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Cartesian echoes in Kant's philosophy of nature

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Abstract

In this paper, we take the cue from a recent observation of Dan Warren about pre-Newtonian elements in Kant's philosophy of nature to argue that there are two puzzles concerning Kant's claim that mechanical laws presuppose dynamical laws in Chapter Three of *Metaphysical Foundations of Natural Science*. We offer responses on Kant's behalf to these puzzles. These responses take us through a journey via Kant's first pre-Critical work, *True estimation of living forces*, and the then lively debate between Cartesians and Leibnizians. We show how some important Cartesian echoes, clearly evident in *True estimation,* seem to have played a role in shaping some seminal ideas of Kant on dynamical forces.

Keywords: Kant, Dortous de Mairan, elasticity, Dynamics, Mechanics.

1. Introduction

In a recent paper on Kant's balancing argument for attractive and repulsive force in the second chapter of *Metaphysical Foundations of Natural Science*[1] (henceforth abbreviated as MAN), Daniel Warren[2] brings to the fore some distinctively pre-Newtonian aspects of Kant's conception of force. Warren argues that Kant can escape the objection that

[1] Kant (1786).
[2] Warren (2010). For an alternative reading of Kant's balancing argument, see Smith (this volume).

repulsion is not required to counteract the attractive force (which—on Kant's view—would tend to make matter collapse into a point), if matter were assumed as endowed with inertial mass and as spinning. This distinctively Newtonian picture of matter as endowed with inertial properties[3] seems at odds both with the dynamical and the mechanical characterization of matter in the *Metaphysical Foundations*. Warren draws attention to some important analogies between Kant's treatment of force and motion in the 1786 Dynamics chapter and the treatment of *dead force* in Kant's earliest pre-Critical work *True Estimation of Living Forces*.[4] In this pre-Critical work, Kant distinguished between two different kinds of motion (1: 28): motion that would maintain itself in the body if no obstacle intervened (for example, bullets being fired), and motion that on the contrary depends on an external impressed force to maintain itself (for example, a ball being gently pushed forward by hand or any other example of gentle pushing and pulling of objects).[5] Kant associated the former type of motion with living force, and the latter with dead force. Warren concludes:

> The idea of motion at work in the "Dynamics" chapter is motion of the second kind. More specifically, Kant is concerned with cases of approach and withdrawal, and he is conceiving of them as the effects of a force, which can be intelligibly discussed even without bringing in the concept of inertia. (…) Force, conceived as the cause of what Kant called "motion of the second kind", presupposes abstraction from the inertial properties of matter, and it is not regarded as something which overcomes inertial resistance and thereby brings about changes of momentum. This is not Newton's conception of force; rather, it is much more pre-Newtonian in character. But neither is the notion of moving force officially introduced in the "Mechanics" chapter to be identified with the Newtonian conception. (…) it is more closely analogous to a Leibnizian conception of living

[3] As Marius Stan pointed us out in private correspondence, at least four distinct concepts are bundled together under the Newtonian picture of matter as endowed with inertial properties: (i.) inertial mass; (ii.) resistance to externally induced acceleration; (iii.) zero self-acceleration; (iv.) conservation of inertial state. Of these four, under Warren's reading, Kant seems to be rejecting only (i.) and (ii.), but not necessarily (iii.) and (iv.).

[4] Kant (1747). In the rest of this paper, we use the standard reference to *Kants gesammelte Schriften* (or Akademie edition), with the volume number, page number, and line numbers.

[5] The first is an example of unconstrained motion in free space, which Kant takes as exemplifying living force. The latter is an example of motion that requires an impressed force, *qua* dead force.

force (though with the appropriate caveats, mentioned earlier), than it is to force on the Newtonian conception.[6]

In this paper, we provide grist to Warren's interpretive mill. Our aim is twofold. First, we expand on Warren's observation by clarifying the sense in which "dynamical forces" such as attraction and repulsion in Chapter Two of the *Metaphysical Foundations* may be legitimately regarded as *dead forces* going back to the debate on *vis viva* between Leibnizians and Cartesians, which was the focus of Kant's *True estimation*. Second, we highlight the distinctively pre-Newtonian nature of these dynamical forces by tracing back Kant's treatment of repulsive force to some important Cartesian assumptions, whose seminal ideas were discussed in *True estimation*. Hence the strategy that gives the title to our article: Cartesian echoes in Kant's philosophy of nature. Our story comes with a twist, though. Far from concluding that Kant's dynamical treatment of repulsive force follows Descartes, we will make the case for a reading of Cartesian and Leibnizian influences on Kant's early philosophy of nature, where it was the idiosyncratic blend of these two traditions (fuelling debates in French intellectual salons between Jean-Jacques Dortous de Mairan and Marquise Du Châtelet) that played a role in shaping some of Kant's early ideas on Dynamics. That is why, after all, we will argue for Cartesian *echoes* only.[7]

2. **Two puzzles about Kant's dynamical forces**

[6] Warren (2010), p. 224. For the sake of accuracy and completeness, it must be said that immediately after this passage, Warren adds (ibid.): "But this certainly does not mean that Kant rejects a Newtonian conception of force or excludes it from his system. For although it is not the conception of force at work in the "Dynamics" chapter itself, and it is not the notion of a "mechanistic" force that is the central focus of the "Mechanics" chapter, it does have a place in the *Metaphysical Foundations*, and this place also is in the "Mechanics" chapter. It corresponds to the role played by the dead forces when inertia *is* added to the picture. (…) The point I have been making is that it is important not to read this step back into the conception of force at work in the "Dynamics" chapter, and that the arguments and lines of thought in that chapter cannot be properly understood or assessed if we do"

[7] In this way we hope to provide a more nuanced picture of the formation of Kant's seminal ideas in dynamics during the pre-Critical period than the received view, which portrays the young Kant of 1747 as an adversary of Cartesianism altogether (see Calinger 1979, p. 350). For an important exception to the received view, see Grillenzoni (1998); and Tonelli (1959), who emphasized the eclectic anti-Wolffianism of the young Kant.

In the Third Chapter of *Metaphysical Foundations of Natural Science*, Kant offers the mechanical definition of matter as "the moveable insofar as it, as such a thing, has moving force" [4: 536.06]. Kant clarifies the mechanical concept of moving force (*bewegende Kraft*) as a force that can *communicate* motion from one piece of matter to another, but not as an originally moving force able to *impart* motion to still matter. This is instead the role that Kant assigns to dynamical forces, namely attraction and repulsion, in Chapter Two. In particular, Kant claims that no matter would be able to impress motion on another matter lying straight ahead of it, if both did not possess repulsion, nor would any piece of matter compel another to follow straight behind it, if both did not possess attractive forces. Hence the conclusion that "all mechanical laws presuppose dynamical laws, and a matter, as moved, can have no moving force except by means of its repulsion and attraction, on which and with which, it acts immediately in its motion, and thereby communicates its own inherent motion to another" [4: 536.23–537.04]. Kant concludes the opening remark of his Chapter on Mechanics with the apologetic note that in the rest of the Chapter, priority will be given to the analysis of repulsive force (instead of attractive force) as an originally moving force communicating motion via pressure "(as by means of tensed springs), or through impact" [4: 537.07-08].

How should we understand Kant's remark that mechanical laws presuppose dynamical laws and that there cannot be communication of motion in the *mechanical* sense, without an originally moving force in the *dynamical* sense? There is something deeply puzzling about it. For, surely, the dynamical forces of attraction and repulsion seem to be neither necessary nor sufficient for communicating motion in the mechanical sense.

That they are not necessary is evident from Newton's mechanics. Newton's laws of mechanics neither have nor require a dynamical underpinning *along Kant's lines*:[8] Newton's laws of motion and the mathematical physics they license stand on their feet without the need of attraction and repulsion as original forces of matter. For example, in Newton's second law, the concept of force at stake is that of an *impressed external* force imparting acceleration to bodies having a certain mass, and it departs in significant ways from

[8] The qualification "along Kant's lines" is relevant here. The above remark is not meant to be a reflection on Newton's *own project*, and how dynamical considerations may (or may not) enter into it (for this issue, see Brading, Janiak, and DiSalle, in this volume). But it is rather a reflection on Kant's *own take* on mechanics and dynamics, under the assumption that Kant's mechanics is somehow meant to follow Newton's one with all the relevant caveats.

Kant's dynamical forces as *original intrinsic* forces of matter.[9] Or, a more poignant example is provided by the difference between Newton's gravitation (which Kant refers to with the term *Gravitation*) and Kant's attractive force (usually referred to with the name *Anziehung*). For Newton, gravitational force is the paradigmatic example of an impressed centripetal force,[10] whose mathematical treatment became the focus of Newton's *Principia* without any further speculation into its metaphysical cause or physical seat (although Newton himself admitted the existence of attraction and repulsion as fundamental forces of nature, most distinctively in the Queries of the *Opticks*).[11] Kant, on his side, interpreted *Gravitation* as the *action* (*Wirkung*) of the universal and original attractive force (*Anziehung*) "immediately exerted by each matter on all other matters, and at all distances", as much as *original elasticity* (*ursprüngliche Elasticität*) was the name given to the *action* of the original repulsive force (*repulsiven Kraft*) of the parts of every piece of matter [4: 518.17–22].

That Newton's mathematical treatment of universal gravitation in the *Principia* did not have the metaphysical–dynamical underpinning that Kant offers in the *Metaphysical Foundations* is also evident from the fact that Newton never used his law of universal gravitation to provide a cosmogony or an explanation of how planets and comets were set into motion at the origin of the universe. Not surprisingly, Kant wrote his *Universal Natural History*[12] with the declared intent of filling this lacuna in Newton's program and providing a cosmogony along the lines of Newton's principles. Thus, there is no overall sense in which Newton's laws of mechanics seem to presuppose or require or demand *dynamical laws along Kant's lines*. Kant's dynamical forces of attraction and repulsion are not necessary conditions for communicating motion in the mechanical sense, from a strictly Newtonian point of view.

But are they sufficient? After all, for Kant's remark at the beginning of the Chapter on Mechanics to be defensible, it would suffice to show that attractive and repulsive forces produce motion in the *mechanical* sense, e.g. the type of motion typically

[9] For an analysis of the differences between Newton's concept of force and Kant's dynamical forces, see Watkins (1998), and Stan (2009), and this volume.

[10] See Def. 5 of Newton's *Principia* (1687). On this point, see also Cohen (2002). And for a recent re-assessment of this central theme within the Newtonian tradition of Maupertuis, see Lisa Downing (2012).

[11] Newton (1704/1717), Query 31, ed. (1952), p. 396.

[12] Kant (1755a). For a discussion of how despite Kant's declared intent, this text betrays some relevant points of divergence from the *Principia*, see Massimi (2011); and Watkins (this volume). For an alternative interpretive line on Kant's *Universal Natural History* and the legacy of the *Principia*, see Schliesser (this volume). For the relevance of mathematical and astronomical riddles in informing Kant's cosmogony and cosmology, see De Bianchi (forthcoming).

exemplified by elastic and inelastic collisions of bodies (to follow Kant's own example of pressure and impact as being produced by the repulsive force). Yet this claim too proves problematic within the resources offered by Kant's Dynamics in Chapter Two. In that Chapter, Kant defines matter as the "moveable insofar as it fills a space" [4: 496.06] via a particular moving force [4: 497.16], namely via a repulsive force (*zurückstoßende Kräfte*) as an original "expansive force of its own" also called "elasticity" [4: 499.06-16; 500.02].

Hence the balancing argument whereby attractive force is introduced as a second essential force of matter, to prevent matter from expanding indefinitely to occupy all the available space in virtue of its expansive force [4: 508.12-13]. Attraction and repulsion are invoked as fundamental forces of matter, whose delicate counterbalance secures matter's stability: it would prevent matter from either expanding *ad infinitum* (because of the repulsive force) or collapsing into a point (under the attractive force). Dynamical forces are introduced as *intrinsic* forces[13] responsible—via their equilibrium—for the stability of matter and its impenetrability. How can they in turn produce motion in a truly Newtonian *mechanical* sense? Kant's examples of a body being impelled to move along a straight line under the repulsive force of another body, or being compelled to follow another one under the attractive force, seem to lack the distinctive features of genuine mechanical motion. They are not examples of motion that would continue after the original moving force has stopped acting on the body.[14] Instead they seem to require the constant action of the moving force, be it repulsion or attraction, as an impulsive force, without which the body would soon come to a halt.[15]

[13] We use the adjective *intrinsic* here to mark the difference from *extrinsic, impressed forces* as those at work in Newtonian mechanics. We leave intentionally open the issue of how to understand in turn the notion of attraction and repulsion in Kant as *intrinsic forces* (for an analysis in terms of causal powers of matter, see Warren 2001a; and 2001b).

[14] "It is clear, however, that the moveable would have no moving force *by means of its motion*, if it did not possess originally moving forces, by which it is active in every place where it is found, prior to any inherent motion of its own" [4: 536.16-18. Underlined emphasis added]. This passage seems to suggest that it is the *activity* of matter, bestowed on it by the original dynamical forces, which ultimately allows communication of motion in the mechanical sense *in every place where active matter is found.*

[15] Take elastic impact as an illustration of Kant's view of a body being impelled to move under the repulsive force of another body. While from a Newtonian viewpoint, the post-collision uniform motion ensues from the inertial masses of the bodies and conservation of momentum (linear or angular, depending on the individual trajectories of the center-of-mass); by rejecting inertial mass (as clarified in footnote 3 above), Kant has to resort to repulsion as a constantly acting dynamical surrogate for Newton's inertial mass. In this sense, the examples he adduces seem puzzling from a strictly Newtonian viewpoint.

How can Kant seriously think of elastic collision between two bodies as ensuing from attractive and repulsive force? It is not just that the ensuing motions would lack distinctive inertial features that we would normally associate with mechanical motions, but it is the very possibility of setting matter in motion via attraction and repulsion that looks *prima facie* suspicious. After all, are not attraction and repulsion primarily responsible for matter's *rest* and its possibility of existence [4: 508.34]?[16] But if so, would not we go back to Kant's original problem of avoiding either matter to expand indefinitely under the repulsive force, or to collapse into a point under attractive force?[17]

16 One may respond on Kant's behalf that attraction and repulsion are *sufficient* to set matter in motion, or to "communicate motion" in a mechanical sense, whenever dynamical forces become 'imbalanced'. The key question here is how one should understand this dynamical 'imbalance' as "communication of motion" in a mechanical sense. One natural option would be to think of two chunks of matter as two perfectly spherical bodies $\mathcal{A}$ and $\boldsymbol{B}$, with centers of mass A and B, and external surface of infinitesimal thickness d*s* moving towards each other such that at the points of contact, call them *a* and *b* respectively, unless there is perfect balance between attraction and repulsion, body $\mathcal{A}$ pushes / communicates motion to body $\boldsymbol{B}$, via point *a* pushing point *b* towards the center of mass *B*. Thus, one may think of point *a* as endowed with mechanical force or momentum sufficient to move *b* towards B, while at the same time B would react against *a,* with higher non-zero repulsion. This mutual imparting of motion may seem to have a perfectly dynamical underpinning, in Kant's account (we thank Marius Stan for raising this point). As we see it, there are two difficulties with this otherwise crisp account. First, that we must assume that point *a* has a mechanical force / momentum responsible for pushing *b* closer to the center of mass B is precisely what Kant needs to explain, as opposed to postulate (since he clearly states that "the moveable would have no moving force *by means of its motion*", see quote in footnote 14). It is not immediately transparent that Kant's *Anziehung* between *a* and *b* can be construed as a mechanical force / momentum / dead force (which was precisely the reason for our first aforementioned puzzle). Second, even granted that Kant's attraction and repulsion can be construed as mechanical forces in the sense above, we would still be left with the question of how to explain not only inelastic impact but mechanical motion more in general (beyond elastic impact).

17 To go back to the example of the two spherical bodies $\mathcal{A}$ and $\boldsymbol{B}$, one may wonder how to understand situations of dynamical imbalance as producing mechanical motion as opposed to falling back onto the original dilemma that triggered Kant's balancing argument. For example, under what conditions and *at what specific material point* (in the straight line that connects centers of mass A and B) does the attractive force (pushing surface point *a* closer to center of mass B) gets arrested by the non-zero repulsive force of B? Let us call *i* the material point on the A–B line, where a new balance is reached between attraction and repulsion (in the imagined scenario of elastic impact with subsequent deformation of elastic body $\boldsymbol{B}$). And let us assume that like any other material point along the A–B line, *i* is infinitesimally small (after all, Kant postulated that matter is divisible to infinity; see Proposition 4, in Chapter Two on Dynamics). Let us call *i** the material point immediately adjacent *i* towards *b*. Kant would owe us an account of how the non-zero repulsion that B exercises towards *a* would produce a new equilibrium of forces at *i as opposed to i**. In the absence of such an account, an unwelcome regress

The dynamical forces of attraction and repulsion seem then not sufficient either for communicating motion in a mechanical sense (in Kant's *own* mechanical sense, needless to say Newton's).

3. A Kantian solution to the first puzzle. Cartesian echoes part I: dead forces and living forces

In the previous Section, we presented two puzzles concerning Kant's treatment of dynamical forces in the *Metaphysical Foundations*. In this Section and the next one, we provide answers to the puzzles on behalf of Kant. As will become apparent, both answers betray some lingering Cartesian echoes in Kant's dynamical theory of matter, from the pre-Critical to the Critical period. How can we then make sense of Kant's claim in MAN that all mechanical laws presuppose dynamical laws, and that there cannot be mechanical motion without dynamical forces?

In the same Chapter on Mechanics, immediately after the aforementioned passage, Kant introduces two central concepts: *quantity of matter* and *quantity of motion*. The former is defined as the "aggregate of the moveable in a determinate space", also called mass when all the parts are considered as acting or moving together. The latter in its mechanical estimation is identified with what we now call momentum, namely the quantity of matter and its speed [4: 537.12–22]. Quantity of motion in a body is in turn associated with the "magnitude of action" (Größe ihrer Wirkung). The passage ends with a famous remark that is worth quoting in some detail:

> Those who merely took the quantity of a space filled with resistance as the measure of entire action (for example, the height to which a body with a certain speed can arise against gravity, or the depth to which it can penetrate into soft matters) came out with another law of moving forces for *actual* motions—namely, that of the compound ratio of the quantity of matters and the squares of their speeds. But they overlooked the magnitude of action in the given time, during which the body traverses its space at a lower speed; and yet this alone can be the measure of a motion that is exhausted by a given uniform resistance. Hence there can be no difference, either, between living and dead forces, if the moving forces

objection may be raised to the effect that at no distinct infinitesimally small material point along the A–B line a new balance of forces is expected to be restored, leaving open the possibility of either attraction or repulsion prevailing on the other force.

are considered mechanically, that is, as those which bodies have insofar as they themselves are moved, whether the speed of their motion be finite or infinitely small (…). Rather, it would be much more appropriate to call dead forces those, such as the original moving forces of dynamics, whereby matter acts on another (…); by contrast, one could call living forces all mechanical moving forces, that is, those moving by inherent motion, without attending to the difference of speed, whose degree may even be infinitely small—if in fact these terms for dead and living forces still deserve to be retained. [4: 539.5–28]

Daniel Warren has, rightly in our opinion, pointed to this passage as evidence for his interpretive analysis of Kant's balancing argument as implying a pre-Newtonian conception of force, and as being more analogous to the Leibnizian concept of living force with some suitable caveats.[18] In this Section, we provide support for Warren's pre-Newtonian reading of Kant's concept of force in this context, by stressing the role of Cartesian echoes, in addition to the Leibnizian ones.

That Kant is siding with the Cartesians against the Leibnizians in the aforementioned passage is clear from what Kant says about the Leibnizians (i.e. those, who came up with the mv^2 law for *actual* motions) overlooking "the magnitude of action in the given time" [4: 539.12]. Why does Kant conclude that *mechanically* there is no difference between the Cartesian dead forces (measured by $m|v|$, or what Kant calls "quantity of motion", following the French *quantité du mouvement*) and the Leibnizian living forces (given by the mv^2 law)? Why does he claim that the Leibnizians overlooked "the magnitude of action in the given time"? Most importantly, how does he justify the identification of living forces with mechanical forces and dead forces with dynamical ones?

In what follows, we take in turn and answer each of these questions, by looking in some detail at Kant's very first work, back in 1747: *Thoughts on the true estimation of living forces*.[19] Kant's aforementioned passage in MAN, and the interpretive problems we have highlighted can receive new light if read from the historical perspective of how the

[18] See Warren (2010), pp. 217–224.
[19] This text has recently been made available in an English translation by Martin Schönfeld, and B. Jeffrey Edwards for the CUP volume on *Natural Science*, edited by Eric Watkins. The quotes here below are taken from this English translation (we are grateful to Eric Watkins for kindly making it available to us in pre-print).

mature Kant reflected back on a topic that had occupied him at the very beginning of his career.[20]

3.1. *True estimation of living forces*. A pro-Cartesian Kant?

In the First Chapter of *True estimation* Kant latched onto a very well-known debate at the time on the nature of forces and motion in nature. Taking the lead from Leibniz's metaphysical dynamics, and in particular from Leibniz's view that every body is endowed with an essential force called 'vis activa',[21] Kant distinguished between two types of motion, as we mentioned above. The first type of motion persists indefinitely if it encounters no resistance: Kant adduced as examples 'fire bullets' and 'all projectiles' [1: 028.19–20]. The second type of motion, on the other hand, is the enduring effect of an external force, and decays as soon as the external force ceases to act on the body, as in the case of a ball gently pushed forward by hand or any body pushed or pulled with moderate velocity [1: 028.20–22]. Kant identified the second type of motion with dead force, and the first type of motion with the Leibnizian *vis viva* as an internal force of a body, striving to sustain its motion.

Leibniz had introduced the concept of 'active force' in the *Specimen Dynamicum*[22] as a first entelechy, in contrast with passive force, which expressed the resistance any body opposes to another. It is within the context of body collisions and transmission of motion that Leibniz discussed Descartes's view,[23] not without some terminological

[20] The analysis we offer in terms of lingering Cartesian echoes in the late Kant's Dynamics needs two qualifications. First, it is certainly not our intention to offer a revisionist account of MAN along Leibnizian-Cartesian lines (against the received view, which has persuasively argued for the Newtonian blueprint of this mature work). Instead, this paper should be read with an eye to disentangling the many philosophical influences skillfully interwoven in Kant's complex account in MAN, and to offering a more nuanced view of his Dynamics, which might help to make sense of some passages that have traditionally belied the Newtonian reading. Second, we want to stress the word "echoes" in our title: we do not intend to track Kant's mature Dynamics back to the debate on *vis viva*, *tourt court,* but to deliver a more nuanced view of Kant's philosophy of nature.

[21] In [1: 18] Kant clarifies why the Leibnizian expression 'vis activa' had to be preferred over the Wolffian expression 'vis motrix', which was regarded as devoid of explanatory power.

[22] Leibniz (1695), pp. 436-7.

[23] Leibniz had criticized Descartes' 'quantity of motion' in his (1686), where by using Galileo's experiments on free fall, he argued against the Cartesians that the quantity of motion necessary to raise a body of 1 pound to the height of 4 yards, was not one and the same with the quantity of motion necessary to raise a body of 4 pounds to a height of

reshuffling. For he called *impetus* Descartes's *quantity of motion* (equivalent to our 'momentum', i.e. $m|v|$), and redubbed as 'quantity of motion' the "integral of the impetuses (whether equal or unequal) existing in the moving body multiplied by the corresponding intervals of time".[24] The distinction between dead and living forces was in turn introduced as follows: dead forces are those where "motion does not yet exist in it but only a solicitation of motion" (such as centrifugal and centripetal forces, the force of gravity and elasticity); living forces, by contrast, "arise from an infinite number of continuous impressions of the dead force" (as in impact).[25] Thus, Leibniz defined living force as ensuing from integrating dead force over space, which results in mv^2 (our current notion of kinetic energy).[26] Kant clearly latched onto this Leibnizian distinction in identifying motion of the first kind (e.g. all projectiles) with living force, and motion of the second kind (e.g. ball pushed forward by a hand) with dead force.

Despite important experiments by Poleni, s'Gravesande and Musschenbroek in support of Leibniz's *vis viva*, in *True estimation* Kant followed the Cartesians in raising objections against living forces. Two experiments by Musschenbroek occupy center stage in Kant's discussion: the first concerns the recoil of springs; the second concerns collisions with soft materials.[27] As Kant reports it, the first experiment [1: 172.22–35; 173.1–22] envisages a hollow cylinder kept in tension with a spring attached to a machine, and with a protruding rod with holes firmly attached to a metal sheet via a peg. By pulling the peg out, the spring recoils and the cylinder moves up to a certain height, with a velocity of say 10 units. Suppose now that the cylinder is twice as heavy. Through the same mechanism, it can be observed to move up with a velocity of 7.07 units. Since the spring was equally compressed and had equal force in both cases, Musschenbroek concluded that the force at work in the experiment should be estimated as the product of

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1 yard. Hence Leibniz's conclusion that force had to be estimated by the "quantity of the *effect* which it can produce", and neither by the velocity it can impress on a body, nor from time either (Leibniz, 1686, pp. 297, 300).

[24] Ibid., p. 437. As Loemker explains in the corresponding footnote 7 on p. 451, by 'quantity of motion', Leibniz meant the integration of the Cartesian $m|v|$ with respect to time.

[25] Leibniz (1695), p. 438.

[26] By latching onto the aforementioned Galilean experiments already discussed in his (1686), Leibniz said: "Yet we have already shown, and will show more fully, that the *force must be calculated in terms of these spaces themselves.* Though he used another name, and indeed, another concept, Galileo began the treatment of living forces and was the first to explain how motion arises from the acceleration of heavy falling bodies" Leibniz (1695), p. 439, emphasis added.

[27] These experiments are discussed in Musschenbroek (1734), pp. 75-81 of the original Latin text.

the mass of the cylinder and the square of velocity, as only the square roots of the velocities 10 and 7.07 are in inverse ratio to the masses of 1 and 2, approximately.

The second experiment (attributed to Riccioli, Poleni, s'Gravesande,[28] and Musschenbroek) concerned the free fall of balls of equal size and mass from unequal heights onto soft materials such as tallow wax, whereby the size of the holes impressed on the wax was estimated as proportional to the heights from which they fell and hence to the square of their respective velocities, assuming that the uniform cohesion of the parts of the soft material provided the same resistance to different balls [1: 176. 20–36].

What is to be said about these two experiments? Kant [1: 175.22–33] objected to the first experiment that the force the spring imparted to the cylinder was greater in the second case where the cylinder weight was doubled, because a greater force would accrue to the spring over the time period (the heavier cylinder would be exposed to the pressure of the expanding spring longer than the lighter cylinder). Puzzling as this remark may sound, Kant's objection is designed to show that time is a key factor in the estimation of the force at work in the recoil of the spring and in allowing the cylinder to overcome the effects of gravity.

But what about the second experiment? Here the Cartesians are said to have applied the same line of reasoning in complaining that the time periods during which the holes were impressed in the tallow wax were not the same [1: 177.17–35]. But this time, Kant sided with the Leibnizians in claiming that the case of soft materials is different from the case of the spring, because the resistance offered by the soft material to the penetration of the body ceased as soon as the parts of the wax were separated, in contrast to the "essentially indestructible spring of gravity", and hence time is not a key factor in the estimation of the force at work in the second experiment [1: 178.1–14].

Apart from Kant's tentative attempt to reconcile the Cartesian and Leibnizian points of view, and ultimately to defend Leibniz's estimation with suitable caveats, we can begin to catch a glimpse of the reasons why forty years later, in the aforementioned passage of MAN, Kant still identified the Leibnizians as "those who merely took the quantity of a space filled with resistance as the measure of the entire action", and by so doing overlooked "the magnitude of action in the given time" [4: 539.5–28]. As Friedman notes in the footnote to this passage, if the quantity $m\mathrm{d}v/\mathrm{d}t$ is integrated over space, it produces the Leibnizian measure ½ mv^2; but if it is integrated over time, it

[28] See Poleni (1718), and s'Gravesande (1722).

produces the Cartesian momentum $m|v|$.[29] But there is more. As Kant's Cartesian take on the experiment with the spring shows, if we take as measure of the force acting on the cylinder simply the space filled with the gravitational resistance that the cylinder has to overcome, and if we neglect the time interval in which—to Kant's eyes—the tensed spring would exercise its pulling action on the cylinder (which is greater, the heavier the cylinder), it is no surprise that Musschenbroek's experiment seemed prima facie to provide support to Leibniz's estimation of living force.[30] But—Kant rejoins—time cannot be overlooked, and if taken in due consideration, Musschenbroek's experiment provided in fact support for Descartes' dead force.

A key assumption lies behind this line of reasoning, an assumption that Kant declares at the start of his discussion of Musschenbroek's experiments:

> No trace of a force measurable by the square will be encountered as long as one presumes to find its origin solely in external causes: genuinely living force is not generated in the body from without, but is rather the consequence of the striving that emerges in the body from its inner natural power, on the occasion of external pushes; and all those who assume only the measure of externally acting and mechanical causes for the determination of the measure of force in the passive body, will therefore never encounter anything but Descartes's estimation, as long as they judge correctly [1: 172.11–20. English translation from Watkins (ed.), forthcoming, p. 219].

Kant's main objection to Leibniz then concerns the way Leibniz defined living force as arising from dead force in a *mechanical* way, i.e. through what Leibniz called the "infinite number of continuous impressions of dead force"[31]—be it in projectiles or free fall—by integrating the 'quantity of motion' (mdv) over space. From the exclusive point of view of externally acting forces (be it gravity, or cannon firing bullets), there is no distinction between living and dead forces. The distinction has to be found instead in what Kant calls an inner natural power or *inner Naturkraft*. If considered from a strictly mathematical-mechanical point of view, there is then no genuine distinction between

[29] See Kant (1786/2004), footnote 41, p. 78.

[30] After all, recall Leibniz' point in *A Brief Demonstration of a Notable Error* (1686, p. 300) about forces being calculated from their effects, and not from time; and same point in *Specimen Dynamicum* (1695, p. 439) that living force "must be calculated in terms of these spaces themselves" (see quotes in footnotes 23 and 26).

[31] See quote in footnote 25.

dead force and living force. To prove this point, Kant resorted to one of Leibniz's key principles, the law of continuity, and articulated a regress objection against the Leibnizian view that living force arises from the "continuous impressions" of dead force.

Kant defined as *actual* [wirklich] any motion that takes place during a time interval AB, where A is the temporal origin and B the temporal end point [1: 35.3–6]. If we take AB as the elapsed time, at A, before motion begins, the body is only endowed with dead force, and according to the Leibnizian account, living force would accrue to the body during the time interval AB, as in the Galilean example of free falling bodies. But—Kant argued—if we now make AB as small as possible and consider an intermediate point C in between A and B, we would find that at C the body would be in a state closer to that of a dead force than a living force, and if we consider yet another point D in between AC, the body will get even closer to the original state A, being endowed with dead force only [1: 35.15–33; 36.1–29]. Hence Kant's conclusion that Leibniz erred in attributing living forces to bodies that have been in motion for a time interval (what Kant called *actual* motion), no matter how long or short the time interval was.[32]

We can now go back to some of the questions with which we concluded Section 2, namely why in MAN Kant claimed that *mechanically* there is no distinction between dead forces and living forces. The answer lies in the regress paradox he set out in *True estimation*, back in 1747, against the Leibnizian account of living forces. In the case of *actual* motions (i.e. motion that spans over a time interval), when only *external mechanical* forces are considered (be it gravity in the case of free fall, or cannon firing bullets in the case of projectiles), no living force can ever result from dead force on pain of regress. Hence the only force we could ever encounter is bound to be the Cartesian dead force. Similarly, if we consider *actual* motions from a mathematical point of view (or what Kant in 1786 would call 'phoronomic' point of view, i.e. motion considered exclusively with respect to velocity and direction), again Descartes's dead force would be confirmed over Leibniz's living force [1: 41. 25–37], because velocity per se "is never a cause of living force" [1: 40.26–27]. Kant's search for a *cause* of living force, which proved elusive from a strictly mathematical-mechanical point of view, brings us to the heart of the *vis viva* debate, and sheds light on Kant's aim to provide a metaphysical account of forces in *True estimation,* which was intended to overcome the Cartesian–Leibnizian standoff, as we are going to see in more detail in the next Section.

32 For Kant's later attack against Leibniz's law of continuity, see Stan (2009).

Let us take stock. At the beginning of this paper, we confronted two puzzles about Kant's treatment of attraction and repulsion in MAN as being neither necessary nor sufficient, to produce mechanical motion. The first puzzle (dynamical forces being not necessary for mechanical motion) arises from a Newtonian reading of the relevant passage. If one interprets Kant's mechanical motion along Newton's lines, i.e. as inertial or non-inertial motion (where the latter is due to the action of an *external impressed force*), it is puzzling to explain how mechanical motion requires or necessitates a dynamical grounding such as the one that Kant offers. The puzzle, however, disappears if Kant's claim is put in the context of the Leibnizian–Cartesian debate on *vis viva*. For it is the Leibnizian idea of inertia as ensuing from a dynamical equilibrium of essential forces, rather than from some natural inertial state of matter as in Newton, which provides the cue for Kant's claim that mechanical motion requires a dynamical grounding *as Kant intends it*.

And yet there are non-negligible Cartesian echoes in Kant's overall Leibnizian claim. Cartesian is Kant's allegation (both in *True estimation* and MAN) that Leibniz's living force simply won't do for mechanical motions, because the Leibnizians neglected time as a measure of the "magnitude of action" in *actual* motions. Once time is taken in due account, as Kant's analysis of Musschebroek's experiments reveals, the force at work in these mechanical motions turns out to be Descartes' dead force. More revealing is Kant's further allegation (in *True estimation* and MAN) that Leibniz's living force won't do for mechanical motions because from a purely *mathematical–mechanical* point of view, the only force one could ever encounter is Descartes' dead force. Kant's use of Leibniz's law of continuity to gerrymander the regress objection against living force attacks the very foundation of Leibniz's metaphysical dynamics in positing dead force as the ultimate ground for any mechanical motion and showing how from a strictly *mathematico-mechanical* point of view (i.e. by appealing to the "continuous impressions of dead force"), living force would never arise from dead force. Kant's proposed remedy was a new metaphysical dynamics,[33] whereby living force does not arise from without (i.e. from the "continuous impressions of dead force"), but from within, namely from an *inner Naturkraft* of bodies.

[33] In *True estimation* Kant repeatedly stressed the need to supplement mathematics with metaphysics: "Mathematics is a science isolated from the medium of genuine knowledge, it does not sufficiently meet the rules of decorum and appropriateness if taken alone, and it must be combined with the tenets of metaphysics, if it is to be perfectly applied to nature" [1: 107.7-30]

But to what extent is Kant's new metaphysical dynamics *truly* Cartesian? A critic may object at this point that Kant's metaphysical project in either *True estimation* or MAN seems far remote from any *bona fide* Cartesian project. After all, are not Descartes' notions of extension and motion alien to Kant's dynamical theory of matter, and in fact the explicit target of Kant's criticism in MAN?[34] And how can the concept of an *inner Naturkraft* resemble any Cartesian account of force? Should not we instead say that Kant's dead force is already at some distance from the original Cartesian concept? After all, in the aforementioned key passage of MAN, although he identified dead forces with his own dynamical forces (attraction and repulsion), he also added the cautionary remark "if in fact these terms for dead and living forces still deserve to be retained" [4: 539.28].[35]

This brings us back to the second puzzle. Why did Kant in MAN identify living forces with mechanical forces and dead forces with dynamical ones? A critic was envisaged at the end of Section 2, challenging such an identification on the ground that however one interprets 'dead force' in Kant (along strict Cartesian lines or not), it is hard to conceive how dynamical forces could even be *sufficient* conditions for mechanical motion. Having introduced dynamical forces in MAN to explain (via their counterbalance) matter's impenetrability and rest, Kant would find himself in the delicate situation of having to explain mechanical motion at the cost of breaking that same balance of forces necessary to guarantee matter's rest in the first instance. In the next Section we take up this second puzzle and offer a response on Kant's behalf. The response betrays a new account of causality originating from Kant's siding with the Cartesians against the Leibnizians in the debate about elastic collisions of bodies.

4. A Kantian solution to the second puzzle. Cartesian echoes part II: causality and elasticity

In the previous Section, Kant was portrayed as accusing Leibniz of providing a purely mathematical treatment of living force, susceptible to an unwelcome regress objection. Yet Kant's final goal in *True estimation* was in fact to vindicate Leibniz's living force by suitably adding a metaphysical dimension to the Cartesian–Leibnizian debate on *vis viva*, which in Kant's eyes could rescue the Leibnizian measure of living force from the objections it fell prey to from a purely mathematical point of view [1: 107.7–31]. Thus,

[34] See MAN [4: 532.38-533.1].
[35] See on this point our caveat in footnote 20.

although the Cartesian measure is said to be more accurate from a mathematical point of view, Kant's objective was ultimately to defend the Leibnizian measure of force via a suitable new metaphysics of forces. In so doing, Kant was following a well-trodden methodological path that Leibniz himself had traced.

Indeed, in the *Specimen Dynamicum*, Leibniz had advocated the necessity to supplement the mathematical treatment of forces, like the one he himself had proposed in earlier works,[36] with proper metaphysical laws such as the law that *causes equal their effects*. By assuming that force is strictly conserved in both elastic and inelastic collisions, Leibniz used the aforementioned causal law to accuse the Cartesians of opening the door to the absurdity of perpetual motion with their estimation of force as $m|v|$.[37]

While following this well-trodden Leibnizian methodological path, in *True estimation* Kant expressed dissatisfaction with the Leibnizian use of causality as an argument against the Cartesians. Not only did he accuse Leibniz of providing an essentially mathematical treatment of force, which proved ultimately congenial to Descartes' dead force; but he also maintained that the Leibnizians were unable to disprove Descartes on the metaphysical ground of causality. Indeed—Kant argued—while Leibniz took the lead from Descartes' principle of conservation of momentum to conclude that forces must be conserved in impact (i.e., effects cannot be greater than their causes), he begged to differ from Descartes as to the measure of such forces acting as causes [1: 58.27–36]. For Leibniz believed that any alternative measure for forces would have resulted in either a violation of causality (as with the Cartesian account), or in some unseemly resort to the 'hand of God' in restoring lost forces in nature (as with the Newtonian account).

Kant set for himself the task of showing that no violation of causality arose from the $m|v|$ estimate of force, and that it was possible in fact to give an account of elastic collisions along Cartesian lines. The Leibnizian error—in Kant's eyes—lay in the assumption that "no motion arises in nature except through matter that is also in actual motion and that therefore motion lost in one part of the world can be restored only by either actual motion or the direct hand of God" [1: 60.14–20]. But—Kant continued—such a false assumption was in turn responsible for the Leibnizians having to introduce

[36] In (1695), p. 440–1, Leibniz referred back to his early (1671) work complaining that it was inadequate to capture some fundamental metaphysical principles such as for example the principle of causality: "that there is *neither more nor less power in the effect than in the cause.* Since this law is not derived from the concept of mass, it must follow from something else which is in the bodies, namely, from *force* itself, which always preserves the same quantity even though it is used by different bodies" (emphases added).
[37] See Leibniz (1695), p. 443–4.

the hypotheses of artificially contrived vortices and "infinitely many strange motions, which are far more wondrous and incomprehensible than everything those very motions are supposed to explain" [1: 60.23–29]. The tacit reference here is of course to Leibniz's fluid vortex theory in the *Tentamen de motuum coelestium causis.*[38]

This is the heart of Kant's metaphysical dispute on living forces: in the attempt to refute Newtonian mechanics and improve on the Cartesian measure of forces, the Leibnizians went astray in thinking that motion can only arise from other matter in actual motion. Leibniz's living force could restore the law of causality (causes equal their effects) on pain of introducing effectively 'mechanical forces' in Kant's own sense: i.e., forces responsible for *communicating* motion from one body to another, but not able to explain the *origin* of such first motions (except by appeal to wondrous, contrived ethereal vortices). Once more, we can catch a glimpse of the complex line of reasoning underlying Kant's famous passage in MAN where living forces are identified with Kant's own 'mechanical forces', as defined in Chapter 3 of MAN. As early as 1747, in *True estimation*, Kant had hinted exactly at such an identification in his criticism of Leibniz's *metaphysical foundations* for elastic impact.

The reader may wonder at this point whether Kant's own solution to the problem, as envisaged in this very early work, pointed to some Newtonian direction instead. The answer is negative. For if the Leibnizians erred in thinking that actual motion originated from matter already in motion, the Newtonians similarly erred in invoking the direct hand of God, who "spares himself as many operations as He can without adversely affecting the mechanical structure of the world, while making nature as active and efficacious as is possible" [1: 62.12–15]. Kant stressed exactly the same point against the Newtonian view eight years later, in *Universal Natural History*, where he complained: "Newton (…) asserted that the immediate hand of God had instituted this arrangement without the intervention of forces of nature" [AK 1: 261.7—262.14].[39] What is to be said about Kant's own *metaphysical ground* for forces? If neither the Leibnizian nor the Newtonian account would do, what else would?

Kant's own *metaphysical ground* takes the lead from the work of George Erhard Hamberger, Prof. of Medicine at Jena—whose main work *Elementa Physices* was published

[38] See Leibniz (1689). In this text, written after seeing the review of Newton's *Principia* in *Acta eruditorum*, Leibniz provided a mechanical explanation for gravity via the fluid vortex.
[39] Kant (1755a), English translation (1968), p. 60.

in 1727, and remained a very influential text in German universities at the time[40]—in asserting that actual motion can originate from matter at rest [1: 60.31–35]. Kant presented this claim as his own *metaphysical foundation*, and with an invitation to the Leibnizians to embrace this view so as to remedy the aforementioned limits. Hamberger is also said to have failed to deliver on his original promise, and Kant presented his own view as succeeding where Hamberger's failed [see especially 1: 142–148]. But how can 'actual motion' originate from matter at rest? How are the Leibnizians supposed to make use of Hamberger's *vis insita* to escape the difficulties of their account? There is something prima facie paradoxical about Kant's suggestion that actual motion can originate from matter at rest: it seems to violate some Eleatic principle to the effect that what there is cannot possibly originate from what there is not. Motion cannot originate from rest as absence of motion.

Yet this is the key to Kant's solution to the Cartesian–Leibnizian stand-off, and an important anticipation of his mature Dynamics: a new notion of *rest*, or matter at rest, had to be introduced as the metaphysical ground of mechanical motion. If the Leibnizians erred in thinking that living forces originate from without (by communication of motion, via the continuous impressions of dead force through an external force), Kant's new metaphysical dynamics was expected to show how living forces originate *from within*, namely from an *inner Naturkraft* of matter at rest.

Hamberger's *vis insita* provided here the cue. Kant called this *inner Naturkraft* 'intension' (Intension), defined it as proportional to the velocity of a body, so that the product of Intension for the external force acting on a body (which was also said to be proportional to the velocity) would give back Leibniz's squared velocity law [1: 142.28–143.30]. The upshot was to provide an *internal ground* for living forces so that when the

[40] See Heilbron *Elements of early modern physics*, p. 133. G. E. Hamberger (1697-1755) published *Elementa physices, methodo mathematica in usum auditorii conscripta* in 1727 (Jena, 5th ed. 1761), where the resistance of bodies is identified with a force, called *vis insita*. But Hamberger's *vis insita* should neither be confused nor identified with Newton's *vis insita* or *vis inertiae*, which is nothing but the inertial mass of a body, distinct from *vis impressa* suh as gravity or any other centripetal force (see Janiak, this volume, ft 25); or better, it is a force that a body exerts when external influences act to change its state of motion or rest (see DiSalle, this volume, end of Section 3.) Instead, Hamberger first defined *vis insita* in §16 as one of the contingent causes of motion (causa motus occasionalis est resistentia), and added that rest is nothing but the result of the equilibrium among forces (§24: Aequales vires directe. i.e. in una linea recta, in se sine motu agentes, sunt in aequilibrio, i.e. in quiete). Hamberger's *vis insita* was not a passive force, but an active principle along Leibnizian lines (§35: resistentia est actio corporis in corpus qua hujus corporis actio quaecunque quocunque modo determinatur; ergo resistentia est a vi insita determinata).

intension was infinitely small and outweighed by the external force, bodies would be subject to dead forces only; whereas, by the law of continuity and by accumulation over *a finite interval of time*, the same intension would eventually be 'vivified' (under the action of external forces) and would result into living forces able to preserve motion, like in projectiles [1: 145.12–36; 146.1–21]. But how can motion originate from rest without violating either Leibniz's metaphysical law that *causes equal their effects,*[41] or some fundamental Eleatic principle?

Kant's metaphysical move did not violate Leibniz's causal law because it presupposed a *new notion of causality*, as Kant's extensive discussion of elastic impact in Chapter 2 of *True estimation* makes it clear. Elastic impact provided indeed the battleground for the Leibnizian attack on the Cartesians. For, the Leibnizians argued, only by the squared velocity measure was it possible to explain why small bodies could produce a motion greater than their original velocity without violating causality. According to the Leibnizian Jacob Hermann,[42] for example, in the collision between a body A of mass 1 and velocity 2 with a body B of mass 3, first, and then a body C with mass 1, causality seems violated as the final force in bodies B and C would be twice as big as the original force of body A [1: 51.2–35].

In defense of the Cartesian measure of force, Kant ingeniously replied that neither a violation of causality nor a miracle takes place in the circumstances, if one thinks of bodies A and B as two elastic spheres, colliding along a plane at a point D, where one can represent the elastic force of body A "which is activated by impact, by spring AD and the elasticity of sphere B by spring DB". The force with which spring DB is compressed by the action of A is said to be equal to the resistance that sphere B in turn opposes to AD, along the lines of a principle of action and reaction. And assuming that velocities are inversely proportional to the masses, Kant concluded that body B has an overall final velocity of 1—by subtracting the half degree of speed along BE conferred by DB from

[41] In defense of Leibniz, one may respond that the causal law is not violated if we assume—under momentum conservation—that the total effect of any elastic collision is in fact equal to the full cause: *causa plena aequat effectus integer* (we thank Marius Stan for making this point). But this response does not address Kant's aforementioned qualm against Leibniz. Namely, that the law of causality can be maintained only by assuming that whatever motion (i.e. momentum) is generated in one direction, an equal quantity of momentum is generated in the other direction, so that the total new motion is zero on pain of introducing 'mechanical forces' in Kant's sense: i.e. forces responsible for *communicating motion* from one body to another, but not able to explain the *origin of motion* (unless Leibniz's vortex theory, or some suitable surrogate, is introduced).

[42] Kant refers here to Hermann (1716).

the 1+1/2 degree of velocity that B allegedly received from spring AD (where the mass of A is a third of the mass of B), exactly as body A would have one degree of velocity, by subtracting from the original 2 degrees, in direction AE after impact with sphere B at point D. Pace Hermann, elastic collisions of bodies confirmed Descartes' $m|v|$ law, in Kant's eyes, without violating causality or resorting to miracles [1: 52].

Kant traced the source of Hermann's mistake in an erroneous view of causality, whereby the Leibnizian principle that *causes equal their effects* translated into forces after elastic impact being solely the effect of the forces before impact [1: 53.13–17]. If understood along the lines of traditional *efficient causes*, the Leibnizian principle enjoins an explanation of elastic impact whereby forces originate from without (e.g. by communication of motion from body A to body B), and, as discussed previously, such mathematico-mechanical treatment was deemed inadequate to capture the metaphysics of nature, according to Kant. A proper metaphysical foundation would require instead a new notion of causality, whereby forces originate from within bodies, through their natural powers. This is what Kant means by saying that "the compression of the spring was not so much a special effect of the force with which A moved forward against B as rather a consequence of the inertial forces [Trägheitskraft] of both bodies" [1: 53.17–23]. Efficient causes must be understood as *inherent* in bodies and unleashed via impact: they must have an immanent basis, or *Grund* in the body itself and its natural powers, if they have to function as first causes, on pain of the same regress that affected the Leibnizian mechanical treatment, as we saw in the previous Section. These are the seeds of Kant's later principle of causality in *New Elucidation,*[43] as a principle of determining ground, whereby "(1) *There is nothing in that which is grounded which was not in the ground itself*" [1: 406. 26–27]. No wonder, in this same context as an elucidation of the new principle of causality, Kant went back precisely to elastic collisions of bodies as he had presented and explained them in *True estimation*:

> The obviousness of this rule is clearly apparent in the changes of bodies. If, for example, body A moves another body B by striking it, a certain force and therefore a certain reality is imparted to the latter body. However, an equal quantity of motion is taken from the body which imparts the blow, so that the sum total of the forces in the effect is equal in magnitude to the forces of the cause. However, in the

[43] Kant (1755b). For an excellent analysis of Kant's broader metaphysics of causality, see Watkins (2005).

> case of the collision of a smaller elastic body with one which is larger the law we have adduced seems to be false. But this is not at all the case. For the smaller elastic body in striking the larger is repelled by it, thereby acquiring a certain force in the opposite direction. (…) The calculation is performed by subtracting from each other the motions which strive in different directions; for these motions will, of course, (…) somehow eventually cancel each other out. (…) as we know from statics, that motion is the same after impact as it was before [1: 407.7–22].

Kant's defense of the Cartesian $m|v|$ law, against the Leibnizian allegation of violating causality, finds then its echoes in Kant's new treatment of causality as originating from an inner ground, and not as mere efficient cause due to external impressed forces. But what is this inner ground, or *inner Naturkraft*? And what, if anything, is distinctively Cartesian about it? Back to the second aforementioned worry: is not Kant dangerously violating some Eleatic principle in conceding the possibility of mechanical motion out of matter at rest?

Here we touch upon a central aspect of Kant's view of nature, for the natural power that bodies would possess in their state of rest—which would explain both their ability to *resist* penetration by other bodies and their ability to *cause* motion in other bodies—is nothing but some primordial *elasticity*. That Kant's view does not violate any Eleatic principle is clear from the identification of the *inner Naturkraft* of bodies with elastic matter, responsible for matter's impenetrability as well as for its ability to produce mechanical motion under an external force. In the aforementioned diagnosis of Hermann's mistaken calculation for elastic impact, Kant referred to this force of elasticity as *Federkraft* or *Elasticität* [1: 54. 32–33]. There follows the most intriguing passage for our story, where in mentioning a controversy on this issue between the Marquise du Châtelet and Jean-Jacques Dortous de Mairan, Kant reported the latter's opinion with the following words:

> Herr von Mairan says: elastic force is a true machine of nature etc., etc.; that if one wishes to observe in particular all the effects of the impact of elastic bodies by summing up as positive what they yield in the two opposite directions, then one must ascribe the new force that appears to arise from this in nature and that manifests itself through the impact, not to the activity of the striking body, as if it transferred this activity to the body struck, but rather to a foreign source of force,

etc. etc.. In a word, one must ascribe it to a certain *physical cause of elasticity*, *whatever it may be, whose efficacy the impact has merely released and triggered*, so to speak, etc. etc. But I say that if Herr von Mairan says this, then Frau von Chastelet responds: it is useless to investigate this until the author of this opinion has taken the trouble to base what he wished to claim here on some proof. I have done myself the honor of undertaking this effort in place of Herr von Mairan, which is the justification by which I excuse my long-windedness in this matter [1: 55.9–34]

Thus, it is ultimately the internal debate in French salons between du Châtelet and de Mairan that proved key to Kant's metaphysical stance against the Leibnizians. Kant's search for an *inner Naturkraft* ended with the identification of elasticity as a primordial force, *whose physical cause had yet to be found*, and whose efficacy would be unleashed by impact among bodies. In the following years, Kant embarked on a journey to investigate the physical cause of elasticity in air (particularly through the experiments of Stephen Hales on vegetable and animal fermentations) as well as in some primordial fine cosmic matter (in *Universal Natural History*).[44] The physical cause or seat of elasticity was ultimately found in an original repulsive force, which many years later, in *Metaphysical Foundations of Natural Science*, became the cornerstone of Kant's Dynamics, both in the treatment of matter's impenetrability, and, via the balancing argument, of matter's rest and possibility of mechanical motion.

Equipped with this textual interpretive analysis, we can now go back to the second puzzle with which we opened this paper. Why did Kant in *MAN* say that dynamical forces (i.e. attraction and repulsion) are more akin to dead forces, if the term 'dead force' still deserved to be used? How could static forces, such as attraction and repulsion, ever produce mechanical motion? The interpretive analysis we have offered so far suggests the following answer on behalf of Kant.

Dynamical forces are akin to Cartesian dead forces, as the first (non-mechanical / metaphysical) *causes* of motion (where motion cannot be generated by other matter in motion as per Leibniz's living forces *qua* mathematical-mechanical forces, on pain of regress; nor can it be generated by resorting to the hand of God, as per Newton's mechanics). The analogy between Kant's dynamical forces of attraction and repulsion and Cartesian dead forces is justified because Cartesian dead forces are regarded as:

[44] For a reconstruction of Kant's analysis of elasticity in air and cosmic matter, and for its sources in Hales's experimental tradition among others, see Massimi (2011).

1. forces that originate from bodies at rest (e.g., a ball gently pushed forward by hand), by contrast with living forces;

2. forces, whose ground is inherent in bodies at rest (e.g. forces unleashed or triggered by impact);

3. forces that are somehow elastic in nature (where elasticity is the first ground of motion, whose physical cause or seat has yet to be determined).

Cartesian dead forces provided the closest candidate for Kant's search for a new dynamics in 1747, within the context of the heated debates on the nature of matter, *vis viva,* and motion in the Paris Academy of Sciences. Fifty years later, when even the echoes of these debates were fading away, Kant built his dynamical theory of matter and mechanics on some metaphysical tenets (causality and elasticity), still somehow lingering from those original debates.

Dynamical forces are *sufficient conditions* for mechanical motion if understood as *causes* of mechanical motion. And they can legitimately be regarded as *causes* (without the risk of either violating causality, pace Leibniz; or resorting to divine intervention, pace Newton) because they ground the possibility of mechanical motion as an *inner Naturkraft*, or natural powers of bodies, such as elasticity (in the case of elastic impact).[45]

But the perceptive reader might insist at this point that there is nothing distinctively Cartesian about this line of reasoning. After all, in the *Specimen Dynamicum,* Leibniz had himself defended elasticity as a natural power of matter, precisely in the context of his criticism of Descartes.[46] Is not Kant ultimately following a well-established Leibnizian tradition here, pace any prima facie defense of Cartesian echoes in *True estimation*? In the next Section, we take a closer look at one specific episode in the *vis viva* controversy, which in our view, lends some credibility to our interpretive line.

[45] One may wonder how to reconcile this characterization of repulsion as a fundamentally elastic force with Kant's Mechanics, which is essentially dedicated to inelastic impact. But this worry is precisely grist to our interpretive mill. If our line is correct, the origin of the conundrum has to be found in the pre-Newtonian echoes of Kant's Dynamics versus the broadly Newtonian tone of his Mechanics.

[46] "There follows also from these matters the view which Descartes attacked in his letters and which some great men are even now unwilling to admit—that *all rebound arises from elasticity,* and a reason is given for many brilliant experiments which show that a *body is bent before it is propelled;* Mariotte has shown this most beautifully. Finally, there follows also that most admirable principle of all—that there is no body, however small, which has no elasticity and is not thus permeated by a still subtler fluid" Leibniz (1695), p. 447.

5. Which Cartesian echoes in Kant's dynamical forces?

Mary Terrall[47] has vividly reconstructed one of the most animated episodes in the *vis viva* controversy: namely, the 1724 essay prize of the Paris Academy of Sciences on the laws of impact, where Johann Bernoulli's Leibnizian account of bodies' collisions in terms of elastic matter lost the prize to the Scottish Colin Maclaurin's Newtonian account. Jean-Jacques Dortuous de Mairan, later Secretary of the Paris Academy, played a key role in this essay prize competition not only by personally liaising with Bernoulli but also by encouraging a close ally of him, Pierre Bouguer, to refute Bernoulli's proofs in the years to follow. In 1728, Mairan went himself public against Leibnizian dynamics by reading his *Dissertation sur l'estimation et la mesure des forces motrices des corps* to the Paris Academy. Twelve years later, Émilie du Châtelet, working closely with the Newtonian Maupertuis and under Bernoulli's influence, published a response to Mairan in her *Institutions de physique* (1740). The ensuing exchange with Mairan was reprinted as an appendix in her *Dissertation sur le feu*.[48]

As we mentioned above, the young Kant was familiar with this debate and referred to it in several places in *True estimation;* in particular, he referred in key places to Mairan's Cartesian defense of dead forces against Leibniz and Du Châtelet, and even presented himself as continuing the work of Mairan on elasticity as the "true machine of nature". In this Section we briefly highlight some of the salient points of this famous dispute so as to provide the answer to the question that gives our paper its title: Cartesian echoes in Kant's philosophy of nature.

In the opening Section of his 1728 *Dissertation,* Mairan declared that the purpose of his work was not to treat forces in bodies from a metaphysical point of view, but to look instead at how geometers use them in mechanics.[49] This was the distinctive way in which the Cartesian tradition had been treating forces as measured by their effects, by the velocity with which hard bodies moved uniformly, for example. Mairan then swiftly presented the opinion of another type of 'geometers' (the Leibnizians), who regarded perfectly uniform motion and empty space without resistance as "pure fiction":

[47] Terrall (2004).
[48] Châtelet (1744).
[49] Mairan (1728), p. 2.

> There is in the majority of bodies a property, which is called springiness [ressort] or elasticity [vertu élastique], which acts by compression and by rendering the parts displaced by collision, and which acts over a finite time only and via a sequence of impulsions or successive impressions.[50]

While acknowledging that the abstractions and idealizations of thinking motion as perfectly uniform in a perfectly empty space does not change anything about the nature of the effects of collision (and it is only designed to make its study simpler), Mairan warned that one should consider whether the collisions between bodies did not nonetheless hide some degree of activity, which was not immediately perceivable to us, and which could not enter into the fictitious hypothesis of hard and rigid bodies. Led by these considerations some geometers (i.e. the Leibnizians) distinguished between dead forces and living forces, where the latter were defined by the squared velocity. There followed a description of the aforementioned experiments with balls released at different heights and leaving impression on clay as a way of measuring the nature of living forces, to which Kant referred in *True estimation* (as we saw in Section 3.1). While these experiments were designed to vindicate the Leibnizian principle of causes being proportional to their effects, Mairan responded that one should not forget to take into account the common measure in these experiments, namely time.[51]

We have already discussed in Section 3.1 how Kant availed himself of precisely this response in terms of time to make his point against the Leibnizians both in *True estimation* and in MAN. Despite this Cartesian defense of dead forces, Mairan proved to be a *sui generis* Cartesian in defending the view that the inertia of matter (or inertial mass) could be ultimately explained in terms of forces.[52]

[50] Ibid., p. 6.

[51] Ibid., p. 23: "the loss of force and velocity of mobiles that traverse different spaces, or that overcome a different number of obstacles, are always proportional to the times employed to traverse each of these spaces, and to overcome each of these obstacles; the reason being that the contrary impulses, the *resistances*, or, so to speak, the contrary forces act, by as much or as little (all else being equal) against those forces that overcome them, and they consume them, in proportion to the more or less time they are applied to them".

[52] Ibid., p. 39: "the inertia of matter, whatever be its cause; this resistance, more or less big, which it exercises against being moved from rest and against receiving a finite movement in virtue of its mass, could not it be conceived eventually as the effect of some movement? (..) should not it be conceived as a real force, acting through some mechanism which we do not know of?".

Mairan's *Dissertation* steered intellectual debates in Parisian salons. The defence of Leibnizian dynamics fell on Gabrielle Émilie Du Châtelet, who under the help of the Newtonian Maupertuis (sympathetic to Bernoulli) and the tutoring of Samuel Koenig (introduced by Maupertuis himself), read Leibniz's *Specimen Dynamicum* and in 1740 wrote the *Institutions de Physique,*[53]where she undertook the ambitious task of defending Leibniz-Bernoulli analysis of elastic impact against the predominant Cartesian orientation of the French intelligentsia. The publication of the *Institutions de Physique* raised Mairan's hackles, by then acting as Secretary of the Paris Academy of Sciences. The reaction was immediate. In 1741, Mairan published a *Lettre à Mme *** sur la question de forces vives,*[54] where he went back to the discussion of elastic collisions between three balls A, B, and C to defend the Cartesian measure of force. It is in this context that Mairan accused Du Châtelet of making a mistake in the calculation of the joint effects of the forces acting on the three balls and concluded:

> The springiness [ressort] is a true machine of nature, of which the effects should be evaluated as those of ordinary machines, by their total action in the direction of the strongest: that these effects consist in doubling those that would have been produced by simple collision in non-elastic matters: that if we want to consider separately all the effects of collisions of elastic bodies, by summing up as positive what they give in two opposite directions, one should not attribute the new force, which seems to result in nature and that manifests itself through the collision, to the energy of the colliding body, as if it were only transmitting it to the receiving body, *but to a foreign principle of force* [principe étranger de Force], where the force apparently produced already was and from which it originates; in a word, *to whichever physical cause of springiness,* to which the collision has done nothing but unleashing the activity.[55]

This is the famous passage, which as we saw above, Kant quoted at some length in *True estimation* [1: 55.9–34]. Kant traced the source of Hermann's mistake in a metaphysically erroneous view of causality, which he saw enacted in Leibniz's principle of *causes equal*

[53] For biography and work of Du Châtelet, and her defense of Leibnizian dynamics, see Iltis (1977); Barber (1967); Terrall (1995).
[54] The letter was followed by Du Châtelet (1741). Both letters were reprinted in the appendix to Du Châtelet (1744).
[55] Mairan (1741), pp. 26-27. Emphases added.

their effects whereby forces are treated in a mathematico-mechanical way as *merely communicating* motion between bodies and hence as arising from without, rather than from within. We mentioned also how Kant's defense of dead forces was going to be part of a larger project that redefined efficient causes as inherent in bodies and being unleashed via impact (project that Kant undertook eight years later in *New elucidation*). We can then appreciate the Cartesian echoes of this larger project in the specific way in which Mairan advanced and advocated a view of causality at work in elastic collisions against the Leibnizian dynamics defended by Bernoulli and Marquise Du Châtelet. Mairan's appeal to the "physical cause of springiness, whatever it be", whose activity has simply been unleashed by the collision, pointed to a new metaphysics of nature whereby the same inertial mass of bodies, responsible for their resistance and impenetrability, was given a dynamical gloss.[56]

Not surprisingly, in her response, Du Châtelet ridiculed Mairan, on the ground that the 'minus' sign through which forces were subtracted in elastic collisions among bodies seemed to correspond somehow to *negative* forces that had to be subtracted to the *positive* forces of bodies.[57] But how could a 'minus' sign have physical meaning, according to the rules of algebra and geometry? The answer to this question was precisely the shift from the Leibnizian mathematico-mechanical way of thinking about forces, still exemplified by Du Châtelet's response to Mairan, to a proper metaphysical approach, which Kant went on to explore, taking its cue from Mairan. The end product of this metaphysical approach eventually was a new dynamics of moving forces, like the one that Kant laid

[56] One may wonder here whether Mairan ends up being a thoroughgoing Leibnizian, *malgré lui.* After all, it was Leibniz, who in Part II of the *Specimen Dynamicum*, postulated elasticity as an essential internal force of bodies, activated or triggered by collisions (we thank Marius Stan for this comment). There is yet a clear sense in which Mairan's self-declared Cartesianism is evident in his defense of *ressort.* For Leibniz, elasticity is a fundamental property of any body "awakened" or "triggered" by collisions and even explained by subtle fluids ("there is no body, however small, which has no elasticity and is not thus permeated by a still subtler fluid", Leibniz 1695, p. 447). For Mairan, vice versa, "the springiness is a true machine of nature, of which the effect should be evaluated as those of ordinary machines". We want to stress the word "machine" here as an important Cartesian signpost. For the Cartesian Mairan, *ressort* was not so much a metaphysical force of matter (as for Leibniz). But it was a *true machine of nature, a machine for the production* of phenomena including, following Descartes, barometric pressure and its exact estimate in barometers (see entry "Barometre" in Diderot's *Encyclopédie: ou dictionnaire raisonné des sciences, des arts et des métiers*). Our interpretive hypothesis is that Kant resorted to Mairan in his defense of elasticity, because he might have found in the Cartesian Mairan a metaphysically more neutral characterization of elasticity, amenable to the reinterpretation that Kant was about to give to it at a distance from Leibniz.
[57] See Du Châtelet (1741), p. 25.

out in MAN fifty years later. But the seeds of this larger metaphysical project are visible in *True estimation,* in the rapidly fading debate on living forces that constitutes its topic, and in Kant's unequivocal partisanship for the Cartesian side, tentative and provisional as it was.

6. Conclusion

In this paper, we have endeavored to accomplish two tasks. First, to clarify in what sense Kant's dynamical forces of attraction and repulsion are *necessary* for Kant's mechanics. Second, to make the case for why Kant's dynamical forces are *sufficient* for Kant's mechanics. The answer we have offered to the first task enjoined us to go beyond the received view of Kant's philosophy of nature as almost emanating from Newtonian mechanics. Following a suggestion by Dan Warren, we argued that the *necessary* role of Kant's Dynamics for his Mechanics forces us to expand the intellectual context against which Kant elaborated his view to include the lively debate between Cartesians and Leibnizians on the nature of forces in the first half of the eighteenth century. Thus, there is a genuine pre-Newtonian element in Kant's Dynamics and its claim that there cannot be laws of mechanics without dynamical laws.

Our answer to the second task expanded on this pre-Newtonian intuition. For dynamical forces can be regarded as *sufficient* for mechanical motion, only if one endorses a view of forces as *inherent causes* of mechanical motion. We made the case for this view of *inherent causality*, or causality from within bodies, in the light of Kant's idiosyncratic take on the *vis viva* debate in *True estimation*. What we found is that what is distinctively Cartesian, or better Mairanian is Kant's idea of elasticity as a *physical cause* of motion being inherent in bodies and being, so to speak, 'unleashed' by impact. Years later, Kant identified the *physical causes* of mechanical motion with dynamical forces of attraction and repulsion.

Yet there is an important caveat to the conclusion we have reached. The Cartesian / Mairanian echoes alluded to in our title are not meant to suggest any radical, revisionist account of Kant's philosophy of nature. Kant is no Descartes. There is no trace of Cartesian *res extensa* and corpuscular philosophy in Kant's Dynamics or Mechanics. The same Mairan, for all his Cartesian allegiances, never held a truly *dynamical* view of elasticity as the one that the mature Kant defended in MAN. Instead, Mairan's 'vertu élastique' or 'ressort' as the "true machine of nature" was still understood in the

Cartesian tradition of collisions among hard bodies. What is new in Mairan's treatment of forces is the critique of the Leibnizian principle of *causes equal their effects* (typically deployed against the Cartesian dead forces), and his hint at the "cause Physique quelconque du ressort".

This is where the young Kant came in, and in his unique way embarked in the long metaphysical journey of searching for the *physical cause* of the springiness of bodies in their original repulsive force as an elastic force. Although elasticity was a common and widespread theme in the literature on collisions at the time (e.g. both Leibniz and Newton had in different ways appealed to it), the specific way in which Kant bestowed on *elasticity* the role of an original metaphysical force capable of engendering mechanical motion via *a new conception of causality*, at odds both with the Leibnizian mechanical dictum of *causes equal their effects* and with the Newtonian resort to the divine hand, speaks, in our view, to the relevance of Cartesian echoes in Kant's natural philosophy.

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