Mathematics Education Research Journal 2009, Vol. 21, No. 3, 36-59

Bilingual Students using their Languages in the Learning of Mathematics

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This paper is about how immigrant bilingual students use their languages in the learning of mathematics. Our research has been with immigrant bilinguals in Catalonia, Spain, who arrived at a young age from South-American countries. We use a critical sociolinguistic approach, which draws on social theory in the analysis of how language is involved in the construction of teaching and learning opportunities. Our data point to the differences in the ways that the Spanish dominant bilingual students use their two languages during their engagement in mathematical activity. The shifts from Catalan to Spanish, and from Spanish to Catalan, coincide with shifts in the complexity of the students' mathematical practices. The students tend to use the two languages for different purposes, depending on the complexity of the mathematical practices, and in relation to different social settings that coexist within the classroom.

What does it mean to learn mathematics in a language that is not your first language in a country that is not your own? This is the situation in many immigrant bilingual mathematics classrooms in Catalonia, Spain. A majority of immigrant students in these classrooms have Spanish as a first language. However, they learn mathematics in Catalan, which is the official language for learning and teaching mathematics in Catalonia. How do these Spanish dominant bilingual students use languages during mathematics teaching and learning in Catalan classrooms? Do they switch languages during mathematical activity? If so, what are some of the factors that promote their language switching within the context of specific lessons?

In this paper we explore those questions. We do this by drawing on a wider study involving immigrant bilingual children attending a Catalan school who were either born in Catalonia or went there at a young age from South-American countries such as Ecuador, Bolivia, Perú and Colombia. Our analysis highlighted two recurrent themes in regard to which language was used by the students: 1) the impact from acquiring specific vocabulary in the second language; and 2) the development of mathematical argumentations in the first language. We argue that the shifts from Catalan to Spanish coincided with shifts in the complexity of the students' mathematical practices. The students tended to use the two languages for different

purposes, depending on the complexity of the mathematical practices, and in relation to different social settings that coexisted within the classroom.

According to Grosjean (1997) bilinguals use their languages for different purposes and in different domains of life. Grojean refers to this practice as the complementarity principle. The alternate use of two languages is not only a question of proficiency of the languages – how well the bilingual knows them – but also a question of the social circumstances of the languages in use – the influence that other persons have on the bilingual. The proficiency of the languages and the social circumstances determine the contexts in which a bilingual has the opportunity to use one of the languages and how s/he is recognised when doing it. The study reported in this paper contributes to on-going discussions concerning the complementarity principle in the case of bilingual learners of mathematics (see, for instance, the work of Moschkovich, 2002, 2007a, 2007b). By studying bilingual learners, we hope to come to a better understanding of how the use of the languages is mediated by the interpretation of the different contexts of mathematical school practices. Although many researchers recognise that the choice of language in bilinguals may vary depending on the context where the use of the language is situated (Civil & Planas, 2008; Daller, Van Hout, & Treffers-Daller, 2003), there is a need for more research on contexts of mathematical practices (Parvanehnezhad & Clarkson, 2008). In our view, bilingualism reflects a richness of the individuals (see Bialystok, 2001, for data on bilinguals doing better on tasks requiring more analysed linguistic knowledge and scoring higher in tests of divergent thinking in comparison to monolinguals) and a positive value for groups.

A Sociolinguistic Approach to Bilingualism

Bilingualism can be defined in different ways. At one end, we have the native-like control of two languages (Bloomfield, 1933) as the classical criterion for bilingualism, and at the other those speakers of one language who can produce meaningful constructions in a second language. We accept that bilingualism is best regarded as a continuum, and assume that even when there is a minimal proficiency in the second language and effective communication results from the ability of the listener, not the speaker, the speaker may be considered bilingual. Moreover, we share Cummins's (2000) interpretation of bilingualism as a result of unequal social power structures rather than differing contexts of socialisation. Our approach is influenced by the perspectives of several researchers on mathematical learning and multilingualism, such as Adler (2001), Barton (2008), Barwell (2003, 2005), Clarkson (2004, 2006, 2009), Moschkovich (1999), and Setati (1998, 2005). They all assume the integrity of the language and culture of minority groups, and emphasise the need for developing a sense of "language

awareness" in the domain of mathematics education research. The researchers have in common: (a) the attention to the on-going struggles over power, and resulting inequalities, in multilingual contexts of mathematical practice; (b) the analysis of both social structure and agency; (c) the need for sociolinguistic work to draw on social theory in our understanding of school mathematics; and (d) the interest in how difference and dominance may be created in face-to-face classroom interactions.

In our current work, we explore issues of language diversity and language contact in settings of mathematics education. We see language diversity as the number of different languages that may be spoken in a given context, and language contact as the effective use of some of them in the same place and at the same time. To further explore these notions, we propose a critical sociolinguistic approach, which draws on social theory in the analysis of how language is involved in the construction of teaching and learning opportunities. We consider the construction and use of the language essentially as a social process embedded in social interaction that takes place in social participation structures where issues of identity and power are structural (Fairclough, 1989, 2001). This process of construction and use of the language is about how the individuals, the groups, and the contexts mutually constitute one another and work – together or separately – to maintain certain power relationships and change some others.

Language diversity and language contact are particularly represented by bilingualism, in relation to the knowledge of and the ability to use two languages (Garcia, 2008). Myers-Scotton (2006) establishes that a bilingual is someone who has acquired or learned to speak and understand the world by means of two languages, though the contents of her/his understanding may not be the same depending on the language that s/he uses. In her book, Myers-Scotton focuses on how bilinguals integrate the understandings from two languages, and how they put together their knowledge to use in communication. The integration of two languages and the construction of joint knowledge are always problematic as language contact involves one kind or another of social imbalance that reflect some degree of tension or conflict among groups. Muysken (2000) suggested that the selection of one of the two languages as well as the maintenance or the eventual shift to the other, are types of imbalance that are related to differences in the knowledge of the languages. The choice of language, however, can also be thought of as strongly influenced by students' perceptions of the value of each language in different situations. Caldas and Caron-Caldas (2002) argue that a bilingual's preference for either of her/his two languages is context sensitive. Together with the similar linguistic structure of Catalan and Spanish, the fact that each language is socially marked points to the classical phenomenon of diglossia; in our context South-American Spanish is often stigmatised and

therefore marks competence differently than might other accents. In general, the shift to the language with a higher status may be favoured by the students' perception of conditions of gaining access to social goods, while the maintenance of the language with a lower status may be associated to the perception of conditions of segregation and marginalisation.

Wal-Pastoor (2005) carried out an ethnographic study in a Norwegian multilingual classroom. In the Norwegian political context, it is assumed that minority children's second language proficiency is a crucial factor for their school success. It has been established that, since most minority students speak their first language at home, the classroom has to be a setting for the learning of their second language. Wal-Pastoor reflects on the fact that language serves a double function: It transmits the subject matter to be learned ("learning through language") and it provides an important source of linguistic input for the second language acquisition ("learning language"). Her main data, concerning the exploration of "learning through language," reveals the existence of "silent students" who do not have a voice in the classroom discourse when it is entirely developed in Norwegian and around cultural meanings taken from the local dominant culture. These "silent students" are, however, high-achieving if we attend to their participation in certain discussions with some of their immigrant peers in languages with a low status in the class.

In her work on bilingualism in the learning of mathematics, Moschkovich (2002, 2007a, 2007b) explores how English-Spanish bilingual working-class Latino students in the USA use language to communicate mathematically. In the current political context of classrooms in the United States, Latino students' instruction is mainly in English but the language in the home (at least with the parents) and with some peers is Spanish. In these classrooms as well as in the Catalan case (Civil & Planas, 2009; Planas, Iranzo, & Setati, 2009), the instructional reform-based materials make language even more salient than in traditionally oriented classes. The nature of homework tasks in reform curricula is quite different from that of tasks in "traditional" textbooks. Although many of the tasks in the reform curricula include computation exercises, they are dominated by language rich activities in the sense of both the English language and the mathematical language. There are typical word problems, but also tasks in which the instructions are complex such as those where the students are asked to find out the mathematics in a short tale. Moschkovich (2007a) points to several aspects of learning mathematics in a bilingual classroom with bilingual students in such a reform context. Here, the knowledge of the two languages is used as a resource to communicate mathematically by means of selecting one of the languages and then shifting to the other depending on the complexity of the mathematical practice, mixing the two of them, or

introducing code-switching for specific words. These learners bring multiple competencies to the mathematics classroom – while they may have difficulties with one of the languages they may use the other language as a resource. A student who is missing English vocabulary, for instance, may be competent in describing mathematical processes and presenting mathematically sound arguments in Spanish.

Planas (2007) summarises data from a multilingual mathematics classroom where different groups of bilingual and trilingual students (Catalan-Spanish, Catalan-Spanish-Arabic, Catalan-Spanish-Urdu) interpret the coexistence of multiple languages and cultures as an obstacle in their learning of the mathematics. The teacher in that study took a strong position in favour of integrating the different home languages represented in the classroom by organising the students to work in linguistically homogeneous small groups. However, the local students who were Catalan-Spanish bilingual had a deficit view of their immigrant bilingual peers and found it problematic that they came to school with a language other than Catalan or Spanish. The immigrant bilinguals also preferred not to use their first language in the class because they accepted that they were supposed to practice the local official languages in their learning. These students' positionings needed to be interpreted within the broader social contexts where the class (labelled with the category of "compensatory" due to the "learning disabilities" of the students, as established by the group of teachers) and the school (classified as "special needs" by the local Administration based on the high number of students from low-income families) are placed.

Context and Method

Catalan is a Romanic language that shares many linguistic structural properties with Spanish. The structural similarities between Catalan and Spanish makes a distinction between this research and research on immigrant bilinguals in Europe that is faced with the problem of comparing the use of language pairs with large structural differences (see, for instance, the work by Wal-Pastoor, 2005, where the two languages are Norwegian and Turkish or Arabic). In Catalonia, Catalan and Spanish are common street languages in the everyday communication, and bilingual conversations – where each speaker uses one of the languages – are frequent. Spanish, however, is not official within the classrooms. In short, this means that it is not the language of teaching: The teacher is required to produce written texts in Catalan and to use Catalan in her/his speech.

Catalonia is an autonomous region with considerable first and second generation immigration. Recent data indicate that 16% of the total population are immigrants or children of immigrant parents. This

percentage is even higher in the urban area of Barcelona, the main city of the country. In this area, 28% of the population have immigrant origins, including 15% who are recent and second generation immigrants from South-America. As a group, South-American students continue to perform lower than the local Catalan students in mathematics on the institutional assessments. The average mathematics scores for sixth graders in the 2008 national test show that the failure rates for South-Americans are about four times higher than for the group of Catalan native speakers. This is despite the fact that the immigrant Spanish speakers in Catalonia – parents and children – still live in basically monolingual Spanish groups with low status in the wider society. At present there is a shift towards more Catalan due to the demands of the school, as well as indications that these groups are progressively interacting in different bilingual settings.

Research Questions and Analysis

As explained earlier, the study reported in this paper was guided by the following questions:

1. Do Spanish-dominant bilingual students in Catalan classrooms switch languages during mathematical activity?
2. If so, what are some of the factors that seem to promote the language switching with a group of these students in the context of specific lessons?

To address these two questions, we developed an interpretive qualitative research, which also incorporated an initial descriptive-quantitative approach. The main data come from regular lessons in a Catalan-Spanish bilingual group of a secondary school in Barcelona, Spain, with 24 students about 12 years-old and a bilingual Catalan native speaker teacher. The study was conducted in five lessons of 50 minutes each. The lessons were planned so that the students spent most of the time working in small groups. The teacher of the class encouraged the use of the first language by grouping students according to their dominant language. She had collaborated with us in previous research. When the study began, she had had more than a decade of experience in teaching mathematics in bilingual classrooms. The stated goals in her class included giving the students the experience of "thinking like mathematicians" in addition to "learning basic facts about the mathematics." Students had been informed that they were expected to "develop some ability to think critically about mathematics in open-ended situations." For the five lessons that were video-recorded, the emphasis was on the theory of geometrical transformations, mainly about translation, rotation, homothety and symmetry, as well as the composition of transformations. These concepts were part of a unit called

"Our dynamic planet", which included a variety of mathematical activities that encouraged students to pose questions and solve problems in real contexts. In the third lesson, for instance, the teacher wanted the students to think about the composition of transformations. The central task in this lesson was the following: "How can you mathematically represent a tornado?" In all the lessons, there was an initial short open-ended question presenting the task that had more than one correct answer.

In the class, all the students had a different bilingual proficiency profile due to the differences in their cultural and language biographies. There were eight students from South-America who were Spanish-dominant bilingual, whereas the other 16 students from Catalonia, mostly from Barcelona, were Catalan-dominant bilingual, except for one, who was a second generation immigrant who came from a Colombian family. Although we assume that bilinguals are not totally "balanced" (Cummins, 2000) because they tend to have a better proficiency in one of their languages, the immigrant students in our study had successfully passed a test on language competencies in Catalan and Spanish the year before. It may be over simplistic to claim that the levels of fluency of bilinguals in their two languages invariably have an impact on their choice of language, therefore we find it reasonable to search for more issues of influence when there is a good knowledge of the languages. The fact that most of them could be seen as almost native-like competent in their second language was an important condition as we wanted to primarily look for the influence of context on the choice of language.

Our research was focused on the nine students who spoke Spanish at home. They all had similar working class backgrounds; most of their parents had not completed high school, had limited Catalan proficiency and immigrated to Catalonia for work reasons. The data for this paper mainly come from one of the regular small groups in the class (WG1), whose members were Máximo – a second generation Colombian boy, Luna – a girl born in Perú, and Nicolás and Eliseo – two boys born in Colombia. The teacher referred to the four students in this group as having an average mathematical competency. The other five students, Daisy, Paola, Ángela, Elber, and Genaro, worked together in another regular small group (WG2) and were also studied. Given that code switching is presumably done as a result of students' perceptions of what is appropriate in different parts of the lesson, it would have been interesting to complement the classroom video-tapes with interviews with the nine students about their choices of language. This was not done because it was decided to focus the analysis on what had been publicly said during the five lessons.

For the five lessons, the teacher and a student in each of the groups wore a wireless microphone. There was also a static camera placed in one corner

to capture the general picture of the entire classroom environment. For the analysis, different portions of the students' interactions within the small groups and with the teacher were first isolated and then transcribed. After having examined that language switching that occured, by quantifying the shifts from Catalan to Spanish and from Spanish to Catalan (see Table 1), we drew on ethnomethodology (Garfinkel, 1967) and interactional sociolinguistics (Goffmann, 1981) to describe the contents of the talk that were observable and interpretable when reading the interactions. The use of Glaser and Strauss's (1967) constant comparative method led to the development of interrelated themes that seemed to be promoting language switching – the same method was used in Planas and Civil (in press) and Civil and Planas (2009). In the following section we discuss two of the main themes: 1) the impact from acquiring specific vocabulary in the second language; and 2) the development of mathematical argumentations in the first language.

Results

Our quantitative data confirm the use of the two languages, Catalan and Spanish, during the five lessons. To obtain quantitative discrete measures of how much Catalan and Spanish each student had spoken during these lessons, and to have some indication of whether or not the shifts in the use of the languages were frequent, we studied the transcripts for the number of individual Spanish and Catalan utterances and examined the number of language shifts. A language shift in a student's discourse refers to changes from Catalan to Spanish or from Spanish to Catalan. We only counted shifts that had been produced within the small group's conversation or within the big group's discussions. In the following example, we find two shifts: Máximo and Eliseo go from Catalan (C) to Spanish (S).

Máximo: [C] Hem de decidir les fletxes que dibuixem i ja està. / We need to decide the arrows that we draw and that's all.

Eliseo: [C] Primer pensem les fletxes, després les dibuixem i després en parlem. / First we think about the arrows, then we draw them and then we talk about it.

Máximo: [S] Esta idea de las flechas no es fácil. Tenemos que imaginar los diferentes movimientos que existen dentro del tornado. / This idea of the arrows is not easy. We have to imagine the different movements that exist within the tornado.

Eliseo: [S] Una flecha tiene que ser una línea recta para que el tornado baje. Tenemos la *t* para la translación. / An arrow needs to be a straight line for the tornado to go down. We have the *t* for the translation.

Due to the similarites between the two languages,[4] we quantified only those utterances that had more than one word. Moreover, the attention to Catalan and Spanish utterances, instead of individual words, makes sense because we very rarely found code switching within sentences. This is different from what Moschkovich (1999) found. This difference could have something to do with the fact that, in everyday communication, the two languages are rarely mixed by one speaker, even though bilingual conversations where each speaker keeps using one of the languages are frequent in the Catalan context. For ease of interpretation, we paid attention to the students' interactions in the whole group and within the small group. We added together the total number of Catalan and Spanish utterances separately for each lesson and for each student in the study, and we did the same for the total number of language shifts in the whole group, and within the small group. Table 1 shows the final quantities for a total of approximately 250 minutes. The C and S mentioned in all the tables refer to Catalan and Spanish. As the first table shows, the students used both languages in the two main classroom social settings - whole group and small group. The table also shows that during the whole group discussions the Spanish-dominant students tended to conform to the linguistic norm of speaking Catalan, the language of teaching and learning, while they changed to their first language in the small group discussions. These results clearly illustrate how different languages dominate different parts of the lesson. The table points to the students' variability in the number of language shifts. Ángela and Elber, for instance, rarely changed the language in their few interactions, while Daisy, with a higher participation, did it more frequently. The students in WG1 did not change the languages often. Their participation in the whole group was also higher than that of Ángela or Elber, but low in relation to that of Catalan native speakers in the class.

Table 2 shows the percentage data concerning the quantitative use of the two languages from the students in this working group – WG1. The monolingual composition of the groups had an influence on which language was more used in the small groups' work. When looking at the two contexts, whole group and small group, one of the main points is that the Spanish-dominant students' switching of languages actively contributed to the classroom dialogue having monolingual components (in different languages) rather than having a more even distribution of language use. The students in WG1 mostly used their first language in the local interaction, and

[4] Monosyllables like "yes," "no," and "well" are said with the same words in Catalan and Spanish. This happens with plenty of words and, therefore, it would have been difficult to identify the language in a very short interaction without a whole verb construction.

Catalan in the public whole group. They rarely spoke Spanish in the whole class except for a few occasions when they talked to each other.

Table 1
Measures Concerning the Frequency of Turns and Shifts

		Whole group		Small group	
		Utterances (C, S)	Shifts	Utterances (C, S)	Shifts
WG1	Máximo	(79, 13)	16	(28, 153)	7
	Luna	(49, 6)	9	(33, 177)	7
	Nicolás	(82, 8)	13	(11, 195)	10
	Eliseo	(55, 6)	22	(30, 246)	15
WG2	Daisy	(34, 26)	24	(102, 198)	56
	Paola	(73, 18)	12	(30, 326)	4
	Ángela	(12, 2)	2	(0, 34)	0
	Elber	(17, 5)	3	(0, 45)	0
	Genaro	(55, 35)	13	(64, 255)	24

Table 2
Measures Concerning the Frequency of Turns in WG1

		Whole group Utterances (C, S)	Small group Utterances (C, S)
WG1	Máximo	(85.9%, 14.1%)	(15.5%, 84.5%)
	Luna	(89.1%, 11.9%)	(15.7%, 84.3%)
	Nicolás	(91.1%, 8.9%)	(5.3%, 94.7%)
	Eliseo	(90.2%, 9.8%)	(10.9%, 89.1%)

After having identified language "preferences" of the students in the whole group and within the small group, we explored whether the use of the languages in these two social settings was related to the content of the talk. In what follows we turn our focus to the qualitative data paying specific attention to WG1. Our data point to the differences that the Spanish-dominant bilingual students have in the use of the languages during their engagement in mathematical activity. As we will see in the next two

sections, the shifts from one language to another, in either of the two directions, coincide with shifts in the complexity of the students' mathematical practices. At the beginning of the lessons, when the students are becoming familiar with the task and the new mathematical vocabulary, and when they devote time to the assessment of the task itself, they tend to use Catalan, both with their small group peers, who are Spanish dominant, and with the teacher. However, when they start reflecting on solving the task and developing mathematical explanations, they tend to use Spanish as if it was easier for them to complete and communicate their mathematical processes in this language. They move from the use of Catalan in a sort of socialisation into vocabulary practices when briefly speaking in the whole group, to the use of Spanish in the mathematical resolution of the task when extensively acting like mathematical learner experts in the small group.

Other research (see, for instance, Adler, 2001; Clarkson, 2006, 2009; Setati & Adler, 2000; Setati, 2005) has pointed to the impact of difficulties in the comprehension of the mathematics on students' switching between languages. We know of findings in relation to English as a second language and with pairs of languages that have many structural linguistic differences such as Vietnamese and English (Clarkson, 2006) or Iranian and English (Parvanehnezhad & Clarkson, 2008). However, to the best of our knowledge, there is no literature on language switching by Catalan and Spanish bilingual students in Catalan mathematics classrooms. On one hand, the specific language context given by the sociopolitical situation in our country, where the Spanish has a low social standing, makes it relevant to pay attention to this group of bilingual students and their efforts towards the public use of Catalan, the language with a higher status. On the other hand, it is interesting to note that, despite the structural similarities between the two languages and their frequent simultaneous use in out-of-school social situations, the students' switching contributes to the coexistence of monolingual settings instead of a more general context of bilingual conversations within the classroom.

The Impact from Acquiring Specific Vocabulary in the Second Language

In the five lessons, during the first minutes the teacher asks the students to begin by telling their small group peers something about the task they have just been reading. Then she gives priority to the language of the task and to the introduction of specific geometrical vocabulary. She asks the students if they know the meaning of a certain word that has been written on the board or orally introduced, and urges them to use it in the context of the resolution of the task. She does not translate the word into Spanish, nor

do the students ask for a translation. In all the lessons, the teacher begins by explaining only vaguely the mathematical meaning of the new word and leaves the students to explore in small groups the underlying concepts in the context of the task. In the interactions with the teacher, the immigrant bilingual students tend to use Catalan, their less proficient language, when they are prompted to introduce the new technical vocabulary.

Excerpt 1 is situated in the third lesson, when the students are told to mathematically represent a tornado. It shows part of the moment when the words "helicoidal", "helicoid", and "helix" are first presented and two students, Máximo and Nicolás, interact with the teacher in their learning of the concepts underlying these words. These two students try to infer the meanings of the new words from the real context suggested by a tornado. Particularly interesting here is the way in which the speakers co-learn individual vocabulary terms by repeating one another's talk, completing the other's turns and providing supportive feedback. In this case, there is no difference between Catalan and Spanish in relation to the three new words except for some accents that do not affect the pronunciation.[5] The teacher models appropriate language behavior by only using the Catalan and referring to the idea of word family (helicoidal, helicoid, and helix):

EXCERPT 1

Teacher: [C] Sabeu què és un moviment helicoïdal? / Do you know what a helicoidal motion is?

Máximo: [C] Bé, sabem el que és un tornado. / Well, we know what a tornado is.

Nicolás: [C] I sabem que un tornado es mou amb facilitat i rapidesa. / And we know that a tornado moves easily and quickly.

Teacher: [C] Un tornado va recte endavant i també gira, oi? Això és un moviment helicoïdal. / A tornado goes straight forward and it also turns around, ok? This is a helicoidal motion.

Máximo: [C] Un tornado va recte i cap avall. Com es diu? Helicoïdal? / A tornado goes straight forward and down. How do you say it? Helicoidal?

Teacher: [C] Es diu igual. Un moviment helicoïdal. Una helicoïde. Una hèlix. És el mateix. / You say it the same. A helicoidal motion. A helicoid. A helix. It is the same.

Nicolás: [C] Llavors un tornado és un moviment helicoïdal. / Then a tornado is a helicoidal motion.

Máximo: [C] És un moviment helicoïdal que va recta avall i gira. / It is a helicoidal motion that goes down straight forward and turns

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[5] The Catalan words for helicoidal, helicoid, and helix are "helicoïdal," "helicoïde," and "helix," and the Spanish are "helicoidal," "helicoide," and "helix." In the Catalan bilingual contexts, this type of subtle differences in the pronunciation tend to be seen like dialectical features in speech.

around.

Teacher: [C] Això mateix, ara resoleu la qüestió. / That's it, now you solve the question.

In the interactions with the Spanish-dominant students in the small group initial discussions, the immigrant bilinguals also tend to use Catalan. Excerpt 2 illustrates the episode when Máximo and Nicolás go on with the use of Catalan in the context of getting familiar with the new words when talking to Luna and Eliseo, even when the teacher is not standing next to them. These students maintain the idea of word family and do not accept incorporating the word "spiral" in their small group talk, although mathematically the helicoid and the spiral are two names that refer to rather similar curvilinear motions.[6] Both the helicoid and the spiral have in common the idea of representing a curve in motion and some of the shared geometrical meanings associated with these words are helpful in the representation of a tornado. Eliseo points to the idea that the understanding of the concept "tornado" is more important than the words used to describe it, but then immediately corrects this approach and rejects talking about spirals. One can sense in this excerpt possible tensions between the focus on the language and the focus on the mathematics, specifically between the idea of practising the new vocabulary ("We are talking about helixs, not spirals") and the idea of exploring geometrically similar mathematical concepts ("It is a bit of a spiral").

EXCERPT 2

Máximo: [C] Hem de dibuixar un tornado i assenyalar el moviment helicoïdal. / We need to draw a tornado and indicate the helicoidal motion.

Luna: [C] Hem de fer l'espiral. / We need to make the spiral.

Nicolás: [C] Una espiral no, una helicoïde, una hèlix. / Not a spiral, a helicoid, a helix.

Eliseo: [C] El que hem de fer de debò és un tornado. I li hem de dir una helicoïde. / What we really need to make is a tornado. And we need to name it a helicoid.

Luna: [C] Hem de fer les fletxes com ahir? / Do we need to make the arrows like yesterday?

Eliseo: [C] El que hem de fer és entendre què és un tornado i després li busquem un nom. / What we need to do is to understand what a tornado is and then we find a name for it.

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[6] The helicoid is a three-dimensional coil that runs along the surface of a cylinder, while the spiral is a curve that emanates from a central point, getting progressively farther away as it revolves around the point.

Nicolás: [C] Però ara són fletxes d'un moviment helicoïdal. És una mica una espiral. / But now they are arrows of a helicoidal motion. It is a bit of a spiral.

Eliseo: [C] Estem parlant d'hèlixs, no d'espirals. / We are talking about helixs, not spirals.

Luna: [C] Estem parlant de fletxes helicoïdals. / We are talking about helicoidal arrows.

Máximo: [C] Hem de decidir les fletxes que dibuixem i ja està. / We need to decide the arrows that we draw and that's all.

Eliseo: [C] Primer pensem les fletxes, després les dibuixem i després en parlem. / First we think about the arrows, then we draw them and then we talk about it.

Máximo: [S] Esta idea de las flechas no es fácil. Tenemos que imaginar los diferentes movimientos que existen dentro del tornado. / This idea of the arrows is not easy. We have to imagine the different movements that exist within the tornado.

Eliseo: [S] Una flecha tiene que ser una línea recta para que el tornado baje. Tenemos la t para la translación. / An arrow needs to be a straight line for the tornado to go down. We have the t for the translation [he draws the arrows in Figure 1].

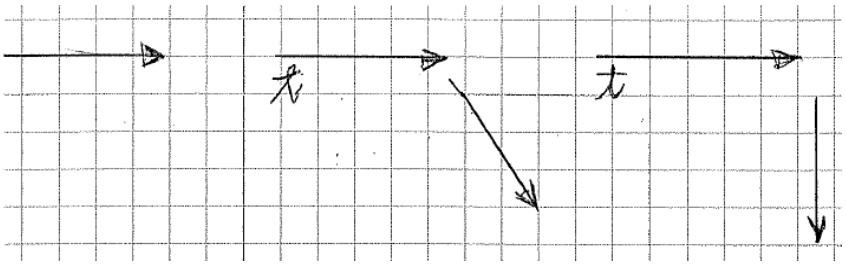

Figure 1. Eliseo's drawings representing different motions.

The students' initial interactions around the notion of helicoidal motion and the use of the related word family are centred on how this notion is represented in the context of the task and in relation to the language priorities that have been established by the teacher. At this stage, the students give priority to the use of the new words instead of making distinctions or stating similiarities between a spiral and a helicoid. The focus on the language rather than the mathematics changes when the students go back to their first language, Spanish. In general, the students' switching in the middle of certain conversations seems to be sensitive to the type of practices. The students tend to switch from Catalan – when "getting familiar with the new vocabulary"– to Spanish – when "solving the task"– and

maintain the switch to Spanish during the time devoted to argumentation in the small group. In Excerpt 2, Máximo uses Catalan when reproducing the new vocabulary and changes to Spanish when starting to develop more sophisticated arguments based on the coexistence of different simple motions within a helicoid. Eliseo goes on with the Spanish in his references to the idea of translation. If we look at the whole excerpt, we see how the students first go from Catalan to Spanish, and then from Spanish to mathematical symbols (see the arrows representing Euclidean vectors in Figure 1). On later occasions (see Figures 2 and 3), the mathematical symbols are also the result of a translation from Spanish.

The Development of Mathematical Argumentations in the First Language

Excerpts 1 and 2 show how well particular mathematical vocabulary is used in the context of the Catalan language. However the extracts do not give information about which of the mathematical meanings for the new technical terms ("helicoidal," "helicoid," and "helix") are already known, nor do they provide information about the process of exploring further some of the geometrical concepts that are being represented by these words. The Spanish-dominant students go back to their first language in the small group when they start experiencing some difficulties in the process of solving the problem. This was the case, for instance, with the group of Máximo, Nicolás, Luna, and Eliseo in the third lesson, as is illustrated by Excerpt 3.

EXCERPT 3

Luna: [S] Sí, dibujar las flechas. Pero, ¿qué flechas? Solo sabemos las de traslación y las de rotación. / Yes, to draw the arrows. But, what arrows? We only know those of translation and rotation.

Nicolás: [S] Puede ser una flecha en diagonal / It can be a diagonal arrow.

Luna: [S] Pero el tornado no sigue una dirección diagonal, va hacia abajo y gira al mismo tiempo. / But the tornado does not follow a diagonal direction, it goes down and it turns around at the same time.

Máximo: [S] La helicoide es como una flecha rota. Tenemos la T para el tornado. / The helicoid is like a a broken arrow, We have the T for the tornado [he makes the drawing on the left in Figure 2].

Eliseo: [S] Un tornado es mucho más complicado. Yo lo haría así. / A tornado is much more complicated. I will do it like this [he makes the drawing on the right in Figure 2].

Luna: [S] Yo no creo que un tornado pueda representarse con flechas. Cuando lo ves, no va a trompicones, ahora esta dirección y luego la otra. / I don't think that a tornado may be represented with arrows. When you look at it, it doesn't go by staggering, now this direction and then the other.

Nicolás: [S] Ninguno de los dibujos que habéis hecho es real. Un tornado

se mueve como un círculo y vosotros habéis hecho solo líneas rectas / None of your drawings are real. A tornado moves like a circle and you have only made rectilinear lines.

Eliseo: [S] Ahora sí que tiene sentido hablar de las espirales. / Now it does make sense to talk about the spirals.

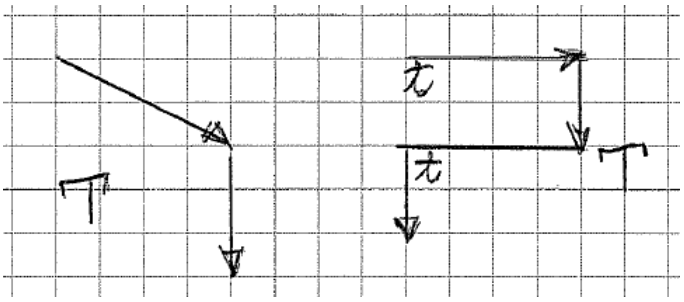

Figure 2. Some of the students' initial drawings representing a tornado.

Excerpt 4 starts with Luna asking for help. Nicolás goes back to Catalan and points to some key terms for the understanding of the statement of the task and its purpose. As soon as Luna shows some understanding of the meaning of the key words in the context of the task, Nicolás goes on with the mathematical explanations of the arrows in Figure 2 and again uses Spanish. Eliseo insists on introducing the notion of spiral, which is now accepted, probably because on this occasion it is not seen as an obstacle in the learning of new vocabulary (see Excerpt 2). The references to the curves of a spiral will help to complete the linear representations in Figure 2 with curves (see Figure 3). It is interesting to note that by manipulating the sociolinguistic dimensions of an initial task, it is possible to construct tasks of different difficulty levels. First, when the approach to the task was focused on the learning of a word family, the notion of spiral is not accepted and the idea of curvilinear lines is not taken into account. Later, when the approach seemed to be centered on the learning of the mathematics, the notion of a spiral is allowed and as a result a more accurate representation of a tornado is achieved.

EXCERPT 4

Luna: [S] La pregunta pide representar un tornado, ¿no? / The question asks to represent a tornado, doesn't it?

Nicolás: [C] Sí, diu que s'ha de representar matemàticament un tornado. / Yes, it says that we need to mathematically represent a tornado.

Luna: [C] No és parlar d'un tornado, és representar-lo matemàticament.

/ It is not to talk about a tornado, it is to mathematically represent it.

Eliseo: [S] Nos puede ser útil dibujar un tornado antes de representarlo. / The drawing of a tornado can be useful before its representation.

Nicolás: [S] Está claro que con una sola flecha no basta, porque un tornado es más que una traslación. / It is clear that only one arrow is not enough, because a tornado is more than a translation.

Eliseo: [S] Hay que pensar en cómo dibujaríamos una espiral. Dibujaríamos curvas. / We need to think about the drawing of a spiral. We would draw curves.

As evident in the above excerpt, these four immigrant bilinguals use their two languages for different purposes. They use Catalan when getting familiar with new vocabulary, when situating the use of this vocabulary in the context of the given task, and when beginning to organise approaches to solving the task. However, they use Spanish, their dominant language and the language that they share with their small-group peers, when arguing and counter-arguing at various degrees of specificity and developing more complex comprehension processes that are not centered on the repetition of some of the teacher's words and sentences. Our findings, concerning the use of the first language when elaborating on an argumentation, fit with Moschkovich's data (2007a), which showed that Latino students use Spanish to justify an answer or elaborate on an explanation and return to English when being asked by the teacher to give priority to the acquisition of new vocabulary.

An important question for research on language socialisation and mathematical competence in a bilingual classroom is to what degree the practice of mathematics in the first language during the small group interactions results in increased engagement with the mathematics during the whole group interactions, where the teacher uses the students' second language. The data from the whole group interactions in the five lessons show that Máximo's group tends not to interact when the teacher asks the groups to present their reasoning. In the small group, their engagement with the mathematics does not lead to an increased participation in the whole group, although they are allowed to use Spanish. Their engagement with the mathematics in Spanish also does not support their increased participation in Catalan outside the context of the small group, although they are allowed to use the Spanish language. They talk mathematics in Catalan with their small-group peers but then tend to remain silent in the whole group, even when they are prompted by the teacher to talk. On the few occasions that the teacher asks these students directly to communicate, they engage in short interactions in Catalan. Conversely, the local bilingual students tend to volunteer information umprompted, even interrupting the teacher to do so. Excerpt 5 illustrates one of the few turns, at the end of the third session,

when one immigrant bilingual, Eliseo, appears as a learner who is mathematically literate in Catalan.

EXCERPT 5

Teacher: [C] Eliseo, per què no dius res? Sé que heu estat treballant en el vostre grup. / Eliseo, why don't you say anything? I know you've been working in your group.

Eliseo: [C] Hem dibuixat algunes fletxes. / We've drawn some arrows.

Teacher: [C] I quina heu triat finalment? Vols dir-ho en castellà. / And which have you finally chosen? Do you want to say it in Spanish?

Eliseo: [C] Sabem que han de ser almenys dues fletxes i una és vertical perquè el tornado va cap avall. / We know that there are at least two arrows and one is vertical because the tornado goes down.

Teacher: [C] Una translació vertical? / A vertical translation?

Eliseo: [C] Una vertical. / A vertical.

Teacher: [C] Els altres possibles moviments? / The other possible motions?

Eliseo: [C] El tornado gira; creiem que una fletxa ha de ser la de la rotació. / The tornado turns around; we think that one of the arrows has to be the rotation [he shows the drawings in Figure 3 from his notebook].

Teacher: [C] I què més? Abans parlàveu molt. / And what else? You were talking a lot.

Eliseo: [C] Res més. / Nothing else.

Teacher: [C] Bé, anem al grup de la Maria. / Well, let's go to Maria's group.

The social conditions of power, distance and imposition are expected to be different in the two settings given by the whole group (with the teacher and the local Catalan bilingual students) and the small group (with the Spanish bilingual students). The social distance between the speakers and the degree of imposition are thought to make the mathematical participation more demanding to perform in certain situations and settings than in others. For instance, in a classroom situation where a student interacts with the teacher, a larger social distance is more likely compared to a situation of peer-peer interaction. Taguchi (2007) shows that the social conditions in the development of tasks affected the use of the two languages in English language learners and the students' participation. In that study, the students actively used their primary language across situations where they had to interact with peers who shared the same first language and where a lesser degree of politeness was required due to the assumption of more equal relationships. Conversely, they tended to use their second language or to remain silent when the teacher was standing next to them and listening to their interactions. Excerpt 5 prompts us to reflect on these findings. When Eliseo, talking in Catalan, says "Nothing else" to the teacher, the discussion moves to another group and Eliseo becomes a "listener" without clarifying the seeming misunderstanding. The teacher could have commented on Eliseo's interpretation of a "translation" and explored confusions in a productive manner. In all his drawings, Eliseo writes the symbol t for

translation when the arrow has a horizontal position from the left side of the paper to the right side. When the teacher refers to the idea of vertical translation, he does not revoice the whole expression but only mentions the relative position.

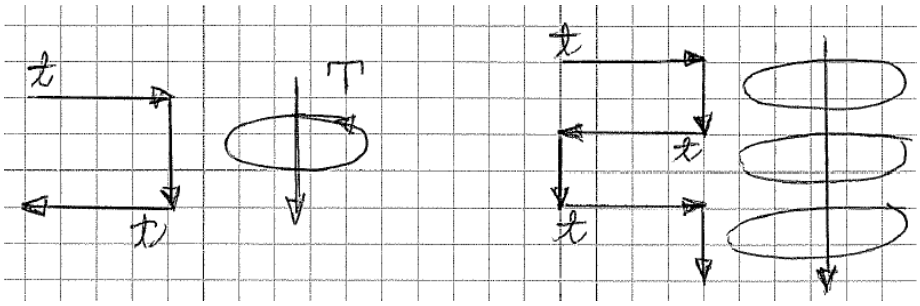

Figure 3. Some of the students' final drawings representing a tornado.

When Eliseo starts explaining the answers from his group to the whole class at the end of the lesson, although he is offered the option of using his first language ("Do you want to say it in Spanish?"), he chooses his second language, Catalan. The fact that the teacher and the other students, who have Catalan as a dominant language, only use Catalan in these discussions is a contributor to this choice. On the other hand, Eliseo's contribution is limited with the teacher having to draw out each of the student's responses ("Let's go to Maria's group"). The positioning of students through the structuring of the lesson and the acceptance by all concerned about who can interact in particular ways is an important outcome of our research. The choice itself of Catalan can be thought of as a contributor to the fact that the South-American students tend to be either silent or provide only minimal explanations to the whole class, even when they have a better understanding of the problem than others who take more initiative even though when not being called on by the teacher.

Conclusion

In this paper we have described certain commonalities in the use of Spanish and Catalan by a group of bilingual students. Our data show that the immigrant bilinguals' social interaction with classroom mathematical practices is accomplished in two languages. However, they tend to use each of the two languages for different purposes, in different domains of mathematical practices, and in relation to different social settings within the

classroom. This data is in line with our expectations based on Grosjean's (1997) complementarity principle. First, when the Spanish dominant bilinguals in our study were prompted by the teacher to become familiar with the task within their small group and learn new mathematical vocabulary, they used Catalan, which was the language in which this vocabulary had been introduced. Second, when these students were more deeply working on the solution to the task within their small group, they changed to Spanish although they did go back to Catalan for certain clarifications. Third, when the time for the whole group discussion began, they contributed only when they were directly prompted by the teacher and, when this happened, they used Catalan. These findings raise the more general issue of how the context of mathematics lessons influences the choice of language, and how the way that the teacher structures the lesson influences the bilingual students' choice to use their second language before returning to their first language.

The immigrant bilinguals' lack of spontaneous participation in the whole-group mathematical discussions, in comparison to their full participation in the small-group context, is an important finding. These students can engage with the whole-group mathematical discussions by using either Catalan or Spanish, but the teacher and the other bilingual students only use Catalan – these language practices may be seen as representing a certain degree of symbolic imposition. The full participation in the whole group would require a transformation of the understanding of and participation in the classroom context, namely the construction of new social knowledge. For example, Eliseo's small participation in the whole group, depending on the teacher's demands and being entirely in Catalan, is very different from this student's full and spontaneous participation in the small group, being almost entirely in Spanish. The development of Eliseo's identity as a learner of mathematics in a classroom seems to be under the influence of the social setting (small group vs. whole group), the language (Spanish vs. Catalan), and the mathematical practices (acquiring vocabulary vs. explaining, arguing, and representing mathematical symbols). Eliseo's participation may be seen as accomplished through repeated interactions with the others, especially with the teacher and the most influential peers. Máximo, Nicolás, and Luna do not interact spontaneously during whole group discussions, nor do they talk in Spanish when they are directly asked by the teacher to explain their reasoning. However, they all have the ability to use Catalan to initiate engagement with the mathematics, Spanish to go on with this engagement, and return to Catalan when helping each other in the clarification of the task and explaining the main ideas to the whole class.

The bilingual students' identities as mathematical learners are framed in terms of their expectations concerning what they might achieve – or lose –

by speaking in one of the two languages, given their different levels of proficiency and the role of each language within the classroom. These expectations are presumably related to the construction of different and simultaneous identities and their orchestration into practice. The immigrant students in our study tended to show a more active learner identity when using Spanish in the small group mathematical discussions and a more passive "listener" identity when using Catalan in the whole group. This distinction is to be interpreted in the social context in which the mathematical talk is produced and in relation to the interactions and expectations that are further promoted. In the whole group, in interaction with the Catalan-dominant students and the teacher, the immigrant bilinguals tended to invoke their identity as "listeners", and, therefore, open up the possibility that other participants orient to them in this way, rather than as "questioners." Máximo, Nicolás, Luna, and Eliseo were, however, the only group that have succeeded in representing the tornado by means of a composition of straight lines and curves. They came close to solving the problem. Their full participation in the whole group would have been valuable because they knew something that the other students did not.

A deeper affirmation of the language identities of the students in the bilingual mathematics classrooms – in our case, through an active promotion of both Spanish and Catalan in the whole class discussions – and a clearer focus on the mathematics might have the power to increase the quality of the mathematical talk and transform the sort of interaction among groups that takes place in these classrooms. The teacher's initial focus on the language, instead of the focus on the language identities and the mathematics, led in our examples to the consideration of a word family (helicoidal, helicoid, and helix) rather than a "concept family" (helix and spiral) that might have helped the students' comprehension of the circular motions represented by a tornado. Only when the students overcame the repetition practices concerning the learning of new vocabulary did they feel free to introduce the idea of a spiral. Moreover, the poor role of the immigrant students' first language in the whole group discussion, where these students privileged the use of the language over the explanation of the mathematics, appeared to be an obstacle in the co-learning of the helicoidal motion because the students who had developed the most accurate representations were the ones who had more difficulties in participating. The introduction of the language identities in the mathematics classroom needs to pay attention to the complexity and unpredictable conditions of these identities, that cannot be simplified on the basis of the cultural origins of the individual. The fact that Eliseo is a Spanish-dominant speaker who was born in Colombia and who is now attending a school in Barcelona is part of this student's language identity but this identity also has to do with his experiences in the

interaction with the others. These experiences may explain in part the reasons for the different uses of the two languages and the difficulties in certain types of social participation structures.

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