of adjustment towards equilibrium. Therefore, eliminating outliers can sometimes lead to the omission of valuable information about the equilibrium.

In this context, the aim of this paper is to analyze whether the characteristics of the adjustment process of the real effective exchange rates for a group of 14 countries can be explained by STAR-type models and, if so, to investigate non-linearities in real effective exchange rates. Our study differs from most of the earlier literature on nonlinear exchange rate modeling at least in two important ways. First, in contrast to previous investigations, we adopt a robust test for STAR type nonlinearity. The main advantage of this robust procedure is that it automatically guards the test against outliers and does not require a priori knowledge concerning their presence and timing (Escribano et al., 1998).[6]

[2] While there are various ways to think about equilibrium exchange rates, our paper centers on the most popular concept of equilibrium in applied exchange rate analysis which is the PPP. The interest for this concept of equilibrium is that its validity as an exchange rate benchmark is still a subject of heated controversy. Yet, it can also be the case that the concept of equilibrium is richer than PPP. For an application of nonlinear adjustment of European exchange rates using a BEER approach see Dufrénot et al. (2006).

[3] It can also be the case that persistent misalignments of the exchange rates from their equilibrium value (i.e., the Purchasing Power Parity or an equilibrium value according to the fundamentals) are due to the long-memory property of the adjustment process towards equilibrium. In this case it would be more natural to study the RER misalignment using fractional cointegration models (see, for example, Dufrénot et al., 2006). This is a relevant point since it is well known that long-memory and nonlinearity can easily be confused.

[4] See Michael et al. (1997), Sarantis (1999), Taylor et al. (2001) who investigate nonlinear adjustment dynamics of exchange rates in major industrial countries.

[5] Although it is not a formal definition, an outlier can be described as an observation (or a subset of observations) which appears to be inconsistent with the remaining of that set of data (Barnett and Lewis, 1994).

[6] Several papers consider testing for unit roots in the presence of structural changes and outliers (for instance, Hegwood and Papell, 1998; Lothian and Taylor, 1996). They argue that shifts in the intercept and/or slope of the trend function of a stationary time series biases standard unit-root tests toward non rejection. However, to our knowledge, no previous investigation exists that tests for a nonlinear mean reversion in real exchange rates in the presence of outliers.

The second advantage of our study is that we not only investigate nonlinearities for a group of industrialized countries, but we also include developing countries, an area that has been largely unexplored.

Our results show that apparent nonlinearity is due sometimes to just some aberrant observations that, once taken into account, considerably reduce the presence of nonlinear features in the data. We also provide evidence that nonlinearity is most common among developing countries, where the transition from one regime to the other is faster than in advanced countries.

The rest of the paper is organized as follows. In the next section we describe the smooth transition autoregressive models and we discuss how these models can capture real exchange rate dynamics. In Section 2 we consider the implications of outliers in the series. Section 3 discusses the methodology. In Section 4 we give the empirical results. Finally, we conclude in Section 5.

2. Nonlinear adjustment of real exchange rates

Formal tests for evidence of PPP as a long-run phenomenon have often been based on an empirical examination of the RER which, in its logarithmic form labeled Q, may be expressed as:

$$q_t = e_t - p_t + p_t^*, \tag{1}$$

where e_t is the logarithm of the nominal exchange rate (domestic price of foreign currency), and p_t and p_t^* denote the logarithms of the domestic and foreign price levels, respectively. If the RER is to settle down at any level at all, including a level consistent with PPP, q_t must display mean reversion. In contrast, if the real exchange rate is the realization of a unit-root process, then although changes in it may be predictable to some extent, the variable may still never settle down at any particular level, even in the long-run.

Hence, in the procedures conventionally applied to test for long-run PPP using Eq. (1)) the null hypothesis is usually that the process generating the real exchange rate series has a unit root, while the alternative hypothesis is that all of the roots of the process lie outside the unit circle. Thus, the hypothesis in the conventional framework assumes a linear autoregressive process for the real exchange rate, implying that adjustment is both continuous and of constant speed, regardless of the deviations from PPP (Taylor et al., 2001).

However, it has been suggested that deviations from PPP may follow a nonlinear process that is mean reverting, with the speed of adjustment toward equilibrium varying directly with the extent of the deviation from PPP. That is, there is a band in which, for small deviations from the equilibrium, there is no tendency of the real exchange rate to revert to its equilibrium value. Outside this band, commodity arbitrage becomes profitable, forcing the real exchange rate towards the band.[7] This implies that deviations from PPP last for a very long time although they do not follow a random walk.

Several models have been suggested to capture the nonlinear nature of the adjustment process of the RER. For instance, threshold autoregressive (TAR) models, as proposed by Tong (1990), and smooth transition autoregressive (STAR) models suggested by Granger and Terasvirta (1993) and Terasvirta (1994) have been advanced. While in the first kind of models the jump to mean-reverting behaviour is sudden, the empirical evidence has suggested that smooth, as in STAR models, rather than discrete adjustment, as in the TAR model, may be more appropriate.

The justification is as follows. In foreign exchange markets with a large number of investors, each switching at different times (probably due to heterogeneous beliefs, varying learning speeds and different investment horizons), a smooth transition between regimes seems more appropriate.[8] Indeed, if an aggregated process is observed, regime changes may be smooth rather than discrete as long as heterogeneous economic agents do not act simultaneously (which is unlikely) even if they individually make dichotomous decisions (Baum et al., 2001).[9]

Indeed, STAR models are a general class of state-dependent nonlinear time series models capable of accounting for deterministic changes in parameters over time, in conjunction with regime switching behavior. A STAR model of order p for the (log) real exchange rate may be written as:

$$q_t = \alpha + \sum_{j=1}^{P} \pi_j q_{t-j} + \left(\alpha^* + \sum_{j=1}^{P} \pi_j^* q_{t-j} \right) F(s_{t-d}; \gamma, c) + \varepsilon_t, \tag{2}$$

[7] See, for example, Benninga and Protopapadakis (1988), Dumas (1992), Sercu et al. (1995), Michael et al. (1997) or Dufrénot et al. (2008).

[8] See Sarantis (1999).

[9] Discrete switching may be more appropriate when considering the effects of arbitrage on disaggregated goods prices (Sarno and Taylor, 2002).

where α and α* are regime constants, qt is assumed to be a stationary ergodic process, ε_t is $\sim iid(0, \sigma^2)$, and F is a transition function which is bounded by zero and one. This function is governed by the parameters γ, which is the speed of transition from one regime to the other, s_{t-d} which is the transition variable (with d the delay parameter) and c, which is the threshold.[10] In these models, nonlinearities arise through conditioning on lagged real exchange rates. The adjustment takes place in every period, but the speed varies with the extent of the deviation from parity.

In Eq. (2) F can be a first-order logistic function, in which case the model is called a logistic STAR model (LSTAR):

$$F(s_{t-d};\gamma,c) = \{1+\exp[-\gamma(s_{t-d}-c)]\}^{-1}, \gamma>0. \tag{3}$$

Here, in Eq. (3) the parameter c can be interpreted as the threshold between the two regimes corresponding to $F(s_{t-d};\ \gamma,\ c)=0$ and $F(s_{t-d};\ \gamma,\ c)=\mathbf{1}$, in the sense that the logistic function changes monotonically from **0** to **1**. The parameter γ determines the smoothness of the change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other. In the LSTAR model, the two regimes are associated with small and large values of the transition variable s_{t-d} relative to the threshold c.

It has been argued that the logistic function (3) is not the most plausible for the modeling of the real exchange rate series, since it is not symmetric for positive and negative deviations from PPP.[11] The need for symmetry leads to the exponential STAR or ESTAR model which can be expressed as:

$$F(s_{t-d};\gamma,c) = 1-\exp\left[-\gamma(s_{t-d}-c)^2\right], \gamma>0. \tag{4}$$

The ESTAR model suggests that the two regimes have rather similar dynamics, while the transition period can have different dynamics. The transition function is U-shaped and symmetric around c meaning that local dynamics are the same for high and for low values of the real exchange rate series.

In the LSTAR (or ESTAR) model, the lower (middle) regime corresponds to $s_{t-d}=c$ when $F=0$ and thus the model (2) becomes a linear AR(p) model. In contrast, the upper (outer) regime for the LSTAR (ESTAR) models corresponds to $s_{t-d}=\pm\infty$ when $F=\mathbf{1}$ and Eq. (2) becomes a different type of AR(p) model:

A drawback of the exponential function (4) is that for either $\gamma\to 0$ or $\gamma\to\infty$, the function collapses to a constant (equal to 0 or **1**, respectively). Hence, the model becomes linear in both cases. This can be solved by using the quadratic logistic function, LSTAR_2:

$$F(q_{t-d};\gamma,c) = \{1+\exp[-\gamma(s_{t-d}-c_1)(s_{t-d}-c_2)]\}^{-1}, c_1\le c_2; \gamma>0. \tag{5}$$

where now $c=(c_1, c_2)'$. In this case, if $\gamma\to 0$, the model becomes linear. If $\gamma\to\infty$, the function $F(s_{t-d};\ \gamma,\ c)=\mathbf{1}$ for $s_{t-d}<c_1$ and $s_{t-d}<c_2$ and equal to zero in between.[12]

Finally, since the STAR models require the dependent variable to be stationary, Eq. (2) can be reparameterized as follows:

$$\Delta q_t = k+\lambda q_{t-1}+\sum\nolimits_{j=1}^{p-1}\phi_j\left[\Delta q_{t-j}\right]+\left(k^*+\lambda^* q_{t-1}+\sum\nolimits_{j=1}^{p-1}\phi_j^*\left[\Delta q_{t-j}\right]\right)F(s_{t-d};\gamma,c)+\varepsilon_t, \tag{6}$$

where k and k^* are regime constants, $\Delta q_t=q_{t-j}-q_{t-j-1}$, $\phi_j=1-\pi_j$ and $\phi_j^*=1-\pi_j^*$.In the previous equation, the crucial parameters are λ and λ^*. The effects of transaction costs suggest that the larger the deviations from PPP the stronger will be the tendency to move back to equilibrium. Therefore, while $\lambda\geq\mathbf{O}$ is admissible (i.e., a unit root for small deviations), we must have $(\lambda+\lambda^*)<0$ (i.e. mean reversion for large deviations from equilibrium).

[10] The transition variable s_{t-d}, can be either lags of the real effective exchange rate in levels, its first difference, an exogenous variable, a function of lagged endogenous variables or a linear time trend, which gives rise to a model with smoothly changing parameters.

[11] See Michael et al. (1997) and Taylor et al. (2001).

[12] Note that for moderate values of γ, the minimum value taken by the function (5). is not equal to zero.

3. The presence of outliers

Recently, regime switching models and, particularly, smooth transition autoregressive models have been applied to study possible non-linearities in the real exchange rates. Based on these nonlinear models, several empirical studies conclude that the behaviour of the real exchange rates depends nonlinearly on the size of the deviation from purchasing power parity.[13]

Certainly, it is possible that empirical evidence for nonlinearity does exist for many series. However, most of the studies do not consider that many linear economic time series are contaminated by occasional outliers. In this case, tests for STAR nonlinearity would tend to reject the correct null-hypothesis of linearity too often. In other words, the apparent nonlinearity could simply be due to structural breaks or outliers in the series.

Indeed, according to Koop and Potter (2000), models that allow for dynamics which vary over the business cycle in a predictable way are the nonlinear models. Still, another possibility is that apparent departures from linearity are due to unpredictable large shocks which have only temporary effects. Models with this property are the "outliers" models. Since these models have very different consequences for characterising and forecasting the real exchange rate dynamics than nonlinear models, it is important to examine whether certain features of the series are caused by genuine nonlinearity or by some outliers.

Two types of outliers are especially interesting in time series. First, the additive outlier (AO), which gives a one time effect on the level of the time series, since only the current observation is affected. Second, the innovative outlier (IO) implies that a shock at time t also influences future observations through the same dynamics as the linear part of the model. It has been shown (van Dijk et al., 1996; van Dijk, 1999; van Dijk et al., 2000) that the presence of both kinds of outliers may distort the results of testing and estimation. In particular the Ordinary Least Squares estimates of the autoregressive parameters, though consistent, are inefficient in the presence of AOs and are biased towards zero in the presence of 10s.

A significant literature now exists suggesting different outlier detection and correction procedures. Here, we follow a robust procedure as suggested by Martin (1981), van Dijk (1999) and van Dijk et al. (2000) among others. This methodology has the advantage that it is not based on an estimation–detection–correction–estimation procedure and therefore it is not subjective nor time consuming.

4. Methodology

Model selection and estimation is based on the procedure suggested by Terasvirta (1994) for single transition models. First, we must specify a linear AR model. Second, based on this model, we test the null hypothesis of linearity against the alternative of nonlinearity. Third, if linearity is rejected, an appropriate transition variable and the form of the transition function are selected. Fourth, the parameters in the selected transition function are estimated. Finally, the model must be evaluated and modified if necessary.

In what follows, we briefly describe these steps while emphasizing the aspects related to linearity tests in the presence of outliers.

4.1. The linear AR model specification

A linear AR(p) model is selected, with the lag length p chosen (to a maximum value of 12) according to the Sehwarz Information Criterium (SIC). In order to anticipate the structure of the STAR model, we specified an AR(p) model in first differences, including a constant and the first lag of the variable in levels. Thus, the estimated linear models can be characterized as follows:

$$\Delta q_t = \beta_0 + \lambda q_{t-1} + \sum\nolimits_{j=1}^{p-1} \phi_j \left[\Delta q_{j-t}\right] + \varepsilon_t. \tag{7}$$

It must be kept in mind that if *linearity* is rejected, the lag order used in the AR model is not necessarily the appropriate lag order in the alternative STAR model, although usually it gives a good first guess.

[13] See, for example, Obstfeld and Taylor (1997) who investigate the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model or Michael et al. (1997) for an application of ESTAR models in real exchange rates.

4.2. Testing linearity

4.2.1. Standard linearity tests

A crucial step in specifying a STAR model is, of course, to test the linearity of the model against the STAR specification. If the null hypothesis of linearity is not rejected, the conclusion is that the real exchange rate can be adequately described by a linear AR model. Conversely, if the null is rejected, we can suggest that the real exchange rate follows a nonlinear adjustment process towards equilibrium.

For this purpose, the null hypothesis of linearity can be expressed as equality of the autoregressive parameters in the two regimes of the STAR model in Eq. (6). That is, under the null hypothesis, H_0: $\phi_j = \phi_j^*$, whereas the alternative hypothesis is H_1: $\phi_j \neq \phi_j^*$ for at least one $j \in (0,\ldots, p)$.

However, the STAR specification shares with many other nonlinear models the property that it is not identified under the alternative hypothesis of nonlinearity. Indeed, the STAR model contains parameters which are not restricted by the null hypothesis, but which nevertheless are no longer present in the model when the null hypothesis holds true. For example, the null hypothesis H_0: $\phi_j = \phi_j^*$ does not restrict the parameters γ and c in the transition function but, when $\phi_j = \phi_j^*$, the function, $F(s_{t-d}; \gamma, c)$ and therefore γ and c drop out of the equation (van Dijk, 1999).

The consequence of the presence of unidentified nuisance parameters is that the conventional statistical theory is not available for obtaining the asymptotic null distribution of the test statistics. Instead the test statistics tend to have non-standard distribution and, therefore, the critical values have to be determined by simulation.

To deal with this problem, Lukkonen et al. (1988) suggested to replace the transition function $F(s_{t-d}; \gamma, c)$ in Eq. (2) with suitable Taylor approximations. The re-parameterized models (i.e. the auxiliary regressions) are no longer associated with an identification problem, and linearity testing proceeds by using regular Lagrange multiplier (LM) with a standard asymptotic χ^2 distribution under the null hypothesis or their F-tests counterparts.[14]

These tests, the LM_1, the LM_3 and the LM_3^e statistics, are used to test against LSTAR with the delay parameter d assumed unknown. On the other hand, Escribano and Jordá (1999) suggested that linearity might be tested against an ESTAR alternative by replacing the exponential transition function in Eq. (4) with a second-order Taylor approximation around $\gamma = \mathbf{O}$. The resulting LM-type statistic, denoted LM_4, has an asymptotic χ^2 distribution with $4(p+1)$ degrees of freedom under the null hypothesis[15].

4.2.2. Outliers robust tests

van Dijk (1999) and van Dijk et al. (2000) suggest that the LM tests discussed above are sensitive to several kinds of misspecification of the model under the null hypothesis. In particular, they show that in the presence of additive outliers and given the properties of OLS, these tests tend to reject the correct null hypothesis of linearity too often, even asymptotically. As a solution, they propose to use outlier-robust estimation techniques.

Therefore, to avoid the deficiencies of the OLS estimator in the presence of outliers, the autoregressive parameter can be estimated robustly using maximum likelihood (M) type or generalized M (GM) estimators. The class of GM estimators are designed to obtain better estimates in the presence of contamination by giving less weight to influential observations such as outliers.

For example, the OLS estimate of the autoregressive parameter $\phi_{i>1}$ in the AR(l) model $y_t = \phi_1 y_{t-1} + \varepsilon_t$ minimizes the sum of square residuals, which can be characterized by the following first order condition:

$$\sum_{t=1}^{T} x_t(y_t - \phi_1 y_{t-1}) = 0. \tag{8}$$

Instead, a robust estimator for the AR(l) model gives less weight to influential observations such as outliers. Thus, these estimators solve the alternative first order condit···

$$\sum_{t=1}^{T} \omega_r(r_t) x_t(y_t - \phi_1 y_{t-1}) = 0, \tag{9}$$

[14] In small samples, van Dijk (1999) recommends to use F-versions of the LM-tests statistics because these have better size and power properties than the χ^2 variants.

[15] For a detail exposition of the testing procedures used for STAR models, the reader is referred to van Dijk (1999).

where r_t are the standardized residuals, $r_t \equiv (y_t - \phi_1 y_{t-1})/(\sigma_\varepsilon \omega_{y\ t-1}(y_{t-1}))$, with σ_ε a measure of scale of the residuals $\varepsilon_t \equiv y_t - \phi_1 y_{t-1}$ and $\omega_{yt-1}(\bullet)$ and $\omega_r(\bullet)$ are weight functions that are bounded between 0 and **1**. This estimator is a type of weighted least squares estimator, with the weight of the tth observation given by the value of $\omega_r(\bullet)$. The functions $\omega_{yt-1}(\bullet)$ and $\omega_r(\bullet)$ should be chosen such that the tth observation receives a relatively small weight if either the regressor y_{t-1} or the standardized residual becomes unusually large. In this way, outliers and influential observations automatically receive less weight. For the OLS estimator, $\omega_{yt-1}(\bullet) \equiv \mathbf{1}$ and $\omega_r(\bullet) \equiv \mathbf{1}$, such that all observations receive the same weight.

Several forms of bounded functions for $\omega_{yt-1}(\bullet)$ and $\omega_r(\bullet)$ are suggested in the literature. Usually, the weight function $\omega_r(\bullet)$ is specified in terms of a function $\psi(r_t)$ as $\omega_r = \psi(r_t)/r_t$ for $r_t \neq 0$ and $\omega_{yt-1}(0) = \mathbf{1}$. Based on van Dijk (1999) we use the following polynomial function $\psi(r_t)$ for $\omega_r(\bullet)$:

$$\psi(r_t) = \begin{cases} r_t & \text{if } |r_t| \leq c_1, \\ \operatorname{sgn}(r_t) g(|r_t|) & \text{if } |c_1| \leq |r_t| \leq c_2, \\ 0 & \text{if } |r_t| > c_2, \end{cases} \tag{10}$$

where c_1 and c_2 are tuning constants, sgn is the Signum function and $g(|r_t|)$ is a fifth order polynomial such that $\psi(r_t)$ is twice continuously differentiable. Therefore, observations receive a weight equal to **1** if their standardized residuals are within $(-c_1, c_1)$ and a weight equal to zero if the residuals are larger than c_2 in absolute value. The polynomial $g(|r_t|)$ is such that partial weighting occurs in-between. The tuning constants c_1 and c_2 are taken to be the square roots of the 0.99 and 0.999 quantiles of the $\chi^2(1)$ distribution, that is, $c_1 = 2.576$ and $c_2 = 3.291$[16]. Finally, a function that uses the Mahalanobis distance for x_t is a good specification for the weight function $\omega_{yt-1}(\bullet)$ for the regressors.[17]

This outlier robust estimation procedure allows us to modify the LM tests discussed before. In particular a robust test can be derived by using a robust estimator to obtain the model under the null hypothesis. Under conventional assumptions, the test statistics retain their standard limiting χ^2 distributions. Therefore, as shown by van Dijk (1999), these robust tests have good size properties in small samples and in the presence of outliers.

4.3. The choice of the transition variable and transition function

If linearity is rejected, we proceed to select the appropriate transition variable s_{t-d}. For this purpose, even though the LM_3 statistic was developed as a test against LSTAR, it has power against ESTAR and $LSTAR_2$ alternatives as well. Therefore, computing LM_3 for the candidate variables, $s_{1t-d}, \ldots s_{mt-d}$ and selecting the one with the smallest p-value gives a good starting point.

To select the transition function $F(s_{t-d}; \gamma, c)$ we limit our choices to that between the first-order logistic function (LSTAR) or the second-order logistic function ($LSTAR_2$). Therefore, when choosing between these two models for the real effective exchange rate series where linearity is rejected, we use a sequence of nested tests applied to the auxiliary regressions. Given the properties of the auxiliary parameters in the auxiliary regressions, we can select between LSTAR or $LSTAR_2$. Alternatively, Escribano and Jordá (1999) propose a selection method which makes use of LM_4 as a test for general STAR nonlinearity.

4.4. The estimation

Once the correct transition function has been selected, the LSTAR or $LSTAR_2$ specifications are estimated by nonlinear least squares (NLS), which gives consistent and asymptotically normal estimates. For this, finding good starting values for the algorithm is important to ensure that a global minimum is achieved. One way of obtaining them is by a two-dimensional grid search over γ and c. When constructing the grid, note that γ is not a scale free parameter. There fore the exponent of the transition function is standardized by dividing it by the kth sample standard deviation of

[16] Other possible functions, as the Huber, the bisquare or the Student t specification can also be used for $\omega_r(\bullet)$.

[17] For a complete explanation, see Lucas (1996).

the transition variable s_{t-d}, named $\hat{\sigma}_s$, where K=1 for the LSTAR and K=2 for the LSTAR$_2$. The transition function becomes:

$$F(\gamma; s_{t-d}, c) = \left(1 + \exp\left\{\left(-\gamma/\hat{\sigma}_s^k\right)\sum_{k=1}^{K}(s_{t-d} - c_k)\right\}\right)^{-1}, \gamma>0. \tag{11}$$

This makes the parameter γ in Eq. (11) scale-free, which in turn facilitates the construction of an effective grid. A significant set of grid values for the location parameter c may be defined as sample percentiles of the transition variable s_{t-d}. For each value of c and γ the residual sum of squares is computed and the values that correspond to the minimum of that sum are taken as starting values. Once good starting points have been found, the unknown parameters c, γ, ϕ,θ can be estimated by using a form of the Newton–Raphson algorithm to maximize the conditional maximum likelihood function.[18]

It is important to notice that a specific numerical problem exists in the estimation of STAR models when γ in Eq. (11) is large and the model is consequently close to a switching regression model (see Terasvirta, 1994). To obtain an accurate estimate of γ, one needs many observations in the immediate neighborhood of c, because even large values in γ only have a small effect on the shape of the transition function. It is unlikely that such clusters can be found in small samples. The estimate of γ may therefore be rather imprecise and often appears to be insignificant when judged by its t-statistic. Yet, this does not suggest redundancy of the nonlinear component.

4.5. Dynamic properties of the STAR models

According to Terasvirta and Anderson (1992), it is usually difficult to interpret the individual coefficients of STAR models, but the roots of the characteristic polynomial associated with these models provide information which is useful for understanding their dynamic properties. This can be done by computing the roots of the STAR(p) model by solving:

$$z^p - \sum_{j=1}^{p}\left(\hat{\theta}_j + \hat{\theta}_j^* F\right)z^{p-j} = 0, \tag{12}$$

It is possible to compute the roots of the characteristic polynomial of the models at each point in the sample period and for several values of the transition function. However, the roots of the respective extreme regimes describe the local dynamics of appreciation and depreciation. In particular, we considered two regimes: first, F=0, which corresponds to the lower (rising q) regime in the LSTAR model and the middle regime in the LSTAR$_2$ model. Second, F=1, which corresponds to the upper (falling q) regime in the LSTAR model, and the outer regime in the STAR model.

5. Data sources and construction of the variables

For the empirical analysis we considered the real effective exchange rate (REER) of the following countries: Argentina, Australia, Brazil, Canada, Euro-zone, India, Indonesia, Japan, Korea, Mexico, Norway, Turkey, UK and the USA. The data are monthly and cover the period January 1980 until December 2005.[19] The REER, based on consumer prices, of the country i was constructed as follows:

$$\mathrm{REER}_{it} = \frac{P_{it}E_{it}}{\Pi_{j\neq i}^{N}\left(P_{jt}E_{jt}\right)^{\omega_{ij}}}, \tag{13}$$

where j is an index of country i's trade partners; N is the number of countries, ω_{ij} is the competitiveness weight put by country i on country j, P_i and P_j are Consumer Price Indices (CPI) in countries i and j, and E_i and E_j represent the

[18] We should keep in mind that one problem with NLS is that the algorithm may give local maxima or minima instead of a global solution.

[19] We retained these countries because of the following reasons. First, it is interesting to contrast our results with previous investigations which conclude that the real exchange rate in industrialized countries is nonlinear mean reverting. Second, regarding the chosen emerging economies, this was mainly due to data availability and the fact that most of these countries are characterized by currency crises and the presence of clusters of outliers which make them interesting to study in our context.

nominal exchange rates of countries i and j's currencies in US dollars, all of them obtained from the IMF's *International Financial Statistics*.[20] We work with the logarithm of the real effective exchange rate and its first difference, denoting these variables as qt and Δqt respectively.[21]

6. Results

A preliminary evaluation of the data shows that real exchange rates for all the countries are non-stationary in levels.[22] Therefore, in order to anticipate the structure of the STAR/LSTAR$_2$ models, the real exchange rates are made stationary by taking first differences of logarithmic transformed data (see Fig. 1).

6.1. Standard linearity tests

Once we specified the linear AR(p) models, we use the auxiliary regressions to test for real exchange rate linearity against LSTAR/LSTAR$_2$ specification. In carrying out these tests, we have considered values for the delay parameter, d, over the range $1 \leq d \leq 12$, and calculated the p-values for the LM$_1$, LM$_3$, LM$_3$ ^{e} and LM$_4$ standard and robust linearity tests in each ease. We also considered the first lag of the level term and the time trend as possible transition variables. We varied the delay parameter in order to provide the strongest probability of nonlinearity. If linearity was rejected for more than one value of d, then the estimate of d was chosen by the lowest p-value of all (or most) of the LM-type tests. Thus, we chose the delay length for which nonlinearity was strongest according to the tests.

Considering only the standard tests, the first interesting result is that in some cases, particularly for advanced economies, evidence against linearity in favor of STAR-form nonlinearity can be found just for a small number of the transition variables considered.[23] In contrast, the evidence of nonlinearity using the standard LM-type tests increases for developing countries. Second, the chosen transition variable also varies considerably among countries. Indeed, the results from the linearity test show that the delay parameter is over the range **1** to **11**. Since we are working with monthly data this implies that close lags (i.e. less than one year) of the real exchange rate are important for predicting contemporaneous ones. Third, only in the case of Canada, we could not find evidence of nonlinearity.

6.2. Robust linearity tests

Next, according to Eq. (10), we computed the observations that had large residuals in the OLS estimation of the restricted linear model (6). These observations, the outliers, receive a weight equal to zero in the robust estimation and are represented as circles in Fig. 1 which also shows the first difference of the RER. As can be seen, outliers were found to exist in all the series with the number and timing varying across countries. Furthermore, while in developing countries outliers show up more or less in groups, indicating that there is significant correlation between them, the isolated outliers are the main characteristic of the series in advanced countries.[24] In addition, the real exchange rates in some countries, such as Argentina, Brazil or Indonesia, are dominated by outliers.

Based on the outlier robust estimation procedure, we modified the previous standard LM-type tests. The outcome from these tests is particularly interesting (see Table 1). Certainly, once controlling for outliers, the evidence of nonlinearity is reduced considerably, particularly for advanced countries. Indeed, we found that for some countries results coincide in the sense that standard and robust based procedures both indicate the presence of nonlinearity for the same transition variables. Yet, in several cases the p-values for the robust tests are considerably larger than those from the OLS-based tests, indicating that the evidence of nonlinearity might be only due to the presence of a few outlying observations in several countries. Also, even when linearity is rejected by both tests, the results sometimes guide to the election of different transition variables.

[20] In Eq. (13) i's N partners are exclusively the members of our group of countries, which had a competitiveness weight in i's trade equal or bigger than 2%, normalizing these weights to sum to one. For the weights we used data from Durand et al. (1998) and correspond to the 1995 matrix.
[21] An increase (decrease) in a country's RER indicates an appreciation (depreciation).
[22] The time series unit root test results are available upon request.
[23] To avoid too many tables, the results of the standard and outlier robust LM-type tests are not reported here, but they are available upon request to the author.
[24] Outliers in advanced countries seem to be better defined as additive outliers whereas outliers in developing countries can be identify as innovative outliers.

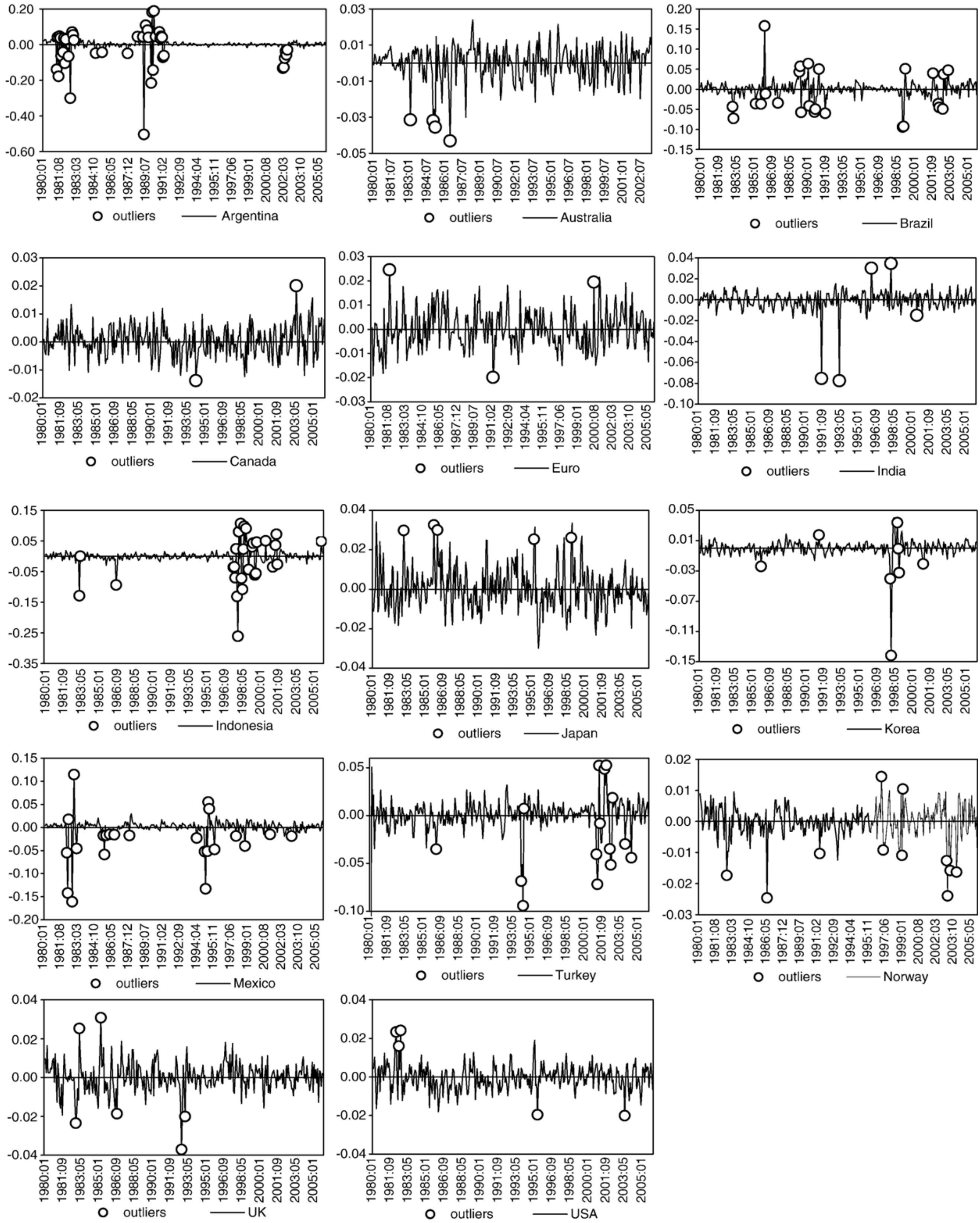

Fig. 1. Monthly growth rates of the real effective exchange rates and outliers according to robust estimation.

Table 1
Model selection by standard and robust tests

Country	P	Standard tests			Outlier rebuts tests		
		s_{t-d}	T	EJ	s_{t-d}	T	EJ
Advanced countries:							
Australia	2	$\Delta\ q_{t-4}$	LSTAR	LSTAR	$\Delta\ q_{t-4}$	LSTAR	LSTAR
Canada	1		Linear	Linear		Linear	Linear
Euro	1	$\Delta\ q_{t-7}$	LSTAR	LSTAR	$\Delta\ q_{t-7}$	LSTAR	LSTAR
Japan	1	$\Delta\ q_{t-6}$	LSTAR	LSTAR		Linear	Linear
Norway	2	$\Delta\ q_{t-3}$	LSTAR	LSTAR	Trend	LSTAR	LSTAR
UK	2	$\Delta\ q_{t-4}$	LSTAR	$LSTAR_2$		Linear	Linear
US	2	$\Delta\ q_{t-3}$	$LSTAR_2$	LSTAR		Linear	Linear
Developing countries:							
Argentina	1	$\Delta\ q_{t-2}$	LSTAR	$LSTAR_2$	$\Delta\ q_{t-11}$	LSTAR	$LSTAR_2$
Brazil	2	$\Delta\ q_{t-1}$	LSTAR	LSTAR	$\Delta\ q_{t-6}$	$LSTAR_2$	$LSTAR_2$
India	1	$\Delta\ q_{t-5}$	$LSTAR_2$	LSTAR		Linear	Linear
Indonesia	1	$\Delta\ q_{t-4}$	$LSTAR_2$	LSTAR	$\Delta\ q_{t-11}$	LSTAR	$LSTAR_2$
Korea	3	$\Delta\ q_{t-9}$	LSTAR	$LSTAR_2$	$\Delta\ q_{t-1}$	LSTAR	$LSTAR_2$
Mexico	2	$\Delta\ q_{t-1}$	LSTAR	$LSTAR_2$	$\Delta\ q_{t-4}$	LSTAR	LSTAR
Turkey	1	$\Delta\ q_{t-2}$	$LSTAR_2$	LSTAR	$\Delta\ q_{t-3}$	$LSTAR_2$	LSTAR

Notes: (1) p is the number of autoregressive terms; (2) s_{t-d} is the transition variable, with d the de: ay parameter. The possible transition variables considered are the first lag of the level term, q_{t-1}, and the lagged values of the first difference of the (log) real effective exchange rate, Δq_{t-j}; (3) T are the sequence of tests suggested by Terasvirta (1994); (4) EJ are the tests by Escribano and Jordá (1999); (5) Δq_{t-j} is the lagged value of the first difference of the (log) real effective exchange rate, with the lag equal to j.

The repercussions of not taking into account the presence of outliers can be exemplified with the case of the United States. According to the standard tests, Δq_{t-3} is the most suitable transition variable. However, once we control for outliers, the p-values for the LM-type tests are bigger than **0.10** in all the cases and the null hypothesis of linearity cannot be rejected at conventional significance levels. Furthermore, we found observations that are down-weighted (outliers) in March, June and August **1982**, April 1995 and May 2003.

The values taken by the transition function $F(q_{t-\mathrm{d}};\ \gamma,\ c)$ which are less than 0.5 in the standard fitted STAR model coincide more or less with these outliers. That is, the regime of the STAR model corresponding to $F(q_{t-d};\ \gamma,\ c)=0$ becomes active for observations around the detected outliers.

In addition, real exchange rates in the Asian countries (Korea and Indonesia) contain a substantial amount of outliers at the end of 1997 and 1998, obviously due to the financial crises. Yet, the standard and robust tests reject linearity but they both point to different suitable transition variables. In contrast, in the cases of Australia and the Euro-zone, the same conclusions can be obtained from the standard and the robust linearity tests in the sense that they both give evidence that real exchange rate can be modeled as a STAR model with Δq_{t-4} and Δq_{t-7} as transition variables, respectively.

6.3. Choice of the nonlinear specification

Once we have tested for linearity, we followed the sequence of tests to discriminate between LSTAR and $LSTAR_2$ following the methodology of Terasvirta (1994) and Escribano and Jordá (1999). A summary of this procedure is presented in Table 1 below. In contrast to conventional wisdom, the real exchange rates for all the advanced countries which rejected linearity according to the robust tests (Australia, Euro-zone and Norway) as well as for Mexico and Turkey, are better specified as LSTAR models. This suggests that the expansion and contraction phases of the exchange rates in these countries may have different dynamics. In contrast, results from both the standard and the robust tests in the exchange rates of the rest of the countries are less conclusive. Based on the decision rule of the procedure of Terasvirta (1994), the outlier robust tests suggest that LSTAR models are appropriate for the candidate transition variables. Nevertheless, the results from the Escribano and Jordá (1999) statistics contradict this suggestion. Only for Brazil, according to the robust tests, there is no doubt that the real exchange rate can be better specified by a $LSTAR_2$ model, implying that exchange rate moves from high or low levels towards the middle ground (or normal level) in a similar fashion.

Based on the results from both the standard and the robust tests, we proceeded to estimate the LSTAR or $LSTAR_2$ models for the first difference of the (log) real effective exchange rate series. In order to do so, we used nonlinear least squares, which, as stated above, provides estimators that are consistent and asymptotically normal.[25] Following the recommendation by Granger and Terasvirta (1993) we standardized the transition parameter (y) by the sample variance or sample standard deviation of the transition variable and we used a grid-search that creates a linear grid in c and a log-linear grid in γ.[26] In order to obtain more parsimonious models, we reduced them by imposing certain restrictions whenever it was possible, and chose the best fitted model according to information criterion.[27]

The results show that the estimated STAR models offer an improvement over the linear specification in terms of the Schwarz Information Criterion (BIC).[28] Also, the residual variances of the nonlinear models are slightly smaller than those of the AR models.[29] Both, the skewness and excess kurtosis are reduced in the STAR models, although normality of the errors is still rejected in most of the countries, probably due to the presence of outliers and residuals corresponding to currency crises.[30]

Second, a key parameter in the STAR models is y. In this respect it is interesting to notice that although the transition parameter varies across countries it is, generally, higher in developing countries than in advanced countries (see Table 2). This implies, in principle, that the transition from one regime to the other is rather slow for the latter group and much faster for developing countries. However, it would be a mistake to judge the significance of this variable by means of its t-value. In fact, as noticed by Terasvirta (1994), the t-statistic of γ does not have its customary asymptotic t-distribution under the hypothesis that $\gamma=0$, due to identification problems.[31] Therefore, we could not conclude, for example, that there is evidence of nonlinearities based on finding a transition parameter, γ, negative and significantly different from zero at standard levels.

Furthermore, according to both the standard and the robust estimation, the estimates of the parameters γ and c are such that the change from the functions from 0 to **1** takes place for very different values and ranges of the transition variable. Yet, the change from one regime to the other is more abrupt in the ease of the standard estimation, indicating that most of the observations remain either in the lower (as in Brazil) or in the upper regime (Indonesia, Korea, Mexico and Norway). This suggests that the estimations based on the results from the robust methodology are more appropriate, since they provide smoother transition functions.[32]

Also, the inference based on the estimated parameters λ and λ^* slightly differs in the standard and robust estimations (see Table 2). Indeed, while the possible existence of a unit root for small deviations from equilibrium (that is $\lambda \geq 0$) was found only for Turkey according to the standard estimation, based on the robust estimations, we found unit roots for Argentina and Indonesia.[33] Conversely, mean reversion for large deviations from the purchasing power parity $((\lambda \cdot t\lambda^*)<0)$ was found in all the countries and in concordance to both estimations, except in the case of Mexico.[34]

The previous results then indicate that there is evidence of asymmetric dynamics in the real exchange rates. This implies that periods of under and overvaluation have different characteristics. For instance, it may be the case that reversion towards equilibrium is faster when the currency is overvalued than when it is undervalued. Indeed, it has been suggested that real exchange rate overvaluation can precede currency crises. If this is the case, the reversion towards equilibrium takes the form of sudden devaluations (which in turn are believed to be contractionary). On the contrary,

[25] Tong (1990) shows that this condition holds for STAR models if series are stationary, ergodic and the error terms are independently and identically distributed.

[26] Note that the grid search is made for c_1 and c_2 in case $LSTAR_2$ was selected and only for c in case of a LSTAR model. For the transition parameter, the grid search is performed for values of $0.5 \leq \gamma \leq 10$.

[27] For example when the intercept in the two regimes is similar in value, we can impose $k=-k^*$ in (6). Also, if certain coefficients are not significant in the linear or nonlinear part, they can be eliminated.

[28] The models are not presented here but are available upon request.

[29] A priori, the R^2 would also favor the nonlinear models. Yet, as noted by a referee, inference based on it may be invalid. Indeed, its higher value in the nonlinear specification compared to the linear counterpart is expected as more explanatory variables are used in the former specification. Thus, based on this criterium, nonlinear specification may be mistakenly concluded as better than the linear counterpart even if the nonlinear component is not statistically significant.

[30] Since the normality assumption is violated, bootstrapped techniques are recommended. We present the bootstrap p-values for the estimated coefficients in Table 2.

[31] Under the null hypothesis, each of the real exchange rates follows a unit root process. Hence, the presence of a unit root under the null hypothesis complicates the testing procedure.

[32] Fig. (2) shows the standard and robust transition functions against the transition variables.

[33] Notice that even if the coefficient λ is positive in the case of Korea and Australia in the standard estimation (as well and the robust in the case of Australia) their bootstrap p-values indicate that they are not significant.

[34] Yet in the case of the standard estimation for Brazil, the null hypothesis $H_0=\lambda=0$, $\lambda^*=0$ cannot be rejected.

Table 2
Selected models and estimated key parameters

	Argentina		Australia		Brazil		Euro-zone		Indonesia		Korea		Mexico		Norway		Turkey	
	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust	Standard	Robust
Model	LSTAR	$LSTAR_2$	LSTAR	LSTAR	LSTAR	$LSTAR_2$	LSTAR	LSTAR	$LSTAR_2$	$LSTAR_2$	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR
$\hat{\lambda}$	−0.196	0.001	0.035	0.035	−0.015	−0.102	−0.008	−0.008	−0.130	0.182	0.001	−2.325	−0.662	−0.011	−1.060	−0.013	−0.046	−0.097
	(0.000)	**(0.967)**	**(0.146)**	**(0.146)**	**(0.081)**	**(0.072)**	**(0.316)**	**(0.316)**	**(0.063)**	**(0.002)**	**(0.858)**	**(0.000)**	**(0.000)**	**(0.171)**	**(0.001)**	**(0.062)**	**(0.003)**	**(0.002)**
λ^*	0.168	−0.061	−0.055	−0.055	−0.028	0.094	−0.030	−0.030	0.117	−0.227	−1.208	2.315	0.662	0.045	1.060	−0.120	0.026	0.071
	(0.001)	**(0.010)**	**(0.060)**	**(0.060)**	**(0.813)**	**(0.122)**	**(0.203)**	**(0.203)**	**(0.126)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.124)**	**(0.001)**	**(0.342)**	**(0.100)**	**(0.098)**
$\hat{\gamma}$	26.19	54.99	2.090	2.090	14.93	11.14	2.976	2.976	251.549	2.976	23.39	4.747	30.204	39.84	0.838	10.738	46.845	6.863
C	−0.019	−0.064;	−0.007	−0.007	0.050	0.015	0.008	0.008	−0.048	0.023	−0.009	−0.024	−0.023	0.013	−0.025	262.123	0.002	−0.019
		0.012				0.014			−0.021	0.023								
F-statistic	**0.003**	**0.015**	**0.057**	**0.057**	**0.169**	**0.000**	**0.057**	**0.057**	**0.006**	**0.000**	**0.000**	**0.000**	**0.000**	**0.179**	**0.000**	**0.022**	**0.009**	**0.002**

Notes: (1) Standard and robust correspond to the estimated parameters according to the standard and robust linearity tests; (2) Bootstrap (parametric) p-values in parenthesis (with 1000 simulations); (3) λ and $\hat{\lambda}^*$ are the estimated coefficients of the lagged level in the linear and nonlinear parts, respectively; (4) $\hat{\gamma}$ is the estimated transition parameter; (5) $\hat{c}$ is the estimated half-way point between the two regimes; (6) The F-statistic is the bootstrap p-value for the null hypothesis H_0: $\hat{\lambda}=0$, $\hat{\lambda}^*=0$. Values in bold are the significance levels of the tests.

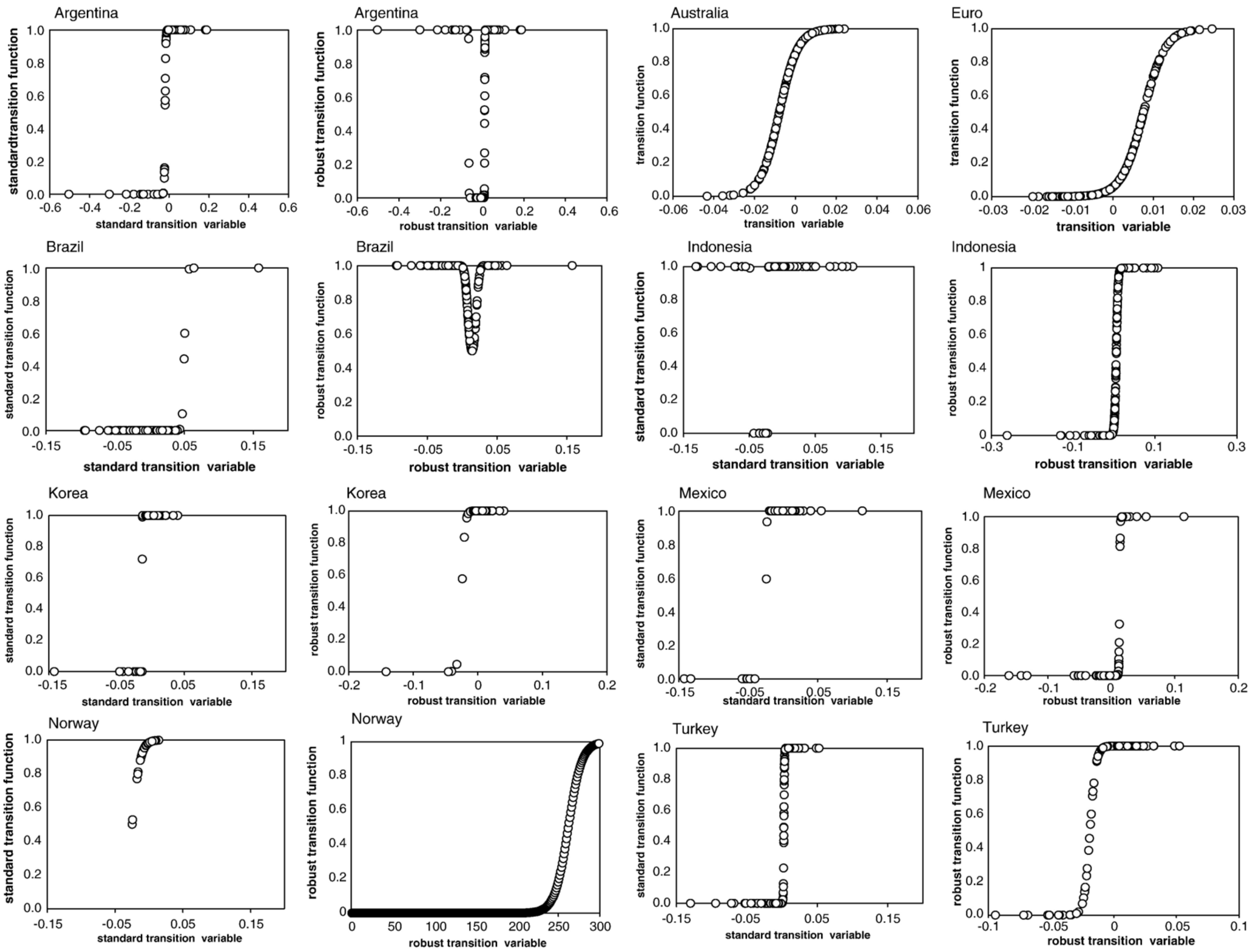

Fig. 2. Transition functions versus transition variables.

Table 3
Roots of the characteristic polynomial in each regime

Country	Regime	Standard		Robust	
		Root(s)	Modulus	Root(s)	Modulus
Argentina	0	−0.0376±0.1612i	0.1655	−0.3095	0.3095
	M	−0.0606±0.4384i	0.4426	0.1213	0.1213
Australia	U	0.3217±0.4604i	0.5619	0.3217±0.46041	0.5619
	L	−0.2731±0.2524i	0.3718	−0.2731±0.25241	0.3718
	L	−0.2532	0.2532	−0.2532	0.2532
Brazil	0	−0.0696±0.1977i	0.2096	0.1862±0.2831i	0.3388
	M	0.3281	0.3281	−2.1820	2.1820
	M			0.6594	0.6594
Euro	U	0.5348	0.5348	0.5348	0.5348
	L	0.2103	0.2103	0.2103	0.2103
Indonesia	0	0±0.1155i	0.1155	−0.0512±0.2062i	0.2125
	M	1.6704	1.6704	1.4417	1.4417
Korea	U	0.3538±0.4808i	0.5969	0.595±0.45611	0.5808
	U	−0.1398±0.1355i	0.1947	−0.1378±0.1065i	0.1742
	L	1.7097	1.7097	1.7104	1.7104
	L	0.0302±0.8143i	0.8148	1.6666	1.6666
Mexico	U	0.3122	0.3122	−0.0457±0.9216i	0.9227
	L	0.4784±.2021i	1.2938	0.1984±0.1682i	0.2601
Norway	U	0.3693	0.3693	0.3443±0.56141	0.6586
	U			−0.3078	0.3078
	L	−4.8584	4.8584	0.2128±0.1706i	0.2727
Turkey	U	0.5405	0.5405	0.5316	0.5316
	L	0.1472±0.1585i	0.2163	−0.0446±0.3088i	0.312

(1) only dominant roots are shown; (2) U: upper regime in LSTAR (F=l), L: lower regime in LSTAR (F=O), 0: outer regime LSTAR$_2$ (F=l), M: middle regime LSTARZ (F=O).

undervalued currencies are seen as less dangerous: when currencies are undervalued, their reversion towards equilibrium is usually not so abrupt.

In the same line, reversion is faster in developing countries. Indeed, most of the emerging economies in our sample are characterized by episodes of currency and financial crisis. On the contrary, the RER in advanced economies is usually more stable.

However, the previous statements must be taken with precaution, given that nonlinear models are difficult to interpret from the estimated coefficients. Instead, the roots of the characteristic polynomials associated with these models provide more useful information.

6.4. Dynamic properties

Once we estimated the STAR models, we can deduce their dynamic properties through the roots of the characteristic polynomial. Table 3 shows the most prominent roots for each regime. Broadly speaking, most of the countries are characterized by complex roots in the upper (or outer) or in the lower (middle) regime, which implies that real exchange rates in these cases display cyclical movements during contraction (depreciation) and expansion (appreciation) phases.

In addition, the models for several countries are stable in both regimes. Yet, according to the robust estimation, in Brazil, Indonesia and Korea, the characteristic polynomials of the lower (or middle) regime contain a large explosive root, whereas the upper (outer) regimes are stationary. Quite the opposite, the largest modulus for the upper regime in Mexico is not far from the unit circle. This suggests that the real effective exchange rates for these countries move very aggressively from a low level into a higher growth rate whereas once they are in the expansionary phase they will tend to remain there.[35]

In contrast, the rest of the countries are completely characterized by stable roots in the outer and middle regime. This implies that there is nothing in the dynamics of the regimes to suggest a rapid change from one to the other. Therefore, only large shocks could cause these changes.

[35] Note that these results remain in the standard estimation, except that in this case Mexico and Norway also contain explosive roots in the lower regime.

7. Summary and conclusions

The emerging literature on real exchange rate determination and purchasing power parity suggests that persistent misalignments in the real exchange rates can be explained by the presence of a nonlinear adjustment process towards the long-run equilibrium. In simple words, this means that the real exchange rate can have different characteristics according to the size of the deviations from PPP.

While it is true that for a number of countries the real exchange rates are well characterized by a process which adjusts nonlinearly towards its long-run equilibrium, this is not always the case. Indeed, we performed robust linearity tests, finding partial evidence of nonlinearity for 9 out of **14** countries. Particularly, we were not able to reject linearity in most of the advanced countries of our sample, which are characterized by the presence of fewer outliers than developing economies.[36] These aberrant observations in the series cause the standard nonlinear tests to point towards nonlinear structures when there are none. Even when the real exchange rate can be characterized by a nonlinear model, the presence of these outliers can lead to wrong decisions, particularly in terms of the transition variable. Therefore, care must be taken when modeling nonlinearities and, particularly, the detection and location of outliers should be made prior to estimation and testing.

This is especially important since it implies that the apparent nonlinear adjustment process in the real exchange rate of advanced economies is caused by only a small number of data points. These aberrant observations do not signal intrinsic nonlinearity in the process generating the real exchange rates, but rather are caused by some exogenous and "irregular" events. In other words, the RER in this group of countries appears to be linear and the series, occasionally, show outliers. In these cases, not being able to reject the null hypothesis of a unit root in the real exchange rate definitely implies that the PPP is not a valid long-run equilibrium value and, as such, it cannot be consider as a benchmark to judge under or over valuations of the RER.

On the other hand, developing countries have experienced stronger devaluations that are usually followed by further movements in the exchange rate, generating clusters of outliers. These shocks may imply, effectively, the existence of different regimes.

For the real effective exchange rates that had some evidence of nonlinearity, we estimated STAR type-models for the period 1980:01 to 2005:12.[37] Most of the exchange rates were found to be better classified by logistic STAR models, implying that the expansion and contraction phases of the exchange rates may have different dynamics. Although the estimated STAR models perform better than the linear autoregressive models, we failed to obtain Gaussian residuals in several cases, probably due to the outlier observations.

The estimates of the transition parameter indicate that the speed of transition from one exchange rate regime to the other is slower for advanced countries than for the developing ones. This may imply that sizeable over or under valuations adjust more abruptly in developing countries. The abrupt devaluations and financial crises experienced in Asian and Latin American countries can be examples of situations when changes from one regime to the other occur in a more aggressive way. Instead, adjustment in industrial countries is smooth.

Regarding the dynamics, the exchange rates are characterized by cyclical movements mainly in the upper or outer regime and less often in the lower or middle regime. The fact that in most of the countries the upper regime (i.e., when the real exchange rate is far from its equilibrium value) is stable, suggests that once the exchange rate is in this phase, it will tend to stay there for long periods of time. That is, when the real exchange rate is far from equilibrium, convergence to long-run PPP is rather slow. There is nothing in the dynamics of the upper regime to suggest a rapid fall into a contraction. Only sufficiently large shocks could cause this.

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[36] This is a clear difference between our findings and previous ones, such as Taylor et al. (2001) who conclude that the real exchange rate in industrialized countries is nonlinear mean reverting.

[37] Notice that the nonlinearity property detected in the paper for some countries is specific to the PPP as a reference equilibrium point. Yet, as noted by a referee, it may be the case that the nonlinearity property is due to a misspecification of the equilibrium process.

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