

# Technological endowments in entrepreneurial partnerships\*

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## Abstract

This paper discusses a novel argument to interpret the importance of thinking of collaborative partnerships in pre-competitive agreements. To do so, we adopt a dynamic iterative process to model technology diffusion between the partners of an agreement. We find that the success of an agreement of a given length hinges around identifying the suitable efficient combinations of the initial technological endowments of partners. As the time horizon of the agreement expands, the probability of identifying a suitable partner decreases, thus justifying the prevalence of short-horizon R&D agreements.

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JEL classification: C61, L13.

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# 1 Introduction.

Consider the following situation: an enterprise has decided to sign a collaborative contract to develop a certain R&D project with a finite horizon. There is a number of potential partners in the market. Which are the ones allowing for a successful completion of the project? Among those, which one should be chosen?

Also, after the (successful) completion of the project, the enterprise may envisage the possibility to extend the collaborative R&D activity. Should it do so with the same partner, or should it start the procedure to select a partner anew, instead?

This paper attempts to answer these questions. We argue that the time horizon and the initial technological endowments are crucial elements in the choice of the collaborative partner even in a perfect information set-up. In other words, we sustain that the failure of a project may well be due to the choice of a wrong partner rather than to the lack of quality of the project. In this respect we complete theoretically the empirical arguments discussed in Lokshin *et al.* (2011) that identify to which extent bad performing partnership outcomes entails a failure of the partnership agreements (often around 30% of signed agreements with R&D purposes). Assuming that the conditions guaranteeing the existence of an optimal contract are fulfilled if the contract is signed, the choice of the right partner is fundamental and we are focusing on this issue.

The relevance of these questions arises in the framework of the recent growth in the tendency of firms to engage in partnership agreements in R&D as a means to increase their competitiveness (an extensive discussion can be found in Gillier *et al.*, 2011). The economic literature devotes quite a lot of attention to this phenomenon from the empirical viewpoint. However, to the best of our knowledge, there is no theoretical contribution that studies how firms select partners with which to sign successful pre-competitive agreements. In this paper, we intend to focus on it by looking at the importance of a firm's initial technological endowments, for instance, when deciding to sign a particular kind of collaborative R&D agreement.

During the period 1991-2001, international technology alliances increased from 339 to 602. U.S. firms were always the most involved in this process, participating

in about 75% of all contracts (UNCTAD, 2005). A common evidence of these collaborative activities is that short contracts are preferred to long contracts. Several arguments have been offered to support this observation. It may be that one party needs to gather information on the other party, particularly regarding its trustworthiness and willingness to cooperate in the future (Aghion *et al.* 2002). Harrigan (1986) stresses that firms engage in different types of R&D partnerships to exploit knowledge in new applications and to enter into new fields. Indeed, these ventures allow firms to share research costs, save on assets, and avoid duplicative laboratories and testing periods (see Hagedoorn, 2002, Roijakkers and Hagedoorn, 2006 (focusing on the pharmaceutical sector) and Frankort *et al.* 2012). These collaborative agreements cover technology and the sharing of R&D between two or more companies in combination with joint development projects. As an illustration, Segrestin (2005) explores the Renault-Nissan alliance as a new way to develop high-risk innovative business opportunities involving the design of a new collective identity. This (successful) alliance had to cope with coordination and cohesion issues in the form of a new managerial organization, and the appropriateness of existing legal frameworks to a new entity. All along the process, both manufacturers could refrain from collaboration if the threat of opportunism outweighed profit expectations. As a second illustration, Iveco-Fiat has a record of research programs containing high technological content with European and non-European partners. One interesting feature of those contracts is that their length changes according to the partners and nature of the agreement and that some programs can be renewed (see the case of the programs Chauffeur I and Chauffeur II<sup>1</sup>). A few studies have already suggested interesting arguments to justify the profitability of collaborative agreements in case of technological similarities among the participating firms (e.g., Chesbrough, 2003, Segrestin, 2005, or Zeller, 2004), but all the other situations are still unexplored. In contrast, our contribution will assess that a successful agreement may also arise between firms with both similar and differentiated technologies.

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<sup>1</sup>See [http://cordis.europa.eu/telematics/tap\\_transport/research/projects/chauffeur.html](http://cordis.europa.eu/telematics/tap_transport/research/projects/chauffeur.html).

Our research question is to examine whether firms' initial technological endowments are relevant for the successful completion of an agreement in a dynamic framework with learning. The length of an agreement turns out to be a crucial element in the cumulation of advantages stemming from the collaboration, along with the selection of the suitable partners. It is the influence of an implicit process of cumulation of experience that eventually allows for the selection of initial technologies that lead to successful collaborations. We address this issue by studying a model of competition of partners in separated markets under long and short partnership agreements and extend the conclusions to the case of interrelated markets. The main contribution of this paper is the modeling of the evolution in time of a firm agreement to study how initial technological conditions determine its success in a dynamic framework.

Formally, we set-up a two-stage model where two (horizontally) differentiated firms compete in prices in the final product market in the second stage, while in the first stage they decide on signing a technological agreement. With this two-stage structure we capture the idea that firms when considering signing the agreement foresee its consequences on the competition in the final product market. In other words, we assume that firms have more flexibility in adjusting prices than in signing technology agreements. More precisely, the price competition stage is modeled following Sing and Vives (1984) approach (see section 2). To induce the possibility of collaboration in the first stage, we assume that firms bear different technologies yielding different levels of technological efficiency. This difference in efficiency generates the opportunity for signing an agreement entailing the development of a joint technology whose outcome is the cut of the production costs in the second stage. Therefore, our focus is to study a dynamic model of price competition whose driving force is the temporal dimension embedded in the development of the joint technology that follows a diffusion process to take into account the learning stemming from the joint action of the two partners. Section 2.1 contains the detailed description of the elements characterizing the content of the agreement: a diffusion process that combines the initial technological endowments of the firms and makes

their available technology evolve across time.

The dynamic mechanism proposed, besides its technical features, gives rationale to the empirically contrasted feature of the increasing difficulty of engaging in long-term R&D partnerships. Using data from several countries, Canepa and Stoneman (2004) identify that the principal determinants of diffusion patterns in manufacturing technologies (whose size is quite relevant) are those generally classified as epidemic and ranks effects (as also discussed in Karshenas and Stoneman, 1993). We focus on agreements where both parties benefit from the advantages of their collaboration in the research stage while maintaining their own identity and independence, either in the commercialization phase of the final product or in the adoption of a new productive process. We also assume that a successful agreement will allow firms to produce more efficiently without closing the initial technological gap between them. These assumptions although extreme try to maintain the analysis clean of other arguments (such as the interest of technologically lagged parties to catch up through collaborative contracts) already in the literature and allow us to point to our novel argument to understand the importance of initial endowments on the success of entrepreneurial partnerships.

This paper is organized as follows. Section 2 presents the main building blocks of the theoretical setting. Section 3 deals with the definitions of the terms of the agreement, and Section 4 presents the initial conditions suitable to ensure successful agreements. Section 5 discusses the implications of such results and Section 6 presents our conclusions.

## **2 The model**

In our theoretical development, we proceed by following a backward approach. We start illustrating the (second-stage) price competition equilibrium between two firms holding different technologies and, then, we will focus on the agreement issue (first-stage). Following Vives (1999) and Singh and Vives (1984), we consider a differentiated duopoly with two firms  $i = 1, 2$  competing à la Bertrand in the final

product market. Market demands are linear and given by

$$q_i = a_i - b_i p_i + c p_j, \quad i, j = 1, 2; \quad i \neq j; \quad a_i > b_i. \quad (1)$$

Firms use constant but different marginal cost technologies given by,

$$C_i(q_i) = \xi_i q_i, \quad i = 1, 2, \quad (2)$$

where  $\xi_i \in (0, 1]$  is an exogenous parameter representing firm  $i$ 's capacity to control (and reduce) production costs.<sup>2</sup> For simplicity, we normalize  $\xi_1 = 1$  and assume  $\xi_2 = \xi \leq 1$ . If firms use the same technology then  $\xi = 1$ , while the more efficient firm 2 is with respect to firm 1, the lower is  $\xi$ .

We introduce the technology transfer process below. Its particular features will redefine the cost functions in accordance with the horizon of the agreement (see section 3).

Let,  $\hat{p}_i$  denote firm  $i$ 's price net of its marginal cost. Solving for the profit maximizing problem, we obtain prices,

$$\hat{p}_1^* = \frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2}; \quad \hat{p}_2^* = \frac{2b_1\hat{a}_2 + c\hat{a}_1}{4b_1b_2 - c^2}, \quad (3)$$

where  $\hat{a}_1 := a_1 - b_1 + c\xi$ , and  $\hat{a}_2 := a_2 - b_2\xi + c$ . Also, we assume  $4b_1b_2 > c^2$ , so that the economic logic requiring positive prices and quantities is satisfied.

In the case of independent goods (i.e.  $c = 0$ ), markets are separated, and we obtain monopoly prices:

$$\hat{p}_1^m = \frac{a_1 - b_1}{2b_1}; \quad \hat{p}_2^m = \frac{a_2 - b_2\xi}{2b_2}. \quad (4)$$

Note that these prices given our assumptions, are strictly positive. From the equilibrium prices (3), we compute the associated equilibrium quantities,

$$q_1^* = b_1\hat{p}_1^*; \quad q_2^* = b_2\hat{p}_2^*. \quad (5)$$

Finally, equilibrium profits are given by,

$$\Pi_1 = b_1 \left( \frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2} \right)^2; \quad \Pi_2 = b_2 \left( \frac{2b_1\hat{a}_2 + c\hat{a}_1}{4b_1b_2 - c^2} \right)^2. \quad (6)$$

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<sup>2</sup>Note that we assume  $\xi_i > 0$ . This is so because  $\xi_i = 0$  would mean that firm  $i$  has already exhausted its possibilities to lower costs, thus pre-empting its participation in any agreement.

For future reference, monopoly profits are,

$$\Pi_1^m = \frac{1}{b_1} \left( \frac{a_1 - b_1}{2} \right)^2 ; \quad \Pi_2^m = \frac{1}{b_2} \left( \frac{a_2 - b_2 \xi}{2} \right)^2 . \quad (7)$$

## 2.1 The terms of the agreement

In the first stage of the partnership process, firms decide to sign an agreement towards improving their efficiency in production. Such an agreement is characterized by five elements:<sup>3</sup> (i) the agreement lasts for a fixed number of periods; (ii) the two parties agree not to use the outcome of their collaboration prior to its completion, so that neither party obtains a competitive advantage in the final goods market; (iii) the benefits stemming from the agreement do not allow the technologically-lagged firm to level up with its partner; (iv) firms compare profits period by period to determine the optimal length of the agreement; and (v) once the agreement is completed, firms implement the reduction of production costs (as also studied in Chipman, 1970).

We simplify the content of the contract by ignoring the penalties incurred when partners fail to honor the contract.<sup>4</sup> Instead, we impose its enforcement.

Features (ii), (iii) and (iv) provide the incentives for collaboration. In particular, (ii) simply states that a contract implies commitment. Feature (iii) avoids the possibility of only the less technologically-developed party being willing to collaborate. Although it may be somewhat restrictive when partners belong to the same market, it eases collaboration that would otherwise be unfeasible. Alternatively, when partners belong to different markets, the assumption is innocuous. Finally, feature (iv) places us in the worst possible scenario for collaboration. Allowing for the possibility of profit transfers across periods only eases the possibilities of collaboration. In this sense, our results have to be understood as limit results. Nevertheless, one of our results states that the negative relationship between the probability of find-

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<sup>3</sup>As it appears, for instance, in the Operating procedures of the HDP Inc at [http://www.hdpug.org/sites/all/files/Documents/HDP\\_Op.Procedures\\_approved\\_1006.pdf](http://www.hdpug.org/sites/all/files/Documents/HDP_Op.Procedures_approved_1006.pdf).

<sup>4</sup>As we will argue in the next section, in that case our results would be even more stringent because of the structure of our approach.

ing partners and the time span of the agreement is robust to inter-temporal profit transfer.

We model the content of the agreement as a technology diffusion process between the two firms. Jayaraman *et al.* (2004) state that a distinguishing feature of a general technology transfer process is that the rate of diffusion in a particular location at time  $t$  is proportional to the present level of diffusion and the level of diffusion to be achieved.<sup>5</sup> Similarly, the rate of assimilation of a technology (in an industry) turns out to be proportional to the existing level of technology and that to be achieved. To study the evolution of a general technological assimilation process, the authors select the logistic function as the most suitable functional form. As the aim of a pre-competitive agreement is to improve the partners' technologies, we follow Jayaraman *et al.* (2004) and also adopt the logistic function. This choice allows us to interpret the dynamics of an agreement as the effort put by the firms (across time) in accomplishing the objective of the contract.<sup>6</sup> One firm plays the role of transferor and the other firm plays the role of transferee, while keeping the common objective to reduce cost in mind.

In doing so, we also recover a very common feature in literature of the development and spreading of a new technology. We follow some well-known models in industrial organization literature, such as Mansfield (1961) or De Palma *et al.* (1991). The rationale of this choice is the following: the adoption of a new technology as well as its development moves along a path at a constant rate. There are differences in adoption time simply because potential adopters are heterogeneous and react differently to the new technology (Baptista, 1999). As stated in Mansfield (1961), a logistic process is the most suitable process to model such a development because it bears the difference in speeding the adoption of a new technology along the development path. This process requires that the returns from the agreement are higher at the beginning of the collaboration (because of the novelty effect), then slowing down before finally reaching a constant motion. The most benefits are re-

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<sup>5</sup>This finds empirical and theoretical support in the studies by Mansfield (1961), Stoneman (1981) Karshenas and Stoneman (1993), or De Palma (1991).

<sup>6</sup>We are implicitly assuming the absence of any kind of free-riding behavior, as anticipated at the beginning of this section.

alized at the moment when the two firms sign the agreement, while the returns reduce proportionally as time passes. In our framework, this is not a crucial point: firms agree just for a particular project running for a short-period, while the criticism addressed above deserves the most attention when considering an adoption process in the long run. Here firms join their efforts to develop a unique technique and the project follows its own path separated from the other activities of both partners, even if each firm can only enjoy the benefits of the partnership at the end the entire production.

Our model is set in discrete time, so we need to specify the logistic sequence mapping of the diffusion process that Jayaraman *et al.* (2004) defined in continuous time. Once the two partners begin collaborating in the common project, they acquire new knowledge, causing their initial common stock of knowledge to evolve. Therefore, we model this evolution as a diffusion process in discrete time. We model the dynamics of the technological implementation process as follows: firms agree in achieving a certain objective  $a$  at time  $t$  by means of a dynamic diffusion process  $\lambda_t$ .

Let us denote by  $\lambda_0$  the stock of knowledge that both partners share at the beginning of the agreement.

**Assumption 1.** *Let  $\lambda_0$  be the initial value of  $\lambda_t$ . We define it as:*

$$\lambda_0 = \xi_1^\alpha \xi_2^\beta \in (0, 1], \text{ with } \xi_i \in (0, 1], \alpha, \beta \in (0, 1) \text{ for } i = 1, 2.$$

The value  $\lambda_0$  represents a composition of the initial levels of technology of the firms. We assume that once a firm subscribes an agreement, it discloses its technological information (embedded in its production function) to the partner. Parameters  $\alpha$  and  $\beta$  stand for the relative weight that each firm has in the agreement. Accordingly, we assume  $\alpha + \beta = 1$ . Modeling the combination of the technology as a Cobb-Douglas function allows for a full capture of the efforts between the two partners by accounting for individual participation by each firm (to the realization of the project) and the externalities that can emerge by the joint action. Addition-

ally, note that from the definition of  $\xi$ , lower levels of  $\lambda$  means higher efficiency to reduce production costs.

**Definition 1.** *Let us consider a pre-competitive agreement between two firms lasting for  $t$  periods. The diffusion process embedded in  $\lambda_t$  is modeled as:*

$$F^t(\lambda_0) = \lambda_t = \mu\lambda_{t-1}(a - \lambda_{t-1}), \text{ for } t = 0, 1, \dots, n, \mu > 0, \lambda_0 > 0. \quad (8)$$

The process just described (see May, 1976 and Li and Yorke, 1975) is a quadratic and concave function in  $\lambda_t$ , that for particular values of  $\mu$  will lead to a chaotic behavior. In the next section, we will precisely define its domain of existence and will define its structure.

Equation (8) states that  $\lambda_t$  increases from one period to the next when it is small, and decreases when it is large. The parameter  $\mu$  is a multiplier of this dynamics and  $a$  represents the horizontal asymptote, ideally the limiting value of the combination of the two technological endowments. This affects the steepness of the hump in the curve. It captures a cumulation process that appears when the agreement lasts for several periods. In terms of our model, this process can be interpreted as follows. By construction,  $\lambda_0 \in (0, 1]$  and equation (8) is built around  $\lambda_0$ . Hence, there is a continuum of possible agreements that spans from cases in which firms participating in an agreement display different technologies ( $\lambda_0$  small) to cases in which firms are similar in technology ( $\lambda_0$  large). The expected benefits of the two extreme types of agreement are different. The maximum is reached at a point where technologies are not identical but still match in an optimal way. This occurs because the law of motion of  $\lambda_t$  given by (8) is quadratic and concave in  $\lambda_t$ .

Taking for granted that the optimal contracts supporting such agreements exist (see Pérez-Castrillo and Sandonís, 1996 and Veugelers and Kesteloot, 1994), our concern is to find the initial technological conditions that allow two firms to sign an agreement that leads to an optimal and successful outcome.

We split our analysis into two parts. First, we study agreements that do not span over time. We characterize the constellation of firms' profiles allowing to mutually benefit from an agreement. Next, we introduce the time dimension. The degree of

differentiation of products supplied by firms may range from independent goods (so that firms serve separate markets) to some degree of substitutability, so that markets will be interrelated. We will consider in detail the former case and provide some insights on the latter.

Finally, a remark. Note that we assume that the bargaining powers  $\alpha$  and  $\beta$  are constant over time (in Assumption 1). Alternatively, it could be envisaged a more general formulation with  $\alpha(t)$  and  $\beta(t)$ . There are two arguments advising against pursuing this line of analysis. In economic terms, if the relative weights of the firms in the negotiation vary with time, firms would be able to renegotiate the terms of the agreement each period. This would lead us to considering incomplete contracts where its stability and optimality is in jeopardy (see Pérez-Castrillo and Sandonís, 1996). Also, a situation where firms can renegotiate at any point in time the terms of the contract goes against the very spirit of a contract. From a technical viewpoint, assuming  $\alpha(t)$  and  $\beta(t)$  generates another type of dynamic process different from the one of the quadratic family we consider in Definition 1 whose full discussion deserves an independent study.

### 3 One-period agreements

Let us first study the (benchmark) case where agreements are signed in a static context. We find that the set of solutions is the union of two disjoint sets. From an economic viewpoint, we are assessing (see Proposition 1) successful agreements that take place either with firms with very similar or very different technological endowments. It may be the case of two competitors with very similar technological production systems, but may also be two very different competitors. An example of the latter would be the case of a firm operating in a developed country that signs an agreement with another firm in a developing country.

#### 3.1 Separate markets

Let us normalize  $a = 1$  in (8). Also, assume  $c = 0$  in (1). This means that goods are independent and firms are (local) monopolists in their respective markets. We

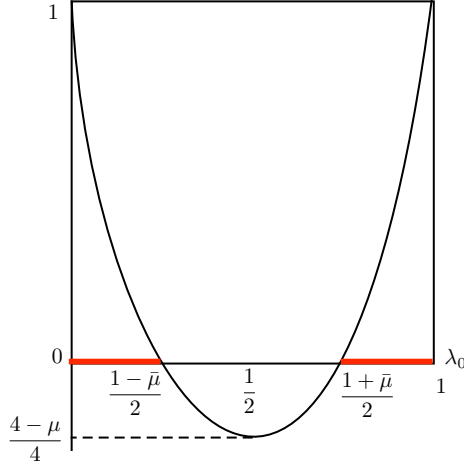


Figure 1: Agreement under static local monopolies and  $\mu > 4$ .

characterize the conditions under which those firms could profitably engage in an agreement to develop a more efficient (cost saving) technology.

**Definition 2.** Consider two firms signing a pre-competitive agreement lasting for one period ( $t = 1$ ). According to (2), the cost function of each firm changes as follows:

$$C_1 = F(\lambda_0)q_1 = \mu\lambda_0(1 - \lambda_0)q_1,$$

$$C_2 = F(\lambda_0)\xi q_2 = \mu\lambda_0(1 - \lambda_0)\xi q_2.$$

To decide whether to sign the agreement, these two firms simply compare profits with and without agreement. The outcome of this comparison is described in the proposition 1 and illustrated in Figure 1.

**Proposition 1.** Assume firms are local monopolies. They are willing to engage in an agreement when:

$$\lambda_0 \in \left[0, \frac{1}{2} - \frac{\bar{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\bar{\mu}}{2}, 1\right], \text{ if } \mu > 4,$$

$$\forall \lambda_0 \in [0, 1], \text{ if } 0 < \mu \leq 4$$

where  $\bar{\mu} = \left(\frac{\mu-4}{\mu}\right)^{\frac{1}{2}} \in (0, 1)$ .

*Proof.* We start by computing the corresponding equilibrium prices, quantities and profits for both firms.

$$\begin{aligned}\tilde{p}_1^m &= \frac{a_1 + \mu\lambda_0(1 - \lambda_0)b_1}{2b_1}; \quad \tilde{q}_1^m = \frac{a_1 - \mu\lambda_0(1 - \lambda_0)b_1}{2}, \\ \tilde{\Pi}_1^m &= \frac{1}{b_1} \left( \frac{a_1 - \mu\lambda_0(1 - \lambda_0)b_1}{2} \right)^2,\end{aligned}\tag{9}$$

$$\begin{aligned}\tilde{p}_2^m &= \frac{a_2 + \mu\lambda_0(1 - \lambda_0)b_2\xi}{2b_2}; \quad \tilde{q}_2^m = \frac{a_2 - \mu\lambda_0(1 - \lambda_0)b_2\xi}{2}, \\ \tilde{\Pi}_2^m &= \frac{1}{b_2} \left( \frac{a_2 - \mu\lambda_0(1 - \lambda_0)b_2\xi}{2} \right)^2.\end{aligned}\tag{10}$$

Note that since  $\lambda_0 \in (0, 1]$ , the equilibrium prices are well defined. Also, note that  $\lambda_0(1 - \lambda_0) < 1/4$ , so that when  $\mu \in (0, 4]$  it follows that  $\mu\lambda_0(1 - \lambda_0) \leq 1$ . Accordingly, the equilibrium quantities take strictly positive values (recall that by assumption  $a_i > b_i$  and  $\xi \in (0, 1]$ ).

Next, when  $\mu > 4$ , equilibrium quantities will be well defined if and only if  $a_i - \mu\lambda_0(1 - \lambda_0)b_i > 0$ , or equivalently, iff  $\mu\lambda_0(1 - \lambda_0) < 1$ . The roots of this inequality are

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\bar{\mu}}{2}\tag{11}$$

with  $\bar{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2}$ . Note that  $0 < \frac{1-\bar{\mu}}{2} < \frac{1+\bar{\mu}}{2} < 1$ . Therefore, equilibrium quantities when  $\mu > 4$  are well-defined for  $\lambda_0 \in \left[0, \frac{1}{2} - \frac{\bar{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\bar{\mu}}{2}, 1\right]$ .

Finally, to assess the incentive to participate in an agreement we need to compare profits. Given the symmetry of the equilibrium values, we can concentrate on firm 1 and extend the conclusions to firm 2. Comparing profits firm 1 gets in (7) and in (9), it is easy to see that firm 1 will participate in the agreement if and only if,

$$\frac{1}{b_1} \left( \frac{a_1 - \mu\lambda_0(1 - \lambda_0)b_1}{2} \right)^2 > \frac{1}{b_1} \left( \frac{a_1 - b_1}{2} \right)^2,$$

that reduces to a quadratic function of  $\lambda_0$ ,

$$b_1[1 - \mu\lambda_0(1 - \lambda_0)] > 0.\tag{12}$$

Given that  $b_1 > 0$  by assumption, we need to verify that  $[1 - \mu\lambda_0(1 - \lambda_0)] > 0$ . As we have seen above, this inequality admits real roots only for  $\mu > 4$ . These

are given by (11). Therefore, inequality (12) is fulfilled for  $\lambda_0 \in \left[0, \frac{1}{2} - \frac{\bar{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\bar{\mu}}{2}, 1\right]$ .

When  $\mu \in (0, 4]$ , the quadratic function (12) does not have real roots and the inequality  $[1 - \mu\lambda_0(1 - \lambda_0)] > 0$  is satisfied for any value of  $\lambda_0 \in [0, 1]$ .  $\square$

### 3.2 Interrelated markets

Assume that goods are substitutes ( $c > 0$ ) so that we have an interrelated market structure. The conditions under which firms may engage in a profitable agreement will be characterized by the degree of substitutability (the value of  $c$  in the demand system) and by the technical similarity between firms (the value of  $\lambda_0$ ). Again, we normalize  $a = 1$  in (8).

As before, firms will engage in an agreement if the agreement yields higher profits than otherwise.

**Proposition 2.** *Assume that goods are substitutes. Firms are willing to engage in an agreement if goods are either poor substitutes or close substitutes. Let  $\bar{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2} \in (0, 1)$ . Then,*

- For  $\mu > 4$ ,
  - (i) *for values of  $c$  small enough, the underlying technological initial conditions supporting a sustainable agreement are  $\lambda_0 \in [0, \frac{1-\bar{\mu}}{2}] \cup [\frac{1+\bar{\mu}}{2}, 1]$ , and*
  - (ii) *for values of  $c$  large enough the agreement is sustainable for  $\lambda_0 \in (\frac{1-\bar{\mu}}{2}, \frac{1+\bar{\mu}}{2})$ .*
- For  $0 < \mu \leq 4$ , *the agreement is sustainable only if  $c$  is small enough.*

*Proof.* See Appendix  $\square$

A symmetric argument will hold when goods are complements ( $c < 0$ ), by construction). Figure 2 illustrates.

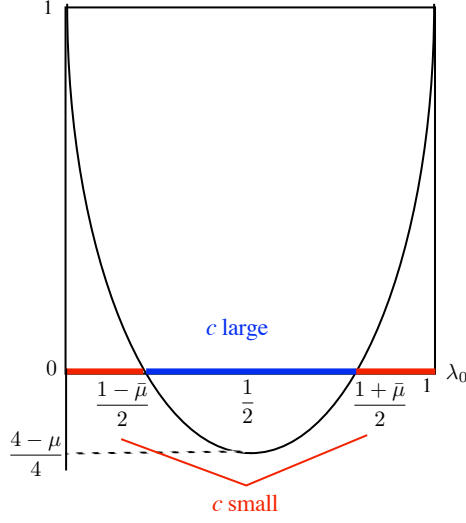


Figure 2: Agreement under static duopoly and  $\mu > 4$ .

In the remainder of this study, we implicitly consider the case of substitute goods. Concentrating the analysis to the case of independent firms has two purposes. On the one hand it simplifies the technical development of the model providing useful intuitions for the case of interrelated markets. On the other hand, looking at the partnerships that have formed along the years, we can identify a good proportion where their participants develop their activities in different sectors of the economy. In this sense, we think our analysis has value.<sup>7</sup> Results are robust to the substitute or complementary nature of final goods. The relevant feature is the degree of competition among firms.

## 4 Multi-period agreements

We extend the results obtained in the previous section by introducing the time dimension. In other words, we assume that when a firm takes a decision, it is aware of the advantages of the agreement, following an iterating process given by (8). We should note here that this diffusion process is well defined only for  $\mu > 4$ . We will maintain this assumption in the remaining of the analysis. With the logic

<sup>7</sup>Examples of these partnerships are agreements like Chamalon, Aider, or Chauffeur involving partners from the rubber, automobile, aircraft, electronics, and telecom industries among others.

developed in the single-period case, firms will now identify those combinations of technologies (embedded in  $\lambda_0$ ), guaranteeing a profitable agreement lasting for more than one period.

We will proceed in two steps. First, we will identify the conditions guaranteeing that subscribing to an agreement lasting for more than one period is profitable for each firm. That is, we examine whether there are combinations of technologies, embodied in the variable  $\lambda_0$ , giving firms the incentive to maintain their collaboration for  $t > 1$  periods (Lemma 1). Narula and Hagedoorn (1999) note that firms signing agreements look for profits in the short run. We transpose this evidence in our setting by imposing the (strict) condition that we only admit agreements that guarantee positive profits period by period (and not allowing for intertemporal monetary transfers). In this first step, we concentrate on a situation in which two local monopolies (i.e. duopoly in separate markets) may decide to extend the length of an existing agreement and we evaluate the conditions under which such a decision may be successful. The second step illustrates by means of an example how the set of solutions depends on the time horizon.

**Lemma 1.** *Consider two local monopolists and let  $\mu > 4$ . For an existing  $(t - 1)$ -period agreement, there is a range of values of  $\lambda_0$  allowing to extend the agreement one additional period. It is given by  $\lambda_0 \in (\frac{3}{4}, 1]$ .*

*Proof.* Given the structure of the iterative function, we can write  $\lambda_1 = \mu\lambda_0(1 - \lambda_0), \dots, \lambda_t = \mu\lambda_{t-1}(1 - \lambda_{t-1}), \lambda_{t+1} = \mu\lambda_t(1 - \lambda_t)$ .

As a consequence, the sequence of profits for, say, firm 1 in every iteration  $t$  is,

$$\tilde{\Pi}_{1t}^m = \frac{1}{b_1} \left( \frac{a_1 - \lambda_t b_1}{2} \right)^2, \quad t = 1, 2, \dots \quad (13)$$

Our local monopolist will be willing to extend the agreement from period  $t - 1$  to period  $t$  if and only if,

$$\tilde{\Pi}_{1t}^m > \tilde{\Pi}_{1t-1}^m. \quad (14)$$

Note that, from the expressions of profits, it follows that  $\text{sign}[\tilde{\Pi}_{1t}^m - \tilde{\Pi}_{1t-1}^m] = \text{sign}[\lambda_{t-1} - \lambda_t]$ . Accordingly, inequality (14) reduces to studying the values of  $\lambda$  satisfying  $\lambda_{t-1} - \lambda_t > 0$ .

Given that  $\lambda_t = \mu\lambda_{t-1}(1 - \lambda_{t-1})$ , the previous expression holds for  $\lambda_{t-1} > 1 - \frac{1}{\mu}$ . Given that  $\mu > 4$ , firms will be willing to extend the agreement from period  $t - 1$  to  $t$  if  $\lambda_{t-1} > \frac{3}{4}$ .  $\square$

Lemma 1 gives the consistency conditions ensuring that given an agreement of length  $t$ , there are no incentives to break it at an earlier period. These conditions involve firms' technologies being sufficiently similar. Note that equation (8), describing the diffusion of the technological change, considers  $\lambda_0$  as the initial (exogenous) condition. This is the description before the agreement of the technological differences between firms. Thus, the Lemma proves that, given some initial conditions, firms will maintain their collaboration period after period as long as the diffusion process maintains their technologies similar enough. Also, note that the degree of feasible similarity is increasing over time, even though the less efficient firm never catches up with its partner. Moreover, according to the expected length of the agreement, the magnitude of benefits over production costs varies.<sup>8</sup>

We illustrate the dynamics of a local monopolist forecasting the impact on profits period by period when planning to sign an agreement lasting for  $t$  periods.<sup>9</sup>

#### 4.1 The two-period agreement

Consider an agreement lasting for two periods. Firm 1 evaluates the profits that it will receive at the end of period two, according to the technology available at that time.<sup>10</sup> They are given by  $\tilde{\Pi}_{12}^m = \frac{1}{b_1} \left( \frac{a_1 - \lambda_2 b_1}{2} \right)^2$ . Then, it compares these profits with the ones in absence of agreement given by (7). It turns out that  $\tilde{\Pi}_{12}^m > \Pi_1^m$  if

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<sup>8</sup>We do not explicitly model punishments for deviations from the agreements. This would go beyond the main of the analysis. Remember that we are assuming that the model satisfies the condition for an optimal contract to exist. Accordingly, the design of the contract already takes into account those penalties.

<sup>9</sup>In general, this is the kind of cost-benefit analysis that firms carry out when they evaluate the convenience of joining an agreement. Firms look at the evolution of profits over a finite horizon from the actual situation by computing the present (discounted) value of the flow of future profits. In addition, we compare stock variables at different moments in time and implicitly discount them at the same rate. It is important to remember that we are considering extreme cases where the agreement must be profitable during every single period. Midler assumptions would consider comparing aggregate discounted profits over a certain number of periods. Then opportunities for successful collaboration should appear more easily.

<sup>10</sup>Remember that firms can exploit the benefits they get from the agreement only at the end of period two.

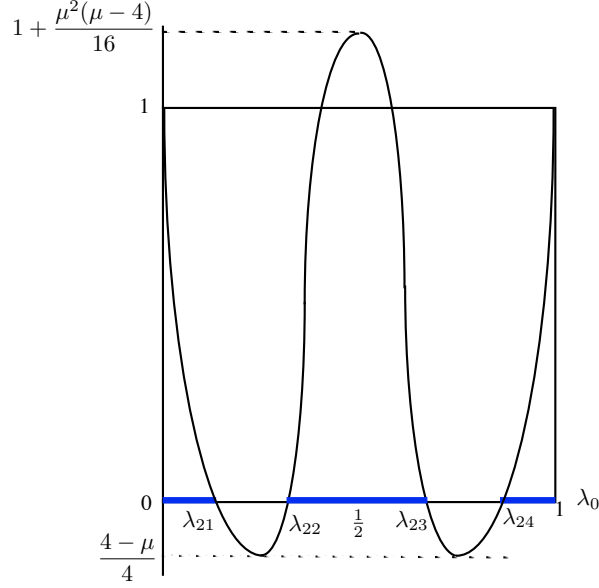


Figure 3: Feasible agreements at  $t = 2$

$b_1(1 - \lambda_2) > 0$ , that is,

$$b_1[1 - \mu^2\lambda_0(1 - \lambda_0)][1 - \mu\lambda_0(1 - \lambda_0)] = b_1[1 - F^2(\lambda_0)] > 0. \quad (15)$$

This is a polynomial of degree four, with real roots only for  $\mu > 4$ . As displayed in Figure 3, for  $\mu > 4$ , inequality (15) admits four strictly positive critical points ( $0 < \lambda_{21} < \lambda_{22} < \lambda_{23} < \lambda_{24} < 1$ ), where

$$\lambda_{2i} = \frac{1}{2} \pm \frac{\sqrt{\mu^2 - 2\mu(1 \pm \bar{\mu})}}{2\mu}, \quad \bar{\mu} = \left(\frac{\mu - 4}{\mu}\right)^{1/2} \in (0, 1), \text{ for } \mu > 4$$

and  $i = 1, 2, 3, 4$  according to the combination of positive or negative signs of the square roots chosen. Therefore, (15) is satisfied for

$$\lambda_0 \in [0, \lambda_{21}] \cup [\lambda_{22}, \lambda_{23}] \cup [\lambda_{24}, 1].$$

Finally, combining the range of admissible values of  $\lambda_0$  just obtained for period 2 with the corresponding ones in period 1 (see Proposition 1), we get the range of values of  $\lambda_0$  for which the two-period agreement is profitable:

$$\lambda_0 \in [0, \lambda_{21}] \cup \left[\lambda_{22}, \frac{1}{2} - \frac{\bar{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\bar{\mu}}{2}, \lambda_{23}\right] \cup [\lambda_{24}, 1].$$

Figure 4 illustrates.

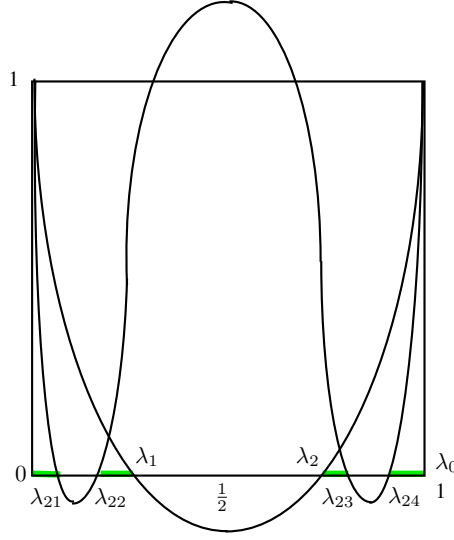


Figure 4: Two-period profitable agreements

## 4.2 The N-period agreement

As shown by this example and illustrated in Figure 4, the different intervals of solutions shrink when the number of iterations increases, i.e., the length of the agreement expands.

Hence, the general question is to determine the values of  $\lambda_0$  for which an agreement can be successful based on its length, knowing that the set of admissible values of  $\lambda_0$  shrinks when the time dimension increases.

The set of values of  $\lambda_0$  we are interested in is defined as the intersection of the sets of values of  $\lambda_0$  supporting agreements lasting for one period (given by equation (12)), lasting for two periods (given by equation (15)), lasting for three periods, etc. Let us rewrite those conditions in the following way:

- For agreements lasting one period ( $t = 1$ ), the possible values of joint efficiency to reduce production costs entailing a successful agreement are  $\lambda_0 \in \Lambda_1 \subset (0, 1)$  such that  $G(\lambda_0) \equiv 1 - F(\lambda_0) \geq 0$ ,
- For agreements lasting two periods ( $t = 2$ ), the possible values of joint efficiency to reduce production costs entailing a successful agreement are  $\lambda_0 \in \Lambda_2 \subset \Lambda_1$  such that  $G^2(\lambda_0) \equiv 1 - F^2(\lambda_0) \geq 0$ .

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- For agreements lasting  $N$  periods ( $t = N$ ), the possible values of joint efficiency to reduce production costs entailing a successful agreement are  $\lambda_0 \in \Lambda_N \subset \Lambda_{N-1}$  such that  $G^N(\lambda_0) \equiv 1 - F^N(\lambda_0) \geq 0$ .

Such behavior is induced by the iterative structure of function  $F^N(\lambda_0)$ . At the limit, when  $t \rightarrow \infty$  we obtain a infinite collection of points as the set of solutions. These points are precisely the (infinite) roots of a polynomial (of infinite degree), resulting from the comparison of profits between signing an infinite horizon agreement and no agreement at all. To clarify this argument, define  $A_t$  as the set of  $\lambda_0$ -points that escape from the interval  $I = (0, 1)$  at iteration  $t + 1$ . That is, those points that were admissible at iteration  $t$  but are no longer solutions after iteration  $t + 1$ . Formally,

$$A_t = \{\lambda_0 \in I \mid G^t(\lambda_0) < 0 \text{ and } G^\tau(\lambda_0) \in I, \tau < t\}.$$

This set of the solutions ( $\Lambda$ ), after an infinite number of iterations, reduces to:

$$\Lambda = I \setminus \bigcup_{t=0}^{\infty} A_t.$$

**Proposition 3.**  $\Lambda$  is a Cantor set.

*Proof.* See Appendix. □

We provide here an informal argument of the proof of proposition 3. Remember that a Cantor set is defined as closed, perfect and totally disconnected.

Intuitively, note that  $A_t$  are open sets. Thus,  $\Lambda$  is formed by (sequentially) suppressing from the interval  $I$  a collection of open sets that are disjoint intervals. In other words,  $\Lambda$  is the union of closed and disjoint intervals, and thus *closed*. Incidentally, note that  $\Lambda$  is not empty because at least it contains the extreme points of the suppressed intervals.

Next, by definition, a set is *perfect* if it does not contain isolated points; in other words, all of its points are limit points. Let us assume on the contrary, that  $x \in \Lambda$

is an isolated point. Then  $x$  must be an extreme point common to two adjacent intervals. As we have previously argued,  $\Lambda$  is a collection of disjoint intervals. Hence, those adjacent intervals do not have points in common. Accordingly,  $x$  cannot be an isolated point.

Finally, a set is *totally disconnected* if it does not contain any open intervals. Again, let us proceed by contradiction. Assume that there exists an open interval  $\delta \in \Lambda$ . Then  $\delta$  has to be contained in one of the open intervals obtained in an iteration  $\tau$ . However, this is not possible since as  $\tau \rightarrow \infty$ , the length of the intervals tends to zero. Thus, at the limit  $\Lambda$  has an infinite number of points.

Indeed, our main conclusion can be described in terms of the probability of the existence of a suitable technology matching between the two firms. Such probability decreases with the length of the contract. In other words, a firm willing to sign a short-term agreement will find it quite likely that the other firm has a technology suitable for a matching. As the commitment term that a firm is willing to engage in increases, it becomes less and less likely that the technology of the other firm will be suitable to sign an agreement. Therefore, our model provides a rationalization of the prevalence of short-run agreements as described by the empirical evidence.

Our diffusion process is set on the discrete logistic map. In this sense, we can obtain the same qualitative results with other functions  $F$  of the quadratic family (Devaney, 1985). Recently, Blackmore *et. al* (2009) have proved that a discrete system similar to the model of the Set-Reset Flip-Flop circuit can also be used with the same purpose. Its expression looks as follows

$$F^t(\lambda_0) = \lambda_t = \lambda[\nu(1 - \lambda_{t-1}) + y]; \quad \lambda = 4.1$$

and, when  $y$  is small the system acts like the discrete logistic map.

## 5 Discussion and conclusions

This paper proposes an iterative dynamic process among firms to introduce a time dimension for partnership in pre-competitive agreements. Every expansion of the time horizon in one period eliminates an open set of  $\lambda_0$ -values that were solutions

in the previous time horizon. The extreme points of those intervals remain in the set of solutions  $\Lambda$  though. A value of  $\lambda_0$  that has been eliminated as a solution after an enlargement of the time horizon, remains out of  $\Lambda$  forever; in other words, it cannot be considered again as a solution as the length of the agreement increases.

From an intuitive viewpoint, the successful combination of the technological endowments are basically of two types: with very similar or very dissimilar technologies. On the one side, the successfulness of agreements with technological similar firms may be supported by the peer status that both the firms may have in the signed agreement, and, as a consequence, by the natural affinities that may arise between them yielding an efficient matching. On the other side, the case of very dissimilar technologies between firms may be associated with the classical leader-follower partner structure. In this case, the successfulness may be due to the right degree of complementarity between the two partners. By contrast, the intermediate values of  $\lambda_0$  leading to unfruitful agreements can be considered as *in-between* situations in which the role of the partners cannot be defined so precisely. These are situations in which contracts are less likely to be profitable because, for instance, some competition effects may be stronger than in the other two situations and, possibly, free rider behaviors may occur more frequently.

Given this dynamic process, as firms envision longer agreements, an increasing number of smaller intervals are excluded as solutions.

Proposition 3 tells us that knowing the length of the agreement, a firm evaluates the advantages prior to signing the agreement. According to initial conditions ( $\lambda_0$ ), it may or may not be able to fulfill its expectations. Moreover, the iteration process imposes that firms need to be very careful when choosing the agreement (a partner and a time horizon), given their initial technologies. In other words, if a firm wants to derive the expected benefits from an agreement, it needs to be extremely precise in choosing the right partner allowing to fulfill its expectations. With an infinite number of iterations, there is a number of *discrete* points  $\lambda_0$  ensuring the success of the agreement. These correspond to the optimal combinations of initial technologies available at the firm level. However, the probability of such matching

to occur is zero.

In order to obtain the intuition contained in this result, imagine a firm that is willing to sign a short-term agreement. It can find a compatible partner almost effortlessly. As the commitment the firm is willing to engage in increases, the difficulty of finding a suitable partner also increases. The reason behind this difficulty is *not* that there are fewer partners available (there are always an infinite number), but rather that getting to identify suitable firms becomes increasingly hard.

Casual empiricism points out that most research projects have a short time horizon (maximum of 5 years). Accordingly, we should expect to observe research projects involving firms with relatively different initial technologies or relatively similar initial technologies. Examples of projects showing these characteristics are the EU-sponsored Carsense and Cartalk projects.<sup>11</sup> Our analysis provides some rationale to these empirical observations.

The relevant feature that we want to stress of our approach is that we are able to propose a novel argument to explain the increasing difficulty in observing long-lasting pre-competitive agreements.

The partnership agreements are mainly perceived as a cost-sharing device (see Nicolini and Artige, 2008). Of course, the major problem faced by firms is the choice of an appropriate partner to avoid the waste of capital in case of failure. Thus, it appears that public policies should target some cost-sharing mechanisms through partnership agreements satisfying two conditions: (i) the most suitable (and effective) policy should target short-term agreements and, (ii) the partnership between two competitors is a feasible contract that yields positive benefits to the two parts upon completing the terms of the contract.

Some extensions deserve attention. We have only considered firms operating in separate markets. Section 3 studying agreements that do not span in time, leads

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<sup>11</sup>Carsense is a consortium of 12 European car manufacturers, suppliers and research institutes, sponsored by the EC to develop a sensor system, that shall give sufficient information on the car environment at low speeds in order to allow low speed driving. See <http://www.carsense.org>. Cartalk is another consortium of 7 European car manufacturers, suppliers and research institutes, sponsored by the EC and focusing on new driver assistance systems based on inter-vehicle communication. See <http://www.cartalk2000.net>.

us to conclude that qualitatively, the outcomes of the local monopolies case and the interrelated market case are similar. The introduction of time in Section 4 involves a dynamic process but it does not change the dynamics of the decision process of firms when markets are separated. Hence, we conjecture that the remaining case of multi-period agreements between firms in interrelated markets will also be characterized by a Cantor set-type of solutions as the number of iterations increases, but the results are more blurred. Other extensions deal with uncertainty and technical development in a full dynamic learning process (giving structure to  $\lambda_0$ ), or allowing for the possibility that a firm can leave the agreement before its completion.

Finally, from an enlarged perspective, our analysis can be extended to problems where the matching condition between two partners is fundamental such as the labor market, or other decisions in a social behavioral context. As in our framework, in these situations the essential feature is the alignment of objectives between the two parties involved.

## Appendix 1: Proof of proposition 2

Here, firms compete à la Bertrand in the market. The equilibrium prices, quantities, and profits are,

$$\bar{p}_1 = \frac{2b_2\bar{a}_1 + c\bar{a}_2}{4b_1b_2 - c^2}; \quad \bar{q}_1 = b_1\bar{p}_1; \quad \bar{\Pi}_1 = b_1 \left( \frac{2b_2\bar{a}_1 + c\bar{a}_2}{4b_1b_2 - c^2} \right)^2, \quad (16)$$

$$\bar{p}_2 = \frac{2b_1\bar{a}_2 + c\bar{a}_1}{4b_1b_2 - c^2}; \quad \bar{q}_2 = b_2\bar{p}_2; \quad \bar{\Pi}_2 = b_2 \left( \frac{2b_1\bar{a}_2 + c\bar{a}_1}{4b_1b_2 - c^2} \right)^2, \quad (17)$$

where  $\bar{a}_1 = a_1 - \mu\lambda_0(1 - \lambda_0)[b_1 - c\xi]$  and  $\bar{a}_2 = a_2 - \mu\lambda_0(1 - \lambda_0)[b_2\xi - c]$ .

As before, given the symmetry of the problem, we concentrate on the behavior of firm 1. Firm 1 evaluates the benefits it can get from the agreement and compares the level of profits with and without the agreement. That is, it compares profits in (6) and (16). Participating in an agreement will be profitable if and only if,

$$b_1 \left( \frac{2b_2\bar{a}_1 + c\bar{a}_2}{4b_1b_2 - c^2} \right)^2 > b_1 \left( \frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2} \right)^2.$$

After some algebraic computations, the previous inequality reduces to,

$$b_1[1 - \mu\lambda_0(1 - \lambda_0)](2b_1b_2 - b_2c\xi - c^2) > 0. \quad (18)$$

Note that (18) differs from (12) in the second term in (round) brackets. Note also that this term is independent of  $\lambda_0$ . The second term in round brackets is concave in  $c$ , and has one positive and one negative root. Therefore, for positive values of  $c$  smaller than the positive root, the term  $(2b_1b_2 - b_2c\xi - c^2)$  is positive, and inequality (18) behaves as (12). Thus, we obtain the same result as in the monopoly case. In contrast, for large enough values of  $c$  (beyond the positive root), the term  $(2b_1b_2 - b_2c\xi - c^2)$  is negative, so that the inequality is fulfilled when  $[1 - \mu\lambda_0(1 - \lambda_0)] < 0$  that is, for  $\lambda_0 \in (\frac{1-\bar{\mu}}{2}, \frac{1+\bar{\mu}}{2})$ , where  $\bar{\mu} = (\frac{\mu-4}{\mu})^{1/2} \in (0, 1)$ , for  $\mu > 4$ .

When  $0 < \mu \leq 4$  the first term of (18) in square brackets is always positive. Hence, a solution exists only if the second term in round brackets is positive, i.e. for positive values of  $c$  smaller than the positive root.

## Appendix 2: Proof of proposition 3

We need to prove that  $\Lambda$  is a Cantor set, namely, that it is a closed, perfect and totally disconnected subset of  $I$ . Following Devaney (1985), we structure the proof in three steps.

### 1. $\Lambda$ is a closed set.

Let us define  $G(\lambda_0) = 1 - F(\lambda_0)$  and re-write it as  $G \equiv 1 - F$ . By construction  $A_i$  is an open interval centered around  $1/2$  (see Figure 4). Let us concentrate on  $A_0$ . In that case, the function  $G$  maps both the intervals  $I_0 = [0, \lambda_1]$  and  $I_1 = [\lambda_2, 1]$  monotonically onto  $I$ . Moreover,  $G$  is decreasing on the first interval and increasing on the second. Since  $G(I_0) = G(I_1) = I$  there is a pair of intervals (one in  $I_0$  and the other in  $I_1$ ) which are mapped onto  $A_0$  by  $G$ . These intervals define the set  $A_1$ . Next, let us consider  $\Lambda_1 = I - (A_0 \cup A_1)$ . This set consists of four closed intervals (see Figure 4) and  $G$  maps them monotonically onto either  $I_0$  or  $I_1$ , but as before, each of the four intervals contains an open subinterval which is mapped by  $G_2$  onto  $A_0$ , i.e., the points of this interval escape from  $I$  after the third iteration of  $G$ . By applying this iterative process, we note that  $A_t$  consists of  $2^t$  disjoint open intervals and  $\Lambda_t = I - (A_0 \cup \dots \cup A_t)$  consists of  $2^{t+1}$  closed

intervals. Hence,  $\Lambda$  is a nested intersection of closed intervals, and is thus a closed set.

## 2. $\Lambda$ is a perfect set.

Note that all endpoints of  $A_t$ , ( $t = 1, \dots$ ) are contained in  $\Lambda$ . Such points are eventually mapped to the fixed point of  $G$  at 1, and they stay in  $I$  under iteration. If a point  $x \in \Lambda$ , were isolated, each nearby point must leave  $I$  under iteration, and therefore these points must belong to some  $A_t$ . Two possibilities arise. We can think of a sequence of endpoints of  $A_t$  converging to  $x$ . In this case the endpoints of  $A_t$  map to 1 and so, they are in  $\Lambda$ . Alternatively, all points in a deleted area nearby  $x$  are mapped out of  $I$  by some iteration of  $G$ . In this case, we may assume that  $G_\tau$  maps  $x$  to 1 and all the other nearby points are mapped in the positive axis above 1. Then  $G_\tau$  has a minimum at  $x$ , i.e.,  $G'_\tau(x) = 0$ . This iterative process ensures that it must be so for some  $t < \tau$ . Hence,  $G_t(x) = 1/2$ , but then  $G_{t+1}(x) \notin I$  and  $G_\tau(x) \rightarrow -\infty$ , contradicting the fact that  $G_\tau(x) = 1$ .

## 3. $\Lambda$ is a totally disconnected set.

Let us focus in the first iteration and assume  $\mu$  is large enough so that  $|G'(x)| > 1$  for all  $x \in I_0 \cup I_1$ . For those values of  $\mu$ , there exists  $\gamma > 1$  such that  $|G'(x)| > \gamma$  for all  $x \in \Lambda$ . Our iterative process yields  $|G'_\tau(x)| > \gamma_\tau$ . We want to prove that  $\Lambda$  does not contain any interval. Let us proceed by contradiction and assume that there is a closed interval  $[x, y] \in \Lambda$ ,  $x, y \in I_0 \cup I_1$ ,  $x \neq y$ . In this case,  $|G'_\tau(z)| > \gamma_\tau$ , for all  $z \in [x, y]$ . Choose  $\tau$  so that  $\lambda_\tau|y - x| > 1$ . Applying the Mean Value Theorem, it follows that  $|G_\tau(y) - G_\tau(x)| \geq \gamma_\tau|y - x| > 1$ , implying that either  $G_\tau(y)$  or  $G_\tau(x)$  lies outside of  $I$ . This contradicts with our main hypothesis, and thus  $\Lambda$  does not contain intervals. It remains to be determined the  $\mu$ -values for which the previous argument holds. Finding the values of  $\mu$  allowing  $|G'(x)| > 1$  means to identify  $\mu$ -values for which  $[-\mu(1 - 2x)]^2 > 1$ . When  $G = 0$ , this inequality holds for  $\mu > 2 + \sqrt{5}$ . Thus, we have proven that  $\Lambda$  is totally disconnected for  $\mu > 2 + \sqrt{5}$ . Recall from Lemma 1 that we already know that  $\mu > 4$ . Hence, we need to verify whether  $\Lambda$  is also totally disconnected for

$\mu \in (4, 2 + \sqrt{5}]$ . We appeal to Kraft (1999) who establishes that  $\Lambda$  is a Cantor set for  $\mu > 4$ . The idea behind the proof is that for  $\mu \in (4, 2 + \sqrt{5}]$  it turns out that  $|G'(x)| \leq 1$ . Kraft argues that the iteration process shrinks some components of  $I$ , and stretches some others. His proof thus consists in showing that in the interval  $(4, 2 + \sqrt{5})$  the stretching is dominated by the shrinking. To this end, he proves that  $\Lambda$  is an hyperbolic set, namely that  $|G'_\tau(x)| > k\delta_\tau > 1$  for  $x \in \Lambda$ ,  $k > 0$ ,  $\delta > 1$ .

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