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The Effect of Aspirations and Habits on the Distribution of Wealth*

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Abstract

We analyze how the introduction of habits and aspirations affects the distribution of wealth when individuals' labor productivity is subject to idiosyncratic shocks and bequests arise from a joy-of-giving motive. In the presence of either bequests or aspirations, labor income shocks are transmitted intergenerationally and this transmission, together with contemporaneous shocks, determines the distribution of wealth. We show that the introduction of aspirations (habits) decreases (increases) the average wealth and increases both its intragenerational variability and the degree of intergenerational mobility. Therefore, a distinction between aspirations and habits is relevant as they involve different implications for the distribution of wealth.

Keywords: bequests; variability; mobility

JEL classification codes: D31; E21; E62

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I. Introduction

In this paper, we analyze how the introduction of aspirations and habits affects the distribution of wealth. When aspirations are present, an individual's utility depends on a comparison between his current consumption and that of his parents, while in the case of habits, the utility associated with a given amount of current consumption depends on a comparison between an individual's current and past experiences of consumption. So, on the one hand, aspirations generate preferences depending on the previous generation's consumption, while habits are an intragenerational phenomenon that leads to greater consumption smoothing throughout the life cycle. In both cases, past consumption is used as a reference with which current consumption is compared and this implies that preferences turn out to be time non-separable. However, as we will see, the implications for the distribution of wealth are quite different depending on whether habit or aspiration motivations are more relevant in the evaluation of the utility delivered by current consumption.

A large number of empirical studies provide evidence for the effect of the level of past consumption on the satisfaction derived from current consumption. In accordance with this evidence, some authors have used preferences displaying habit formation to improve the predictions made under time separable preferences in different economic scenarios.[1] Moreover, there is also empirical evidence for the existence of aspirations associated with the involuntary transmission of tastes from one generation to the next. For instance, Cox et al. (2004) estimate that parental preferences explain between 5 to 10 percent of their children's preferences after controlling for their respective incomes.[2]

Our analysis will be conducted in the framework of an overlapping generations (OLG) economy where individuals' preferences display "joy of giving". This means that individuals' utility will be an increasing function of the amount that they bequeath to their children, as in Yaari (1965) and Abel (1986). Several alternative motives for

[1] See, among others, Abel (1990, 1999), Ljungqvist and Uhlig (2000), Carroll et al. (1997, 2000) and Alonso-Carrera et al. (2004, 2005).

[2] Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) provide surveys of the evidence on intergenerational transmission of tastes. Among the theoretical studies on the macroeconomic implications of aspirations, we could mention those by de la Croix (1996, 2001), de la Croix and Michel (1999, 2001), and Alonso-Carrera et. al (2007).

intergenerational transfers have been proposed in the literature. Among these, we could mention strategic behavior (Bernheim et al., 1985), the existence of incomplete annuity markets (Abel, 1985), and pure intergenerational altruism (Barro, 1974). However, the empirical evidence is not conclusive regarding the reasons why individuals make intergenerational transfers and a combination of all those motives probably lies at the core of the mechanism governing the intergenerational transmission of wealth.

When individuals care about their children's total income, bequests play an equalizing role since individuals then tend to compensate for the differences in the idiosyncratic realizations of the random income of their direct descendants. This compensation principle has been used to argue against inheritance taxation since it could have a disequalizing effect due to the distortion of the optimal risk sharing between two consecutive generations (Becker and Tomes, 1979; and Davies, 1986). In our framework of joy-of-giving preferences, this compensation principle does not come into play since individuals do not seek an optimal allocation of family income between them and their children but rather an optimal allocation of individual income between their own consumption and bequests. Bossmann et al. (2007) have shown that under joy-of-giving preferences, the introduction of bequests results in a reduction in the value of the coefficient of intragenerational variation of wealth. This is so because the average stock of capital grows due to the increase in saving induced by the bequest motive, which offsets the modest increase in the variance of wealth associated with the intergenerational transmission of income shocks through bequests.

Our framework will also be suitable for the study of intergenerational mobility, which is characterized by the correlation between the wealth of parents and their children. Obviously, the introduction of bequests has a negative effect on mobility since they facilitate the intergenerational transmission of wealth status.

We will show that habits and aspirations affect both the size of aggregate bequests and the level of the capital stock in the economy in a similar direction to that obtained by de la Croix and Michel (2001), Jellal and Wolff (2002), and Alonso-Carrera et al. (2007), who conducted the analysis under the assumption of altruistic preferences à la Barro (1974). Using the coefficient of variation of wealth as a measure of intragenerational wealth inequality, we also show that the introduction of aspirations increases wealth inequality since aspirations make the shocks in labor income more persistent. This is so because aspirations make an individual's adult consumption (and his saving) more dependent on the income shocks of his family predecessors, which results in a stronger propagation of those shocks within a dynasty. However, the introduction of habits decreases the intragenerational inequality of wealth when aspirations are present. Even if saving becomes more sensitive to labor income shocks as a result of the stronger desire for consumption smoothing brought about by habits, the increase in the average amount of savings results in a smaller value of the coefficient of variation of savings.

Note that an environment populated by infinitely lived agents does not make any distinction between aspirations and habits possible. However, this distinction proves fundamental in an OLG setting since, while habits tend to reduce wealth inequality, aspirations tend to exacerbate it. Therefore, the claim that the introduction of past consumption references to the utility function does not help to generate more wealth inequality (see, for instance, Cagetti and De Nardi, 2008, and Díaz et al., 2003) should be qualified for an OLG economy where we can effectively distinguish between the two aforementioned features affecting individual preferences.

We also evaluate the effects of habits and aspirations on intergenerational mobility. We measure this mobility using the autocorrelation coefficient of asset holdings within a family. We show that, due to the induced reduction in the amount of bequests, aspirations tend to enhance intergenerational mobility. However, habits make saving more correlated with contemporaneous wages and this leads in turn to a larger intergenerational correlation of savings when aspirations are present.

Our results for an OLG economy with preferences displaying 'joy of giving' differ in many respects from those of the related paper by Díaz et al. (2003), who considered an economy with infinitely lived agents. In order to make a proper comparison, we should consider the version of their model in which the elasticity of intertemporal substitution is not adjusted when habits are introduced. First, as we have already said, our model enable us to introduce the phenomenon of intergenerational transmission of tastes, which cannot be accommodated in non-OLG economies. Furthermore, our simple model allows us to obtain closed form expression for the comparative statics exercises when aspirations and habits are marginally introduced. Concerning the results, while Díaz et al. do not obtain a definite sign for the change in aggregate savings when habits are introduced, our life-cycle specification makes it possible to obtain an unambiguous increase in aggregate savings due to the induced shift in income from the adult to the old period of life. Finally, our demographic structure permits a sharp characterization of the effects of habits on intergenerational mobility within a family.

The paper is organized as follows. Section II presents the general model with habits and aspirations. Section III characterizes the optimal individual decisions. Section IV analyzes some dynamic stability issues of the intragenerational distribution of wealth. In Section V, we characterize the measure of intergenerational mobility in wealth. In Section VI, we conduct the comparative statics analysis of changes in the intensity of habits and aspirations on the steady-state values of the moments of the distribution of wealth and on intergenerational mobility. We conclude the paper in Section VII.

II. Preferences and Technology

Let us consider a small open OLG economy with a continuum of dynasties, where individuals live for three periods and a new generation is born in each period. Each individual has offspring in the second period of his life and the exogenous number of children per parent is $n \geq 1$. We assume that agents make economic decisions only during the last two periods of their lives.

Each agent inelastically supplies one time unit of labor in the second period of his life and is retired in the third period. Let us index each generation by the period in which its members work (i.e. when they are adults). In the initial period $t = 0$, the mass of dynasties is N_0, there is a continuum of individuals of mass 1 per dynasty and dynasties are uniformly distributed on the interval $[0, N_0]$. The mass of dynasties N_0 is constant for all periods but the size of each dynasty grows and is equal to n^t in period t. The individuals belonging to each dynasty $j \in [0, N_0]$ are uniformly distributed on the interval $[0, n^t]$. Therefore an individual i who is an adult in period t is fully described by his dynasty $j \in [0, N_0]$ and his position $m \in [0, n^t]$ within the dynasty he belongs to. Therefore, we can write the individual index i as a two-dimensional vector, $i = (j, m) \in [0, N_0] \times [0, n^t] \equiv P_t$. Note that the Lebesgue measure of the population set P_t in period t is $N_t = N_0 n^t$.

There is a single commodity, which can be devoted to either consumption or investment. An adult individual $i \in P_t$ of generation t distributes his net labor income and inheritance between consumption and saving. The budget constraint faced by this worker i in period t is

$$w_t^i + b_t^i = c_t^i + s_t^i, \tag{1}$$

where w_t^i is the wage compensation of this worker, c_t^i is his amount of consumption (hereinafter, adult consumption), b_t^i is the amount of inheritance he has received from parents and s_t^i is the amount of saving.

When individuals are old, they receive a return on their savings, which is distributed between own consumption and bequests for their children. Therefore, the budget constraint of an old individual i belonging to generation t will be

$$R_{t+1} s_t^i = x_{t+1}^i + n b_{t+1}^i, \tag{2}$$

where R_{t+1} is the gross rate of return on saving, b_{t+1}^i is the amount of bequests the individual leaves to each of his descendants (who where born in period t) and x_{t+1}^i is the amount of consumption of the old individual i in period $t+1$ (hereinafter, old consumption). Note that we are implicitly making an equal-treatment assumption so that all the direct descendants of the same individual i receive the same amount of inheritance.

We will assume that in each period individuals derive utility from the comparison of their consumption with a consumption reference. Note that during their first period of life individuals neither work nor consume. However, as in de la Croix (1996), the member i of the generation born in period $t-1$ inherits a certain level of aspirations a_t^i in period t. These aspirations are based on the standard of living achieved by his parents. More precisely, we assume that the inherited aspiration of an individual i of generation t is

$$a_t^i = c_{t-1}^i, \tag{3}$$

where c_{t-1}^i is his parent's amount of consumption when the parent was an adult (second period of life). We posit the following additive specification for the aspiration adjusted consumption $\hat{c}_t^i$ of an adult individual i belonging to generation t:

$$\hat{c}_t^i = c_t^i - \delta a_t^i, \quad \text{with } \delta \in [0,1)\,. \tag{4}$$

We set a common initial level of aspirations for all individuals at the initial date, $a_t^i = a_0,$ for all $i \in P_t$. Therefore, when aspirations are present, the same amount of adult consumption will give rise to different levels of utility depending on the standards of living set by parental consumption. The adult individuals who have acquired higher aspirations due to their parents' experience of consumption will require a larger amount of consumption in order to achieve the same level of utility.

Preferences will also exhibit habit formation and, hence, the consumption reference of an old individual i of generation t is given by the consumption level reached in the previous period. Like with aspirations, we assume that the habit adjusted consumption $\hat{x}_{t+1}^i$ of an old individual i in period $t+1$ is given by the following additive function:

$$\hat{x}_{t+1}^i = x_{t+1}^i - \gamma c_t^i, \quad \text{with } \gamma \in [0,1) \,. \tag{5}$$

As occurs with aspirations, the old individuals who have experienced a larger amount of consumption when adults will need a larger amount of consumption when old in order to obtain the same level of utility as those individuals with less accumulated habits.

The individual i belonging to generation t derives utility from both aspiration adjusted adult consumption and habit adjusted old consumption. We posit the following time-additive and homothetic utility function representing the preferences of the individual i belonging to generation t:

$$U(\hat{c}_t^i, \hat{x}_{t+1}^i, b_{t+1}^i) = u\left(\hat{c}_t^i\right) + \beta u\left(\hat{x}_{t+1}^i\right) + \rho u\left(b_{t+1}\right), \tag{6}$$

where β and ρ are positive. Note that we are generating positive bequests through a "joy-of-giving" motivation (as in Yaari, 1965; or Abel, 1986) so that the amount of bequests enters directly as an argument in the utility function. There are other motives for intergenerational transfer, such as altruistic preferences à la Barro (1974) and Becker (1981) where individuals derive utility from their children's indirect utility function, or through paternalistic preferences where individuals care about their offspring's level of consumption (Pollak, 1988). Under altruistic preferences, the last term in the utility (6) would be replaced by the indirect utility function of one's children, which is an increasing function of the amount of inheritance received by descendants. If preferences were paternalistic, the last term in the utility (6) would be replaced by the offspring's adult consumption, which in turn would be an increasing function of the amount b_{t+1} bequest. In both cases, the results would be qualitatively similar to those obtained under a joy-of-giving specification. However, a problem posed by these two alternative types of preferences is the potential existence of corner solutions when the bequest motive is not operative, i.e. when the amount of bequest in equilibrium is equal to zero. We will avoid this problem by assuming joy-of-giving preferences displaying an Inada condition when the amount b_{t+1} of bequest tends to zero. In particular, for tractability, we assume a logarithmic utility, i.e.

$$u(z) = \ln z.$$

Clearly, our results will be qualitatively similar if we assume instead a isoelastic utility,

$$u(z) = \frac{z^{1-\eta} - 1}{1-\eta}, \quad \text{with } \eta > 0,$$

exhibiting a value for the parameter η sufficiently close to 1.

An alternative functional form found in the literature to introduce past consumption references to the utility function is the multiplicative. According to this formulation (see Abel, 1990 or Díaz et. al, 2003), individuals derive utility from the ratio between current consumption and the past reference rather than from their difference. Thus, the aspiration adjusted young consumption and the habit adjusted old consumption of the individual i belonging to generation t will be as follows:

$$\hat{c}_t^i = \frac{c_t^i}{\left(a_t^i\right)^{\delta}}, \quad \text{with } \delta \in (0,1). \tag{7}$$

and

$$\hat{x}_{t+1}^i = \frac{x_{t+1}^i}{\left(c_t^i\right)^{\gamma}}, \quad \text{with } \gamma \in (0,1), \tag{8}$$

respectively. It is obvious that the additive formulation is much more tractable than the multiplicative. In particular, one of the problems with the multiplicative formulation is that the objective function faced by individuals could fail to be jointly concave with respect to the consumption vector. To see this, note that the amount of young consumption c_t^i of an individual i belonging to generation t will appear both in the numerator and in the denominator of the first two terms of the sum of utilities (6) and this sum of concave functions of fractions is generically non-concave (see Alonso et al., 2005). Moreover, the qualitative results of the model remain unchanged under these two alternative formulations as they rely exclusively on the fact that a larger intensity of habits and aspirations (measured by the values of the parameters γ and δ, respectively) results in a larger marginal utility of current consumption. This effect on the marginal utility holds under both specifications. The multiplicative functional form should be adopted in stochastic environments in order to prevent the instantaneous utility function u from displaying a negative argument, which is a possibility under the additive formulation given in (4) and (5) when aggregate consumption fluctuates exogenously and cannot be accommodated through positive or negative saving. However, since we will allow poor individuals when adult to borrow (negative saving) and rich individuals to lend (positive saving), the problem of obtaining negative values for the argument of the utility function u readily disappears under the assumption of pure idiosyncratic shocks that we next introduce.

Let us assume that the good of this economy is produced by means of a production function displaying constant returns to scale in capital and efficient labor. In our small open economy, capital is fully mobile and labor is not mobile. Under competitive input markets this implies that the rental price of a unit of capital is constant and equal to its international level r. Therefore, the gross rate of return on savings satisfies $R_{t+1} = 1 + r \equiv R$. Moreover, the equilibrium capital to efficient labor ratio becomes constant and, thus, the marginal productivity of an efficiency unit of labor (which is equal to the competitive real wage per efficiency unit) is also constant. This means that the labor income w_t^i received by individual i can be interpreted as the realization of the random number of efficiency units of labor owned by worker i in period t multiplied by the constant wage per efficiency unit.

We assume that the number of efficiency units of labor, and thus the wage w_t^i received by the worker i of generation t, is the realization of a random variable that is identically and independently distributed (i.i.d.) across individuals of the same generation and across time with a finite fourth moment, $\mathbb{E}\left[(w_t^i)^4\right] < \infty$. Finally, we assume that the random variable w_t^i has the mean $\bar{w}$ and the variance σ^2 for all i and t. Therefore, we are assuming that all workers experience idiosyncratic productivity shocks that are cross-sectionally (i.e. across all individuals of the same generation) and intergenerationally (i.e. across individuals belonging to different generations irrespective of the dynasty they belong to) identical and independent. These shocks on labor income are assumed to be uninsurable.

The assumption of serial independence of wages will allow us to highlight the contribution of bequests and aspirations to explaining the correlation of wealth among members of the same family belonging to two consecutive generations. Serial correlation of labor income within a dynasty will automatically generate a dependence on consumption and wealth among different generations of the same dynasty. We thus abstract from this trivial, exogenous mechanism to link generations and focus exclusively on the contribution of inherited tastes and endogenous wealth transmission through bequests.

III. Optimal Consumption and Bequest

Individual i of generation t maximizes (6) with respect to $\{c_t^i, x_{t+1}^i, b_{t+1}^i, s_t^i\}$ subject to (1), (2), (4) and (5), taking as given a_t^i, b_t^i, w_t^i and R_{t+1}. If we plug the competitive gross rate of return $R_{t+1} = R$ into the solution to this individual problem, we find the following linear demand functions for consumptions, bequest, and saving in equilibrium:

$$c_t^i = A\left(w_t^i + b_t^i\right) + Ba_t^i, \tag{9}$$

$$x_{t+1}^i = C\left(w_t^i + b_t^i\right) - Da_t^i, \tag{10}$$

$$b_{t+1}^i = E\left(w_t^i + b_t^i\right) - Fa_t^i, \tag{11}$$

and

$$s_t^i = G\left(w_t^i + b_t^i\right) - Ba_t^i, \tag{12}$$

where

$$\begin{gathered} A = \frac{R}{(R+\gamma)(1+\beta+\rho)}, \quad B = \frac{\delta(\beta+\rho)}{(1+\beta+\rho)}, \\ C = \frac{R[\beta(R+\gamma)+\gamma]}{(R+\gamma)(1+\beta+\rho)}, \quad D = \frac{\delta(R\beta-\rho\gamma)}{(1+\beta+\rho)}, \\ E = \frac{\rho R}{n(1+\beta+\rho)}, \quad F = \frac{\delta\rho(R+\gamma)}{n(1+\beta+\rho)}, \\ G = \frac{R(\beta+\rho)+\gamma(1+\beta+\rho)}{(R+\gamma)(1+\beta+\rho)}. \end{gathered} \tag{13}$$

Clearly, optimal consumption, bequest, and saving of the individual i depend on the realization of his productivity shock w_t^i, on the amount of inheritance b_t^i he has received and, finally, on his aspiration level a_t^i, which is equal to the adult consumption of his parent c_{t-1}^i. Note also that adult and old consumption and bequest depend positively on both w_t^i and the amount b_t^i of inheritance. However, while adult consumption is increasing in the aspiration level a_t^i, saving and bequest are decreasing in a_t^i as aspirations force a shift in income towards adult consumption so as to mimic the parents' consumption experience. The effect of the aspiration level a_t^i on old consumption is generally ambiguous although it becomes negative for low values of either the level of altruism parametrized by the value of ρ or the intensity of habits parametrized by the value of γ (see the expression for the coefficient D in (13)).

From the previous expressions for consumption at different ages, bequest, and saving we can easily obtain the effect of changes in the preference parameters characterizing the strength of aspirations and habits on these variables. We immediate observe from (9) that, for given values for aspirations a_t^i, wage w_t^i, and inheritance b_t^i, adult consumption c_t^i increases with the value of aspiration intensity δ and decreases with the value γ of habit intensity. This is so because aspirations push adult consumption above the standards of living established by the parents whereas habits push adult consumption down so as to decrease the stock of future habits. Concerning the amount of saving and old consumption, we obtain the following partial derivatives from (12) and (10), respectively:

$$\frac{\partial s_t^i}{\partial \gamma} = \frac{R\left(b_t^i + w^i\right)}{\left(R+\gamma\right)^2\left(1+\beta+\rho\right)} > 0 \tag{14}$$

and

$$\frac{\partial x_{t+1}^i}{\partial \gamma} = \frac{R^2\left(b_t^i + w^i\right) + \delta\rho\left(R+\gamma\right)^2 a_t^i}{\left(R+\gamma\right)^2\left(1+\beta+\rho\right)} > 0.$$

The sign of the partial derivative of saving and old consumption with respect to δ is immediate from (10), (12), and (13). Obviously, due to the aforementioned effects on adult consumption, aspirations reduce individual saving, while habits increase them. Moreover, since the initial wealth at the beginning of the last period of life decreases (increases) with aspiration (habit) intensity, the amount of old consumption x_{t+1}^i becomes smaller (larger) accordingly. Finally, it is plain to see from (11) and the expression for the coefficient F that both, aspirations and habits, reduce the amount of intergenerational transfer. On the one hand, aspirations lower the amount of saving and thus the wealth available for bequest. On the other hand, habits raise the amount of old consumption at the expense of lower bequest.

Another property of the above consumption, bequest, and saving functions refers to their sensitivity with respect to the idiosyncratic wage (or productivity) shocks. To this end, we only have to analyze how the coefficients of w_t^i in these functions vary with the preference parameters, δ and γ. We observe that none of these coefficients depend on the aspiration intensity parameter δ, which means that the conditional variances of consumption, bequest, and saving given b_t^i and c_{t-1}^i, $Var\left(c_t^i|b_t^i, a_t^i\right)$, $Var\left(x_{t+1}^i|b_t^i, a_t^i\right)$, $Var\left(b_{t+1}^i|b_t^i, a_t^i\right)$, and $Var\left(s_t^i|b_t^i, a_t^i\right)$ are all independent of the aspiration intensity, since these conditional variances are equal to the square of the coefficients of w_t^i in (9), (10), (11) and (12), respectively. Note that the parameter δ only affects the sensitivity of these functions with respect to parental consumption, which is assumed constant in this comparative statics analysis. On the contrary, changes in the value of the habit parameter γ affect the sensitivity of those functions with respect to wage shocks. Note that the value of γ refers to the importance of adult consumption in evaluating the utility arising from old consumption. Since adult consumption is decided by the individual under consideration and not by his parents, all decisions throughout the life-cycle will be affected by the variation in the value of habit intensity. As habit intensity rises, individuals shift adult consumption towards the future so as to lower the stock of future habits. Therefore, old consumption will be more sensitive to wage shocks when habits are present, while the opposite will hold for adult consumption. A by-product of the previous effect is that saving is more sensitive to wage shocks. Finally, the coefficient of w_t^i in the bequest function (11) is independent of the habit parameter γ. This means that the sensitivity of bequests with respect to wage shocks is not affected by habit intensity either since the induced changes in this preference parameter are all accommodated throughout the life-cycle.

The first four columns in Table 1 summarize the previous comparative static exercises. As we have already said, the sensitivity of consumption, saving, and bequest with respect to wage shocks is measured by the corresponding conditional variance given the amount of inheritance b_t^i and aspirations a_t^i. Since the average amount of consumption, saving, and bequest is also affected by the changes in the values of preferences parameters, we also include in the table this effect on the coefficient of variation, $CV(\cdot) = [Var(\cdot)]^{1/2} \Big/ \mathbb{E}(\cdot)$, which is a more appropriate measure of variability than variance when the mean is changing. Note that in order to compute this measure we only need to know the mean $\bar{w}$ and the variance σ^2 of the i.i.d. stochastic process of wages w_t^i for all i and t. This measure is so simple that we can easily perform easily our intended comparative statics exercises. Other measures of variability, such as the Gini coefficient, will require full knowledge of the distribution of wages and will make it impossible to obtain comparative statics results in closed form. Thus, since we are able to sign the effect on the coefficient of variation of the introduction of both habits and aspirations, we will stick to this measure of inequality throughout our analysis. Moreover, as González-Abril et al. (2010) argue, the coefficient of variation and the Gini coefficient are in some sense similar measures of the variability of a random variable.

[Insert Table 1]

IV. The Dynamics of Consumption, Wealth, and Bequests

The dynamics of the economy within each dynasty is entirely governed by the system of stochastic difference equations formed by (9) and (11) after making $a_t^i = c_{t-1}^i$. Once the realization of the stochastic processes of young consumption and bequest have pinned down from those two equations, we immediately obtain the realization of the stochastic process of saving and old consumption from (10) and (12).

To analyze the aggregate behavior of the economy we should first obtain the aggregate levels per capita of the endogenous variables. To this end, note that the number of dynasties N_0 is constant for all periods but the size of each dynasty varies and is equal to n^t in period t. Note also that, due to the presence of aspirations and bequests, the labor income shocks are transmitted intergenerationally within a dynasty. However, those shocks are not transmitted across dynasties since the amounts of consumption, bequest, and saving of two individuals belonging to two different dynasties depend on the i.i.d. stochastic process of wages within their own dynasties. So, let us consider the set of individuals $i = (j, m) \in P_t$ placed in position $m \in [0, n^t]$ within their respective dynasties in period t. Let us fix the value m and assume that the generic random variables $y_t^{(j,m)}$, $j \in [0, N_0]$, are i.i.d., with mean $\mathbb{E}\left(y_t^{(j,m)}\right) = \bar{y}_t$ and finite variance $Var\left(y_t^{(j,m)}\right) = Var\left(y_t\right)$ for all j and m. This assumption agrees with our scenario with i.i.d. stochastic processes of wages across individuals belonging to different dynasties and with a common initial aspiration level for all dynasties, i.e. $a_0^i = a_0$ for all i. Note that two individuals belonging to the same dynasty in period t have a partially common history of realizations of wages. In particular, two individuals who are brothers will share the same history of wages until period $t-1$ and an independent realization of the wage in period t. Therefore, these two individuals will not exhibit independent consumption, bequest, and saving when aspirations and bequests are present. By fixing the position $m \in [0, n^t]$

within each dynasty we ensure that the i.i.d. assumption holds for the generic endogenous variables $y_t^{(j,m)}$, $j \in [0, N_0]$. Then, the law of large numbers for large economies (see Theorem 2 in Uhlig, 1996) implies that the average (or empirical mean) of the random variable $y_t^{(j,m)}$ in period t for a given position m satisfies

$$\frac{1}{N_0}\int_{[0,N_0]} y_t^{(j,m)} dj = \bar{y}_t, \quad \text{with probability 1.}$$

The previous expression gives us the average value for the individuals in the same position m in each dynasty. To find the average value of $y_t^{(j,m)}$ for the total population, we must average the previous values across all the positions $m \in [0, n^t]$. Formally, we can use Fubini's theorem to get

$$\frac{1}{N_t}\int_{P_t} y_t^i di = \frac{1}{N_t}\int_{[0,N_0]\times[0,n^t]} y_t^{(j,m)} d(j,m)$$
$$= \frac{1}{N_0 n^t}\int_{[0,n^t]}\left[\int_{[0,N_0]} y_t^{(j,m)} dj\right] dm = \frac{1}{n^t}\int_{[0,n^t]}\left[\frac{1}{N_0}\int_{[0,N_0]} y_t^{(j,m)} dj\right] dm$$
$$= \frac{1}{n^t}\int_{[0,n^t]} \bar{y}_t dm = \frac{n^t \bar{y}_t}{n^t} = \bar{y}_t, \quad \text{with probability 1.} \tag{15}$$

Since the term c_{t-1}^i in the aspirations equation (3) refers to the adult consumption in period $t-1$ of the parent of individual i belonging to generation t, the average stock of aspirations in period t is equal to the average consumption of the adult individuals in period $t-1$, $\bar{a}_t = \bar{c}_{t-1}$. Moreover, for every two pairs of individuals belonging to distinct dynasties, their demand functions (9), (10), (11) and (11) will depend on the realizations of their corresponding i.i.d. stochastic processes of wages. Then, according to our previous discussion, we can use the law of large numbers for this large economy to compute the following average values of adult consumption and bequests within a generation by merely computing the expectation in both sides of equations (9) and (11),

$$\bar{c}_t = B\bar{c}_{t-1} + A\bar{b}_t + A\bar{w} \tag{16}$$

and

$$\bar{b}_{t+1} = -F\bar{c}_{t-1} + E\bar{b}_t + E\bar{w}, \tag{17}$$

where $\bar{c}_t$ and $\bar{b}_t$ are the average amounts of adult consumption, bequests, and aspirations in period t, respectively.[3] It should be understood that the previous two equations hold with probability 1. The dynamic system composed of the difference equations (16) and (17) fully describes the evolution of the average values of adult consumption and bequests.

We could also analyze the dynamics of the second moments of the endogenous variables of our economy. It should be noted that in this large economy with i.i.d. labor income shocks, $Var\left(c_t^i\right)$, $Var\left(x_t^i\right)$, $Var\left(s_t^i\right)$ and $Var\left(b_t^i\right)$ coincide with the empirical intragenerational variances of adult consumption, old consumption, saving and bequest at date t. To see this, we fix again position m within each dynasty and assume that the continuum of generic i.i.d. random variables $y_t^{(j,m)}$, $j \in [0, N_0]$, is i.i.d. with mean $\mathbb{E}\left(y_t^{(j,m)}\right) = \bar{y}_t$ and variance $Var\left(y_t^{(j,m)}\right) = Var\left(y_t\right)$ and has a finite fourth moment $\mathbb{E}\left[\left(y_t^{(j,m)}\right)^4\right] = \mathbb{E}\left[\left(y_t\right)^4\right] < \infty$, for all pairs (j,m). Then the empirical variance of the continuum of random variables y_t^i , $i = (j,m) \in P_t$ is given by

$$\frac{1}{N_t}\int_{P_t}\left(y_t^i - \bar{y}\right)^2 di = Var\left(y_t\right), \quad \text{with probability 1,} \tag{18}$$

which follows again from Theorem 2 in Uhlig (1996) and from replicating the same steps as in (15) since the random variables $\left(y_t^{(j,m)} - \bar{y}\right)^2$, $j \in [0, N_0]$, are also i.i.d. for a given m.

We can use a similar argument to show that the covariance $Cov\left(c_t^i, b_{t+1}^i\right)$ coincides with the empirical covariance between adult consumption and the amount bequeathed to each descendant by individuals of generation t. Let us fix again position m within each dynasty. The random vectors $\left(c^{(j,m)}, b_{t+1}^{(j,m)}\right)$ are i.i.d. for $j \in [0, N_0]$,with $Cov\left(c^{(j,m)}, b_{t+1}^{(j,m)}\right) = Cov\left(c_t, b_{t+1}\right)$ for all pairs (j,m), then the empirical covariance satisfies

$$\frac{1}{N_t}\int_{P_t}\left(c_t^i - \bar{c}_t\right)\left(b_{t+1}^i - \bar{b}_{t+1}\right) di = Cov\left(c_t, b_{t+1}\right), \quad \text{with probability 1,}$$

as the random variables $\left(c_t^{(j,m)} - \bar{c}_t\right)\left(b_{t+1}^{(j,m)} - \bar{b}_{t+1}\right)$, $j \in [0, N_0]$, are also i.i.d. for a given position m.

[3] Note that, as wages w_t^i, $i \in P_t$, are cross-sectionally and serially i.i.d. for all individuals irrespective of their dynasty, the average wage of the economy in period t is

$$\frac{1}{N_t}\int_{P_t} w_t^i di = \bar{w}, \quad \text{with probability 1, for all } t.$$

We have just seen that $Var\left(c_t^i\right)$, $Var\left(x_t^i\right)$, $Var\left(s_t^i\right)$ and $Var\left(b_t^i\right)$ coincide with the empirical intragenerational variances of c_t^i, x_t^i, s_t^i and b_t^i in period t, which are equal to $Var\left(c_t\right)$, $Var\left(x_t\right)$, $Var\left(s_t\right)$ and $Var\left(b_t\right)$, respectively. Moreover, $Cov\left(c_t^i, b_{t+1}^i\right)$ coincides with the empirical covariance between the amount of adult consumption and bequests left by the same individuals, which is equal to $Cov\left(c_t, b_{t+1}\right)$. From now on we will suppress the superindex referred to the individual when we refer to the empirical second moments. Therefore, we can compute the variances of equations (9) and (11) to get

$$Var\left(c_t\right) = B^2 Var\left(c_{t-1}\right) + A^2 Var\left(b_t\right) + 2AB\ Cov\left(c_{t-1}, b_t\right) + A^2\sigma^2, \tag{19}$$

and

$$Var\left(b_{t+1}\right) = F^2 Var\left(c_{t-1}\right) + E^2 Var\left(b_t\right) - 2EF\ Cov\left(c_{t-1}, b_t\right) + E^2\sigma^2, \tag{20}$$

where we have again used the aspirations equation (3).[4] In order to close the system referred to the dynamics of the second-order moments, we need to compute the covariance $Cov\left(c_t, b_{t+1}\right)$ between the amount of adult consumption and the amount bequeathed to each of his descendants by a generic individual. This covariance is immediately obtained from the expressions (9) and (11),

$$Cov\left(c_t, b_{t+1}\right) = -BF\ Var\left(c_{t-1}\right) + AE\ Var\left(b_t\right) + \left(BE - AF\right)\ Cov\left(c_{t-1}, b_t\right) + AE\ \sigma^2. \tag{21}$$

We next find the steady-state values of the first and second empirical moments of the endogenous variables of our economy and characterize their dynamics. The steady-state value of average adult consumption and bequest is given by

$$\bar{c} = \frac{nR\bar{w}}{\left[n + \left(n\left(\beta + \rho\right) - \rho R\right)\left(1 - \delta\right)\right]\left(R + \gamma\right)} \tag{22}$$

and

$$\bar{b} = \frac{\rho\left(1 - \delta\right)R\bar{w}}{n + \left[n\left(\beta + \rho\right) - \rho R\right]\left(1 - \delta\right)}, \tag{23}$$

where these values are obtained by merely making $\bar{c}_t = \bar{c}_{t-1} = \bar{c}$ and $\bar{b}_{t+1} = \bar{b}_{t+1} = \bar{b}$ in equations (16) and (17) and then solving for $\bar{c}$ and $\bar{b}$ and using the expressions for

[4] As wages w_t^i are cross-sectionally and serially i.i.d. and $\mathbb{E}\left[\left(w_t^i\right)^4\right] = \mathbb{E}\left[w^4\right]$ is finite for all $i \in P_t$, the empirical variance of wages in this economy in period t is

$$\frac{1}{N_t}\int_{P_t}\left(w_t^i - \bar{w}\right)^2 di = \sigma^2, \text{ with probability 1, for all } t.$$

$A, B, E,$ and F given in (13). Moreover, taking expectations in both sides of (12) and evaluating the resulting equation at the steady-state average values of adult consumption and bequest, we obtain the steady-state average amount of savings,

$$\bar{s} = \frac{\left[(1-\delta)(\beta+\rho)(R+\gamma)+\gamma\right] n\bar{w}}{\left[n+(n(\beta+\rho)-\rho R)(1-\delta)\right](R+\gamma)}. \tag{24}$$

Similarly, using equations (19), (20) and (21), the values of the coefficients given in (13), and making $Var\left(c_t\right) = Var(c)$, $Var\left(b_t\right) = Var(b)$, and $Cov\left(c_t^i, b_{t+1}^i\right) = Cov\left(c, b'\right)$ for all pairs (i, t), we can compute the steady-state values of the variances of adult consumption and bequest, $Var\left(c\right)$ and $Var\left(b\right)$, and the corresponding steady-state value of the covariance $Cov\left(c, b'\right)$ between adult consumption c of an individual and bequest b' left by this individual to each descendant,

$$Var\left(c\right) = \frac{n^2R^2\left[n(1+\beta+\rho)+R\delta\rho\right]\sigma^2}{\left[n(1+\beta+\rho)-R\delta\rho\right]\left[n+(n(\beta+\rho)+\rho R)(1+\delta)\right]\left[n+(n(\beta+\rho)-\rho R)(1-\delta)\right](R+\gamma)^2}, \tag{25}$$

$$Var\left(b\right) = \frac{\rho^2R^2\left[n\left((\beta+\rho)\left(1-\delta^2\right)+1+\delta^2\right)-R\delta\rho\left(1-\delta^2\right)\right]\sigma^2}{\left[n(1+\beta+\rho)-R\delta\rho\right]\left[n+(n(\beta+\rho)-\rho R)(1-\delta)\right]\left[n+(n(\beta+\rho)+\rho R)(1+\delta)\right]}, \tag{26}$$

and

$$Cov\left(c, b'\right) = \frac{n^2\rho R^2\left[\left(1-\delta^2\right)(\beta+\rho)+1\right]\sigma^2}{\left[n(1+\beta+\rho)-R\delta\rho\right]\left[n+(n(\beta+\rho)+\rho R)(1+\delta)\right]\left[n+(n(\beta+\rho)-\rho R)(1-\delta)\right](R+\gamma)}. \tag{27}$$

Moreover, from equation (12) and (3) we obtain the variance of saving,

$$Var\left(s_t\right) = G^2 Var\left(b_t\right) + B^2 Var\left(c_{t-1}\right) - 2BG\ Cov\left(c_{t-1}, b_t\right) + G^2\sigma^2,$$

so that the steady-state value of the variance of saving becomes

$$Var\left(s\right) = G^2 Var\left(b\right) + B^2 Var\left(c\right) - 2BG\ Cov\left(c, b'\right) + G^2\sigma^2, \tag{28}$$

where $Var\left(c\right)$, $Var\left(b\right)$, and $Cov\left(c, b'\right)$ are given in (25), (26), and (27), respectively.

We next proceed to find the conditions under which the first and second central moments of the joint distribution of the endogenous variables of our model converge to their steady-state values. The following lemma provides a sufficient condition for the dynamic stability of the first moments of the intragenerational distribution of adult consumption and bequests:

Lemma 1 *If $\rho R/n\left(1+\beta+\rho\right) < 1$ and the aspiration intensity δ is sufficiently small, then the dynamic system formed by equations (16) and (17) converges monotonically to the steady-state value for average adult consumption and bequest given by (22) and (23), respectively.*

For sufficiently high values of aspiration intensity, i.e. when

$$\delta > \frac{\rho R}{n}\left(\frac{1-(1+\beta+\rho)^{1/2}}{\beta+\rho}\right)^2 \equiv K,$$

the first moments of aggregate consumption and bequests exhibit cycles. If the condition $\rho R/n\,(1+\beta+\rho) \in (0,1)$ for dynamic stability is imposed, it is plain to see that the bifurcation value K for the aspiration intensity is strictly smaller than one. The role of aspirations in the emergence of cycles has already been exhaustively analyzed by de la Croix and Michel (1999) for an economy with no intergenerational transmission of wealth. Note that when aspirations are absent, $\delta = 0$, the condition $\rho R/n\,(1+\beta+\rho) < 1$ is necessary and sufficient for the convergence of the means of the endogenous variables of our economy. As occurs in the traditional Ak models of growth, under the linear demand functions (16) and (17), it is necessary to impose an upper bound in the return R on savings in order prevent the dynamic system from exhibiting an unbounded growth path of the average values of consumption and bequest per capita.

Concerning the stability of the dynamic system driving the evolution of second order moments formed by equations (19), (20) and (21), we can proceed in a similar fashion. The following lemma provides a sufficient condition for dynamic stability of the second moments of the intragenerational distribution of consumption and bequest:

Lemma 2 *If $\rho R/n\,(1+\beta+\rho) < 1$ and the aspiration intensity δ is sufficiently small, then the dynamic system formed by equations (19), (20) and (21) converges to the steady-state value for the variance of adult consumption, variance of bequest, and covariance between adult consumption and the amount bequeathed to each descendant given by (25), (26), and (27), respectively.*

The condition on the aspiration intensity δ for convergence of the second moments is more stringent than for convergence of means. This is so because random variables with finite second moments have finite first moments, but the converse is not true. For the remainder of the paper, we will maintain the assumption $\rho R/n\,(1+\beta+\rho) < 1$, which together with a sufficiently small value of the parameter δ, ensures the dynamic stability of the first and second moments of the intragenerational distribution of adult consumption and bequests.

V. Intergenerational Mobility

To perform an analysis of intergenerational mobility in our economy we should analyze the behavior of the correlation coefficient between s_{t+1}^i and s_t^i, $Corr(s_{t+1}^i, s_t^i)$, i.e. between the amount s_t^i of assets held by a generic individual i and the amount s_{t+1}^i held by one of his children. If bequests and aspirations were absent, this autocorrelation would be equal to zero and thus we would obtain perfect mobility as wages are i.i.d. If we had perfect correlation of asset holdings, i.e. $Corr(s_{t+1}^i, s_t^i) = 1$, then intergenerational mobility would be null.

It is important to note that, even if altruism is absent, the presence of aspirations induces a wealth correlation across the members of consecutive generations within the same family. This is so because aspirations induce a correlation between the amount of parental consumption and the profile of consumption and saving of their descendants.

As before, we should point out that the covariance of the stochastic process of savings between two consecutive individuals belonging to the same family coincides with the empirical autocorrelation. To see this, let us fix again the positions m and m' within each dynasty so that the individual (j, m) is the parent of individual (j, m'). As labor income shocks are i.i.d. across individuals belonging to different dynasties, the random vectors $\left(s_t^{(j,m)}, s_{t+1}^{(j,m')}\right)$ are i.i.d. for $j \in [0, N_0]$, with $Cov\left(s_t^{(j,m)}, s_{t+1}^{(j,m')}\right) = Cov\left(s_t, s_{t+1}\right)$ for all j, m, and m'. Therefore, the empirical covariance satisfies

$$\frac{1}{N_t} \int_{P_t} \left(s_t^i - \bar{s}\right) \left(s_{t+1}^i - \bar{s}\right) di = Cov\left(s_t, s_{t+1}\right), \quad \text{with probability 1,} \tag{29}$$

as the random variables $\left(s_t^{(j,m)} - \bar{s}\right)\left(s_{t+1}^{(j,m')} - \bar{s}\right)$, $j \in [0, N_0]$, are also i.i.d. for given positions m and m', with the individual (j, m) being the parent of individual (j, m').

Concerning the autocorrelation of the stochastic process of savings, the following holds:

$$Corr(s_{t+1}^i, s_t^i) = \frac{Cov(s_{t+1}^i, s_t^i)}{\left(Var(s_{t+1}^i)\right)^{1/2} . \left(Var(s_t^i)\right)^{1/2}} = \frac{Cov(s_{t+1}, s_t)}{Var(s_t)},$$

as follows (18) and (29). Therefore, the autocorrelation coefficient of the stochastic process of savings between parents and children coincides with its empirical autocorrelation coefficient. In the next lemma we provide the formula for the empirical covariance of $Cov(s_{t+1}, s_t)$ and establish its convergence towards its steady-sate value, $Cov(s', s)$, where s are the savings of a generic individual and s' are the savings of a direct descendant of that individual.

Lemma 3 *The empirical intergenerational covariance of wealth in period t is given by*

$$Cov(s_{t+1}, s_t) = GH \ \left[\sigma^2 + Var(b_t)\right] + BI \ Var(c_{t-1}) - (BH + GI) \ \ Cov\left(c_{t-1}, b_t\right),$$

where

$$H = EG - AB \ , \quad I = B^2 + FG,$$

and A, B, E, F, and G are the coefficients whose values are given in (13). Moreover, if $\rho R/n\,(1 + \beta + \rho) < 1$ and the aspiration intensity δ is sufficiently small, then $Cov(s_{t+1}, s_t)$ converges to its steady-state value

$$Cov(s', s) = GH \ \left[\sigma^2 + Var(b)\right] + BI \ Var(c) - (BH + GI) \ \ Cov\left(c, b'\right), \qquad (30)$$

where $Var\,(c)$, $Var\,(b)$, and $Cov\,(c, b')$ are given in (25), (26), and (27), respectively.

Since expression (28) gives us $Var(s)$, we can use (30) to find an explicit expression for the steady-state value of the autocorrelation coefficient of wealth,

$$Corr(s', s) = \frac{Cov(s', s)}{Var(s)}.$$

Using a similar procedure we can find the expressions for the autocorrelation coefficient of adult consumption $Corr(c', c)$, of old consumption $Corr(x', x)$, and of bequests $Corr(b', b)$. In the next two sections we will conduct the corresponding comparative statics exercise on the steady-state value of these autocorrelation coefficients to characterize the effects on intergenerational mobility of changes in the intensity of habits and aspirations.

VI. Effects of Habits and Aspirations on the Intragenerational Distribution of Wealth and Intergenerational Mobility

In this section, we will characterize the effect of habits and aspirations on the steady-state values of the first two moments of the distribution of wealth. Note that those properties of individual preferences affect the amount of saving since they modify the evaluation of the utility derived from consumption in the two periods of life. Moreover, in our economy the individuals' savings are equal to the their asset holdings at the beginning of their last period of life.

By differentiating (24) and (23) with respect to the parameters δ and γ measuring the intensity of aspirations and habits, respectively, we get the following effects on the average amount of savings and bequests:

$$\frac{\partial \bar{s}}{\partial \delta} = -\frac{\left(n\left(\beta+\rho\right)+\gamma\rho\right) Rn\bar{w}}{\left[n+\left(n\left(\beta+\rho\right)-\rho R\right)\left(1-\delta\right)\right]^{2}\left(R+\gamma\right)} < 0, \tag{31}$$

$$\frac{\partial \bar{s}}{\partial \gamma} = \frac{Rn\bar{w}}{\left[n+\left(n\left(\beta+\rho\right)-\rho R\right)\left(1-\delta\right)\right]\left(R+\gamma\right)^{2}} > 0, \tag{32}$$

$$\frac{\partial \bar{b}}{\partial \delta} = \frac{-Rn\rho\bar{w}}{\left[n+\left(n\left(\beta+\rho\right)-\rho R\right)\left(1-\delta\right)\right]^{2}} < 0, \tag{33}$$

and

$$\frac{\partial \bar{b}}{\partial \gamma} = 0. \tag{34}$$

The derivative $\partial \bar{s}/\partial \gamma$ is positive since it has a negative numerator and a positive denominator. To see the latter, note that

$$n+\left(n\left(\beta+\rho\right)-\rho R\right)\left(1-\delta\right) = n\left(\left(1-\delta\right)\left(1+\beta+\rho\right)+\delta\right)-\rho R\left(1-\delta\right)$$

$$> n\left(1-\delta\right)\left(1+\beta+\rho\right)-\rho R\left(1-\delta\right) = \left(n\left(1+\beta+\rho\right)-\rho R\right)\left(1-\delta\right) > 0 \tag{35}$$

where the last inequality is a consequence of the dynamic stability assumption, $\rho R/n\left(1+\beta+\rho\right) < 1$.

An increase in the value of the aspiration intensity δ increases the marginal utility of an extra unit of adult consumption since individuals are more sensitive to their parents' level of consumption when evaluating their own adult consumption. Therefore, individuals optimally increase their adult consumption and so the amount of saving, old consumption, and inheritance received by their children should go down.

Concerning the effect of an increase in the value of the habit formation parameter γ, we notice that individuals experience an increase in the marginal valuation of their old consumption as they more intensely internalize their past adult consumption and so they optimally decide to shift consumption from adult to the old age by saving more (see (32)). On the one hand, the reduction in adult consumption lowers the stock of habits and, on the other hand, the increase in old consumption is the optimal response to the increase in the marginal utility of old consumption. We also note that the aggregate effects of stronger habits are accommodated throughout the individuals' life cycle since the aggregate amount of bequests remains unchanged (see (34)).

We could now also analyze how the changes in the values of the parameters δ and γ affect the intragenerational variability of wealth. We will first concentrate our analysis on the variability of saving, which fully determines the amount of assets held by individuals at the beginning of the last period of their lives. Since the average amount of saving is also affected by those changes, it seems appropriate to perform our comparative statics exercise on the coefficient of variation, $CV(s) = (Var(s))^{1/2}/\bar{s}$. As we have already said, we choose this measure of variability because it is simple enough for us to get explicit signs for our intended comparative static exercises. Moreover, due to the linearity of the saving function (12) we can easily compute the coefficient of variation from the knowledge of the values of the first two moments of the distribution of wages. Other measures of inequality, such as the Gini coefficient, require a complete description of the distribution of wages and, moreover, with these measures, the comparative statics results will not arise from closed form expressions but from simulations (see Bossmann et al., 2007).

By combining (25), (26), (27) and (28), and after some algebra, we get the following derivative of the coefficient of variation:

$$\left.\frac{\partial CV(s)}{\partial \delta}\right|_{\delta=\gamma=0} = \frac{n^2(1+\beta+\rho)\sigma}{\left[n^2(1+\beta+\rho)^2 - \rho^2 R^2\right]^{1/2}\left[n(1+\beta+\rho)+\rho R\right]\bar{w}} > 0, \tag{36}$$

where the sign of the previous expression follows immediately under our maintained condition of dynamic stability, $\rho R/n(1+\beta+\rho) < 1$. Our comparative statics exercise on the coefficient of variation of saving is conducted in a fairly restrictive scenario, which enables us to unambiguously sign the effects of stronger aspirations. We evaluate the derivative of $CV(s)$ with respect to δ at point $\delta = \gamma = 0$, that is, we analyze the effect of the marginal introduction of aspirations to an economy when habits are absent or present on a small scale. Thus, the marginal introduction of aspirations increases the variability of wealth in this case. When aspirations are introduced, the variability of asset holdings of an individual is magnified since it will not depend on the fluctuation of his own wage only but also on the shocks in the wages of his predecessors. This effect combined with the decrease in the amount of savings (see (31) results in a larger value of the coefficient of variation of savings.

The evaluation of the partial derivative (36) for arbitrary values of δ and γ cannot be explicitly signed. In order to evaluate the robustness of the sign of that derivative, we conduct a numerical analysis. We choose the values of the preference parameters $\beta = 1/2$, and $\rho = 1/2$. Moreover, following Iacoviello (2008), we choose the value of the average wage $\bar{w} = 2/3$ and make the cross-sectional standard deviation of the log of earnings equal to 0.5173. Therefore, we set the associated variance of wages equal to $\sigma^2 = \left(\frac{2}{3}\right)^2 \left(\exp\left(0.5173^2\right) - 1\right) = 0.13637$, which amounts to a cross-sectional coefficient of variation of wages equal to $\sigma/\bar{w} = 0.55392$. We assume constant population, $n = 1$. Finally, we choose an interest rate per year of 4% and we consider that each period lasts for 30 years so that $R = (1.04)^{30} = 3.2434$. We maintain these parameter values for the remaining numerical exercises unless otherwise specified. In Figure 1, we observe that the positive effect of aspirations on the coefficient of variation of asset holdings is preserved for all the values of δ and for different combinations of values for the habit parameter. Note that we restrict the domain of the aspiration parameter δ to lie in the interval (42) so that $\delta \in [0, 0.27824)$.

[Insert Figure 1]

Concerning the implications for the intragenerational variability of wealth of changes in habit intensity, it can be shown that

$$\left.\frac{\partial CV(s)}{\partial \gamma}\right|_{\delta=0} = 0.$$

Therefore, we observe that habits cannot affect the level of intragenerational variation of wealth if aspirations are not present. To understand this result note that, if aspiration are not present ($\delta = 0$), the effect of a change in habit intensity γ results exclusively in an adjustment in the allocation consumption throughout the life cycle and parental consumption plays no role in this allocation of consumption (see (9), (10) and the expressions for the constants B and D given in (13)). Moreover, the optimal amount devoted to bequests remains unchanged after this reallocation of consumption (see (11) and the expressions for E and F given in (13)) so that the distribution of bequest b_t^i is left unchanged. Note that the saving function (12) when aspirations are absent becomes simply $s_t^i = G\left(w_t^i + b_t^i\right)$, where the coefficient G is increasing in the habit intensity γ (see (14)). Therefore, an increase in habit intensity results in a proportional increase in the steady-state values of both the standard deviation and the mean of savings, which leaves the corresponding coefficient of variation unchanged. This means that changes in habit intensity affect the intergenerational transmission of productivity shocks only through inherited tastes. Moreover, we can compute the following derivative:

$$\left.\frac{\partial CV\left(s\right)}{\partial\gamma}\right|_{\gamma=0,\delta>0}= \tag{37}$$

$$-\frac{n\delta(1+\delta)[n+(n(\beta+\rho)-\rho R)(1-\delta)]^{3/2}\sigma}{(1-\delta)^{2}Rn(\beta+\rho)\left[n\left(1+\delta^{2}\right)+(n(\beta+\rho)-R\delta\rho)\left(1-\delta^{2}\right)\right]^{1/2}[n+(1+\delta)(n(\beta+\rho)+\rho R)]^{1/2}[n(1+\beta+\rho)-\rho R\delta]^{1/2}\bar{w}}<0,$$

whose negative sign follows again from (35) under the assumption $\rho R/n\left(1+\beta+\rho\right)<1$. Note also that the previous derivative (37) gives us the effect on the variability of asset holdings of the marginal introduction of habits when aspirations are present. Clearly, when habits are introduced, a shock in wages is more evenly distributed among adult and old consumption since habits enhance consumption smoothing throughout the life cycle and this results in a larger variance of saving. However, the increase in the average amount of saving (see (32)) leads to a reduction in the coefficient of variation $CV\left(s\right)$ of saving as shown in (37). In Figure 2, we observe that the negative sign of the derivative (37) is preserved for strictly positive values of both the habit parameter γ and the aspiration parameter δ. We have also checked the robustness of the sign of the derivative for different values of the aspiration intensity δ in the interval $[0, 0.27824)$.

[Insert Figure 2]

We have thus observed that, while aspirations tend to increase inequality in wealth distribution, habits tend to decrease it. The latter result agrees with the observation made by Cagetti and De Nardi (2008), who claim that the introduction of habit formation does not help to explain the large empirical inequality in the distribution of wealth. However, our model with overlapping generations enables the introduction of aspirations as another channel through which the amount of past consumption affects the utility delivered by current consumption. In this way, the introduction of aspirations does result in larger wealth inequality. Therefore, the distinction between these two features in individual preferences has dramatic consequences for the comparative statics exercises involving the characteristics of the distribution of wealth.

In Table 2, we summarize the effects of changes in aspiration and habit intensity on the steady-state values of the first and second moments of all the endogenous variables in the model. Note that the changes in the coefficients of variation of bequests and savings are mostly driven by the changes in the mean of these variables rather than by those of the variances, which means that whenever the variance and the mean move in the same direction, the resulting effect on the coefficient of variation will be the opposite of that of the variance. Thus, the effects on the variability of wealth (savings and bequests) depend crucially on the statistical measure we use in this respect. Another lesson that we can infer from that table is that the effects of aspirations on the wealth at the end of the last period (i.e. the amount of bequest) are the same as those on the wealth at the beginning of the last period of life (i.e. the amount of savings). Finally, the effect of habits on the variance and mean of adult consumption is of the same magnitude and thus the coefficient of variation remains unchanged when habit intensity varies.

[Insert Table 2]

We can now proceed with an analysis of the effects of habit and aspiration intensities γ and δ on the level of intergenerational mobility within a family, which is characterized in the long-run by the steady-state value of the autocorrelation coefficient of asset holdings, $Corr(s', s)$. To this end, we compute the derivatives of the autocorrelation coefficient $Corr(s', s)$, which was obtained in Section V., with respect to the parameters representing the aspiration and habit intensities. The corresponding partial derivatives are

$$\left.\frac{\partial Corr(s', s)}{\partial \delta}\right|_{\delta=\gamma=0} = -\frac{1}{(1+\beta+\rho)} < 0 \tag{38}$$

and

$$\left.\frac{\partial Corr\,(s', s)}{\partial \gamma}\right|_{\gamma=\rho=0,\delta>0} = \frac{\delta\left(1-\delta^2\right)\left[(1+\beta)^2-\beta^2\delta^2\right]}{R\beta\left[1+\delta^2+\beta\left(1-\delta^2\right)\right]^2} > 0. \tag{39}$$

Again these derivatives characterize the effects of the marginal introduction of either aspirations or habits. Moreover, the derivative of the correlation coefficient $Corr\,(s', s)$ with respect to the habits parameter γ can only be explicitly signed when the degree of altruism, which is parametrized by ρ, is sufficiently low. Note that if we evaluate the derivative (39) when there are no aspirations, we clearly obtain

$$\left.\frac{\partial Corr(s', s)}{\partial \gamma}\right|_{\delta=\gamma=0} = 0$$

so that, again, changes in habits only affect the level of intragenerational mobility through the transmission of tastes across the generations within the same family. We thus observe that the introduction of aspirations and habits have opposite effects on intergenerational mobility. On the one hand, the introduction of aspirations raises the degree of mobility. As the marginal utility of adult consumption increases when aspirations are introduced, workers tend to increase their consumption by reducing both their saving and the amount of bequests they plan to leave to their children (see (33)). Obviously, this results in a smaller correlation between the assets of parents and their direct descendants.

On the other hand, when habits are introduced, workers wish to smoothen their consumption more throughout their life cycle. Hence, a positive shock in their labor income results in a larger increase in their saving, which is aimed at shifting adult consumption towards old consumption. In the presence of aspirations, the saving levels of individuals belonging to consecutive generations become correlated and this correlation does indeed become larger as each individual's saving becomes more sensitive to productivity shocks. We thus conclude that the introduction of habits results in an increase in the correlation of wealth between members of consecutive generations within the same family.

In Figures 3 and 4, we conduct a numerical analysis to check the robustness of the signs of the derivatives (38) and (39). When aspiration intensity increases, individuals raise their adult consumption in response to the larger parental consumption by reducing their saving. This negative effect of aspirations on asset autocorrelation is preserved through the numerical examples in Figure 3 with $\delta \in [0, 0.27824)$. Note that since we have introduced altruism to these examples, bequests are positive and, hence, the autocorrelation $Corr(s, s')$ is positive as a consequence of the intergenerational transmission of wealth.

[Insert Figure 3]

In Figures 4 an 5, we consider strictly positive values of the aspiration parameter and two values for the altruism parameter, $\rho = 0$ (Figure 4) and $\rho = 1/2$ (Figure 5). When there is no altruism, $\rho = 0$, we observe in Figure 4 that the autocorrelation of asset holding is negative. Obviously, if aspirations are present in an economy with no habits and no altruism, then adult individuals seek to mimic the consumption level of their parents and, since labor income is uncorrelated across generations, the savings of two consecutive members of the same family become negatively correlated. Obviously, as individuals seek to acquire the same standard of living as their parents, the direct descendants of rich individuals save very little on average in order to finance their adult consumption. The relationship between the autocorrelation $Corr\,(s, s')$ and the value of the habit parameter γ is positive, which agrees with (39). In Figure 5, we consider the case with $\rho = 1/2$ and, then, the wealth autocorrelation becomes positive due to the introduction of altruism and the corresponding positive bequests. Here, the positive relationship between $Corr\,(s, s')$ and the intensity of habits is preserved.

[Insert Figures 4 and 5]

In Table 3, we summarize the effects of changes in aspiration and habit intensity on the autocorrelation coefficients of all the endogenous variables in the model. We observe that the sign of the effects on old consumption is the same as that on savings. On the one hand, when aspirations are introduced only the autocorrelation of adult consumption increases as this consumption determines the level of aspirations that individuals have to overcome, while the autocorrelations of old consumption, saving and bequest decrease. On the other hand, the introduction of habit formation only affects the transfer of wealth from the adult to the old period of life, which leaves the autocorrelation of both bequest and adult consumption unchanged.

[Insert Table 3]

VII. Conclusion

We have developed a simple model that enables us to study the effect of the introduction of habits and aspirations on the intragenerational distribution of wealth. Our results show that the introduction of habits and aspirations has opposite effects on both the average amount of assets accumulated by individuals and the level of wealth inequality measured by the coefficient of variation of savings.

Concerning mobility in wealth within the same family, we find that the introduction of aspirations increases intergenerational mobility as the amount of bequests tends to be lower. However, the introduction of habits to preferences results in an increase in the autocorrelation between the amount of assets held by parents and children since the amount bequeathed by individuals becomes more correlated with their wealth.

Our model is simple enough to obtain explicit characterizations of other policy experiments, such as the introduction of social security systems, and taxes on either capital income or consumption. The latter tax is especially relevant since it would directly affect the reference that individuals take into account when they evaluate their current consumption. Another potential extension of our model would be the introduction of either idiosyncratic or aggregate risks affecting the return on savings. This will create a source of volatility in the income of old individuals which will give rise to precautionary saving. How this saving will be affected by the presence of habits and aspirations is a topic for future research.

References

Abel, A. (1985), Precautionary Saving and Accidental Bequests, *American Economic Review 75*, 777-791.

Abel, A. (1986), Capital Accumulation and Uncertain Lifetimes with Adverse Selection, *Econometrica 54*, 1079-1097.

Abel, A. (1990), Asset Prices under Habit Formation and Catching up with the Joneses, *American Economic Review 80*, 38-42.

Abel, A. (1999), Risk Premia and Term Premia in General Equilibrium, *Journal of Monetary Economics 43*, 3-33.

Alonso-Carrera, J., Caballé, J. and Raurich X. (2004), Consumption Externalities, Habit Formation and Equilibrium Efficiency, *Scandinavian Journal of Economics 106*, 231-251.

Alonso-Carrera, J., Caballé, J. and Raurich X. (2005), Growth, Habit Formation, and Catching Up with the Joneses, *European Economic Review 49*, 1665-1691.

Alonso-Carrera, J., Caballé, J. and Raurich X. (2007), Aspirations, Habit Formation, and Bequest Motive, *Economic Journal 117*, 813-836.

Barro, R.J. (1974), Are Government Bonds Net Wealth?, *Journal of Political Economy 82,* 1095-1117.

Boyd, R. and Richerson, P. (1985), *Culture and the Evolutionary Process*, University of Chicago Press, Chicago, IL.

Becker, G. (1981), *A Treatise on the Family*, Harvard University Press, Cambridge, MA.

Becker, G. and Tomes, N. (1979), An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility, *Journal of Political Economy 87*, 1153-1189.

Bernheim, B., Schleifer, A. and Summers, L. (1985), The Strategic Bequest Motive, *Journal of Political Economy 93* 1045-1075.

Bossmann, M., Kleiber, C. and Waelde, K. (2007), Bequests, Taxation and the Distribution of Wealth in a General Equilibrium Model, *Journal of Public Economics 91*, 1247-1271.

Cagetti, M. and De Nardi, M. (2008), Wealth Inequality: Data and Models, *Macroeconomic Dynamics 12 (S2)*, 285-313.

Carroll, C., Overland, J. and Weil, D. (1997), Comparison Utility in a Growth Model, *Journal of Economic Growth 2*, 339-367.

Carroll, C., Overland, J. and Weil, D. (2000), Saving and Growth with Habit Formation, *American Economic Review 90*, 341-355.

Cavalli-Sforza, L. and Feldman, M. (1981), *Cultural Transmission and Evolution: A Quantitative Approach*, Princeton University Press, Princeton, NJ.

Cox, D., Ng, S., and Waldkirch, A. (2004), Intergenerational Linkages in Consumption Behavior, *Journal of Human Resources 32*, 355-381.

Davies, J. (1986), Does Redistribution Reduce Inequality?, *Journal of Labour Economics 4*, 538-559.

de la Croix, D. (1996), The Dynamics of Bequeathed Tastes, *Economics Letters 53*, 89-96.

de la Croix, D. (2001), Growth Dynamics and Education Spending: The Role of Inherited Tastes and Abilities, *European Economic Review 45*, 1415-1438.

de la Croix, D. and Michel P. (1999), Optimal Growth when Tastes Are Inherited, *Journal of Economic Dynamics and Control*, vol. 23, pp. 519-37.

de la Croix, D. and Michel P. (2001), Altruism and Self-Restraint, *Annales d'Economie et de Statistique 63-64*, 233-260.

Díaz, A., Pijoan-Mas, J. and Rios-Rull, J.V. (2003), Precautionary Savings and Wealth Distribution under Habit Formation Preferences, *Journal of Monetary Economics 50*, 1257–1291.

González-Abril, L., Velasco, F., Gavilán Ruiz, J.M. and Sánchez-Reyes, L.M. (2010), The Similarity Between the Square of the Coefficient of Variation and the Gini Index of a General Random Variable, *Journal of Quantitative Methods for Economics and Business Administration 10,* 5-18.

Iacoviello, M. (2008), Household Debt and Income Inequality, 1963-2003, *Journal of Money, Credit and Banking 40*, 930-965.

Jellal, M. and Wolff, F.C. (2002), Altruistic Bequests with Inherited Tastes, *International Journal of Business and Economic 1*, 95-113.

Ljungqvist, L. and Uhlig, H. (2000), Tax Policy and Aggregate Demand Management under Catching Up with the Joneses, *American Economic Review 90*, 356-366.

Pollak, R. A. (1988), Tied Transfers and Paternalistic Preferences, *American Economic Review 78*, 240-244.

Uhlig, H. (1996), A Law of Large Numbers for Large Economies, *Economic Theory 8*, 41-50.

Yaari, M. (1965), Uncertain Lifetime, Life Insurance, and the Theory of the Consumer, *Review of Economic Studies 32*, 137-150.

A Appendix

Proof of Lemma 1. To analyze the stability of the dynamic system formed by the equations (16) and (17) determining the evolution of the average values of adult consumption and bequests in each period, we can rewrite the system in matrix form,

$$\begin{pmatrix} \bar{c}_t \\ \\ \bar{b}_{t+1} \end{pmatrix} = \mathbb{P} \times \begin{pmatrix} \bar{c}_{t-1} \\ \\ \bar{b}_t \end{pmatrix} + \begin{pmatrix} A\bar{w} \\ \\ E\bar{w} \end{pmatrix}, \tag{40}$$

where the coefficient matrix $\mathbb{P}$ is given by

$$\mathbb{P} = \begin{pmatrix} B & A \\ \\ -F & E \end{pmatrix}$$

Using the values of the A, B, E, and F given in (13), we find that the coefficient matrix $\mathbb{P}$ appearing in (40) has the determinant

$$Det(\mathbb{P}) = \frac{R\delta\rho}{n\left(1+\beta+\rho\right)} > 0$$

and the trace

$$Tr(\mathbb{P}) = \frac{\rho R + n\delta\left(\beta+\rho\right)}{n\left(1+\beta+\rho\right)} > 0.$$

The corresponding characteristic polynomial is

$$Q\left(\lambda\right) \equiv \lambda^2 - \left[\frac{\rho R + n\delta\left(\beta+\rho\right)}{n\left(1+\beta+\rho\right)}\right]\lambda + \frac{R\delta\rho}{n\left(1+\beta+\rho\right)}, \tag{41}$$

so that the values λ_1 and λ_2 solving the equation $Q\left(\lambda\right) = 0$ are the eigenvalues of the coefficient matrix $\mathbb{P}$. The discriminant $\Delta(\delta)$ of the quadratic equation $Q\left(\lambda\right) = 0$ is

$$\Delta(\delta) = \frac{\left(\rho R + n\delta\left(\beta+\rho\right)\right)^2 - 4R\delta\rho n\left(1+\beta+\rho\right)}{n^2\left(1+\beta+\rho\right)^2}.$$

It can be checked that $\Delta(\delta) > 0$ if and only if

$$\delta \in \left[0, \frac{\rho R}{n}\left(\frac{1-\left(1+\beta+\rho\right)^{1/2}}{\beta+\rho}\right)^2\right). \tag{42}$$

Let us assume for the remainder of the proof that δ lies on that interval so that the two eigenvalues are real. We know that $\lambda_1 + \lambda_2 = Tr(\mathbb{P}) > 0$ and $\lambda_1\lambda_2 = Det(\mathbb{P}) > 0$.

Therefore, as the two eigenvalues are real, their sign must be positive. Moreover, if δ tends to zero, the two eigenvalues converge to $\lambda_1 = \rho R/n\,(1+\beta+\rho) \in (0,1)$ and $\lambda_2 = 0$ since the polynomial (41) tends to $\lambda^2 - [\rho R/n\,(1+\beta+\rho)]\,\lambda$, and $\rho R/n\,(1+\beta+\rho) \in (0,1)$ by assumption. As the eigenvalues are continuous functions of the parameter δ representing the aspiration intensity, we conclude that, for a sufficiently small value of the parameter δ, both eigenvalues are real, positive, and smaller than 1, which proves the desired monotone convergence property. *Q.E.D.*

Proof of Lemma 2. We first rewrite the dynamic system formed by equations (19), (20) and (21) in matrix form,

$$\begin{pmatrix} Var\,(c_t) \\ \\ Var\,(b_{t+1}) \\ \\ Cov\,(c_t, b_{t+1}) \end{pmatrix} = \mathbb{W} \times \begin{pmatrix} Var\,(c_{t-1}) \\ \\ Var\,(b_t) \\ \\ Cov\,(c_{t-1}, b_t) \end{pmatrix} + \begin{pmatrix} A^2\sigma^2 \\ \\ E^2\sigma^2 \\ \\ AE\sigma^2 \end{pmatrix},$$

where the coefficient matrix $\mathbb{W}$ is given by

$$\mathbb{W} = \begin{pmatrix} B^2 & A^2 & 2AB \\ \\ F^2 & E^2 & -2EF \\ \\ -BF & AE & BE - AF \end{pmatrix}.$$

Using (13), we find that the characteristic polynomial of the matrix $\mathbb{W}$ is

$$T\left(\hat{\lambda}\right) \equiv \hat{\lambda}^3 - d\hat{\lambda}^2 + f\hat{\lambda} - g\hat{\lambda}, \tag{43}$$

with

$$d = \frac{\rho R\,(\rho R + n\delta\,(\beta+\rho-1)) + n^2\delta^2\,(\beta+\rho)^2}{n^2\,(1+\beta+\rho)^2},$$

$$f = \frac{\left[\rho R\,(\rho R + n\delta\,(\beta+\rho-1)) + n^2\delta^2\,(\beta+\rho)^2\right] R\delta\rho}{n^3\,(1+\beta+\rho)^3} = \left[\frac{R\rho\delta}{n\,(1+\beta+\rho)}\right] d$$

and

$$g = \left[\frac{R\delta\rho}{n\,(1+\beta+\rho)}\right]^3.$$

If δ approaches zero, then the three eigenvalues of the coefficient matrix $\mathbb{W}$ tend to $\hat{\lambda}_1 = [\rho R/n\,(1+\beta+\rho)]^2 \in (0,1)$, $\hat{\lambda}_2 = 0$, and $\hat{\lambda}_3 = 0$ since the coefficient d of the characteristic polynomial converges to $[\rho R/n\,(1+\beta+\rho)]^2$ while the coefficients f and g tend to zero, and $\rho R/n\,(1+\beta+\rho) \in (0,1)$ by assumption.. Therefore, the characteristic polynomial (43) tends to $\hat{\lambda}^3 - [\rho R/n\,(1+\beta+\rho)]^2\,\hat{\lambda}^2$. Finally, since the eigenvalues are continuous functions of the parameter δ, it follows that the three eigenvalues will lie in the interior of the unit circle for a sufficiently small value of δ. *Q.E.D.*

Proof of Lemma 3. To compute $Cov(s^i_{t+1}, s^i_t)$ we use the functions (9), (11), and (12) making $a^i_t = c^i_{t-1}$ for all t. Then, the saving s^i_{t+1} of the direct descendent of the individual i belonging to generation t is

$$\begin{aligned} s^i_{t+1} = Gw^i_{t+1} + Gb^i_{t+1} - Bc^i_t &= Gw^i_{t+1} + G\underbrace{\left(Ew^i_t + Eb^i_t - Fc^i_{t-1}\right)}_{b^i_{t+1}} - B\underbrace{\left(Aw^i_t + Ab^i_t + Bc^i_{t-1}\right)}_{c^i_t} \\ &= Gw^i_{t+1} + (EG - AB)w^i_t + (EG-AB)b^i_t - \left(B^2 + FG\right)c^i_{t-1} \\ &= Gw^i_{t+1} + Hw^i_t + Hb^i_t - Ic^i_{t-1}, \end{aligned} \tag{44}$$

where

$$H = EG - AB, \text{ and } I = B^2 + FG.$$

Using the fact that $a^i_t = c^i_{t-1}$, we can combine (12) with (44), to obtain

$$Cov(s^i_{t+1}, s^i_t) = GH\,\sigma^2 + GH\,Var(b^i_t) - GI\,Cov\left(c^i_{t-1}, b^i_t\right) - BH\,Cov\left(c^i_{t-1}, b^i_t\right) + BI\,Var(c^i_{t-1}).$$

As wages w^i_t are cross-sectionally and serially i.i.d., the law of large numbers applied to our large economy implies that $Var(b^i_t) = Var(b_t)$, $Var(c^i_{t-1}) = Var(c_{t-1})$, and $Cov\left(c^i_{t-1}, b^i_t\right) = Cov\left(c_{t-1}, b_t\right)$ so that we immediately obtain the following empirical value of the covariance of savings between parents and their children:

$$Cov(s_{t+1}, s_t) = GH\ \left(\sigma^2 + Var(b_t)\right) + BI\ Var(c_{t-1}) - (BH + GI)\ Cov\left(c_{t-1}, b_t\right).$$

In Lemma 2 we have proved that, under the conditions appearing in the statement, $Var(b_t)$, $Var(c_t)$ (and, thus, $Var(c_{t-1})$), and $Cov\left(c_{t-1}, b_t\right)$ converge to $Var(b)$, $Var(c)$, and $Cov\left(c, b'\right)$, respectively. Therefore, $Cov(s_{t+1}, s_t)$ converges to the expression $Cov(s', s)$ given in (30). *Q.E.D.*

	$\frac{\partial}{\partial\delta}$	$\frac{\partial}{\partial\gamma}$	$\frac{\partial Var\left(.\vert b_t^i,a_t^i\right)}{\partial\delta}$	$\frac{\partial Var\left(.\vert b_t^i,a_t^i\right)}{\partial\gamma}$	$\frac{\partial CV\left(.\vert b_t^i,a_t^i\right)}{\partial\delta}$	$\frac{\partial CV\left(.\vert b_t^i,a_t^i\right)}{\partial\gamma}$
s_t^i	< 0	> 0	$= 0$	> 0	> 0	> 0 (if $\delta = 0$)
b_{t+1}^i	< 0	< 0	$= 0$	$= 0$	> 0	> 0
c_t^i	> 0	< 0	$= 0$	< 0	< 0	< 0
x_{t+1}^i	< 0	> 0	$= 0$	> 0	> 0	< 0

Table 1. Comparative statics of demand functions, conditional variances, and coefficients of variation.

	$\frac{\partial \mathbb{E}(\cdot)}{\partial \delta}$	$\frac{\partial \mathbb{E}(\cdot)}{\partial \gamma}$	$\left.\frac{\partial Var(\cdot)}{\partial \delta}\right\rvert_{\delta=\gamma=0}$	$\left.\frac{\partial Var(\cdot)}{\partial \gamma}\right\rvert_{\delta=\gamma=0}$	$\left.\frac{\partial CV(\cdot)}{\partial \delta}\right\rvert_{\delta=\gamma=0}$	$\left.\frac{\partial CV(\cdot)}{\partial \gamma}\right\rvert_{\gamma=0}$
s	< 0	> 0	< 0	> 0	> 0	< 0
b	< 0	$= 0$	< 0	$= 0$	> 0	$= 0$
c	> 0	< 0	> 0	< 0	< 0 (if $\rho = 0$)	$= 0$
x	< 0	> 0	< 0	> 0	< 0 (if $\rho = 0$)	< 0

Table 2. The effects of changes in aspiration and habit intensity on the steady-state values of the moments.

	$\left.\frac{\partial Corr(\cdot)}{\partial \delta}\right\vert_{\delta=\gamma=0}$	$\left.\frac{\partial Corr(\cdot)}{\partial \gamma}\right\vert_{\gamma=0}$
s	< 0	> 0 (if $\rho = 0$)
b	< 0	$= 0$
c	> 0 (if $\rho = 0$)	$= 0$
x	< 0	> 0 (if $\rho = 0$)

Table 3. The effects of changes in aspiration and habit intensity on the autocorrelation coefficient.

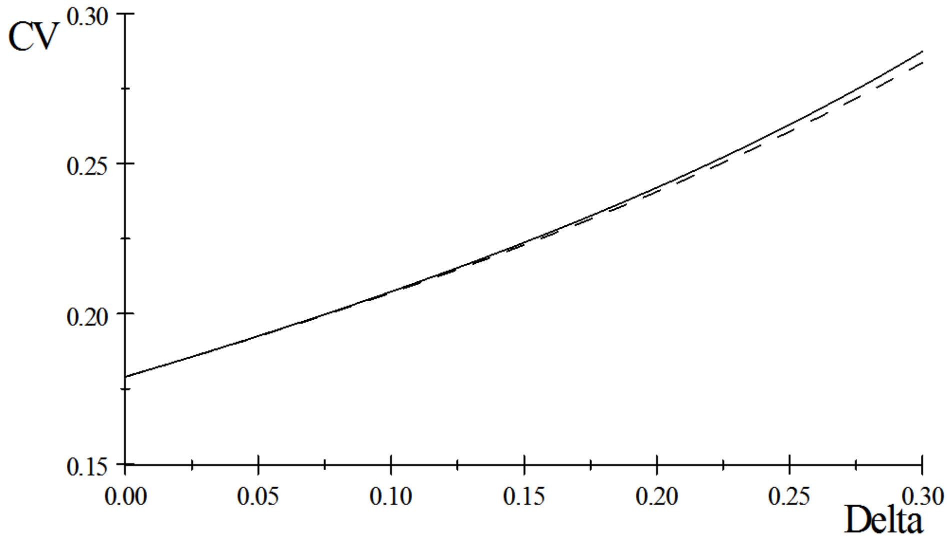

Figure 1. The effects of δ on the coefficient of variation $CV(s)$ of wealth. Solid line: $\gamma = 0$. Dash line: $\gamma = 1/4$.

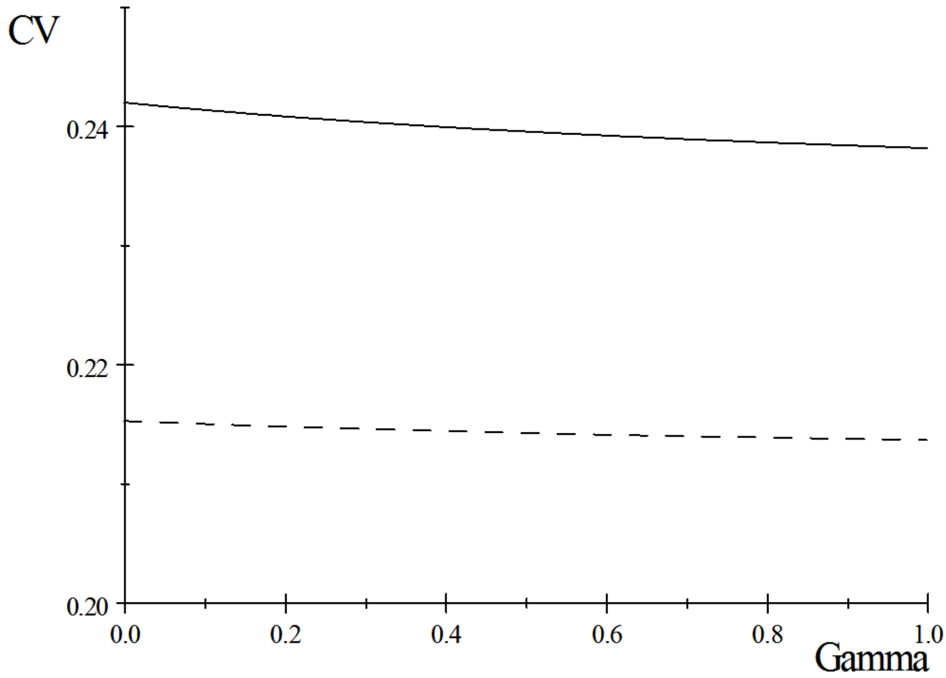

Figure 2. The effects of γ on the coefficient of variation $CV(s)$ of wealth. Solid line: $\delta = 1/5$. Dash line: $\delta = 1/8$.

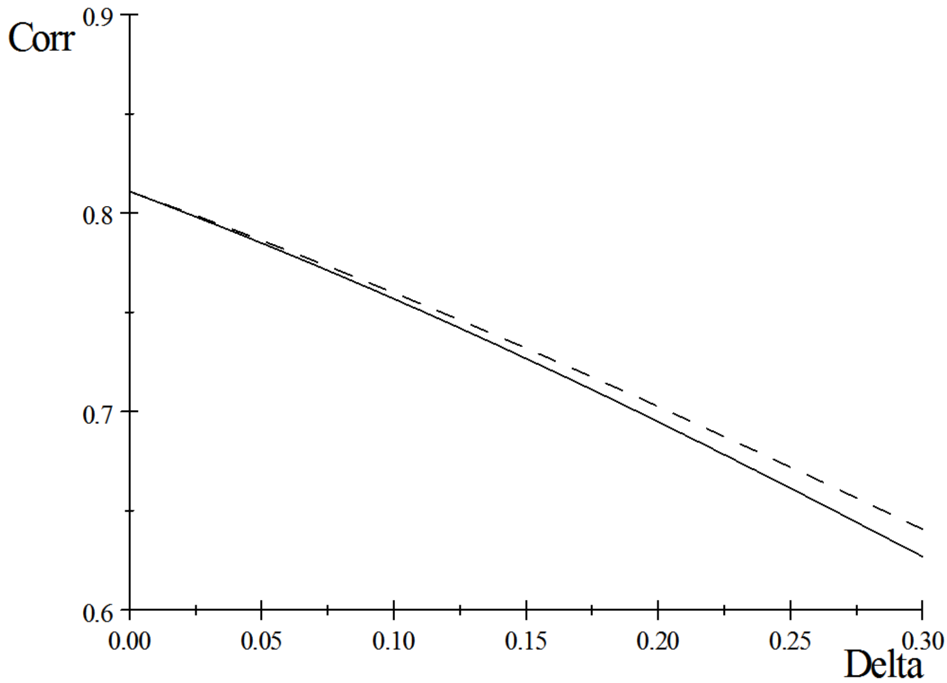

Figure 3. The effects of δ on the autocorrelation coefficient $Corr(s, s')$ of wealth. Solid line: $\gamma = 0$. Dash line: $\gamma = 1/4$.

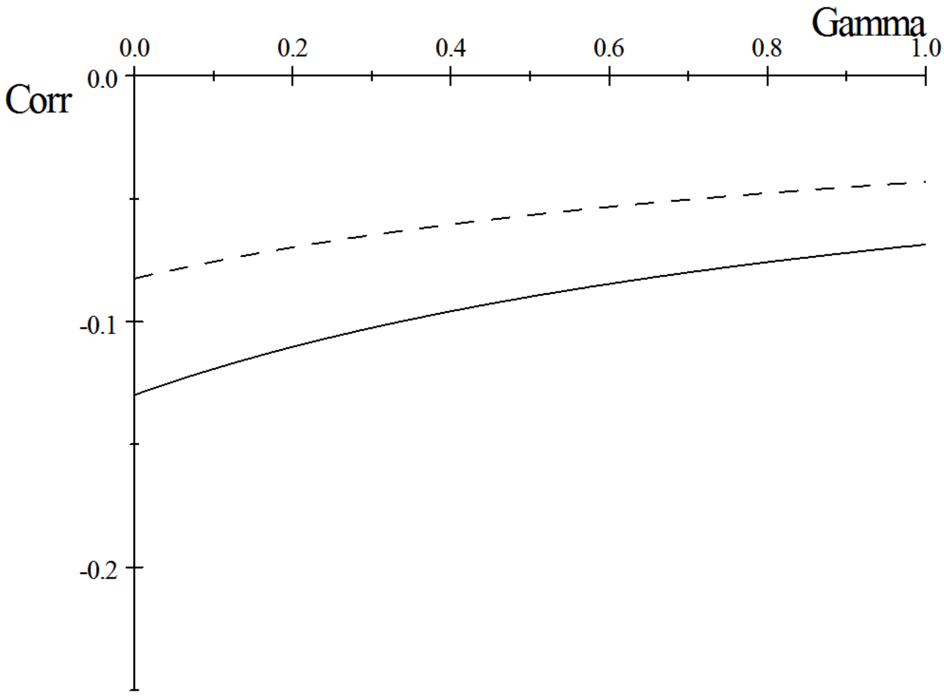

Figure 4. The effects of γ on the autocorrelation coefficient $Corr(s, s')$ of wealth. Solid line: $\rho = 0, \delta = 1/5$. Dash line: $\rho = 0, \delta = 1/8$.

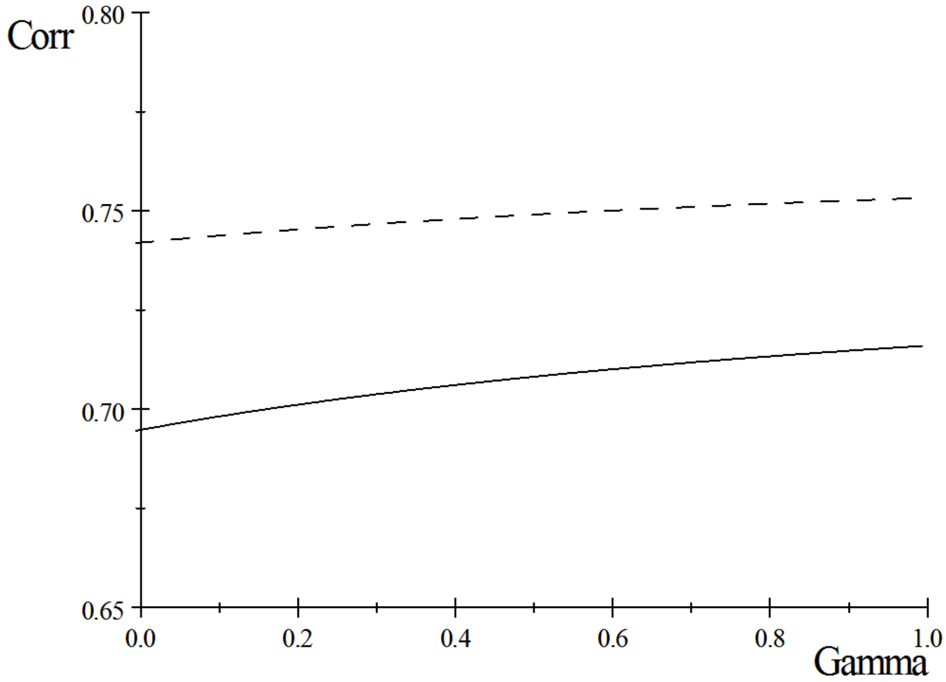

Figure 5. The effects of γ on the autocorrelation coefficient $Corr(s, s')$ of wealth. Solid line: $\rho = 1/2, \delta = 1/5$. Dash line: $\rho = 1/2, \delta = 1/8$.