

Complete snapshot Stokes polarimeter based on a single biaxial crystal

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We propose a new snapshot complete Stokes polarimeter based on the conical refraction (CR) phenomenon. This is the first CR based polarimeter able to provide complete polarimetric information by only using a single biaxial crystal and a CCD camera. Compared with existing CR polarimeters, the proposed optical scheme is simpler and reduces costs. In addition, all the benefits of CR based polarimeters (snapshot, large data redundancy, etc.) are also present in the proposed architecture. The polarimeter is experimentally implemented and analyzed in terms of accuracy and sensibility. © 2015 Optical Society of America

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Polarimetric information is used in a large number of applications, as for instance, in astronomy [1], materials characterization [2], bio-medical applications [3], etc. To cover this need, different polarimeter architectures have been proposed in literature [4]. In the case of real-time applications, snap-shot polarimeters [5] are used to operate at high velocity rates, they being commonly based on front-division or amplitude-division architectures. A new type of amplitude-division polarimeter, based on the conical refraction (CR) phenomenon, was proposed in Ref. [6]. These polarimeters take benefit of the CR phenomenon occurring in bi-axial crystals (BC) [7, 8]. In fact, when a focused Gaussian beam passes through one of the optical axes of a BC, at the focal plane we observe two concentric bright rings separated by a dark (Poggendorff) ring under conditions of $R_0 \gg \omega_0$, R_0 being the radius of the ring (it depending on the crystal length and on the principal refractive indices n_1 , n_2 and n_3) and where ω_0 is the waist radius of the focused input beam. Moreover, the polarization distribution observed at the ring plane is always linearly polarized, in a way that each two light spots being diametrically placed on the ring present orthogonal linear polarizations. In this situation, the effect of the BC on the input light at the focal plane can be described by the input polarization projection on a basis of linear analyzers [6].

Polarimeters based on the CR phenomenon are quite repeatable instruments because they provide very large data redundancy, this redundancy data allowing us to decrease experimental noise from raw intensity measurements. Moreover, unlike most polarimeters, which require time-sequential measures to decrease statistical noise, CR based polarimeters do that instantaneously because redundancy data is spatially obtained from intensity images on a CCD camera [6, 9]. This feature was applied in Ref. [10], where the suitability of CR based polarimeters under low-intensity conditions was highlighted.

By contrast, biaxial crystals (BC) for themselves only provide linear information of input beams, so the polarimetric information is incomplete [10]. If elliptically polarized light is of interest, a second arm combining a BC with an optical retarder (OR) is required, this allowing us to include elliptical polarization analyzers at the measuring configuration. That is the reason why architectures of complete CR polarimeters present two optical arms (i.e. two BCs as well as two CCD cameras), which raises the polarimeter price and imposes a proper synchronization between the two CCD cameras in the system.

We propose a new complete CR based polarimeter that only requires one biaxial crystal and a single CCD camera. It is based on a simple set-up (CCDs synchronization is avoided) that reduces costs when compared with existing prototypes [9, 10]. The optical scheme is sketched in Fig. 1(a). A monochromatic light source is focused by the convergent lens (L1). The focused beam enters into an Optical Module (OM, Fig. 1(b)) where it is divided in two sub-beams by means of a non-polarizing Beam-Splitter (BS1). At this point, the reflected beam (RB) is totally reflected on a prism (P1) and then partially transmitted on a second beam-splitter (BS2), while the transmitted beam (TB) is totally reflected on a second prism (P2) and partially reflected on the BS2. Therefore, two convergent and parallel beams with the same intensity are obtained at the exit of the OM. In addition, the beams separation can be controlled by performing linear displacements Δd of the prism P2. Note that between the prism P1 and the BS2 elements, we have also set a linear retarder (LR), so the input polarization is modified by this element. Once the two convergent light beams exit from the OM, they pass through one of the optical axis of a bi-axial crystal and thus, two rings-like intensity distributions are obtained at the focal plane of the lens L1. The intensity distribution of these two light rings corresponds to the projection of the input polarization on: (i) a system of linear analyzers with different orientations due to the biaxial crystal (BC) effect (configuration related to the TB in Fig. 1(b));

and (ii) a system of elliptical analyzers with different ellipticity, they being set due to the combination of the linear retarder LR and the BC (configuration related to the RB in Fig. 1(b)).

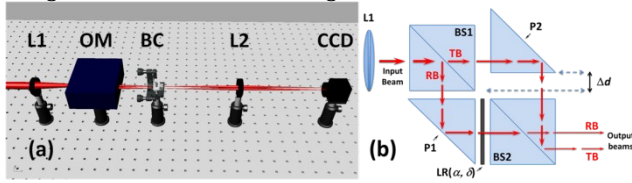


Fig. 1. (a) Complete Stokes polarimeter scheme based on a single bi-axial crystal; and (b) detail of the Optical Module (OM) for the generation of two parallel and convergent beams.

Following, the polarization analyzers (PAs) provided by our polarimeter are determined. First, we calculate the PAs related to the Transmitted Beam (TB) (see Fig. 1(b)). This TB is not affected by the linear retarder and its corresponding PAs are those directly provided by the CR phenomenon –i.e. a set of linear analyzers. By using the Mueller-Stokes (M-S) formalism, those analyzers are given by,

$$a_{TB}(\varphi) = \frac{1}{2} \begin{pmatrix} 1 & \cos \varphi & \sin \varphi & 0 \end{pmatrix}^T, \quad (1)$$

where φ is the angular position of the different linear analyzers at the ring and the subscript T denotes the vector transpose. The intensity distribution measured by the CCD camera for the TB is the projection of the incident polarization over the different analyzers a_{TB} in Eq. (1),

$$I_{TB}(\varphi) = \frac{1}{2} [S_0 + S_1 \cos \varphi + S_2 \sin \varphi], \quad (2)$$

where S_0 , S_1 , S_2 and S_3 are the unknown Stokes parameters of the input polarization. Note that Eq. (2) does not give information about the S_3 parameter; and thus, the Stokes information encrypted in Eq. (2) is incomplete.

Regarding to the Reflected Beam (RB), it is affected by the combination of the linear retarder and the linear analyzers set by the conical refraction phenomenon. By taking into account the general expression for the Mueller matrix of a linear retarder with an orientation α and a retardance δ [11], the PAs related to the RB are,

$$a_{RB}(\varphi, \alpha, \delta) = \frac{1}{2} \begin{pmatrix} 1 \\ \cos \varphi (\cos^2 2\alpha + \cos \delta \sin^2 2\alpha) + \sin \varphi (1 - \cos \delta) \sin 2\alpha \cos 2\alpha \\ \cos \varphi (1 - \cos \delta) \sin 2\alpha \cos 2\alpha + \sin \varphi (\sin^2 2\alpha + \cos \delta \cos^2 2\alpha) \\ \cos \varphi \sin \delta \sin 2\alpha - \sin \varphi \sin \delta \cos 2\alpha \end{pmatrix} \quad (3)$$

and the intensity distribution recorded by the CCD camera –this time related to the RB– is the projection of the incident polarization over the different analyzers a_{RB} ,

$$I_{RB}(\varphi) = \frac{1}{2} \begin{pmatrix} S_0 + S_1 [\cos \varphi (\cos^2 2\alpha + \cos \delta \sin^2 2\alpha) + \sin \varphi (1 - \cos \delta) \sin 2\alpha \cos 2\alpha] \\ S_1 [\cos \varphi (1 - \cos \delta) \sin 2\alpha \cos 2\alpha + \sin \varphi (\sin^2 2\alpha + \cos \delta \cos^2 2\alpha)] \\ S_2 [\cos \varphi \sin \delta \sin 2\alpha - \sin \varphi \sin \delta \cos 2\alpha] \end{pmatrix} \quad (4)$$

Thus, complete polarimetric information of the input beam is obtained by considering the intensity distributions of the RB and TB beams. At this point, different strategies could be applied to extract the encrypted input polarization from Eqs. (2) and (4), as applying the synchronous detection method [10] or some inversion method. We use this last approach by applying the following matrix system,

$$S_{input} = \hat{A}^\dagger I_{RB-TB}, \quad (5)$$

where S_{input} is the Stokes vector of the input polarization and $\hat{A}^\dagger$ is the pseudo-inverse of the matrix A (i.e. the so-called instrumental matrix), the A matrix being a $n \times 4$ matrix whose rows contain the n polarization analyzers of the system –both a_{TB} and a_{RB} analyzers in Eqs. (1) and (3). Likewise, I_{RB-TB} is the radiometric vector, which is constructed from the projections of the input polarization on the different polarization analyzers (i.e. ring intensities at different angles φ).

There are different possibilities to implement a complete polarimeter with the optical scheme proposed in Fig. 1. One choice to be done is to select the specific linear retarder to be placed into the optical module in Fig. 1(b). Two different situations are here analyzed: (a) a quarter-waveplate (QWP) oriented to a certain angle α ; and (b) a variable retarder oriented at 0 degrees of the laboratory vertical. Each proposed configuration lead to a different polarimeter with different polarimetric performance. To estimate the quality of a polarimeter there are different parameters that can be revised, as its sensibility, accuracy, robustness or noise amplification, the last being a crucial property of polarimeters. There are well-known quality indicators, as the condition number (CN) [12] or the equally weighted variance (EWV) [13] that provide the performance of polarimeters in terms of noise amplification from noise present at the intensity measurements to the final Stokes parameters measurements. By considering the form of the CN and the EWW indicators, one realizes that such noise amplification is closely related with the specific arrangement of PAs present in a polarimeter; or in other words, with the way in which the matrix inversion present in Eq. (5) amplify noise in the radiometric vector I_{RB-TB} .

Next, the performance of the optical configurations (a) and (b) described above, is analyzed in terms of the CN indicator. CN estimates how close a certain matrix is from singularity. In fact, the farther the matrix is from singularity (i.e. the smallest the CN), the lower the noise amplification of the system. First, we study the configuration (a) –i.e. a quarter-waveplate (QWP) with an orientation α as linear retarder. Under this scenario, $\delta = 90^\circ$ and Eq. (3) becomes,

$$a_{RB}(\varphi, \alpha) = \frac{1}{2} \begin{pmatrix} 1 \\ \cos \varphi \cos^2 2\alpha + \sin \varphi \sin 2\alpha \cos 2\alpha \\ \cos \varphi \sin 2\alpha \cos 2\alpha + \sin \varphi \sin^2 2\alpha \\ \sin(2\alpha - \varphi) \end{pmatrix} \quad (6)$$

Figure 2(a) shows the PAs related to the configuration (a) over the Poincaré sphere [11], which have been calculated according to Eq. (1) (red line) and Eq. (6) (blue line). The different blue curves correspond to different orientations α of the QWP (from 0° to 180° in steps of 10°). We see as a rotation of the QWP leads to a rotation of the a_{RB} analyzers curve around the S_3 axis. For each angle α selected, we have calculated the corresponding CN, this being equal to 2.0 in all the cases (see inset in Fig. 2(a)). Therefore, the orientation of the QWP is not a relevant parameter in terms of system conditioning. In addition, the obtained CN is very close to the theoretical minimum in polarimetric systems (i.e. $\sqrt{3} = 1.73$ [14]), this showing that the configuration (a) leads to polarimeters with very low noise amplification.

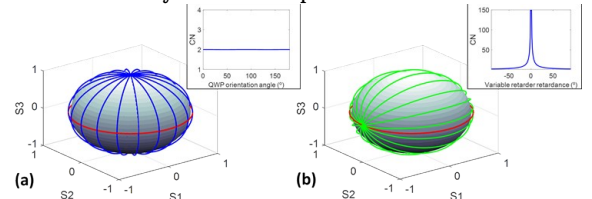


Fig. 2. Polarization analyzers as a function of: (a) the QWP orientation α (configuration (a)); and (b) the variable retarder retardance δ (configuration (b)). In both cases, the corresponding CN values are shown as an inset image.

Second, we study the configuration (b) –i.e. a variable linear retarder oriented to an angle $\alpha = 0^\circ$. In such situation Eq. (3) becomes,

$$a_{RB}(\varphi, \delta) = \frac{1}{2} \begin{pmatrix} 1 & \cos \varphi & \sin \varphi \cos \delta & -\sin \varphi \sin \delta \end{pmatrix}^T, \quad (7)$$

Figure 2(b) shows the PAs related to the configuration (b). Whereas the red curve shows the linear analyzers (a_{TB}), calculated according to Eq. (1), the green curve shows the polarization analyzers (a_{RB}),

calculated according to Eq. (7). The different green curves correspond to different retardances δ of the variable retarder (from -90° to 90° in steps of 20°). Note that this element could be experimentally implemented, for instance, by using a parallel aligned liquid crystal panel. A change of the retardance δ leads to a rotation of the a_{RB} analyzers curve over the S_1 axis. Therefore, unlike the configuration (a) above discussed (Fig. 2(a)), this configuration (b) leads to different CN values depending of the retardance δ selected (see inset in Fig. 2(b)). Thus, the noise amplification of the system strongly depends on this parameter. Two limit situations are observed for $\delta=0^\circ$ and $\delta=\pm 90^\circ$. If $\delta=0^\circ$, both a_{RB} and a_{TB} analyzers are at the sphere equator and there is no S_3 information (incomplete polarimeter; $CN=\infty$). By contrast, when $\delta=\pm 90^\circ$, the configuration (b) turns into the configuration (a) (for the case $\delta=90^\circ$ and $\alpha=0^\circ$), and the CN reaches the minimum possible value ($CN=2.0$), so the noise propagation is minimized.

It is also interesting to analyze the configurations (a) and (b) in terms of the specific Stokes variances (SV) [13, 15], they providing information of the noise amplification at each particular Stokes channel. The SV components can be calculated by error propagating Eq. (5), this leading to [13, 15],

$$\Delta S_i = \Delta I^2 \sum_{k=1}^n q_{ik}^2, \quad (8)$$

with $i=(0,1,2,3)$, where n is the number of elements of the radiometric vector I_{RB-TB} , q_k are the coefficients of the pseudo-inverse matrix $\mathcal{X}^\phi$ and where ΔI is an statistical noise that has been considered uniformly distributed on the components of the vector I_{RB-TB} .

Figure 3 shows the SV values, calculated according to Eq. (8), as a function of the orientation α of the QWP (configuration (a); Fig. 3(a)) and as a function of the selected retardance δ for the variable retarder (configuration (b); Fig. 3(b)). In both cases, we set an arbitrary value for the statistical noise ΔI in Eq. (8) (i.e., $\Delta I=1$).

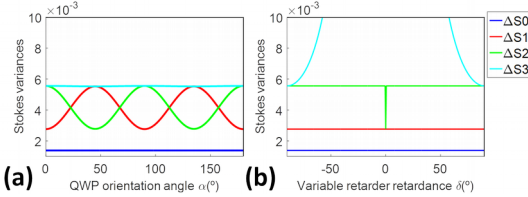


Fig. 3. SV as a function of: (a) the QWP orientation α (configuration (a)); and (b) the retardance δ for the variable retarder (configuration (b)).

For the configuration (a) (Fig. 3(a)), the four Stokes variances remain very small ($>6 \cdot 10^{-3}$) as a function of the orientation α of the QWP. This is logical because in all the cases the system is optimized (see Fig. 2(a)) and thus, the noise amplification must be always small. However, whereas ΔS_0 and ΔS_3 remain constant (0.0014 and 0.0056 respectively), ΔS_1 and ΔS_2 follow a sinusoidal behavior as a function of α . Therefore, users interested in measuring some specific polarimetric information can take profit of the data in Fig. 3(a). In this sense, if variances for the linear content want to be equally balanced, we have to select the orientations α {22.5°, 67.5°, 112.5° or 157.5°}, these values being related with the ΔS_1 and ΔS_2 crossings in Fig. 3(a) (variances of 0.0041). By contrast, if noise amplification in ΔS_1 or ΔS_2 wants to be minimized, the QWP orientations to be selected are {0°, 90° or 180°} and {45° or 135°}, leading to variances of 0.0028 for ΔS_1 and ΔS_2 , respectively. In the case of the configuration (b) (Fig. 3(b)), ΔS_0 and ΔS_1 remain constant as a function of the retardance δ – whereas ΔS_0 shows the same value than that in configuration (a), ΔS_1 remain constant to the minimum value reached for the configuration

(a) (i.e. 0.0028). For its part, ΔS_3 shows a minimum value (0.0056) for $\delta=\pm 90^\circ$, when the variable retarder acts as a QWP and the configuration (b) turns into configuration (a). By contrast, as lower the value for δ , as larger ΔS_3 , it tending to ∞ when δ tends to 0 (see as well $CN(\delta)$ in the inset of Fig. 2(b)). It is interesting to note that ΔS_2 remains constant (0.0056, this being the maximum value reached in configuration (a)) for all the δ values, except for $\delta=0$, where the variances ΔS_1 and ΔS_2 simultaneously show a minimum (0.0028, see Fig. 3(b)). Thus, configuration (b) with $\delta=0^\circ$ is the best configuration for linear polarization metrology in terms of noise amplification –this is because both a_{RB} and a_{TB} PAs are on the Poincaré sphere equator. This situation arises very interesting as PAs curves can be rotated in a controlled way by using linear retarders properly placed at the optical module (Fig. 1). However, the aim of this work is to perform complete polarimetry under optimal conditions, and thus, the configuration (a) is chosen for the subsequent experimental implementation.

The optical arrangement in Fig. 1 was implemented for the configuration (a). A Gaussian beam (He–Ne laser; 633 nm) was used as light source. The beam reaches a Polarization State Generator (PSG) formed by a linear polarizer and two Parallel Aligned (PA) Liquid Crystals panels (LCC1223T-A provided by Thorlabs). This system, analogous to that described in Ref. [15], allows us to calibrate and to test the CR based polarimeter. In addition, as the PSG is static (i.e. different polarizations are digitally addressed), it also minimize beam wander effects [16] and permits us to arbitrarily generate any fully polarized State of Polarization (SoP). After that, the Optical Module is placed (Fig. 1(b)). The beam is focalized by a convergent lens (L1) (focal length of $f_1=16$ cm) and it is divided in two sub-beams by using 10 mm beam splitters and prisms. The two exiting beams reaches a biaxial crystal (made of KGd(WO4)2, fabricated by CROptics) with a length of $L=23.38$ mm and conicity $c=16.9$ mrad. As a consequence, each light beam form a ring-like structure at the focal plane of the L1 lens, both with a radius of $R_0=397$ μ m. Finally, a second convergent lens (L2) images the obtained rings to a CCD camera (6-bits with a resolution of 1000x1000 pixels distributed by Basler) with certain magnification.

Before testing the polarimeter performance, it was experimentally calibrated. The instrumental matrix A can be experimentally determined by setting different known input polarizations and measuring the corresponding ring intensity distributions (see Eq. 5). To this end, six controlled SoPs were generated with our PSG: four equally spaced linear states (horizontal, vertical, 45° and 135° linear polarizations) and two circular states (right-hand and left-hand circular polarizations). In addition, to link the calibration ring-like intensities with the corresponding pixels at the CCD camera, the ring-centering and ring-correlation algorithm detailed in Ref. [10] was applied. Under this scenario, we calibrated the polarization analyzers (PAs) of our system, with the QWP oriented to an arbitrary angle. The obtained PAs are represented in Fig. 4(a) over the Poincaré sphere, as blue and red curves for the a_{RB} and the a_{TB} analyzers respectively. The differences observed between the theoretical prediction (Fig. 2(a)) and the experimental implementation (Fig. 4(a)) are mainly related with extra retardations introduced by total internal reflection occurring in prisms as well as by reflections at metal thin film in beam splitters (see Fig. 1). For a better understanding of these differences, the a_{RB} case (blue curve) is following analyzed. The ideal situation is represented in Fig. 4(b). The linear PAs set by the biaxial crystal (red empty spots in Fig. 4(b)) are transformed by the QWP, which performs a rotation (with a value of $\delta_1=\pi/2$) around its fast axis (FA), the FA being set in the plane S_1 - S_2 . In the case represented in Fig. 4(b), the FA is oriented at 0 degrees, so the QWP fast axis coincides with the S_1 axis. However, due to the effect of prisms and beam-splitters, which act as an

equivalent retarder, the resultant PAs are rotated once again, this time an angle δ_2 around the equivalent retarder FA (back dashed line in Fig. 4(c)), which will also be in the S_1 - S_2 plane, but in general, to a certain angle of the QWP fast axis. This new transformation defines the final PAs (orange spots in Fig. 4(c)), they being displaced from the desired ones (blue spots). Contrary to expectations from data in Fig. 2(a), the QWP orientation is not negligible in terms of CN and it must be optimized in the experimental implementation. To achieve the optimal configuration (Fig. 4(d)), the FA of the equivalent retarder has to be coincident with the S_2 axis, so PAs rotations produced by the equivalent retarder do not affect the system in terms of CN, independently of the δ_2 value (Fig. 4(d)). This is achieved when the QWP and the equivalent retarder FAs form a relative angle of 45° (angles on the Poincaré sphere are doubled).

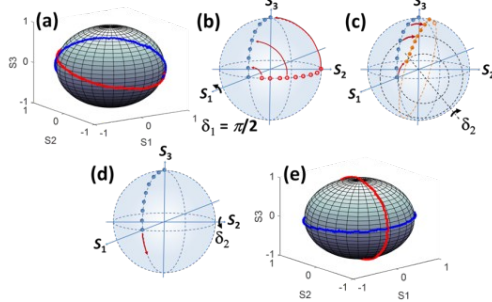


Fig. 4. (a) Experimental analyzers over the Poincaré sphere; (b) Scheme of the theoretical analyzers for the QWP-biaxial crystal system; (c) Effect of the extra retardance introduced by prisms and BS; and (d) Analyzers obtained when the relative angle between the QWP and the equivalent retarder fast axes is optimized.

We performed a new experimental analyzers calibration but this time the QWP and the equivalent retarder were oriented to 45° one to each other. By following the same geometrical argument discussed above, an extra QWP was also set in the transmitted beam to properly obtain the desired linear analyzers a_{TB} . Experimental results are given over the Poincaré sphere in Fig. 4(e) (blue line for a_{RB} and red line for a_{TB}). The calibrated PAs are now very close to the theoretical ones (Fig. 2(a)), they leading to an experimental CN equal to 2.21 (the optimal CN for CR based polarimeters is 2.0).

The implemented polarimeter quality was tested by analyzing its accuracy and repeatability. First, the accuracy was studied. We generated different input SoPs with our PSG system. In particular, we set a fixed voltage (2.25 V) to the second PA panel in the PSG, while the first PA voltage was varied from 0.9-8.5 V. By doing this, the generated polarizations describe a cut over the Poincaré sphere and they were measured both by using our proposed polarimeter (for the configuration shown in Fig. 4(e); CN = 2.21) and by means of a commercial polarimeter (PAN 5710VIS, S/N: M60217605; distributed by Thorlabs). The obtained results in terms of azimuth (left-axis) and ellipticity (right-axis) are shown in Fig. 5(a). To the sake of clarity, the absolute error between those values is also provided in Fig. 5(b). The maxima absolute errors obtained for the azimuth and ellipticity measures were of 2.82° (1.56%) and 2.35° (1.30%) respectively, these accuracy values being comparable with commercial instruments.

We also studied the repeatability of the polarimeter by measuring some SoPs 20 times, and the corresponding standard deviation (SD) was calculated. As our optical configuration permits to arbitrary select the number of PAs (i.e., the number N of analyzers used from the array shown in Fig. 4(e)), the SV was calculated for different N into a range between 6 and 360 for each light ring. The result for one particular SoP linear polarization at 25 degrees—is given in Fig. 5(c). Whereas azimuth SD values are represented by green squares, ellipticity ones are

represented by red circles. In all the cases, the SD values are lower than 0.3° , but redundancy decreases this value by following the statistical law $1/\sqrt{N}$. Note that with our proposed polarimeter the SD values decrease without a measuring time increase, because data redundancy is achieved spatially, not from time-sequential measurements.

In conclusion, we provided a new optical scheme for a snapshot and complete CR based polarimeter. To perform complete polarimetric measurements, such polarimeters are nowadays based on bulky systems (they require two biaxial crystals and two CCD cameras properly synchronized) [6, 9]. Other proposal based on more faceable optical arrangements are restricted to perform linear polarization metrology [10, 17]. Unlike this, the proposed optical set-up leads to a much cheaper (one biaxial crystal and one CCD camera) and simpler (in-line) set-up. The polarimeter was optimized by following the condition number criteria (CN=2 and CN=2.21 for the theoretical and experimental configurations, respectively). Finally, the CR polarimeter was experimentally implemented and tested by measuring different controlled input SoPs. Regarding to the accuracy, the maximum absolute errors, in terms of the polarimetric angles azimuth and ellipticity, are of 1.56% and 1.30% respectively. By contrast, the repeatability was tested by calculating the standard deviation (SV) for a linear polarization measured 20 times, this leading (in the case of N=720) to a SV of 0.04% both for the azimuth and ellipticity. These parameters provide the optimized performance of the proposed polarimeter.

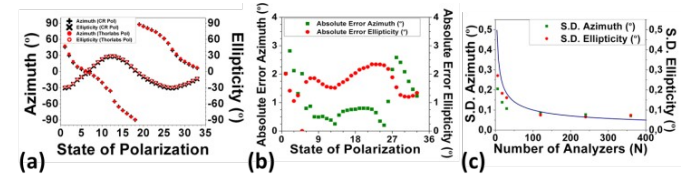


Fig. 5. (a) Azimuth (left-axis) and ellipticity (right-axis) values for the CR polarimeter (black crosses) and for the commercial one (red circles) related to different SoPs; (b) Azimuth (green squares) and ellipticity (red circles) absolute errors for the same SoPs; and (c) Azimuth (green squares) and ellipticity (red circles) SV as a function of the analyzers number (N), for the measure of a linear polarization at 25° .

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