Firm size distortions under duopoly*

Distorsiones del tamaño empresarial en duopolio

Miguel González-Maestre**
Diego Peñarrubia***

Abstract

Motivated by the fact that some regulations involve extra costs for those firms at a size beyond a critical threshold, this paper contributes to the analysis of the welfare distortions due to these regulations. In the context of a duopoly, our results show that social welfare is not monotonic with the regulatory threshold. In particular, we obtain the paradoxical result that a policy decision of increasing the threshold might involve a dramatic decrease in welfare in some markets. An interesting consequence of this result is that the positive discrimination towards small firms is a rather subtle issue. Our results suggest that the relevant regulatory thresholds should differ across industries. Apparently, this is taken into account in some countries (e.g., USA), but not in many other countries.

Key words: *Duopoly, welfare, firm size, strategic effects.*

JEL Classification: *L11, L13, L52.*

Resumen

Motivado por el hecho de que con frecuencia algunas regulaciones implican un coste extra para las empresas cuyo tamaño supera un cierto umbral, este trabajo analiza los efectos sobre el bienestar de estas regulaciones. En el contexto de un duopolio, nuestros resultados muestran que dichos efectos no son monótonos en el umbral regulatorio. En concreto, obtenemos el resultado paradójico de que la decisión regulatoria de elevar un poco el umbral puede generar reducciones abruptas del bienestar. Una consecuencia interesante de

* We acknowledge financial support from the Spanish Ministerio de Economía y Competitividad under projects MINECO/FEDE, EU ref ECO2015-63679-P, and ECO2014-53419-R. We are very grateful to a referee for useful comments which helped us to improve the paper.

** Departamento de Fundamentos del Análisis Económico, Universidad de Murcia, 30100 Murcia, Spain. Fax: +34-868 883758. E-mail: mmaestre@um.es.

*** Departamento de Fundamentos del Análisis Económico, Universidad de Murcia, 30100 Murcia, Spain, E-mail: diegope@um.es

Received: April, 2015. Accepted: March, 2017.

este resultado es que la discriminación positiva hacia las empresas pequeñas es un asunto muy sutil. Los resultados sugieren que los umbrales regulatorios relevantes deben diferir entre sectores. Aparentemente, esto se tiene en cuenta en algunos países (como EE.UU.) pero no en muchos otros.

Palabras clave: *Duopolio, bienestar, tamaño empresarial, efectos estratégicos.*

Clasificación JEL: *L11, L13, L52.*

1. Introduction

Given that firms' size distribution seems to be one of the main determinants of productivity, some recent literature has focused on the analysis of the direct effect of market regulations on the distribution of firms' size. A substantial part of this literature has been based on the seminal paper by Lucas (1978) on the firm size and productivity distribution. In particular, García-Santana and Pijoan-Mas (2014), Guner *et al.* (2006), and Guner *et al.* (2008) have evaluated the welfare effects of size-dependent policy regulations. More recently, Garicano *et al.* (2016) have investigated the impact of labour regulation in France, which involves an extra cost for the firms with more than 50 employees[1]. They find that the distortion in the distribution of firms' size, associated with the presence of this critical threshold of a firm's size, can explain approximately 0,6% of GDP. Those authors assume that each firm's productivity is exogenous and determines its optimal size. Therefore, the social cost associated to the threshold regulation of firm size arises because it prevents many firms from reaching their optimal size. Garcia-Santana and Ramos (2015) have contributed to the detailed empirical analysis of the economic relevance of this type of distortion using comparable data across 104 developing countries.

We remark that Garicano *et al.* (2016), as well as the other mentioned contributors, analyze a general equilibrium model without strategic interaction between firms. Consequently, they only consider the direct effect of the threshold regulation. In contrast, the objective of our paper is to investigate the indirect strategic effects of such regulation. To this end, we consider a duopoly model where those strategic effects appear. Our results show that the social welfare is not monotonic with this threshold. In particular, we obtain the paradoxical result that a policy decision of increasing the threshold from intermediate-low levels to intermediate-high levels might involve a dramatic decrease in welfare. Therefore, our analysis shows that the strategic interactions, involved in concentrated oligopolistic markets, might yield counterintuitive policy implications.

[1] Other authors have focused on different relevant thresholds affecting firms' size. In particular, Almunia and Lopez-Rodriguez (2012) have pointed out the distortions associated with thresholds of receipts, from a tax avoidance perspective, in the case of Spain. There are also examples of threshold regulations related to gender issues. In particular, Chilean childcare regulation establishes, in its Article No. 203, that every firm with 20 or more female workers has to provide childcare facilities within firm premises. Therefore, Chilean regulation imposes, theoretically, an additional cost to firms after a certain number of female workers. We are grateful to a referee for pointing out this interesting example.

In particular, the welfare level might change in a non-monotonic way when we move from more to less restrictive regulations.

Therefore, our analysis notes that a positive discrimination towards small firms is a rather subtle issue. As a consequence, our results suggest that the relevant regulatory thresholds should differ across industries. Apparently, this is taken into account in some countries but not in many other countries. In particular, the US Small Business Administration (2009) establishes a very detailed methodology which defines the different size standards at the 6-digit NAICS (North American Industry Classification System) level. Therefore, the USA size standards, which are established for public subsidy purposes, rely on very specific industry definitions. However, the USA regulatory sensitivity to the particular features of each industry contrasts with the general threshold prevailing in most European countries. In this article, we have considered a simple model to illustrate the impact of this type of positive discrimination towards small firms in a framework of strategic interaction. In particular, the existence of multiple equilibria and the non-monotonic relationship between the critical threshold and welfare can be associated, in our model, with a perverse firm incentive to remain small. In other words, there might be a type of *Peter Pan Syndrome*[2] which is socially harmful. Therefore, one important insight of our analysis is that the impact of any policy intending to favor small firms (like the establishment of critical regulatory thresholds) should be analyzed in a separate way for each industry.

2. The Model

We will assume a duopoly Cournot model where two firms compete in the market of a homogeneous good with inverse demand function $p = a - X/T$, where T is interpreted as market size, p is the price of the good, and X is the total demand. Each firm has to pay a fixed cost F_i and a constant marginal cost c. Hence, the cost function of firm i is given by $C(x_i) = F_i + cx_i$, where x_i is the production of firm i. Without loss of generality, we assume, for simplicity, that $c = 0$. Moreover, we assume $F_i = 0$ if $x_i \leq z$, and $F_i = \varepsilon > 0$ if $x_i \geq z$,where ε is the extra cost associated with having a size greater than the threshold z. In the rest of the paper, this threshold will be interpreted as the regulatory critical firm's size such that beyond this level, the firm has to pay an extra cost ε.

Formally, the profit function of firm i (where $i=1,2$) is given by

$$(1) \qquad \Pi_i(x_i, x_j) = x_i\left(a - \frac{x_i + x_j}{T}\right) - C(x_i), \text{ where } C(x_i) = \begin{cases} 0 & \text{if } x_i \leq z \\ \varepsilon & \text{if } x_i > z \end{cases}.$$

[2] This expression was first used, with this meaning, in the article entitled "The Peter Pan Syndrome", *The Economist* (May 17, 2014). This article contains an interview of Manuel Milano of the Mexican Competitiveness Institute.

3. Equilibrium Analysis

In the absence of regulatory restrictions (that is, $\varepsilon=0$, or $z=\infty$) firm i 's reaction function would be given by

$$\frac{\partial \Pi_i}{\partial x_i} = 0 \rightarrow x_i(x_j) \equiv \frac{aT - x_j}{2}. \tag{2}$$

However, if ε is sufficiently large and z is sufficiently small, then we might have corner solutions in each firm's optimal decision, which affects the shape of the reaction functions.

To investigate this issue, let us define the following critical values of x_j:

First, x_j^H is defined by

$$x_i(x_j^H) = z \rightarrow \frac{aT - x_j^H}{2} = z \rightarrow x_j^H \equiv aT - 2z. \tag{3}$$

Second, x_j^B is defined by

$$\Pi_i(x_i(x_j^B), x_j^B) = \Pi_i(z, x_j^B) \rightarrow T\left(\frac{a - x_j^B / T}{2}\right)^2 - \varepsilon = z\left(a - \frac{z + x_j^B}{T}\right) \rightarrow \tag{4}$$

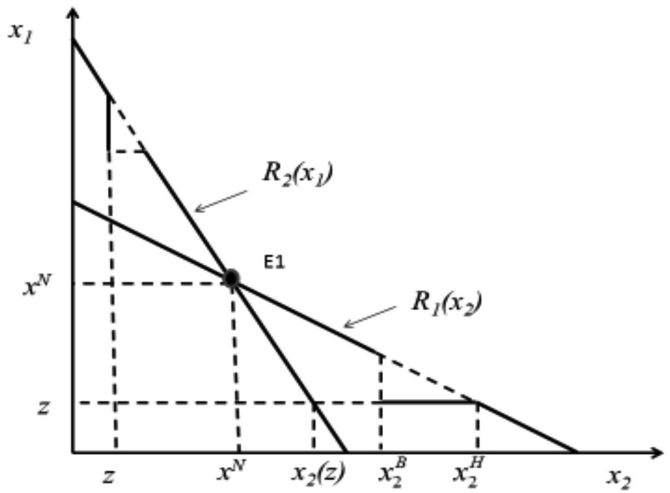
$$x_j^B \equiv aT - 2z - 2\sqrt{\varepsilon T}.$$

Therefore, x_j^H is the critical x_j such that above x_j^H, the optimal firm i's output is lower than z. Similarly, x_j^B is the critical x_j, such that below x_j^B the optimal firm i's output is larger than z (which implies the payment of the extra cost ε). Because $x_j^B < x_j^H$, there is an intermediate interval of x_j, satisfying $x_j^B < x_j < x_j^H$, such that the optimal firm i's output is a corner solution given by the threshold z. This corner solution is the result of analyzing firm i's decision at the margin, when making the decision whether to go beyond the threshold. In particular, expression (4) provides the critical x_j such that firm i is indifferent about producing the interior solution $x_i(x_j)$ or the corner solution given by z. Note that, in the absence of the extra cost ε, for these intermediate levels of its rival's output, firm i's best-response would involve an output level above the threshold z. In conclusion, the firm i's reaction function is given by

$$R_i(x_j) = \begin{cases} x_i(x_j) \equiv \dfrac{aT - x_j}{2} & \text{if } x_j < x_j^B \\ z \quad \text{if } x_j^B \le x_j \le x_j^H \\ x_i(x_j) \equiv \dfrac{aT - x_j}{2} & \text{if } x_i > x_j^H \end{cases}, \tag{5}$$

which is illustrated in Figure 1.

FIGURE 1
CASE 1, UNIQUE EQUILIBRIUM IS E1



In the following analysis, we will show that, depending on the parameters, there might be three main types of equilibria:

a) A symmetric equilibrium where the regulations do not affect firms' decisions and both firms produce the same output level. In some cases, both firms produce over the critical threshold z and in some other cases both produce below this critical level.
b) An asymmetric equilibrium where one of the firms produces the threshold z (a corner solution), while the other produces beyond z. Obviously, the ex-ante symmetry implies that, in this case, we have two equilibria: one with firm 1 restricted to the threshold z and another one with firm 2 restricted to z.
c) A symmetric equilibrium where both firms produce the threshold output.

Moreover, we will show that for some parameters' values, we have the symmetric equilibrium mentioned in (a) as well as the asymmetric equilibria mentioned in (b).

By combining the reaction functions of both firms and taking into account the rest of the parameters, we will show that there are 5 cases regarding equilibrium outcomes, depending on the regulatory threshold z:

I) For very small levels of z, defined by $z < z^I \equiv \frac{aT}{3} - \frac{4}{3}\sqrt{\varepsilon T}$, there is a unique symmetric equilibrium given by

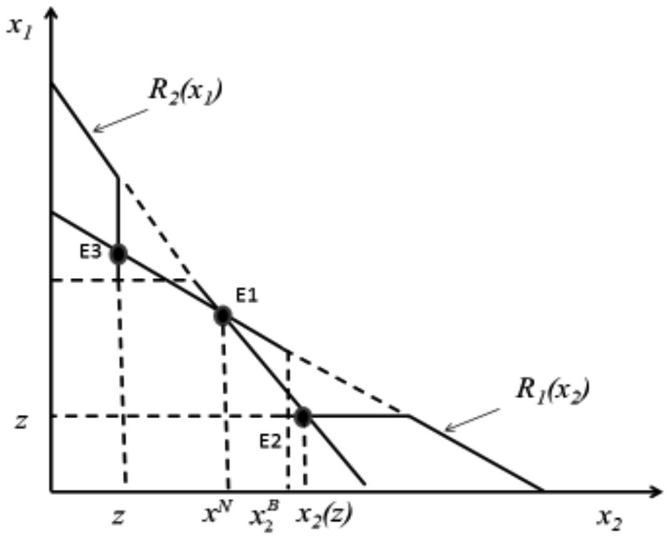
(6) $$x_i^* = x_j^* = x^N \equiv aT/3 > z, \quad X^* = \frac{2}{3}aT, \; p^* = a/3, \quad \Pi_i^* = \Pi_j^* = \frac{a^2}{9}T - \varepsilon.$$

This case is illustrated in Figure 1. Note that the following condition must hold:

$$z < x^N \equiv aT/3 < x_j(z) \equiv \frac{aT-z}{2} < x_j^B \equiv aT - 2z - 2\sqrt{\varepsilon T} \leftrightarrow z < z^I \equiv \frac{aT}{3} - \frac{4}{3}\sqrt{\varepsilon T}, \tag{7}$$

which confirms that this case is satisfied for very small values of z.

FIGURE 2
CASE 2, THREE EQUILIBRIA



II) For intermediate-low levels of z, defined by $z^I < z < z^{II} \equiv \frac{aT}{3} - \sqrt{\varepsilon T}$, there are 3 equilibria, illustrated in Figure 2. One of these equilibria is similar to the one calculated in case (I): both firms produce the same output which is greater than z. At each of the other two equilibria, one of the firms' best reply to the threshold output by the other firm is to produce above that threshold. In particular, if firm i is the one that chooses the threshold output then the associated asymmetric equilibrium is given by

$$x_i^{**} = z,\ x_j^{**} = x_j(z) \equiv \frac{aT-z}{2};\ X^{**}(z) = \frac{aT+z}{2}. \tag{8}$$

According to Figure 2, the following condition must be satisfied in this case:

$$z < x^N \equiv aT/3 < x_j^B \equiv aT - 2z - 2\sqrt{\varepsilon T} < x_j(z) \equiv \frac{aT-z}{2} \leftrightarrow z^I < z < z^{II}, \tag{9}$$

(10) $$\text{where } z^I \equiv \frac{aT}{3} - \frac{4}{3}\sqrt{\varepsilon T}, \text{ and } z^{II} \equiv \frac{aT}{3} - \sqrt{\varepsilon T}.$$

Note that $z^I < z^{II}$, which implies that there is a non-empty interval of intermediate-low levels of z associated with this case.

III) For intermediate-central levels of z, defined by $z^{II} < z < z^{III} \equiv \frac{aT}{3} - \frac{2}{3}\sqrt{\varepsilon T}$, only the two asymmetric equilibria remain. This is illustrated in Figure 3. At each of the two asymmetric equilibria, one of the firms chooses the critical threshold in order to avoid the extra cost, and the other one takes advantage of this restrictive behaviour of its rival to expand its output. Note that, compared with the equilibrium in the case of no regulations, the asymmetric equilibrium associated with the regulation implies that the overall output decreases.

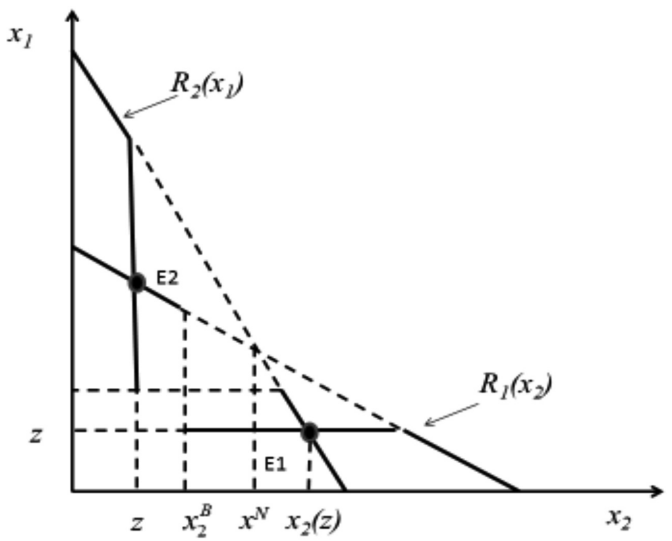
According to Figure 4, the following condition must be satisfied in this case:

(11) $$z < x_j^B \equiv aT - 2z - 2\sqrt{\varepsilon T} < x^N \equiv aT/3 < x_j(z) \equiv \frac{aT - z}{2} \leftrightarrow z^{II} < z < z^{III},$$

(12) $$\text{where } z^{II} \equiv \frac{aT}{3} - \sqrt{\varepsilon T}, \text{ and } z^{III} \equiv \frac{aT}{3} - \frac{2}{3}\sqrt{\varepsilon T}.$$

Note, that $z^{II} < z^{III}$, which ensures a non-empty interval of intermediate-central values of z consistent with this case.

FIGURE 3
CASE 3, TWO EQUILIBRIA



IV) For intermediate-high z, defined by $z^{III} < z < x^N \equiv \frac{aT}{3}$, the unique Nash equilibrium is a corner solution where both firms produce the threshold output:

$$x_i^{***} = x_i^{***} = z, \quad X^{***}(z) = 2z \tag{13}$$

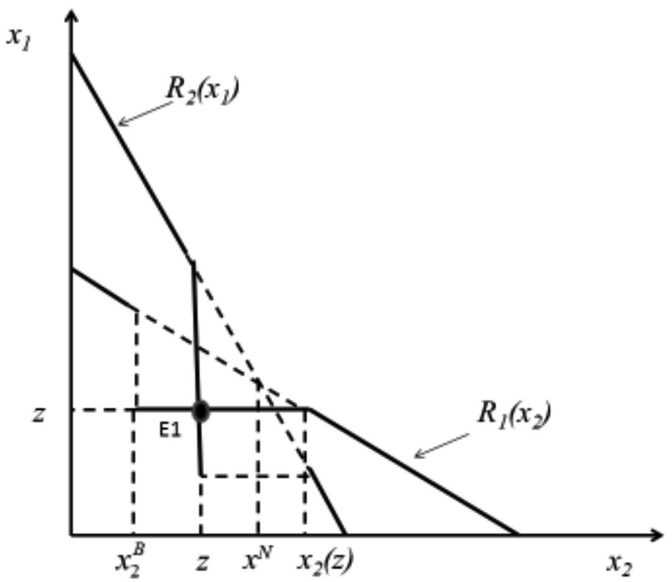
This case is illustrated in Figure 4 and implies the following condition:

$$x_j^B \equiv aT - 2z - 2\sqrt{\varepsilon T} < z < x^N \equiv aT/3 < x_j(z) \equiv \frac{aT - z}{2} \leftrightarrow z^{III} < z < x^N. \tag{14}$$

where $z^{III} \equiv \frac{aT}{3} - \frac{2\sqrt{\varepsilon T}}{3}$, and $x^N \equiv \frac{aT}{3}$.

Given that $z^{III} < x^N$, it turns out that there is a non-empty interval of intermediate-high levels of z corresponding to this case.

FIGURE 4
CASE 4, ONE EQUILIBRIUM



V) For very high levels of z, defined by $z > x^N \equiv \frac{aT}{3}$, this threshold is so large that it becomes irrelevant. In this case, the equilibrium quantities are the same as in case (I), but now profits will be greater because the firms do not

pay the extra fixed cost. This case is illustrated in Figure 5 and is associated with the following condition:

$$z > x^N \equiv \frac{aT}{3}. \tag{15}$$

Therefore, a very high level of z is the necessary and sufficient condition for this case to hold.

FIGURE 5
CASE 5, ONE EQUILIBRIUM

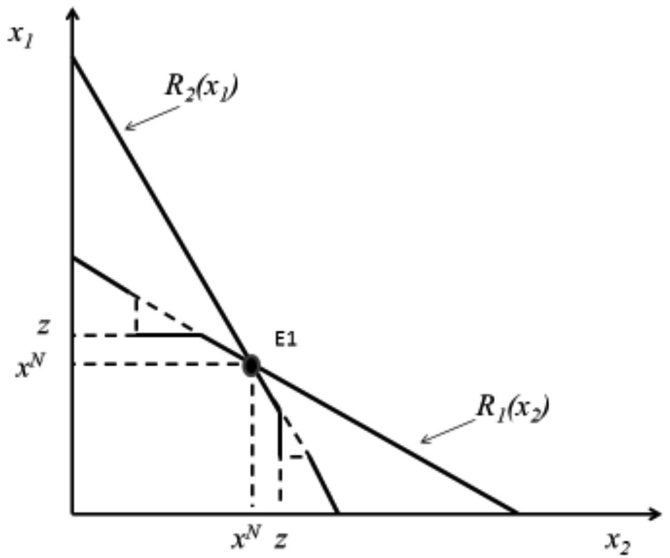
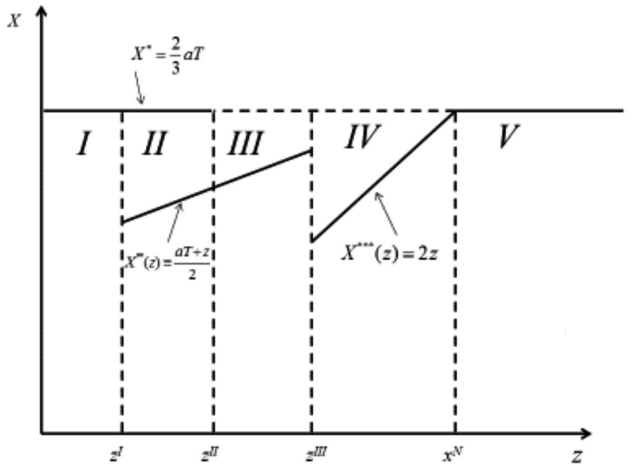


Figure 6 illustrates the comparative statics of the previous cases, focusing on the interplay between the critical threshold z and the equilibrium levels of total output X. In this figure, each of the regions labelled from (I) to (V) corresponds, respectively, to each of the cases from (I) to (V), previously explained. Recall that there are three types of equilibria in our model:

a) The symmetric equilibrium with both firms producing above z, which appears in regions (I), (II), and (V) and involves a total output given by $X^* = \frac{2}{3}aT$.
b) The asymmetric equilibrium with one firm producing z and the other one producing above z, which appears in regions (II) and (III) and is associated with total equilibrium output $X^{**}(z) \equiv \frac{aT + z}{2}$.
c) The symmetric equilibrium with both firms producing z, which is associated with the total equilibrium output $X^{***}(z) \equiv 2z$.

FIGURE 6
TOTAL OUTPUT AS A FUNCTION OF THRESHOLD Z



The previous analysis can be summarized in the following result:

Proposition 1: *As the critical threshold z increases, the following properties hold regarding the equilibrium quantities:*

i) For very low z ($z < z^I$), there is a unique symmetric equilibrium where both firms produce above z (Region I in Figure 6).

ii) For intermediate-low z ($z^I < z < z^{II}$), besides the previous symmetric equilibrium, two new asymmetric equilibria appear. In each of those asymmetric equilibria, one firm produces above z, and the other one produces below z (Region II in Figure 6).

iii) For intermediate-central z ($z^{II} < z < z^{III}$), only the two asymmetric equilibria exist (Region III in Figure 6).

iv) For intermediate-high z ($z^{III} < z < x^N$), at the unique symmetric equilibrium both firms produce the critical threshold z (Region IV in Figure 6).

v) For high z ($z > x^N$), at the unique symmetric equilibrium both firms produce below the critical threshold z (Region V in Figure 6).

4. Welfare Analysis

Let us investigate the welfare associated with each of the cases previously considered. Total welfare is obtained by adding the consumer surplus and firms' profits, which yields:

$$W = pX + (a - p)X / 2 - n_U \varepsilon = aX - X^2 / 2 - n_U \varepsilon, \tag{16}$$

where n_U is defined as the number of firms for which the output is greater than the threshold z. Therefore, ε is interpreted as a social cost (e.g., a red tape cost).

Nevertheless, as it is shown in the Appendix, our basic results holds if ε is re-interpreted as a tax transfer from the firm to the government (See Appendix).

Let us define $W^r(z)$ as the welfare level in each of the previous cases, where $r = I, II, III, IV, V$. Easy calculations yield:[3]

$$W^I(z) = T\left(\frac{2a}{3}\right)^2 - 2\varepsilon, \tag{17}$$

$$W^{II}(z) = \{2z(a - z/T),\ T\left(\frac{2a}{3}\right)^2 - 2\varepsilon\}, \tag{18}$$

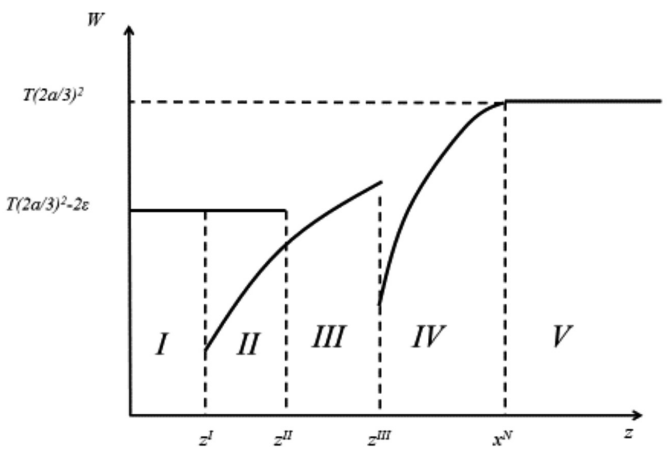
$$W^{III}(z) = \frac{3a^2T^2 + 2aTz - z^2}{8T} - \varepsilon, \tag{19}$$

$$W^{IV}(z) = 2z(a - z/T), \tag{20}$$

$$W^V(z) = T\left(\frac{2a}{3}\right)^2. \tag{21}$$

The comparative statics about welfare is illustrated in Figure 7, which focuses on the relationship between welfare and the threshold output z. Again, the regions from (I) to (V) correspond to the previously explained cases from (I) to (V).

FIGURE 7
TOTAL WELFARE AS A FUNCTION OF THRESHOLD Z



[3] The first of the two values of $W^{II}(z)$ corresponds to the asymmetric equilibrium with one firm producing z, while the second value corresponds to the symmetric equilibrium similar to case *I*.

The following remarks are in order:

i) It is easy to see that welfare is constant at the two extreme cases (Regions I and V in Figure 7) and it is greater in case (V). Therefore, when the critical threshold is either very small ($z < Z^I$) or very large ($z > x^N$), small changes to this regulatory variable do not have welfare effects.

ii) If, as a result of increasing z, there is a transition from Region I to Region II, then there might be a discontinuous fall in welfare. To see this, as illustrated in Figures 6 and 7, there are two types of equilibria in Region II. In addition to the symmetric equilibrium (similar to the one in Region I in which both firms produce above z) there are two asymmetric equilibria with lower production and welfare than at the symmetric equilibrium. By choosing the value of $W^{II}(z)$ corresponding to the asymmetric equilibrium (see footnote 3), we have

$$W^I(z) - W^I(z) = T\left(\frac{2a}{3}\right)^2 - 2\varepsilon - 2z(a - z/T), \tag{22}$$

which evaluated at $z^I \equiv \frac{aT}{3} - \frac{4}{3}\sqrt{\varepsilon T}$ yields

$$W^I(z^I) - W^I(z^I) = \frac{1}{9}(30\varepsilon + 8a\sqrt{\varepsilon T}) > 0. \tag{23}$$

Therefore, if the transition from Region I to Region II involves a shift from the symmetric equilibrium to the asymmetric equilibrium, then as a result of this small increase in z, the total welfare falls.

iii) For intermediate-high levels of z (Region IV in Figure 7), any increase in z increases welfare, which reaches its maximum level once z becomes irrelevant. In this case, relaxing the regulation on the threshold is welfare-enhancing because each firm's output is determined by the threshold.

iv) The transition from case (III) to case (IV) involves a discontinuity:

$$W^{III}(aT - 2\sqrt{\varepsilon T})/3) - W^{IV}((aT - 2\sqrt{\varepsilon T})/3) = \frac{2a\sqrt{\varepsilon T} - \varepsilon}{6} > 0. \tag{24}$$

From the previous inequality, it follows that a small increase in z that shifts the equilibrium from case (III) to case (IV) implies a discontinuous fall in the level of welfare.

According to remarks (ii) and (iv), one interesting conclusion arising from our analysis is that the total welfare is not monotonic with respect to the critical threshold regulation. This conclusion can be summarized as follows:

Proposition 2: *The following properties hold regarding the relationship between the threshold regulation, z, and welfare:*

i) A moderate increase in z from small to intermediate levels might decrease welfare by shifting the market from a symmetric equilibrium to an asymmetric equilibrium.

ii) For intermediate levels of z, a small increase in the threshold might decrease welfare by shifting the equilibrium from an asymmetric equilibrium to a symmetric equilibrium.

To see the intuition of part (i) of Proposition 2 note, first, that increasing the regulatory level of z enhances a firm's incentives to expand its output at the expense of its competitor. However, the net effect on welfare is negative because the competitor's best response is to remain small at z in order to avoid the extra cost. Note that this effect works when we depart from relatively small levels of z. This is because the expanding firm's incentive to increase its output relies on the small size of its competitor. Therefore, one consequence of this part of the results is that if we depart from relatively restrictive regulation rules (given by small z), then a moderate increase in the level of flexibility might have perverse welfare results.

In the case of part (ii) of Proposition 2, the increase in z decreases the welfare as well. However the explanation for the decrease is very different. For intermediate levels of flexibility in the degree of regulation (intermediate z), relaxing this regulation reduces output because the (previously) large firm decides to reduce its production to z. This yields a symmetric corner equilibrium. Intuitively, once z is beyond some intermediate critical level, the residual demand for the large firm is too small to justify producing beyond this threshold.

Related with our previous insights, one interesting aspect of our model is that the degree of concentration in the market is not monotonic with the regulatory threshold: according to part (i), relaxing the regulatory threshold from low values tends to increase the degree of concentration, but, according to part (ii), the same policy tends to reduce the degree of concentration for greater levels of regulation. In other words, in the first case, the perverse effect of relaxing the regulation is associated with an increase in the level of concentration, while in the second case it is due to the reduction in the degree of concentration.

Therefore, an important policy implication of our previous results is that regulation changes in the threshold on firms' size might have welfare consequences which are not monotonic with respect to this regulatory variable. Conversely, as the regulations generating size distortions are usually the same for all industries (they are associated with threshold levels for employment or revenues), their effects might be very different across industries. Nevertheless, as noticed in the introduction, there are some countries, like the USA, where the design of regulatory thresholds are industry-sensitive and account for the differential evolution of each industry.

Interestingly, our analysis is connected with some previous literature dealing with the social convenience of helping minor firms. In particular, Lahiri and Ono (1988) noted that, even in the case of constant returns to scale, a more competitive market (with extra number of firms) might imply a lower level of welfare. In our model, the idea of "helping minor firms" can be interpreted as a policy implying an increase in the critical threshold below which firms do not have to pay the extra cost. To see this, note that in the asymmetric equilibrium arising in our model, this policy "helps" the minor firm (which is the one that produces the threshold output) to produce more output. However, according to Proposition 2(ii), this might involve a fall in the welfare level. This effect resembles the one noted by Lahiri and Ono (1988). However there are two important differences:

first, contrary to these authors' model, consumer surplus decreases in our model because total production decreases. Second, our result holds even if both firms are symmetric ex-ante, while in the case of Lahiri and Ono (1988), the minor firm is more inefficient (it has higher marginal costs).

5. Conclusions and final comments

One important policy implication of our results is that changes in the regulatory thresholds of firms' size might have counterintuitive, drastic, and non-monotonic welfare effects. This conclusion contrasts with most of the previous literature dealing with the welfare effects of this type of regulations. In particular, it seems that this previous literature has been mainly based on general equilibrium models that neglect the important strategic effects that usually appear in relevant oligopolistic markets. Therefore, our contribution notes that taking into account these strategic effects might have crucial implications for the welfare consequences of moderate changes in regulations affecting firms' size decisions.

We remark that our results have been obtained under rather simplified conditions e.g., the number of firms, the linear form of demand, and constant marginal costs. However, the main results can be easily extended. In particular, the counterintuitive idea that relatively small changes in the regulatory framework might have drastic welfare effects is very likely to hold under more general oligopoly models. In fact, the discontinuity in the strategic responses by oligopolistic firms is a usual aspect of concentrated industries. Therefore, it seems that, despite of its simplicity, our model provides some basic insights regarding the effects of firm-size dependent regulations.

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Appendix

In this appendix, we extend the welfare analysis in Section III to the case in which the fixed cost ε is interpreted as a tax (conditional to firm's output being greater than the threshold z). In this case, ε is a transfer from the firm to the government, and the welfare is given by

$$W = pX + (a - p)X / 2 = aX - X^2 / 2, \quad \text{(A.1)}$$

which is a reformulation of expression (16) under the new interpretation of ε. Because (A.1) is strictly increasing in X, for the relevant values of this variable, the qualitative properties of X , as a function of z, are the same as those of W.

Easy computations show the following properties:

a) $X^{*} - X^{**}(z) = \frac{1}{2}(\frac{aT}{3} - z) < 0 \leftrightarrow z < \frac{aT}{3}$,

b) $X^{**}(z) - X^{***}(z) = \frac{1}{2}(aT - 2z) > 0 \leftrightarrow z < \frac{aT}{2}$,

In particular, property (a) holds for z^{I} and z^{II} because $z^{I} < z^{II} < \frac{aT}{3}$. Similarly, property (b) holds for z^{III} because $z^{III} < \frac{aT}{2}$. Those properties are reflected in Figure 6. Therefore, if the fixed cost ε is reinterpreted as a transfer from the firm to the government, then the welfare conclusions are very similar to those in the main text. To see this, note the following:

i) The welfare might fall as z shifts from Region I to Region II (from small to intermediate-low levels of z) and when z shifts from Region II to Region III (from intermediate-low to intermediate-central levels of z).
ii) The welfare might also fall as z shifts from Region III to Region IV (from intermediate-high to very high levels of z).

Note that those conclusions resemble those obtained in Proposition 2.