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Monopolistic Competition and Different Wage Setting Systems.

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Abstract

In this paper we present a disequilibrium unemployment model without labor market frictions and monopolistic competition in the goods market within an infinite horizon model of growth. We consider different wage setting systems and compare wages, the unemployment rate and income per capita in the long run at firm, sector and national (centralized) level. The aim of this paper is to determine under which conditions the inverted-U hypothesis between unemployment and the degree of centralization of wage bargaining, reported by Calforms and Driffill (1988), is confirmed. Our analysis shows that a high degree of market power normally produces the inverted-U shape for unemployment. Moreover, we also illustrate that this inverted-U shape can be reversed when the ability of trade unions to internalize the provision of social services is great enough at sector level.

Keywords: Disequilibrium Unemployment, Monopolistic Competition, Growth, Wage Setting Systems.

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1 Introduction

The financial and economic crisis that started in 2008 has generated strong growth in the unemployment rate in many OECD countries. More specifically, the average unemployment rate increased by 3 percentage points in OECD countries between 2007 and the first quarter of 2010. The increase in the unemployment rate has been dramatic in countries like Spain, where it rose from 8.3% to 19%.

These results have begun to encourage important debates, at political and academic levels, on possible reforms of the labor market in the OECD countries most affected by this problem. It has been suggested to modify the system of collective bargaining in those countries characterized by wage bargaining at sector level to models of negotiation that generate higher wage moderation, and thus higher employment.

These proposals are based on the seminal article by Calmfors and Driffill (1988), where the worst result, in terms of employment, was obtained in a model where the wage was negotiated at sector level. They pointed out that highly centralized (at national or multi-industry level) and decentralized (at the firm level) bargaining systems perform better than intermediate ones (at sector/industry level) on wage demands. This inverted-U shape between unemployment and the degree of centralization of the wage bargaining arises from Calmfors and Driffill's assumption that, as centralization increases, the goods produced by sectors whose unions set the wage together are closer substitutes. We present a different explanation for this inverted-U shape: the role that market power and other institutional characteristics have on the unemployment rate when the wage is set at sector level. For simplicity, we consider only three different wage setting systems: Firm (the wage is set in each firm), sector (one same wage is set for all firms in one sector) and national level (one same wage is set for all sectors). Moreover, we introduce growth in the model because we consider that it is important to check whether the inverse U hypothesis holds also for other economic variables, for example, long run income per capita.

We develop a disequilibrium unemployment model, without labor market frictions, in a monopolistic competition set up with the infinite horizon model of growth (see, for example Galí (1996)). Monopolistic competition in the goods market with non frictional unemployment has received a lot of attention in the recent literature see, for example, Arnsperger and De la Croix (1990), Layard, Nickell and Jackman (1991), Dutt and Sen (1997), Blanchard and Giavazzi (2003) and Spector (2004)¹. However, there has been little research on the interaction between reforms in the wage bargaining system and product market power in order to increase the employment level. Moreover, we also consider how these reforms may affect income per capita in the neoclassical growth model.

¹Ebell and Haefke (2003), Delacroix (2006) and García-Sánchez and Vázquez-Méndez (2008) investigated models with different wage setting systems, imperfect competition and *frictional unemployment* due to matching problems.

Models with non frictional unemployment, perfect competition in the goods market and growth are used for analyzing various issues². For example, Daveri and Tabellini (2000), with an OLG model, show how labor taxes affect employment and long run growth when there is an externality in production. Doménech and García (2008), using an infinite horizon growth model, introduce some of these institutional characteristics presented in this paper and analyze how they affect the employment rate. All these papers make assumptions that imply a constant unemployment rate derived via the wage equation and assuming that the unemployment benefit grows with income per capita³. In a monopolistic competition set up with an OLG model for growth, Brauning (2000) studied the effects of unemployment on income per capita in the long run. However, this paper does not take into account different wage bargaining systems.

We compare the wages set at firm, sector and national (centralized) levels, their unemployment rates and growth in economic variables, for the Cobb-Douglas production function, in order to see under which conditions the hump shaped hypothesis between the unemployment rate and the degree of centralization, postulated by Calmfors and Driffill (1988), holds for the unemployment rate and income per capita.

We use the monopolistic competition set up because it is the natural framework for obtaining different labor demand elasticities with respect to the real wage when wages are set at firm, sector and national levels. More specifically, the elasticity is lower at sector level than firm and centralized levels and higher elasticity, as we will see, provides strong incentives to moderate wage demands. The idea that changing from sector to centralized level results in an increase in labor demand elasticity appears in Layard, Nickell and Jackman (1991). Moreover, they also justify the existence of full employment when wages are set at national level. Hoel (1990) extended the well-known union model developed by Layard, Nickell and Jackman adding at firm level wage setting and justifying the hump-shaped hypothesis. In this paper we upgrade Hoel's (1990) framework to obtain the labor demand function from economic fundamentals.

The existence and consideration of product market power is one of the reasons that produces the inverted-U shape in this paper. We introduce other institutional characteristics that also affect wage determination and employment rates, such as the size and structure of social expenditures, public sector inefficiencies, the degree of internalization of the contribution of labor income to the provision of social services and labor taxes. This study may be used to analyze under what circumstances these variables may change the inverted-U effect.

Our analysis shows that a high degree of market power normally produces the inverted-

²Raurich and Sorolla (2011) present a survey about this topic.

³However, Kaas and von Thadden (2003) present an OLG growth model with perfect competition, disequilibrium unemployment and a CES production function. In this framework, the change of production function produces that the employment rate depends on capital. For a more thorough discussion of these assumptions see Raurich and Sorolla (2011).

U form for unemployment. Moreover, we also illustrate that this inverted-U form can be reversed when the ability of trade unions to internalize the provision of social services is great enough at sector level. In general, the existing literature assumes that there is only complete internalization at national level (Layard, Nickell and Jackman (1991)). We extend internalization considering that there is some degree of internalization of fiscal externalities at sector level, that does not only depend on the wage setting system but also has a demographic or institutional component⁴.

The findings of this study are important for two reasons. First, as we will see in the next section, the values of some of the institutional variables that we introduce in the model are really different across countries. This means that trying to check the inverse U hypothesis by looking only at the level of centralization, without controlling for all the other variables, may result in the inverse U hypothesis not appearing in the empirical evidence (for an excellent survey see Aidt and Tzannatos (2008)).

Second, analysis of the wage set at sector level allows us to discuss alternative strategies for changing the factors that determine this wage rather than changing the wage setting system in order to reduce unemployment. More specifically, our results suggest that for countries with sector wage-bargaining, where unions internalize social services, it may be better to decrease market power or to change other institutional characteristics rather than change the wage setting system from a sector wage-bargaining structure to a decentralized system. This type of result gives alternative or complementary strategies to those presented in the OECD Jobs Study (1994) for improving employment levels.

The remainder of the paper is organized as follows: Section 2 shows the labor market performance of OECD countries, over the period 1998-2008, and relates this performance to labor market institutions and relevant characteristics that appear in the theoretical model. In section 3 we characterize the equilibrium employment rate at firm and sector level. The model is built on three basic assumptions: monopolistic competition in the product market, wage setting by bargaining between firms and unions subject to firms labor demand curve and, finally, the ability of trade unions to partially internalize the social service provided by the government. Section 4 focuses on the equilibrium employment rate at national level under different assumptions made by trade unions. Section 5 adds a simple neoclassical

⁴The idea of the internalization of fiscal externalities in wage-bargaining at sector level is present in the literature in a very disperse and slightly systematic form. In spite of this, there are articles where this issue appears with a more or less formal argument. For example, Pohjola (1992) presents a model where the externality can be internalized through cooperative behavior by a trade union in a framework of wage bargaining at sector level. The paper also shows, with informational imperfections, that the cooperative solution can be a sustained equilibrium when economic activity is low. The OECD Employment Outlook (2007) has included an explicit recommendation to increase the link between taxes and social protection benefits in order to reduce the negative impact that the overall tax wedge may exert on employment. In his review of Calforms and Driffill (1988) paper, Driffill (2006) highlights the need to "take account of informal coordination of bargaining across groups". In theoretical terms, our paper is most closely related to Mares (2004). She presents a model with an explicitly internalization by unions of social expenditure and takes into account the composition of social policy transfers.

model of growth to analyze the relationship between income per capita and the wage bargaining system. The paper concludes with a summary of the main results.

2 Stylized facts for some OECD countries.

Many of the articles that have been written in the last few decades about unemployment focus on explaining the substantial differences in the level and evolution of the unemployment rate across OECD countries. The poor performance of the unemployment rate is explained by shocks and differences in institutions or the interaction of both⁵. It is important to note that the collective bargaining system appears as a key element in all this literature. More specifically, a large body of empirical research has confirmed that wage bargaining at sector level produces more unemployment than more decentralized or centralized wage bargaining systems.

Table 1 presents the classification of many OECD countries by their collective bargaining system in three groups that we have named ANGLO, EUCON and NORDIC. For this country classification we use the product of bargaining level, union density and bargaining coordination relative to the value for Finland⁶. It is important to be clear that coordination of wage bargaining is different from bargaining centralization. The bargaining centralization tends to be used to refer to the level of wage bargaining (firm, sector or central level) while bargaining coordination has been applied to situations where the parties are able to internalize the implications of wage settlements in the unemployment rate. Thus, coordination can also be achieved even if bargaining is conducted at firm or industry level.

According to our theoretical classification between firm, sector and national wage setting systems we identify ANGLO with firm, EUCON with sector and NORDIC with national⁷.

The first variable presented in Table 2 is the harmonized unemployment rate from OECD statistics (U). As mentioned above, Calmfors and Driffill (1988) found a hump-shaped relationship between an index of coordination and the unemployment rate. We examine these issues below taking into account our relative index of collective bargaining.

⁵An excellent survey on these issues can be found in Blanchard (2006).

⁶The Bargaining level is an index of bargaining centralization with a range {1,3}. More specifically, 1= Company/plant level, 2= sectorial level and 3 = Central level. Union Coverage refers to the number of workers covered by collective agreements normalized on employment. Coordination is an index of bargaining coordination with a range {1,5} between employers and employees. Source: Database Nickell (2006).

⁷Calmfors and Driffill (1988) have ranked countries according to similar criteria. More specifically, in his paper the ranking is based on the sum of bargaining level and Coordination. The difference between our classification and that elaborated by Calmfors and Driffill (1988), within the same dates, is slight. However, we include union coverage in our classification due to the importance of extensions to collective contracts in Europe whether unionized or not. For more details, see chapter 3 of the OECD Employment Outlook (2004).

Table 1
Some labour markets institutional indicators in OECD countries.

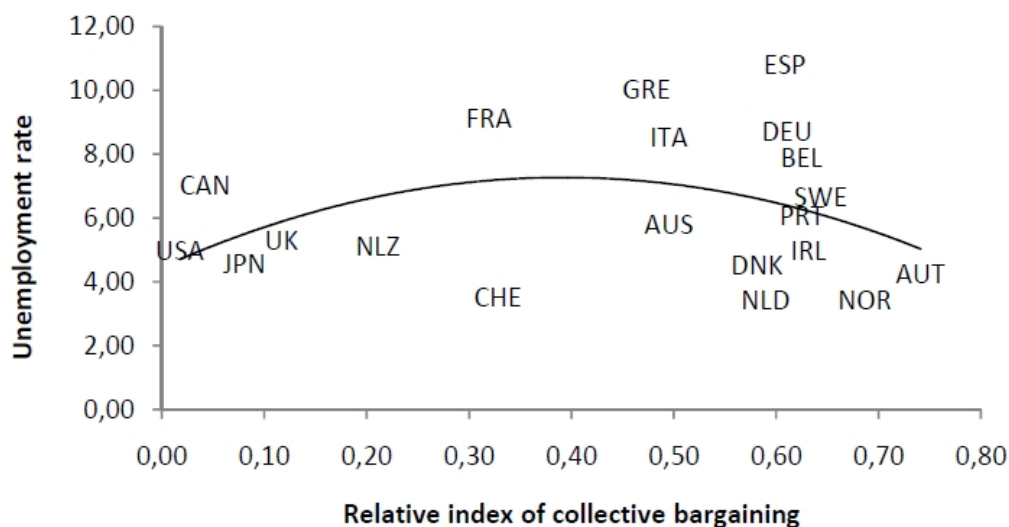
	Bargaining level	Union Coverage	Coordination	Relative Product
<i>ANGLO</i>				
USA	1	0.2	1	0.02
Canada	1.1	0.37	1.3	0.04
Japan	1	0.23	4	0.08
UK	1.3	0.57	1.8	0.12
New Zealand	1.6	0.5	3	0.21
<i>EUCON</i>				
France	2	0.91	2	0.32
Switzerland	1.8	0.52	4	0.33
Greece	2	0.9	3	0.47
Australia	2	0.83	3.4	0.50
Italy	2.4	0.84	2.8	0.5
Denmark	2.3	0.72	4	0.58
Netherlands	2.1	0.8	4	0.59
Spain	2.5	0.75	3.7	0.61
Germany	2	0.87	4	0.61
Belgium	2	0.89	4	0.62
Portugal	2.5	0.75	3.8	0.63
Ireland	2.5	0.9	3.2	0.63
<i>NORDIC</i>				
Sweden	2.48	0.87	3.4	0.64
Norway	2.6	0.7	4.3	0.69
Austria	2	0.96	4.4	0.74
Finland	2.5	0.95	4.8	1

Source: Database Nickell (2006).

Figure 1 plots a scatter diagram of the unemployment rate against our relative index. In this period of time there is no evidence in favor of the hump hypothesis. Consequently, we find practically no relationship between an ordinary least squares regression of unemployment using our relative index and its square root ($R^2 = 0,03$). However, if we omit the date of Finland, we obtain an inverted-U shaped curve and a better regression ($R^2 = 0.11$)⁸.

⁸Finland suffered particularly negative external shocks from the collapse of the Soviet Union in 1990 and a severe financial crisis. The unemployment rate jumped from 3.2% in 1990 to more than 16.7% just four years later.

Figure 1. Unemployment vs relative index of collective bargaining.



This paper gives rigorous theoretical underpinnings for the link between the unemployment and economic growth rates with collective bargaining systems at firm, industry and national levels. However, our analysis goes further, because we add other institutional features, rigidities and macroeconomic parameters that also affect wage bargaining. We include the size and structure of social expenditures, public sector inefficiencies, the labor force participation rate, labor taxes and the degree of competition in the output market. We show that the employment rate depends on all these variables together, which means that empirical research based only on changes in one variable usually yields poor results.

Evidently, the omission of these variables may offer an explanation for the lack of robustness of the hump-shaped hypothesis predicted by Calmfors and Driffill (1988) in the empirical literature⁹. In other words, the group of countries that belong to a certain wage bargaining system may differ in the composition of labor taxes, the inefficiency of their governments, the degree of competition in the goods market, etc.. and we show later that all these variables also affect wage determination and, therefore the unemployment rate.

In Tables 2 and 3 we show the values of the different institutional indicators that we introduce in the model for the period 1998-2008. The theoretical study below investigates more closely the mechanism through which these variables affect the unemployment rate.

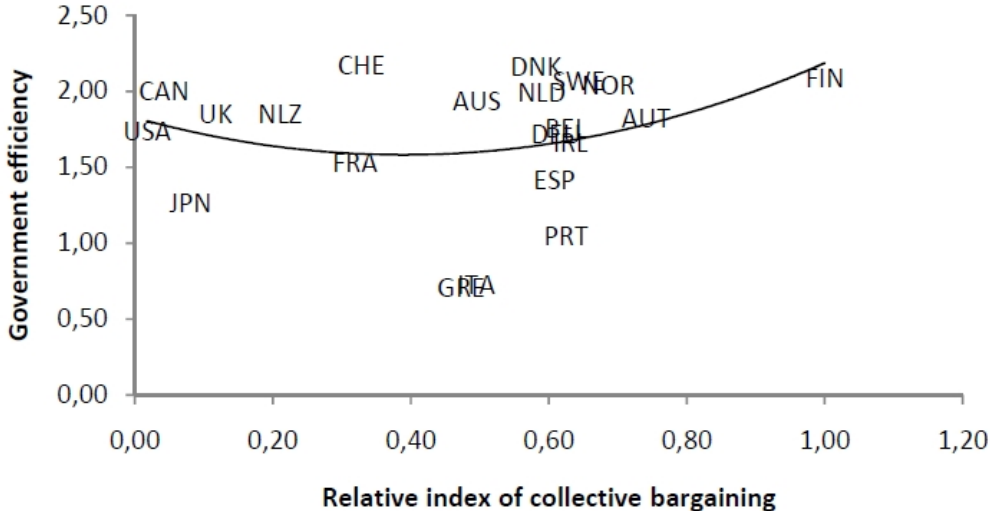
The second column in Table 2 presents the degree of efficiency of the public sector (GE)¹⁰. This variable has been constructed by Kaufmann et al. (2009). These authors define government efficiency as an aggregate governance indicator that measures perceptions of the quality of public service provision, the quality of bureaucracy and the competence of civil servants among other elements related to the government. This variable is relevant

⁹See Aidt and Tzanatos (2008), for an excellent survey of these issues.

¹⁰See Doménech and García (2008) for an in-depth discussion of this variable.

for wage determination when the government finances a given level of social expenditure. The more inefficient the government is, the higher the tax rates necessary to finance a given government expenditure and, therefore, the greater the effects on employment. In Figure 2 we present the scatter diagrams of efficiency of the public sector plotted against our relative index ($R^2 = 0.09$) as in Figure 1.

Figure 2. Government efficiency vs relative index of collective bargaining.



Empirical findings suggest that there is a hump-shaped relationship between bargaining structure and the degree of efficiency.

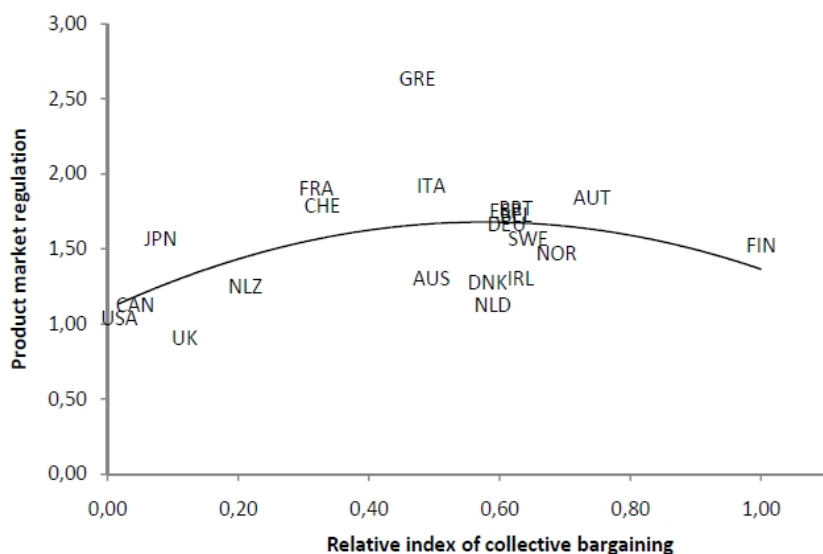
The third column in Table 2 provides the degree of rigidity in the goods market (PMR)¹¹. Rigidities in the product markets comprise all the factors that reduce competition. Many authors point out the relationship between rigidities in the goods markets and wage setting¹². It has argued that when the price elasticity of product demand is low, firms have more price-setting power. As labour demand is derived, the elasticity of labor demand with respect to wages is also low. According to this context, the employed workers will have the opportunity to exercise upward pressure on wages to obtain a proportion of the surplus profits derived from the price-setting power. Thus, although we assume that product market competition does not have a direct influence on union bargaining power, it does have an indirect impact through the elasticity of labor demand and, thereby on the resulting wage rate.

A scatter plot of product market rigidity against our relative index ($R^2 = 0.22$) is shown in Figure 3

¹¹The indicator of product market regulations (PMR) is defined in Conway et. al. (2005). The source of the database is the webpage <http://www.oecd.org/eco/pmr>.

¹²For a more detailed discussion of these issues see, for instance, Nickell (1999), Boeri et.al (2000), Blanchard and Giavazzi (2003) and OECD (2002) chapter five.

Figure 3. Product market regulation vs relative index of collective bargaining.



From the graph above we can see that there is also a hump-shaped pattern between bargaining structure and the degree of rigidity in the goods market.

Finally, the fourth column in Table 2 shows the average labor force participation rate elaborated by the OECD (LBPR). In our theoretical model, we assume that this variable affects the amount of social services that an active worker may receive. A similar analysis also produces similar findings: the LBPR displays a hump-shaped relationship with the bargaining structure.

What do imply all these empirical regularities for the relationship between the unemployment rate and the Calmfors and Driffill 's hypothesis? It is possible that the apparent relationship between the unemployment rate and bargaining structure is caused by a third factor (for example, rigidity in the goods market), which significantly affects unemployment and also registers an inverted u-shaped relationship respect to bargaining structure¹³. The theoretical section provides a more detailed discussion about all the factors that determine the unemployment rate when the wage is negotiated at sector level.

Social security systems and tax structure are other institutions that affect the unemployment rate. For this reason Table 3, Column 1 shows social expenditures with respect to GDP (SE). These expenditures are basically financed by social security contributions paid by workers and employers. Imposition on labor revenues and other taxes plays a minor role (See OECD (2007))¹⁴. Columns 2 to 5 report average tax wedges (TW), income tax (IT) and employees' and employers' social contributions (WSC and ESC respectively). The tax

¹³When one analyzes, the influence of taxes on unemployment, it seems that other elements must be taken into account in order to explain the data. The empirical evidence presented by Daveri and Tabellini (2000) supports the view that in more corporate and decentralized countries, labor taxes are less distortionary than in countries with an intermediate level of wage bargaining. However, this paper does not take into account, for example, rigidity in the goods market.

¹⁴Only Australia, Denmark and New Zealand do not finance social policy expenditure with social security contributions.

Table 2
Rigidities and institutions.

	U	GE	PMR	LBPR
<i>ANGLO</i>				
USA	4.98	1.74	1.04	78.78
Canada	7.05	2.01	1.13	78.59
Japan	4.57	1.27	1.57	78.69
UK	5.30	1.86	0.91	77.72
New Zealand	5.12	1.86	1.25	78.41
<i>average</i>	<i>5.41</i>	<i>1.75</i>	<i>1.18</i>	<i>78.44</i>
<i>std desv</i>	<i>0.96</i>	<i>0.28</i>	<i>0.25</i>	<i>0.42</i>
<i>EUCON</i>				
France	9.12	1.54	1.91	69.05
Switzerland	3.55	2.18	1.79	83.27
Greece	10.04	0.71	2.63	66.28
Australia	5.80	1.93	1.30	76.08
Italy	8.55	0.73	1.93	62.38
Denmark	4.55	2.17	1.28	80.98
Netherlands	3.44	2	1.13	76.21
Spain	10.84	1.42	1.75	69.20
Germany	8.74	1.72	1.66	73.65
Belgium	7.89	1.76	1.73	65.43
Portugal	6.11	1.05	1.77	76.89
Ireland	4.99	1.67	1.31	70.11
<i>average</i>	<i>6.97</i>	<i>1.57</i>	<i>1.68</i>	<i>72.46</i>
<i>std desv</i>	<i>2.55</i>	<i>0.51</i>	<i>0.4</i>	<i>6.43</i>
<i>NORDIC</i>				
Sweden	6.67	2.07	1.57	80.69
Norway	3.44	2.05	1.48	81.27
Austria.	4.29	1.83	1.85	72.94
Finland	8.79	2.09	1.52	75.48
<i>average</i>	<i>5.80</i>	<i>2.01</i>	<i>1.61</i>	<i>77.60</i>
<i>std desv</i>	<i>2.42</i>	<i>0.12</i>	<i>0.17</i>	<i>4.05</i>

wedge is computed as the sum of labor income tax and social security contributions paid by workers and employers¹⁵.

As can be seen from Table 3, there are large differences in the composition of the tax wedge across OECD countries over the period 1988-2008. In general, countries with the highest labor tax are also those that tend to have the highest social contributions paid directly by employers.

The most striking results that emerge from the data for EUCON countries, with respect to the rest of countries, are the following: First, and foremost, the EUCON countries are, on average, the most inefficient (Table 2, GE average 1.57), have a more regulated goods

¹⁵All the effective tax rates have been computed, as suggested in Boscá, García and Taguas (2005), using the methodology proposed by Mendoza et. al (1994).

Table 3
Social expenditure and the tax structure.

	SE	TW	IT	WSC	ESC
<i>ANGLO</i>					
USA	15.36	0.25	0.13	0.06	0.06
Canada	16.91	0.30	0.20	0.04	0.06
Japan	17.28	0.24	0.07	0.08	0.09
UK	20.07	0.27	0.14	0.05	0.07
New Zealand	18.92	0.21	0.21	0.0	0.0
<i>average</i>	<i>17.71</i>	<i>0.25</i>	<i>0.15</i>	<i>0.05</i>	<i>0.06</i>
<i>std desv</i>	<i>1.83</i>	<i>0.03</i>	<i>0.06</i>	<i>0.03</i>	<i>0.03</i>
<i>EUCON</i>					
France	28.68	0.49	0.12	0.10	0.27
Switzerland	19.20	0.24	0.13	0.06	0.06
Greece	19.71	0.39	0.07	0.15	0.18
Australia	17.40	0.18	0.18	0.0	0.0
Italy	23.88	0.51	0.16	0.07	0.27
Denmark	26.81	0.40	0.38	0.02	0.0
Netherlands	20.61	0.34	0.10	0.15	0.1
Spain	20.62	0.38	0.11	0.05	0.22
Germany	26.59	0.41	0.12	0.14	0.15
Belgium	26.12	0.51	0.21	0.10	0.2
Portugal	20.41	0.33	0.09	0.08	0.16
Ireland	14.9	0.27	0.17	0.04	0.07
<i>average</i>	<i>22.08</i>	<i>0.37</i>	<i>0.15</i>	<i>0.08</i>	<i>0.14</i>
<i>std desv</i>	<i>4.27</i>	<i>0.1</i>	<i>0.08</i>	<i>0.05</i>	<i>0.1</i>
<i>NORDIC</i>					
Sweden	29.60	0.58	0.28	0.06	0.23
Norway	22.67	0.43	0.2	0.08	0.14
Austria.	26.93	0.45	0.15	0.14	0.16
Finland	25.42	0.55	0.27	0.06	0.23
<i>average</i>	<i>26.23</i>	<i>0.5</i>	<i>0.23</i>	<i>0.08</i>	<i>0.19</i>
<i>std desv</i>	<i>2.78</i>	<i>0.07</i>	<i>0.06</i>	<i>0.04</i>	<i>0.05</i>

market (Table 2, PMR average 1.68) and the lowest labor participation rate (Table 2, LBPR average 72.46), compared to Anglo and Nordic countries; second, in regard to the financing of social expenditure, we find that the social security contribution paid by employers (Table 3 ESC) in some countries is prominent (e.g. France, Italy and Spain).

As shown below, more inefficiency, more regulated good markets and less internalization driven by a lower participation rate produces more unemployment in a sector level wage setting system. So, these characteristics reinforce the higher mean unemployment rate of this group, besides the centralization system, and suggest that EUCON countries have the institutional characteristics that can generate a higher unemployment rate than the rest of countries.

Finally, Table 4 reports the simple correlation using cross-country data between all

Table 4

Correlations between institutions and social expenditure.

	GE	LBPR	PMR	SE	TW	ESC
U	-0.503*	-0.684**	0.546*	0.29	0.469*	0.663**
GE		0.676**	-0.636**	0.136	-0.065	-0.434*
LBPR			-0.561**	-0.168	-0.393	-0.601**
PMR				0.337	0.442*	0.613**
SE					0.860**	0.607**
TW						0.831**

* The correlation is significant at 0.05 level

** The correlation is significant at 0.01 level

relevant variables over the period 1998-2008. Since a correlation does not imply causality in any sense, the existence of significant correlation suggests a non simple mechanism of relationship. Table 4 shows that the tax rates paid by employers seem to be positively related to the unemployment rate (0.663). At the same time, there is a positive correlation between government efficiency and labor force participation rates in OECD countries (0.676). It is interesting to notice that there is a strong negative correlation between government efficiency and product market regulation (-0.636). Finally, social expenditure and tax paid by employers are highly correlated (0.613). All these correlations also reinforce that the specific variable considered in the regression may explain the relatively good/poor empirical estimations found by different authors over time.

A more accurate study requires the use of multivariate regression analysis. Unfortunately, these series are relatively recent and it is not possible to obtain a longer sample period that allows regressions to be performed. However, the different values that we show for these variables may explain the relatively good/poor empirical estimations found, depending crucially on the specific variable considered in the regression.

3 Employment rate at sector and firm level.

In this section, we present the short run general equilibrium when wages are set at the sector and firm level. Its main elements are monopolistic competition in the goods market and wage bargaining in the labor market. Our innovation lies in taking into account that the trade union partially internalizes the effect of wages on the social services provided by government. Moreover, we add other institutional features that also affect wage bargaining. In this framework we solve the symmetric Nash-solution, at sector level and firm level of bargaining, to determine the wage and employment rate.

3.1 Labor demand with monopolistic competition.

We assume $J \in [0, 1]$ sectors with one firm¹⁶ per sector that produces a different good, $Y_j(t)$, using the same production Cobb-Douglas technology, that is:

$$Y_j(t) = AK_j(t)^\alpha L_j(t)^{1-\alpha}. \quad (1)$$

The demand function facing firm J is

$$Y_j(t) = \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} (\bar{Y}(t)), \quad (2)$$

where $\xi > 1$ is the constant elasticity of demand of product J with respect to its price, $\bar{Y}(t) \equiv \frac{Y(t)}{P(t)}$ is total real expenditures on consumption and investment, $P_j(t)$ is the price of product j , $P(t)$ is a price index with the usual properties and $Y_j(t)$ is the corresponding quantity demanded of the consumption and investment good produced by firm j ¹⁷.

The firm in sector j maximizes the wealth of its shareholders subject to the demand function (2). Each firm pays a payroll tax, τ^f , in order to finance social services. The first order condition in terms of the real wage is¹⁸:

$$(1 - \alpha)AK_j^\alpha L_j^{-\alpha} A^{-\frac{1}{\xi}} K_j^{-\frac{\alpha}{\xi}} L_j^{-\frac{1-\alpha}{\xi}} = \frac{(1 + \tau^f)m\omega_j}{\bar{Y}(t)^{\frac{1}{\xi}}}, \quad (3)$$

where the parameter m represents the degree of monopoly or the (price) markup $m \equiv \frac{1}{(1-\frac{1}{\xi})} \geq 1$ and $\omega_j(t) \equiv \frac{W_j(t)}{P(t)}$ denotes wage in sector j .

Labor demand, in terms of the real wage, is then:

$$L_j^d(t) = \tilde{L}_j^d(\omega_j(t)) = A^{\frac{\xi-1}{1+\alpha(\xi-1)}} K(t)^{\frac{\alpha(\xi-1)}{1+\alpha(\xi-1)}} \bar{Y}(t)^{\frac{1}{1+\alpha(\xi-1)}} \left(m \frac{(1 + \tau^f)}{(1 - \alpha)} \omega_j(t) \right)^{\frac{-\xi}{1+\alpha(\xi-1)}}, \quad (4)$$

where the elasticity of labor demand with respect to the wage is constant and given by:

$$\varepsilon_{L_j^d, \omega_j} \equiv \frac{\partial \tilde{L}_j^d(\omega_j)}{\partial \omega_j} \frac{\omega_j}{\tilde{L}_j^d(\omega_j)} = \frac{-\xi}{1 + \alpha(\xi - 1)} = \frac{-1}{\alpha + \frac{(1-\alpha)}{\xi}}. \quad (5)$$

Note that the elasticity of labor demand depends positively on product market elasticity ξ , with the property that the greater ξ is, the higher $\varepsilon_{L_j^d, \omega_j}$, and always $\varepsilon_{L_j^d, \omega_j} < -1$. Therefore, an increase in the real wage always decreases the wage bill $\omega_j \tilde{L}_j^d(\omega_j)$. In the particular case of perfect competition we have $\xi = \infty$ and labor demand elasticity is equal

¹⁶Alternatively one may assume $K \in [0, 1]$ firms per sector having, obviously, the same result instead of considering only one firm per sector.

¹⁷For more details see Appendix A.

¹⁸This expression comes from equation (42) in the appendix.

to $-\frac{1}{\alpha}$ ¹⁹.

3.2 Government budget constraint.

Before describing wage bargaining, we need to introduce the government budget constraint. The government finances the unemployment benefits paid to unemployed workers and social services. To generate revenue, at each period t , the government imposes a flat-rate tax. More specifically, τ^L denotes the tax rate paid by employees on wages. This tax includes income tax plus the social security contribution paid by employees.

We assume that, given a level of taxes collected, more inefficient governments will produce a lower level of transfers and social services. It can be assumed that this level of inefficiency will be proportional to the administrative cost of managing tax revenues. The parameter η stands for the level of inefficiency of the government to finance its public expenditure²⁰. From all these assumptions, it follows that the government's flow budget constraint in real terms is:

$$(1 + \eta) (S(t) + (N(t) - L(t))B(t)) = (\tau^L + \tau^f) \omega(t)L(t), \quad (6)$$

where $S(t)$ are social services in real terms, $B(t)$ the unemployment benefit in real terms and $N(t)$ the inelastic labor supply (active population).

We consider that part of tax revenues is used to finance social services (such as education, the social security system, pensions etc.) and another part is channeled to financing the unemployment benefits of unemployed workers in each period, so that the following equalities hold:

$$(1 + \eta)S(t) = \phi (\tau^L + \tau^f) \omega(t)L(t), \quad (7)$$

$$(1 + \eta)B(t)(N(t) - L(t)) = (1 - \phi) (\tau^L + \tau^f) \omega(t)L(t), \quad (8)$$

where the parameter ϕ captures the relative weight of the expense in social services, decided by the government, with respect to tax revenues. Rewriting the last two equations we get:

$$S(t) = \phi \frac{(\tau^L + \tau^f) \omega(t)L(t)}{(1 + \eta)} \quad (9)$$

and

¹⁹Alternatively, forcing the model to assume only one firm per sector, we can consider the perfect competition situation where the firm takes $P_j(t)$ as given. Then, the first order condition in terms of $\frac{W_j(t)}{P_j(t)}$ is $F_L = (1 + \tau^f) \frac{W_j(t)}{P_j(t)}$, labor demand is: $L_j^d = \tilde{L}_j^d \left(\frac{W_j}{P_j} \right) = \left(\frac{(1-\alpha)A}{(1+\tau^f) \frac{W_j}{P_j}} \right)^{\frac{1}{\alpha}} K_j$ and the elasticity with respect to the real wage is $-\frac{1}{\alpha}$.

²⁰A similar assumption is made by Doménech and García (2008).

$$B(t) = (1 - \phi) \frac{(\tau^L + \tau^f) \omega(t) L(t)}{(1 + \eta)(N(t) - L(t))}. \quad (10)$$

Note that, because we assume τ^L and τ^f are invariant, the last two equations imply that an increase in the wage always reduces $S(t)$ and $B(t)$, when employment is given by labor demand, because its elasticity with respect to the real wage is less than -1 .

We include the level of public services, $S(t)$, in the utility function of trade unions, reflecting the fact that the welfare of workers depends on the level of social services they receive²¹. If we add equation (9), which expresses how public services are financed, we also assume that workers have perceptions about how changes in the wage affect the amount of public services.

3.3 Wage setting at sector and firm level.

We assume a three-stage game for employment decisions. In the first stage, the firms decide the level of capital stocks anticipating their effects on the wage setting and labor demand. In stage two, the wage rate is determined through a process of bargaining between employers and trade unions. Finally, in stage three, the firm unilaterally determines the employment level once the conditions of the wage negotiations and investment decisions have been settled.²²

There are j unions in the economy (one for each industry) and, as is common in the literature when there is a wage setting at sector level, we assume that the labor force N_t is symmetrically divided between sectors. Moreover, we assume that all the workers in one sector are members of the sector union that takes care of them. In our continuous context, the workers in one sector will, of course, be N_t ($\int_0^1 N_t = N_t$) and then the utility function of the j th union is:

$$V_j = (1 - \tau^L) \omega_j(t) L_j^d(t) + R_j(t)(N(t) - L_j^d(t)) + \theta_s \phi \frac{(\tau^L + \tau^f) \omega_j(t) L_j^d(t)}{(1 + \eta)}, \quad (11)$$

where $R_j(t)$ is the alternative income that a worker receives if he is not employed in the sector j . Additionally, we assume that the union takes into account that social services affect the welfare of workers and that the revenues obtained from the wage bill of the sector may contribute to finance social services²³. We introduce the parameter θ_s which measures the ability of the trade union in sector j to internalize the contribution of the wage bill in sector j , $\omega_j L_j^d$, to the provision of social services²⁴. It is reasonable to assume

²¹For a more extensive discussion, see Mares (2004).

²²For more details over this issue see, for example, Koskela et al. (2009).

²³Details of these hypotheses are given in Mares (2004).

²⁴Alternatively, one can assume that the union only cares for workers that work in sector j and its

that this parameter is determined by two factors. The first is the degree of centralization of wage bargaining. It is very usual in the literature to classify wage setting regimes by their degree of centralization into three types. Highly centralized systems, such as national level bargaining, intermediate levels of centralization, where the bargaining process is carried out at industry level and, finally, negotiation at firm level. We assume that the degree of internalization θ_s is positively related to the level of centralization of wage bargaining because the proportion of revenue that finances social services increases. It should be noted that the value of this parameter will, of course, be not the same for different countries, samples of years and economic performance (crisis or expansion)²⁵.

The second factor that will affect the value of parameter θ_s is the share of active labor force with respect to the inactive population in the economy²⁶. We assume that the same level of social services is available to the labor force and the inactive population. Note that the social services for the non active population are financed by taxes levied on the active population. Therefore, if unions only care about the social services of the active population, the higher the inactive population receiving social services, the lower the ability of trade unions to internalize the provision of services and thus the lower the value of parameter θ_s ²⁷. We also assume that the union considers no effect of the wage bill in sector j on alternative income because, although it is true that the wage bill of the sector will finance the unemployment benefit, the weight of the unemployment benefit on alternative income is small, because it also comprises the wages of all the other sectors.

Turning to wage bargaining, we assume that employers negotiate the sector wage with the trade union, taking into account that firms retain their right-to-manage power and determine employment (and capital) after the wage has been set. The outcome of wage bargaining is determined by the Nash-bargaining solution, which maximizes the Nash product $(V_j - \bar{V}_j)^\beta (\Pi_j - \bar{\Pi}_j)^{1-\beta}$, where β denotes the bargaining power of the trade union. The fall-back position for the union is $\bar{V}_j = R(t)N(t)$ and for the firm is given by $\bar{\Pi}_j = -r_t K_j$ ²⁸.

For an interior solution, the maximization of the logarithm of the generalized Nash criterion gives the first-order condition:

objective is to maximize the income of a worker that works in sector j with respect to the alternative income of working outside the sector, $R_j(t)$, times employment, the utility function in this case being $V'_j = [(1 - \tau^L)\omega_j(t) - R_j(t)] L_j^d(t) + \theta_s \phi \frac{(\tau^L + \tau^J)\omega_j(t)L_j^d(t)}{(1+\eta)}$. Nevertheless, as argued below, the result when using any of these two functions V_j or V'_j is the same.

²⁵See, for example, Pohjola (1992).

²⁶Prior studies have noted the importance of demographic factors in the utility function of trade unions. See, for example, Alesina and Perotti (1997) and Mares (2004).

²⁷In a more formalized way: our active population is $N(t)$ and total population $P(t) = (1 + \lambda)N(t)$. Then social services per person are $\frac{S(t)}{P(t)}$ and the social services of active population (the term that enters the union utility function) is equal to $\frac{S(t)}{P(t)}N(t) = \frac{S(t)}{(1+\lambda)}$.

²⁸If readers prefer the V' utility function for the j union then they must consider no fall-back position for the j th union, $\bar{V}'_j = 0$, which gives the same expression for V_j and $(V'_j - \bar{V}'_j)$.

$$\beta \frac{(V_j - \overline{V}_j)_\omega}{(V_j - \overline{V}_j)} + (1 - \beta) \frac{(\Pi_j - \overline{\Pi}_j)_\omega}{(\Pi_j - \overline{\Pi}_j)} = 0, \quad (12)$$

where

$$\frac{(V_j - \overline{V}_j)_\omega}{(V_j - \overline{V}_j)} = \beta \left[L_j^d + \frac{\partial L_j^d}{\partial \omega_j} \omega_j \right] \left[(1 - \tau^L) + \theta \phi \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right] - \beta R_j \frac{\partial L_j^d}{\partial \omega_j} \quad (13)$$

and

$$(1 - \beta) \frac{(\Pi_j - \overline{\Pi}_j)_\omega}{(\Pi_j - \overline{\Pi}_j)} = (1 - \beta) \frac{1}{\omega_j (m(1 - \alpha) - 1)}. \quad (14)$$

Substituting expressions (13) and (14) into the first-order condition (12) yields, after some rearrangement, the following Nash bargaining solution for the wage rate set by union J

$$\omega_j(t) = \frac{[(1 - \beta) + \beta \frac{m}{1 - \alpha}]}{\left[(1 - \tau^L) + \theta_s \phi \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right]} R_j(t). \quad (15)$$

Equation (15) defines the bargained real wage curve under the assumptions made here. The real wage bargained is a mark-up on the reservation wage $R_j(t)$. Higher market power increases this mark-up, and thus the wage, as pointed out by Layard, Nickell and Jackman (1991) (P. 27) and Spector (2004). It is interesting to note that an increase in θ , ϕ and τ^f produces a reduction in the mark-up, and thus wage moderation. Alternatively, an increase in τ^L and η increases the mark-up and thus wage demands.

If the firm behaves competitively, we have, on the one hand, $m = 1$, which gives the labor demand elasticity $-\frac{1}{\alpha}$; on the other hand, we assume the internalization parameter θ_s to be zero because there are many firms and many sectors, the wage bill of one firm being negligible respect to the total wage bill that finances social services. Then the wage is:

$$\omega_f(t) = \frac{[(1 - \beta) + \beta \frac{1}{1 - \alpha}]}{(1 - \tau^L)} R_j(t), \quad (16)$$

having that $\omega_f(t) < \omega_j(t)$ if m is high enough with respect to $\theta_s \phi$ ²⁹.

This is because in (16) m decreases to 1 and θ_s disappears when comparing with the expression (15), thus the positive effect of the reduction in monopoly power will dominate the negative effect of the elimination of the internalization parameter. If we interpret the wage set in the competitive case ($m = 1$) as wage setting at firm level, because the union takes into account the labor demand of a small competitive firm³⁰, then we have wage

²⁹The corresponding condition that guarantees $\omega_f(t) < \omega_j(t)$ is $m > 1 + \left[(1 - \beta) + \beta \frac{1}{1 - \alpha} \right] \frac{(1 - \alpha)}{\beta(1 - \tau^L)} \theta_s \phi \frac{(\tau^L + \tau^f)}{(1 + \eta)}$.

³⁰In this case the utility function of the union would be $V_{f,j} = \left[(1 - \tau^L) \frac{W_j}{P} - R_j \right] \tilde{L}_j^d \left(\frac{W_j}{P} \right)$ where the union chooses W_j , and, one can show that the solution is the above expression.

restraint at firm level if m is high enough with respect to $\theta_s \phi$.

3.4 Short run general equilibrium at sector and firm level.

In the short run partial equilibrium presented above, the wage bargaining process takes the alternative income of working outside the sector (or the firm) as given. Nevertheless, in short run general equilibrium, all sectors set the same price and wages in all firms and sectors are set in a similar way. As a result $R(t)$ becomes endogenous. We assume that the alternative income a worker receives, if he does not work in firm or sector j , is given by³¹

$$R_j(t) = l(t)(1 - \tau^L(t))\omega^e(t) + (1 - l(t))B(t), \quad (17)$$

where $l(t)$ is the employment rate of the economy i.e. $l(t) \equiv \frac{L(t)}{N(t)}$, where $L(t) = \min(L^d(t), N(t))$

and $L^d(t) = \int_0^1 L_j^d(t) dj$, $\omega^e(t)$ is the alternative wage of working outside (firm) sector j and $B(t)$ is the unemployment benefit that an unemployed worker receives.

In a symmetric equilibrium $\omega_j(t) = \omega^e(t) = \omega(t)$. We assume that the unemployment benefit is financed by employed workers' revenues and determined by the budget constraint of the government once it has decided the constant tax rates. Therefore, from (10), it is:

$$B(t) = \frac{(1 - \phi)(\tau^L + \tau^f)}{(1 + \eta)} \frac{\omega(t)l(t)}{(1 - l(t))}, \quad (18)$$

taking into account that $\omega^e(t) = \omega(t)$ and combining equations (15), (17) and (18) we obtain:

$$\omega(t) = \omega(t)l(t) \left[(1 - \beta) + \beta \frac{m}{1 - \alpha} \right] \frac{\left[(1 - \tau^L) + (1 - \phi) \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right]}{\left[(1 - \tau^L) + \theta_s \phi \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right]}. \quad (19)$$

Then, the employment rate when wages are set at sector level is:

$$l(t) = \frac{\left[(1 - \tau^L) + \theta_s \phi \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right]}{\left[(1 - \beta) + \beta \frac{m}{1 - \alpha} \right] \left[(1 - \tau^L) + (1 - \phi) \frac{(\tau^L + \tau^f)}{(1 + \eta)} \right]} = l_{SL}^*. \quad (20)$$

This means that the wage equation plus the unemployment benefit budget constraint equation gives, for the Cobb-Douglas production function, a constant employment rate. Braüninger (2000), in a similar set up, and Layard, Nickell and Jackman (1991) P. 27 also derive a constant unemployment rate using a wage equation plus a constant exogenous replacement rate $\frac{B}{\omega}$. There are other ways of obtaining constant employment rates: with perfect competition and a Cobb-Douglas production function, Daveri and Tabellini (2000)

³¹Similarly, Romer (2006) p.454 assumes: $R_t = (1 - bu_t)\omega_t$.

assume $B(t) = \sigma \frac{Y(t)}{L(t)}$ and Doménech and García (2008) do the same. Raurich and Sorolla (2011) discuss different ways of obtaining a constant employment rate when the wage is a mark-up over the reservation wage. This constant unemployment rate depends crucially on the use of (10), which, as we said before, means that wage increases produce a reduction in the unemployment benefit³². Papers that assume a constant unemployment benefit are, for example, Pissarides (1998), but in our opinion this assumption in an economy with growth is worse than assuming constant taxes.

Looking at l_{SL}^* , it is easy to see that there is unemployment when m is high enough or $\theta_s \phi$ is low enough, that is, higher monopoly power or a lower proportion of social services or lower perceptions produce unemployment. Note that $\frac{\partial l_s^*}{\partial m} < 0$ and $\frac{\partial l_s^*}{\partial \theta_s} > 0$, that is, the lower the level of market power and the higher the degree of internalization, the higher the employment rate. We also have that $\frac{\partial l_s^*}{\partial \phi} > 0$, that is, an increase in the weight of social services, ϕ , increases the employment rate and $\frac{\partial l_s^*}{\partial \eta} < 0$ when $\theta_s > \frac{1-\phi}{\phi}$, that is, an increase in inefficiency, reduces employment when internalization is high enough.

The effect of the imposition on employment in both cases depends also on the relationship between θ_s and ϕ . More specifically, $\frac{\partial l}{\partial \tau^L} > 0$ and $\frac{\partial l}{\partial \tau^f} > 0$ when $\theta_s > \frac{1-\phi}{\phi}$.

Finally note, that neither changes in capital K nor in total factor productivity A affect employment. Therefore, capital and productivity are neutral with respect to unemployment or, in other words, growth does not affect employment. The reason is that with this wage setting rule an increase in K or A decreases unemployment, but then the unemployment benefit increases and also the wage, completely crowding out the positive effect of K or A on labor demand. Kaas and von Thadden (2003) with a CES production function and Koskela, Stenbacka and Juselius (2009), with a particular production function, obtain an employment rate that depends on capital. There is also empirical evidence that K affects employment on the short run (Karanassou et. al. (2008) and Driver and Muñoz-Bugarin (2009)).

When the wage is set at firm level, the employment rate is equal to

$$l(t) = \frac{(1 - \tau^L)}{\left[(1 - \beta) + \beta \frac{1}{1-\alpha} \right] \left[(1 - \tau^L) + (1 - \phi) \frac{(\tau^L + \tau^f)}{(1+\eta)} \right]} = l_{FL}^* < 1 \quad (21)$$

and there is always unemployment. Note that the employment rate does not depend on m . This may seem strange because from (16), we find it is the wage that does not depend on m . The explanation is that an increase in m does not initially change the wage and, via labor demand, increases unemployment, but, if unemployment increases, the unemployment benefit is reduced, implying, via the wage equation, a decrease in the

³²One may argue that real governments do not reduce the unemployment benefit when unemployment increases, but, as we said, a similar result is obtained using $B(t) = \sigma \frac{Y(t)}{L(t)}$.

wage in such a way that, employment is not affected in the end.

Note finally that, as we argued before, if either m is high enough or $\theta_s\phi$ low enough³³, we will have a higher employment rate at firm level, that is $l_{FL}^* > l_{SL}^*$. This result gives the condition for the first part of the inverse U hypothesis to be true, if either market power is high enough or the degree of internalization of the contribution of labor income to the provision of social services is low enough. Thus the unemployment rate will be higher if wages are set at sector level than if they are set at firm level³⁴.

4 Employment rate at national level.

Following Layard, Nickell and Jackman (1991), P.51, in a symmetric equilibrium $P_j(t) = P(t)$ for all j and with, the aggregate price index $P(t)$, labor demand in sector j becomes³⁵:

$$F_L(K_j(t), L_j(t)) = m(1 + \tau^f) \frac{W_j(t)}{P(t)} = (1 + \tau^f)m\omega_j(t),$$

Moreover, in this symmetric equilibrium, $K_j(t) = K(t)$, $L_j(t) = L(t)$ and then the aggregate labor demand, $\int_0^1 L_j(t) dj$, is also $L_j(t)$ and $\omega_j(t) = \omega(t)$. This means that aggregate labor demand is given by the equation:

$$F_L(K(t), L(t)) = (1 + \tau^f)m\omega(t), \quad (22)$$

This expression implies the aggregate labor demand function $L^d(t) = \tilde{L}^d((1 + \tau^f)m\omega(t), K(t))$ where $\tilde{L}_{(1 + \tau^f)m\omega} < 0$ and $\tilde{L}_K > 0$. More specifically, for the Cobb-Douglas production function aggregate labor demand is:

$$L^d(t) = (1 - \alpha)^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} ((1 + \tau^f)m\omega(t))^{\frac{-1}{\alpha}} K(t),$$

with elasticity with respect to the wage equal to $-\frac{1}{\alpha} < -1$, that also does not depend on market power, m , as is the case when the firm acts competitively.

Now, we assume that, in a centralized wage setting system, the national union maximizes the utility function given by:

³³The corresponding condition that guarantees that $l_{FL}^* > l_{SL}^*$ is $\frac{m-1}{\theta_s\phi} > \left[\frac{(\tau^L + \tau^f)}{(1+\eta)} \right] \frac{1-\alpha+\alpha\beta}{\beta(1-\tau^L)}$.

³⁴A more sophisticated situation is to consider that centralization increases as the proportion of sectors θ that sets the same wage for the entire sector increases. In this situation it is not difficult to prove that the employment rate decreases (increases) monotonically with θ from l_{FL}^* ($\theta = 0$) to l_{SL}^* ($\theta = 1$) when $l_{FL}^* > l_{SL}^*$ ($l_{FL}^* < l_{SL}^*$).

³⁵This assumption implies that product demand and then market power disappears from the program of the firm.

$$V = (1 - \tau^L)\omega(t)L(t) + (N(t) - L(t))B(t) + \theta_n\phi\frac{(\tau^L + \tau^f)\omega(t)L(t)}{(1 + \eta)} \quad (23)$$

where

$$L(t) = \min(N(t), L^d(t)).$$

We assume, as argued previously, that $\theta_n > \theta_s$. At centralized level it also seems reasonable to assume that the national union has perceptions about how the wage will affect the unemployment benefit in a similar way to how it considers it affects social expenditures. The most plausible assumption is that the union internalizes what the government really does, that is: the national union considers that changing the wage, the wage bill and the amount of employment will change and then also the unemployment benefit according to the equation:

$$B(t) = \varphi\frac{(1 - \phi)(\tau^L + \tau^f)\omega(t)L(t)}{(1 + \eta)(N(t) - L(t))} \quad (24)$$

where $0 < \varphi \leq 1$ measures the degree of internalization of the effect of the wage on the unemployment benefit, that is, the union internalizes, in part, exactly what the government does. With a Cobb-Douglas utility function, this means that the union now considers that an increase in the wage bill reduces the unemployment benefit because $\omega(t)L(t)$ decreases and $(N(t) - L(t))$ increases. Substituting (24) in (23), the utility function of the union becomes:

$$V_B = \left(1 - \tau^L + \frac{[\varphi(1 - \phi) + \theta_n\phi](\tau^L + \tau^f)}{(1 + \eta)}\right)\omega(t)L(t) \quad (25)$$

and in this case it is obvious that it chooses the competitive wage because labor demand elasticity with respect to the wage is equal to $-\frac{1}{\alpha}$ and an increase in the wage always reduces the wage bill, $\omega(t)L(t)$. In this case, therefore, there is full employment, that is, $l_{NL}^* = 1$ and $l_{NL}^* > l_S^*$ if m is high enough or $\theta_s\phi$ is low enough.

Alternatively, we may assume that the union internalizes a different effect of the wage. For example, it may consider that by changing the wage, the wage bill and the amount of employment will change and then the government will maintain the unemployment benefit and the tax rate on employers constant, changing the tax rate on workers³⁶. In this case, one can show that it may be unemployment at national level and, as usual, the employment level is higher at the national level if m is high enough or θ_s is low enough. The other alternative assumption is that the union does not internalize the effect of the wage on the unemployment benefit. In this case, one can show that there is always unemployment and,

³⁶Layard, Nickell and Jackman (1991) on P. 130 consider the case where an increase in the wage is completely transferred to an increase in workers' taxes which results in the union settings the competitive wage.

again as usual, the employment level is higher at national level if m is high enough or θ_s is low enough.

Therefore, obtaining unemployment or full employment in the centralized wage setting system depends heavily on what the union assumes is going to happen when the wage it sets increases. If it thinks that an increase in the wage will decrease the unemployment benefit, then we have full employment. Another option is to assume that an increase in the wage will increase workers' taxes resulting, in general, unemployment. Finally, the trade union may assume that neither the unemployment benefit nor taxes on employed workers will change, and so, there is unemployment.

Note also that a higher degree of market power does not affect any of the employment rates obtained when wages are set at national level for the same reason as when they are set at firm level. In all three cases we have a higher employment rate when wages are set at national level than when wages are set at sector level if m is high enough or θ_s is low enough.

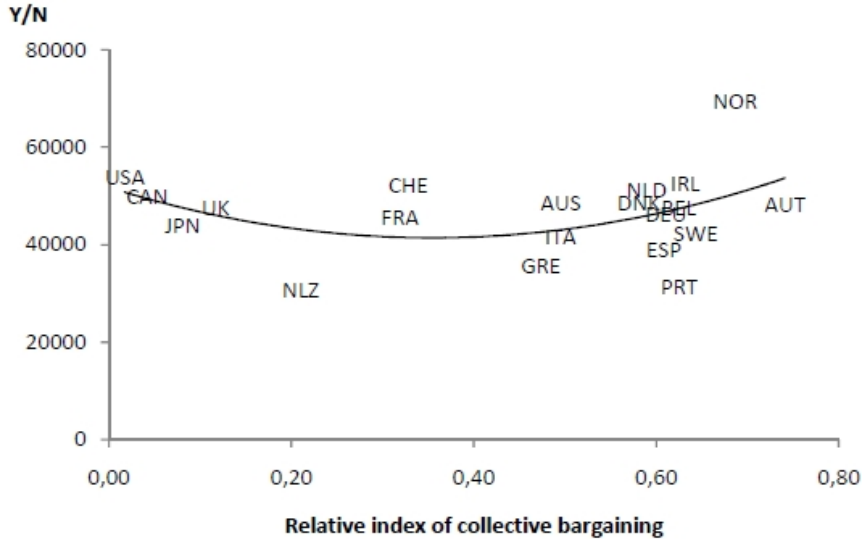
From all this, we should expect an inverse U relationship between unemployment and the degree of centralization of wage setting under two circumstances. The first is when the degree of market power in the product market is high enough. The second is when the degree of internalization of the contribution of labor income to the provision of social services is small enough at sector level. All these results appear in a scenario where market power produces lower labor demand elasticity at sector level. The last argument is similar to Calmfors and Driffil's assumption that the elasticity of labor demand decreases with the degree of centralization, but we have a strong argument for this assumption: the consideration of market power when wages are set at sector level.

Note finally, that this relationship occurs when all the other parameters that appear in the model: ϕ , τ^L , τ^f , ... do not change. Therefore, it is not surprising that if one checks for the inverse U hypothesis without controlling for the other parameters that affect the employment rate, the relationship does not appear.

5 Households and equilibrium

In this section we investigate, theoretically and empirically, the relationship between income per capita and the bargaining system. We provide the empirical evidence through a scatter plot of income per capita against our bargaining index ($R^2 = 0.14$) in Figure 4.

Figure 4. PIB per capita vs Relative index of collective bargaining



From the Figure above we can see that a U-shaped pattern also appears between both variables (Finland is excluded).

The theoretical framework for this relationship can be illustrated briefly through a simple growth model. More specifically, we use the infinite horizon model where there is a representative family with $N(t)$ members growing at the constant rate n , with an inelastic labor supply equal to $N(t)$ that (see Galí (1996) section 2.1) chooses aggregate consumption per capita, $c(t) \equiv \int_0^1 c_j(t) dj$, where $c_j(t) \equiv \frac{C_j(t)}{N(t)}$, in order to maximize:

$$\int_0^{\infty} e^{-(\rho-n)t} \left[\frac{c(t)^{1-\theta} - 1}{1-\theta} \right] \quad (26)$$

subject to:

$$\dot{a}(t) = (1 - \tau)w(t)l(t) + s(t)(1 - l(t)) - e(t) + \int_0^1 (d_j(t) + \dot{q}_j(t))s_j(t) dj - na(t). \quad (27)$$

where $s_j(t)$ is the number of shares per capita in firm j held at time t by the family. A share in firm j trades at price $q_j(t)$ and generates a dividend flow $d_j(t)$ at time t . Financial wealth of the family is thus given by $A(t) = \int_0^1 q_j(t)N(t)s_j(t) dj$ and then $a(t) \equiv \frac{\dot{A}(t)}{N(t)}$.

Note that the revenues of this family accrue from total labor income because we assume the family is so large that it considers all workers, employed and unemployed. Daveri and Maffezzoli (2000), Eriksson (1997) and Raurich, Sala and Sorolla (2006) also use the large family assumption. If we have heterogeneous agents, instead of a large family, the solution does not change as long as we assume complete competitive insurance markets

for unemployment or that the union pursues a redistributive goal, acting as a substitute for insurance markets (Maffezzoli (2001) and Benassy (1997)).

In market equilibrium we obtain (see Galí (1996)):

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} \left(\frac{1}{m} F_K(k(t), l(t)) - (\rho + \delta) \right), \quad (28)$$

$$\dot{k}(t) = f(k(t), l(t)) - c(t) - (n + \delta)k(t). \quad (29)$$

Where $k(t)$ is capital per capita and $F(k, l)$ is the production function per capita (see appendix). Under the assumptions established in the section above, we obtain a constant employment rate $l_t = l^*$. Taking into account this result, the above expressions may be written as follows:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} \left(\frac{1}{m} F_K(k(t), l^*) - (\rho + \delta) \right), \quad (30)$$

$$\dot{k}(t) = F(k(t), l^*) - c_t - (n + \delta)k(t). \quad (31)$$

As $l^* < 1$, it is clear from (31) that the rate of growth of capital per capita is lower for a given level of c and k in a model with unemployment, that is, employment affects growth in the short run. It is also clear, from (30) and (31), that consumption and capital per capita converge to a steady state with a zero rate of growth of capital per capita and consumption per capita. That means that there is no relationship between growth and unemployment in the long run: the constant rate of unemployment is given by l^* and the rate of growth in income per capita is zero, or x , if we introduce exogenous technological progress growing at the constant rate x . It is also easy to see, by drawing a phase diagram, that a decrease in l^* decreases the long run level of consumption, capital and income per capita, that is, there is a positive relationship between income, capital and consumption per capita and the employment rate in the long run. In other words, all other parameters equal, economies with a higher employment rate will record higher income, capital and consumption per capita in the long run.

On the other hand, the level of capital per worker and income per worker in the long run does not depend on the employment rate because we can rewrite (30) as

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} \left(\frac{1}{m} f(\hat{k}(t)) - (\rho + \delta) \right), \quad (32)$$

where \hat{k} is capital per unit of labor and $f(\hat{k})$ the production function in intensive form, and, hence, in the long run \hat{k} is given by:

$$0 = \frac{1}{\theta} \left(\frac{1}{m} f(\hat{k}) - (\rho + \delta) \right). \quad (33)$$

Then, all other parameters remaining equal, we also have a U relationship between long run income, capital and consumption per capita and the degree of centralization of wage setting when the degree of market power, in the product market, is high enough and there is no relationship between capital and income per worker and the degree of centralization of wage setting.

Finally, as we saw, an increase in market power increases unemployment when the wage is set at sector level, but has no effect when it is set at firm and national level. However, in all three systems it produces a decrease in long run income, capital and consumption per capita and in capital and income per worker.

6 Main Results

This paper analyzes the effects of collective bargaining on the employment rate in OECD countries focusing our attention, principally, on the intermediate level of bargaining. This sample of countries has a higher unemployment rate than other countries characterized by highly centralized and decentralized wage bargaining structures. It follows from this result that the relationship between the unemployment rate and wage bargaining systems can be hump-shaped (Calmfors and Driffill 1988). In this paper, we open the framework elaborated by Calmfors and Driffill in three directions: there is monopolistic competition in the product market, we include some key variables that determine labor market performance and, finally, we assume that trade unions at sectorial level are able to partially internalize the effect of the wage on social services provided by the government. It is worthwhile stressing that there are two types of variables. The first group includes variables under the direct control of policy makers: such as the size and structure of social expenditures and labor taxes. The second group includes variables describing the socioeconomic structure, such as public sector inefficiencies, the degree of internalization of social services and rigidities in goods markets.

The results obtained show that a high degree of market power normally produces the inverted-U form for the unemployment rate. Moreover, we also illustrate that this inverse U form can be reversed when the ability of trade unions to internalize the provision of social services is great enough at sector level. The market power effect produces higher labor demand elasticity at sector level than at other levels of wage bargaining. It should be noted that this relationship occurs when all the other parameters that appear in the model under the direct control of government or describing the socioeconomic structure are the same. One possible empirical implication of these results is that, if one checks for the inverted-U hypothesis without controlling for all the parameters that affect the employment rate, the relationship does not appear. Thus, the paper offers an explanation for the weak relationship between the wage bargaining system and the employment rate

that has been found in a large number of studies. As we described above in section 2, the empirical evidence reveals strong heterogeneity for the parameters that determine the rate of unemployment in the theoretical model of this paper for the sample of countries presented. This results in the unemployment rate obtained being very heterogeneous.

Our analysis also finds a U-shaped relationship between long run income, capital and consumption per capita and the degree of collective bargaining. Finally, an increase in market power increases unemployment, when the wage is set at sector level, but has no effect when it is set at firm and national level. However, in all three systems it produces a decrease in long run income, capital and consumption per capita and in capital and income per worker.

These results have important implications for policy makers who plan to implement labor market reforms in order to reduce unemployment in countries characterized by collective bargaining at sector level. First, based on the empirical evidence of section 2 and the model presented, our (modest) advice for improving the employment rate of the EUCON countries in general is to reduce government inefficiencies and market power. However, for more refined advice, one must analyze the specific characteristics for every country of the variables that determine the unemployment rate in the model, such as social expenditure structure, government efficiency, etc.

The second policy implication is that, if sector unions internalize the effect of the wage on the social services offered by government to the labor force, one obtains a higher employment rate than when the collective bargaining is at firm level. It should be noted that it seems easier to coordinate bargaining between trade unions during an economic crisis than during an expansion (see, Pohjola (1992)). On the other hand, it is worthwhile stressing that the share of the active labor force with respect to the inactive population is a limiting factor for internalizing social services.

It is important to emphasize that the empirical evidence presented only shows associations, not causal effects. Further data collection is required in order to perform conventional multivariate regressions, this issue being our future line of research.

7 References

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7.1 Appendix 1

We introduce the monopolistic competition set up in a growth model (Galí (1996)) having $j \in [0, 1]$ sectors with one firm per sector that produces product $Y_j(t)$. Production functions at sector and firm level are characterized by function

$$Y_j(t) = F(K_j(t), L_j(t)), \quad (34)$$

with constant returns to scale with respect to K and L , $F_K > 0$, $F_L > 0$, $F_{KK} < 0$, $F_{LL} < 0$ and the Inada conditions: $\lim_{K \rightarrow 0} F_K = \infty$, $\lim_{K \rightarrow \infty} F_K = 0$, $\lim_{L \rightarrow 0} F_L = \infty$, $\lim_{L \rightarrow \infty} F_L = 0$. The production function in terms of output per worker or unit of labor, $\frac{Y_j(t)}{L_j(t)} \equiv \hat{y}_j(t)$, and capital per worker or the capital labor ratio, $\frac{K_j(t)}{L_j(t)} \equiv \hat{k}_j(t)$, that is, in intensive form, is:

$$\hat{y}_j(t) = f(\hat{k}_j(t)), \quad (35)$$

where $f' > 0$ and $f'' < 0$.

Finally we also rewrite the production function in per capita terms $\frac{Y_j(t)}{N(t)} \equiv y_j(t)$, $\frac{K_j(t)}{N(t)} \equiv k_j(t)$, $\frac{L_j(t)}{N(t)} \equiv l_j(t)$, where $N(t)$ is population a time t . In this case, we have:

$$y_j(t) \equiv \frac{Y_j(t)}{N(t)} = F\left(\frac{K_j(t)}{N(t)}, \frac{L_j(t)}{N(t)}\right) \equiv F(k_j(t), l_j(t)), \quad (36)$$

with $F_k = F_K$ and $F_l = F_L$.

The stock of capital for firm j evolves according to the equation:

$$\dot{K}_j(t) = I_j(t) - \delta K_j(t), \quad (37)$$

where $I_j(t)$ is a composite of the flow of purchases by firm j of the good produced by firm h , $I_{j,h}(t)$ ³⁷.

The firm in sector j maximizes the wealth of its shareholders subject to the demand function. The demand function in sector j is the sum of the demands of consumers and firms (Galí (1996) equation (2.7)):

$$Y_j(t) = \left(\frac{P_j(t)}{P(t)}\right)^{-\sigma} \left(\frac{E(t)}{P(t)}\right) + \left(\frac{P_j(t)}{\Pi(t)}\right)^{-\eta} \left(\frac{Z(t)}{\Pi(t)}\right), \quad (38)$$

where $P(t)$ is the aggregate price index $P(t) \equiv \left(\int_0^1 P_j(t)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}$, $E(t)$ is the flow of ex-

³⁷As defined below $I_j(t) \equiv \frac{Z_j(t)}{\Pi(t)}$.

penditure in consumption goods $E(t) \equiv \int_0^1 P_j(t)C_j(t)dj$, $Z(t)$ is the flow of expenditure in

investment goods, $Z(t) \equiv \int_0^1 Z_j(t)dj$, that is the sum for all sectors of expenditures in $I_j(t)$,

$Z_j(t) \equiv \int_0^1 P_h(t)I_{j,h}(t)dh$, $\Pi(t)$ is the aggregate price index $\Pi(t) \equiv \left(\int_0^1 P_j(t)^{1-\eta}dj \right)^{\frac{1}{1-\eta}}$ and

σ and η are the constant price elasticity of the consumer and firms demand functions. Finally, $\eta > 1$ denotes the (exogenously given) elasticity of substitution between different goods from the viewpoint of the firm which uses them as inputs (Galí (1996), P.255). Assuming that the price elasticity of the demands of consumers and firms is equal to ξ ³⁸, equation (38) becomes.

$$\begin{aligned} Y_j(t) &= \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} \left(\frac{E(t)}{P(t)} \right) + \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} \left(\frac{Z(t)}{P(t)} \right) = \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} \left(\frac{E(t) + Z(t)}{P(t)} \right) \\ &= \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} \left(\frac{Y(t)}{P(t)} \right) = \left(\frac{P_j(t)}{P(t)} \right)^{-\xi} (\bar{Y}(t)) \end{aligned} \quad (39)$$

where $\bar{Y}(t) \equiv E(t) + Z(t)$ and $\bar{Y}(t) \equiv \frac{Y(t)}{P(t)}$ is total real expenditures on consumption and investment. The aggregate price index is now $P(t) \equiv \left(\int_0^1 P_j(t)^{1-\xi}dj \right)^{\frac{1}{1-\xi}}$.

Defining $m \equiv \frac{1}{(1-\frac{1}{\xi})} > 1$, as the monopoly degree or the markup, from the solution to the program of the firm, we obtain the following first order condition for firm j (see again Galí (1996), equation 2.11) with the payroll taxes properly added:

$$F_L(K_j(t), L_j(t)) = (1 + \tau^f)m \frac{W_j(t)}{P_j(t)}, \quad (40)$$

and then

$$\begin{aligned} F_L(K_j(t), L_j(t)) &= (1 + \tau^f)m \frac{\frac{W_j(t)}{P(t)}}{\frac{P_j(t)}{P(t)}} = (1 + \tau^f)m\omega_j(t) \frac{Y_j(t)^{\frac{1}{\xi}}}{\bar{Y}(t)^{\frac{1}{\xi}}} \\ &= (1 + \tau^f)m\omega_j(t) \frac{F(K_j(t), L_j(t))^{\frac{1}{\xi}}}{\bar{Y}(t)^{\frac{1}{\xi}}} \end{aligned} \quad (41)$$

³⁸The complication of the monopolistic competition set up in a growth model arises from the fact that both consumers and firms demand product i due to the demand of capital of each firm. On principle the price elasticity of both types of demand may be different, this is the point of Galí's paper, and this opens the door for multiplicity of equilibria. The assumption that ξ is constant is the $\sigma = \mu$ case in Galí's paper.

where $\omega_j(t) \equiv \frac{W_j(t)}{P(t)}$ is the real wage in sector j . We can rewrite equation (41) as:

$$F_L(K_j(t), L_j(t))F(K_j(t), L_j(t))^{-\frac{1}{\xi}} = \frac{(1 + \tau^f)m\omega_j(t)}{\bar{\mathbf{Y}}(t)^{\frac{1}{\xi}}} \quad (42)$$

and from the last equation³⁹ we get the "labor demand" function for sector j :

$$L_j^d(t) = \tilde{L}_j((1 + \tau^f)m\omega_j(t), K_j(t), \bar{\mathbf{Y}}(t)), \quad (43)$$

where $\tilde{L}_{j,m\omega_j} < 0$ and $\tilde{L}_{j,\bar{\mathbf{Y}}} > 0$.

Because $F_L(K_j(t), L_j(t)) = f(\hat{k}_j(t)) - \hat{k}_j(t)f'(\hat{k}_j(t))$, equation (41) can also be rewritten in terms of the production function in intensive form as:

$$f(\hat{k}_j(t)) - \hat{k}_j(t)f'(\hat{k}_j(t)) = m\frac{W_j(t)}{P_j(t)}, \quad (44)$$

which gives the capital labor ratio function:

$$\hat{k}_j(t) = \tilde{k} \left(m\frac{W_j(t)}{P_j(t)} \right), \quad (45)$$

with $\tilde{k}' > 0$.

³⁹We can also rewrite this condition in terms of the capital labor ratio as: $\left[f(\hat{k}_j) - \hat{k}_j f'(\hat{k}_j) \right] f(\hat{k}_j)^{-\frac{1}{\xi}} = \frac{m\omega_j}{\left(\frac{\bar{\mathbf{Y}}}{L}\right)^{\frac{1}{\xi}}}$.