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the date of receipt and acceptance should be inserted later

After a presentation of the paper cited above at a workshop on Dynamic Networks at the Isaac Newton Institute for Mathematical Sciences, Cambridge, prof Frank Ball in discussions explained two potential errors in our analysis. After further discussions this was indeed confirmed. One mistake was an oversight, whereas the second one was more subtle. It turns out that the first mistake has impacts on the results of the paper, whereas the second one can be repaired and hence has no effect on the results.

The oversight appears in Section 4.1 where the basic reproduction number $R_{BA}^0$ for the SEIR-ω model is derived, and it only affects the case $αω_{EI} > 0$. There the probability for an exposed but not yet infectious individual to rewire away from its infector, and reconnect to a new (susceptible) individual, is computed. The competing events are that the exposed individual rewires (at rate $ω_{EI}$ and only with probability $α$ does the individual reconnect to a new individual), that the individual becomes infectious (when he/she stops rewiring), but also if the infector stops being infectious and recovers, because then the exposed individual stops rewiring according to the model. This last possibility was forgotten and the rate $γ$ at which the infector recovers is missing in the denominator of the probability. So, the last term in Equation (5) should be $αω_{EI}/(ϕ + γ + ω_{EI})$, and the correct expression for the basic reproduction number is

$$R_{BA}^0 = \frac{ϕβ}{(β + ω_{SE})(β + γ + ω_{SI})} \left( E(\tilde{D}) - 1 + \frac{αω_{EI}}{ϕ + γ + ω_{EI}} \right).$$

(1)

This corrected term $αω_{EI}/(ϕ + γ + ω_{EI})$ should also replace the old expression (without $γ$) in the last term on the right of Equation (7) when deriving the exponential growth rate $r$ (the event that the infector recovers is also left out in the integrals preceding this equation). The corrected version of Equation (7) is

$$\frac{βϕ}{(r + ω_{SI} + β + γ)(r + φ + ω_{SE})} \left( E(\tilde{D}) - 1 + \frac{αω_{EI}}{r + φ + γ + ω_{EI}} \right) = 1.$$

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Finally, according to expression (1) of $R_{0}^{BA}$, the right panel of Figure 2 which compares both basic reproduction numbers now becomes

The reason why this mistake has consequences on the results is that after this correction has been made, $R_{0}^{BA}$ now has the same threshold value as $R_{0}^{PA}$, the basic reproduction number obtained from the pair approximation. The two reproduction numbers are not identical, but traverse the threshold value $R_{0} = 1$ for the same parameter set-up, as is evident from comparing the new $R_{0}^{BA}$ with last equation of the Appendix. One of the general conclusions of the paper, for example stated in the last sentence of the abstract, is hence wrong. The conclusion is not that the two approximations have different thresholds for the SEIR-ω model when $\alpha \omega_{IE} > 0$, but instead that they give the same threshold value (although the reproduction numbers remain different from each other in other parts of the parameter space as correctly stated).

Similarly as before, the second mistake applies only to the situation where $\alpha \omega_{IE} > 0$. In Section 4 we approximate the initial phase of the epidemic by a branching process assuming a large initial population. A basic assumption for branching processes is that siblings give birth to new children independently. However, in the SEIR-ω model when $\alpha \omega_{IE} > 0$ this is not the case as the following example shows. Suppose an individual infects two of its neighbours who then are siblings in the approximating branching process. These two siblings then infect their other neighbours independently. However, they each may also infect one individual using the edge of their common infector in the case when they rewire away from their common infector; these two events both depend on the duration of the infectious period of their common infector (if he/she has a very long infectious period it is more likely that both rewire and infect the new neighbour) and are thus not independent. This implies that the defined limiting process is not really a branching process.
However, it is possible to approximate the initial phase of the epidemic by a multitype branching process (where the type is the number of neighbors at the end of the latent period). This process has a next generation matrix, say $M = (m_{ij})$, which has rank 1 since $m_{ij} = im_{1j}$ for all $i, j$. As a consequence, it can be shown that the basic reproduction number of the "correct" multitype branching process equals the basic reproduction number $R_0^{BA}$ given in Equation (1) above.

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