Coplanar Waveguides Loaded with Symmetric and Asymmetric Multi-Section Stepped Impedance Resonators (SIRs): Modeling and Potential Applications

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Abstract

This paper is focused on the analysis and modeling of coplanar waveguide (CPW) transmission lines loaded with multi-section stepped impedance resonators (MS-SIRs), transversely etched on the back substrate side. The considered structure consists of a CPW loaded with a 5-section SIR (5S-SIR) with wide (capacitive) central and external sections cascaded with narrow (inductive) sections. The general case of a 5S-SIR with arbitrary lengths and widths of the different sections is considered. The structure is described by a pair of inductively coupled grounded series resonators coupled to the line through the capacitance of the central 5S-SIR section. If the structure is symmetric, the transmission coefficient exhibits a single transmission zero. Hence, these structures can be used as notch filters exhibiting wide bandwidths, provided the inductance of the 5S-SIR can be made small, and the capacitance can be enhanced by virtue of the broadside coupling. However, if symmetry is broken, two notches separated a distance that depends on the level of asymmetry and inductive coupling appear. Therefore, these structures are also useful for the implementation of differential sensors and comparators. The proposed model is validated through parameter extraction and experiment, and a proof-of-concept of a comparator is reported.

KEYWORDS: Microwave sensors, circuit modeling, stepped impedance resonator (SIR), CPW technology.

1. Introduction

Stepped impedance resonators (SIRs) are electrically small resonators useful for the implementation of planar filters [1-3]. Coupled to planar transmission lines, such resonators have found applications in the design of elliptic lowpass filters [4], or frequency-domain (spectral signature) barcodes [5], due to the transmission zeros that appear at their fundamental resonance frequency. Although SIRs are usually implemented as metallic resonators, the complementary counterparts (slotted SIRs, also called dumbbell defected ground structures) are also of interest for microwave circuit design [6].

In this paper, we study in detail the electromagnetic behavior of CPW transmission lines loaded with multi-section stepped impedance resonators (MS-SIRs), transversely etched on the back substrate side, and we propose a lumped element equivalent circuit model of the structure. Such model accounts not only for the different semi-lumped planar (inductive and capacitive) elements of the MS-SIR, but also for the main coupling mechanisms between the line and the resonator and between the resonator elements, essential for an accurate model. The specific MS-SIR considered is a 5-section resonator (5S-SIR) with wide (capacitive) central and external sections cascaded with narrow (inductive) sections. The central section is placed below the central strip of the CPW transmission line, whereas the external sections are allocated below the ground planes. This provides a large resonator capacitance that makes the particle electrically small. The inductive sections are etched beneath the slot region. All the sections must be electrically short for a correct description of the structure by means of a lumped element circuit.

In the modeling, the general case of a 5S-SIR with arbitrary lengths and widths of the different sections is considered. Although for the application of these structures as wideband notch filters, or as part of elliptic filters, there is no reason to choose asymmetric 5S-SIRs (note that by using symmetric resonators the slot mode is prevented and air bridges or backside strips and vias are avoided), the fact that disruption of symmetry in the 5S-SIR gives rise to two transmission zero frequencies (frequency splitting [7]), opens the path to the implementation of differential sensors or comparators (e.g., for analysis of biological cells and tissues, defects or abnormalities in materials, etc. [8]). These sensors based on frequency splitting are robust against variable environmental conditions. Hence, the modeling of the 5S-SIR-loaded CPW with asymmetric resonators is of interest. To the best authors' knowledge, it is the first time that a lumped element circuit model describing a CPW transmission line loaded with an arbitrary 5S-SIR, including inductive coupling between resonator elements (the two 5S-SIR inductances), is proposed. Nevertheless, a microstrip structure loaded with pairs of coupled SIRs was recently investigated as a potential candidate for differential sensing [8], and a prototype sensor (implemented in CPW technology) operative at Q-band was presented. As compared to the structures of that work, the 5S-SIR-loaded CPW structure presented in this paper exhibits more design flexibility as long as it is described by a circuit model where the resonant element (5S-SIR) is electrically coupled to the line (by contrast, in the structure reported in [8] the pair of SIRs are in direct contact with the line).

This work, dedicated to the analysis and modeling of CPWs loaded with MS-SIRs, represents a clear progress on the topic of transmission lines loaded with resonant elements, and points out the potential of these structures as notch filters (symmetric case) and microwave sensors and comparators based on frequency splitting (asymmetric case).

2. Topology and equivalent circuit model

In this section, the 5S-SIR-loaded CPW topology and its lumped element equivalent circuit model are presented, and the resonance frequencies (transmission zeros) for the general asymmetric case are calculated analytically. Model validation through parameter extraction and comparison to electromagnetically simulated and measured responses is left for next sections.

Fig. 1(a) shows the typical topology of a CPW loaded with a 5S-SIR. The equivalent circuit model is depicted in Fig. 1(b), where L and C are the inductance and capacitance of the CPW line section, and L_i and C_i (with $i = 1,2$) describe the inductances and capacitances of the narrow (inductive) and external sections, respectively, of the 5S-SIR. Two coupling mechanisms are considered: (i) electric coupling between the 5S-SIR and the line, through C_c (the broadside capacitance between the central strip of the CPW and the central section of the 5S-SIR), and (ii) magnetic coupling between the two inductances of the resonator, through M (such coupling is negative because the currents in the inductances flow in opposite directions). Losses are not considered in the model.

Although for sensing purposes the structure should be symmetric and symmetry disruption would modify the element values of the equivalent circuit, we directly consider the more general case of a circuit model corresponding to an arbitrary asymmetric structure. Note that the considered structure is electrically short and hence it is reasonable to assume, to a first order approximation, that the slot mode is not generated (the ports in the electromagnetic simulation and the connectors in the measurement act as air bridges effectively connecting the two ground plane regions).

From the analysis of the circuit model of Fig. 1(b), it follows that the transmission zeros, given by those frequencies that null the reactance of the shunt branch, are:

$$\omega_{\pm}^2 = \frac{A \pm \sqrt{B}}{D} \tag{1}$$

with

$$A = C_1C_2(L_1 + L_2 - 2M) + C_c(L_1C_1 + L_2C_2)$$

$$B = C_1^2C_2^2(L_1 + L_2 - 2M)^2 + C_c^2(L_1C_1 - L_2C_2)^2 + 2C_cC_1C_2 \times$$

$$\left\{L_1C_1(L_1 - L_2 - 2M) + L_2C_2(L_2 - L_1 - 2M) + 2M^2(C_1 + C_2 + C_c)\right\}$$

$$D = 2C_cC_1C_2(L_1L_2 - M^2)$$

If the structure is symmetric (i.e., $L_1 = L_2 \equiv L_r$ and $C_1 = C_2 \equiv C_r$), the mathematical solutions of (1) are:

$$\omega_+ = \frac{1}{\sqrt{(L_r + M)\cdot\frac{C_cC_r}{C_c + 2C_r}}} \tag{2a}$$

$$\omega_- = \frac{1}{\sqrt{(L_r - M)\cdot C_r}} \tag{2b}$$

However, ω_- is not actually a physical solution since it nulls the denominator of the reactance. Thus, the mutual coupling between the two inductors of two SIRs has the effect of increasing the notch frequency (symmetric case).

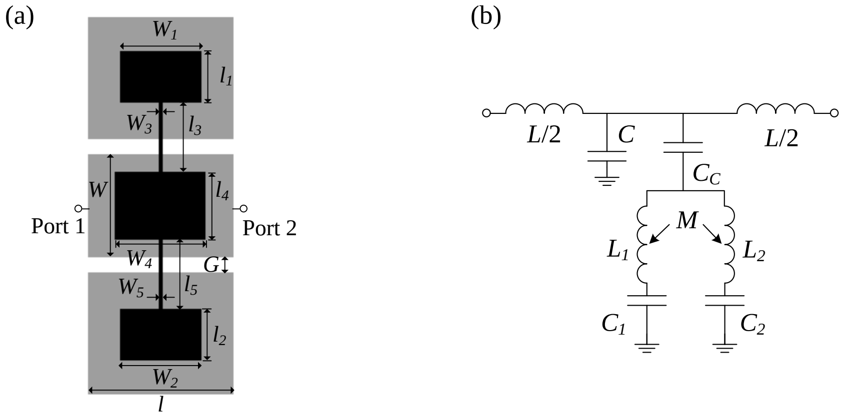

Fig. 1. (a) CPW loaded with a 5S-SIRs and (b) circuit model. Relevant dimensions are indicated.

3. Model validation through parameter extraction

Let us now demonstrate the validity of the proposed circuit model by providing a method to extract the parameters of that model and by comparison of the lossless electromagnetic simulation with the circuit simulation of the model with the extracted parameters. As a case example we will consider a set of three 5S-SIR-loaded CPWs, one symmetric and the other two asymmetric, where the two asymmetric structures are derived from the symmetric one by increasing or decreasing the area of one of the external patch capacitors, while the other external patch capacitors for these two asymmetric structures keep the same dimensions as in the symmetric one. The corresponding layouts are depicted in Fig. 2 (dimensions and substrate parameters are indicated in the caption).

Due to the large number of elements in the circuit model, parameter extraction is a multi-step process. First of all, one of the capacitances and inductances of the 5S-SIR (one half) is removed, and the resulting central patch is short circuited to ground by means of vias, as indicated in Fig. 3(a). The corresponding model, indicated in Fig. 3(b), has a reduced number of parameters (i.e., four) that can be inferred by means of the following four conditions. The first one is the transmission zero frequency, given by:

$$\omega_z = 2\pi f_z = \frac{1}{\sqrt{L_r C_r}} \tag{3}$$

A pair of additional conditions are derived from the frequency that opens the shunt branch, i.e.,

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{L_r \cdot \frac{CC_r}{C+C}}} \tag{4}$$

which can be inferred from the intercept of S_{11} with the unit resistance circle in Smith chart. From this intercept, the reactance of the series branch, χ_s, can be directly obtained, and hence L, i.e.,

$$\chi_s = L\omega_0 \tag{5}$$

Finally, the fourth condition is given by the frequency where the phase of S_{21} is -90^o. At this frequency, $\omega_{-\pi/2}$, the series and shunt reactances verify [9]:

$$\chi_s(\omega_{-\pi/2}) = -\chi_p(\omega_{-\pi/2}) \tag{6}$$

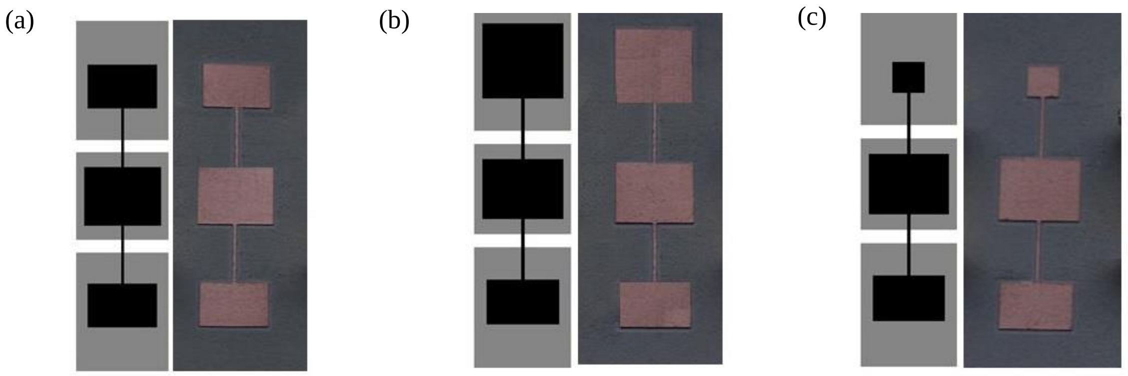

Fig. 2. Considered 5S-SIR-loaded CPWs: (a) symmetric; (b) asymmetric with one external patch capacitor larger; (c) asymmetric with one external patch capacitor smaller. Dimensions are (in reference to Fig. 1): for CPW lines: l = 6 mm, W = 6 mm and G= 0.96 mm, corresponding to 50 Ω. For 5S-SIRs: (a) $W_1 = W_2$ = 4.5 mm, $l_1 = l_2$ = 3 mm, $W_3 = W_5$ = 0.2 mm, $l_3 = l_5$ = 4 mm, W_4 = 5 mm, l_4 = 4 mm. For (b) $W_1 = l_1$ = 5 mm and (c) $W_1 = l_1$ = 2 mm, with the other dimensions the same as in (a). The considered substrate (*Rogers RO3010*) has thickness of h = 0.635 mm and dielectric constant of ε_r = 11.2.

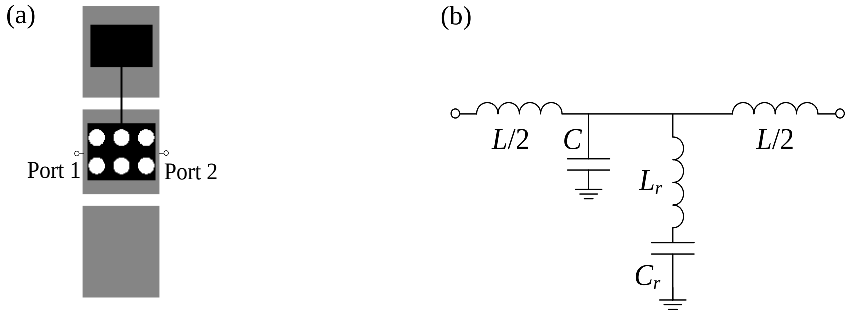

Fig. 3. Layout and circuit model of the symmetric structure of Fig. 2(a) after removing one of the halves of the 5S-SIR and adding vias to the resulting central patch (a), and equivalent circuit model (b).

Using (3)-(6), the parameters of the model of Fig. 3(b) are extracted, namely, L = 3.49 nH, C = 1.21 pF, L_r = 4.20 nH, C_r = 2.65 pF. Once the inductance and capacitance of the halved 5S-SIR and the line parameters are known, the next step is to determine the mutual coupling between the two inductors of the SIRs, M, and the capacitance of the central patch in Fig. 2(a). For the extraction of M, we add vias in the central patch capacitor and short it to ground (see Fig. 4). Then, from this model we can determine M by curve fitting, i.e., $M = -1.08$ nH. Finally, we remove the vias in the central patch capacitor, the whole symmetric structure is simulated, and C_c, the remaining parameter, is determined by curve fitting, i.e., C_c = 3.62 pF. The comparison of the electromagnetic simulation (using *Keysight Technologies*) with the circuit simulation of the symmetric structure is shown in Fig. 5 (the measurement data is included as well), where good agreement can be appreciated, pointing out the validity of the proposed model.

To determine the modified patch capacitances of the two considered asymmetric cases, the procedure is as simple as removing the opposite half of the 5S-SIR. From the resulting transmission zero frequency, the capacitance and inductance of the halved 5S-SIR can be obtained. Then, by simulating the whole structure with and without vias respectively, the mutual inductance M and the middle patch capacitance C_c can be inferred.

By means of this procedure, the small external patch capacitor and narrow inductive section of 5S-SIR have been found to be 0.97 pF and 4.30, and the big external patch capacitor and narrow inductive section of 5S-SIR have been found to be 4.53 pF and 4.26 nH. The mutual inductances for these two asymmetric cases, i.e., in Fig. 2 (b) and Fig. 2(c), have been found to be –1.16 nH and –1.10 nH respectively, i.e., similar values, and also similar to the value corresponding to the symmetric structure. This indicates that *M* is scarcely dependent on the dimensions of the patch capacitances of the 5S-SIR, as expected. The agreement among the electromagnetic simulation, circuit simulation and measurement for the two asymmetric cases (Figs. 6 and 7) is reasonable.

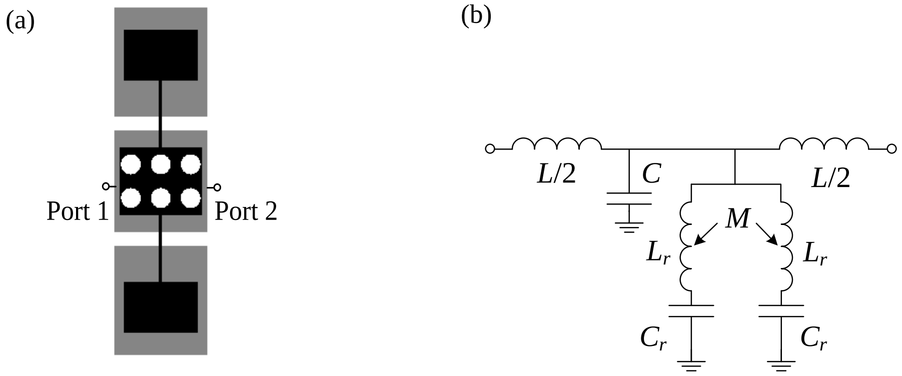

Fig. 4. Layout and circuit model of the symmetric structure of Fig. 2(a) after adding vias to the central patch of the 5S-SIR (a), and equivalent circuit model (b).

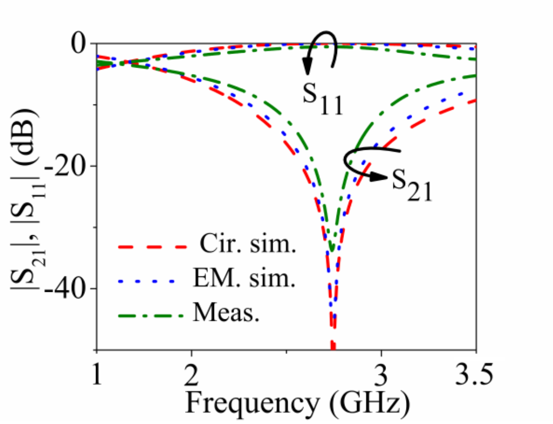

Fig. 5. Electromagnetic simulation, circuit simulation and measurement response for the symmetric structure of Fig.2(a).

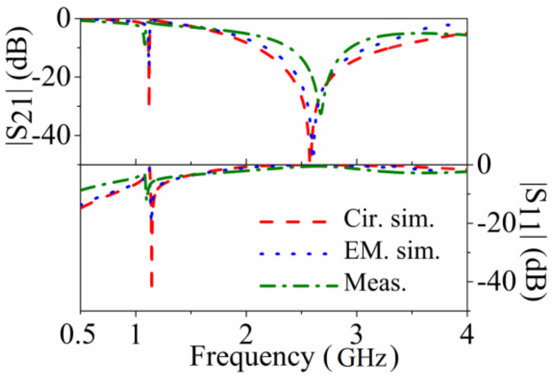

Fig. 6. Electromagnetic simulation, circuit simulation and measurement response for the asymmetric structure of Fig.2(b).

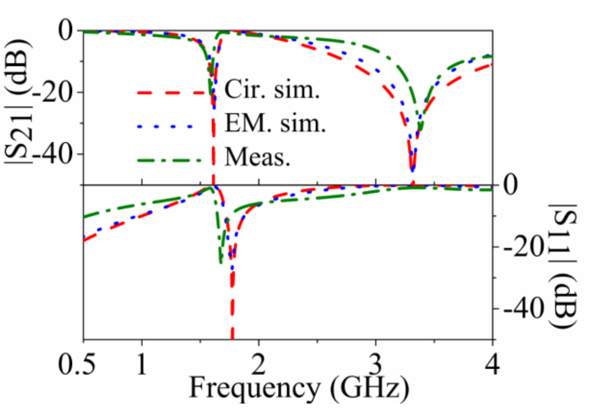

Fig. 7. Electromagnetic simulation, circuit simulation and measurement response for the asymmetric structure of Fig.2(c).

4. Experimental validation

The structures of Fig. 2 have been fabricated by means of a milling machine *LPKF HF100*, and have been measured by means of the *Keysight N5221A* vector network analyzer. The measured responses are depicted in Figs. 5, 6 and 7, where it can be appreciated that the asymmetry splits and modifies the position of the main transmission zero, shifting it to the left and right, respectively, for the big and small patch capacitances, in agreement with the theory. There is some shift between the simulated responses and the measured responses, which is attributed to fabrication related tolerances. Nevertheless, it is experimentally verified that the asymmetry produces two notches, and the structure can be used for sensing purposes, as demonstrated in the next section.

5. Demonstration of the sensing principle

A proof-of-concept of the sensing principle is reported in this section. To this end, we have loaded the symmetric structure of Fig. 2(a) with a dielectric load (consisting of a small piece of *Rogers RO3010* substrate with the copper removed from both substrate sides) placed on top of one of the patch capacitances. The measured response, shown in Fig. 8, exhibits two notches, indicative of the asymmetric loading. Then, we have repeated the experiment by using the same piece of substrate but keeping the metal layers (metallic loading). The measured response is also included in Fig. 8, where it can be seen that the depth of the first notch is superior (as compared to dielectric loading), since the structure is more sensitive to the effects of a metallic layer placed on top of one of the patch capacitances.

6. Conclusion

In conclusion, the modeling of CPW transmission lines loaded with multi-section (specifically 5-section) SIRs of arbitrary dimensions (including thus asymmetric structures) has been carried out for the first time. The model takes into account the negative magnetic coupling between the two inductive elements of the 5S-SIR. A parameter extraction procedure, able to provide the element parameters of the proposed circuit model, considering both symmetric and asymmetric structures, has been reported and explained in detail. The validity of the model has been verified by comparing the electromagnetic responses of the considered structures with circuit simulations. Furthermore, we have validated the model through experiment. Finally, the sensing principle has been demonstrated by asymmetrically loading a symmetric 5S-SIR-loaded CPW with dielectric and metallic loads.

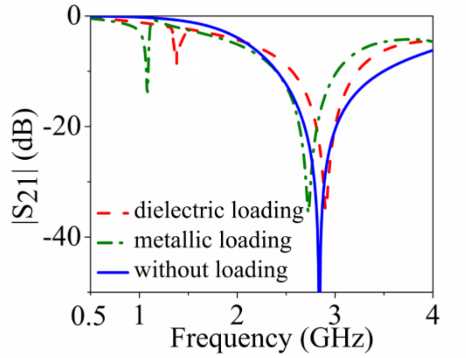

Fig. 8. Measured responses of the structure of Fig. 2(a) with asymmetric dielectric loading and metallic loading.

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