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SURVEY PAPER

Research frameworks for the study of language in mathematics education

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Abstract
In the context of refinement of frameworks over the past decades within the domain of mathematics education research on language, the development of more nuanced theories is a challenge. In this issue of ZDM, a number of researchers present their work of exploration and elaboration of theories for the study and understanding of language in mathematics education. Since various relevant frameworks are present in the collection of papers, we use them to consider and evaluate the existing ontology. We aim to answer the following questions: What theories and concepts are visible in the papers? What are the works of some of the authors and terms that seem to be interpreted differently? What does this complexity imply for research in mathematics education? From the answers to these questions, we conclude that the domain can be characterised by its complexity, diversity, and contention. All three phenomena together seem to have the potential to be a strength for the progress of the domain.

Keywords Mathematics education and language · Research frameworks · Complexity · Diversity · Contention

1 Introduction

Language and language processes are researched in various scientific fields such as linguistics, psychology, sociology, semiology, anthropology, philosophy and cognitive sciences. Hence development of knowledge on language and language processes depends significantly on progress in those fields and their subfields (applied linguistics, sociolinguistics, psycholinguistics, cultural and social psychology, discourse studies, semiotics, etc.). A crucial challenge of mathematics education research on language is the integration of knowledge from diverse fields and sources for the creation of meaning with implications for the understanding of language in mathematics teaching, learning and thinking. There are two sides of the coin. On the one hand, there is the study of out-of-the-field knowledge, and on the other, the production of robust knowledge that is field-specific for the learning of mathematics. At this point in history, some researchers in mathematics education view the sides of the coin as full of possibilities with respect to the study of language, its foundations (Presmeg and Radford 2008) and conceptualizations (Morgan 2013).

Despite the successful integration and development of knowledge at many levels, the question of what exactly language is, is subject to extensive debate. This is because, rather than attempting to produce definite answers, mathematics education researchers have largely prioritized the investigation of what language does and/or what can be done by means of language. Four years ago, the issue of ZDM entitled *Language and communication in mathematics education: An overview of research in the field* provided a collection of frameworks and tools used in the domain. In the introductory paper regarding the role of language in the learning, teaching and doing of mathematics, Morgan et al. (2014) claimed that there was a lack of agreement about "how best to describe this role (or these roles), about which language-related practices should be encouraged, or even about what the term *language* [emphasis added] itself encompasses" (p. 843). In addition, Planas et al. (2018) point to the relationship between continuity and complexity, suggesting that, within the domain, one can consider a 'continuum of complexity': "Continuity is accompanied by a phenomenon of increasing complexity in the ways of

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understanding language" (p. 197). Following mainly Morgan et al. (2014) and Planas et al. (2018), but also Radford and Barwell (2016) and various other surveys of subdomains (e.g., Barwell et al. 2017; Herbel-Eisenmann et al. 2017; Radford et al. 2017), in this paper we aim at supporting the building of the domain by addressing three questions:

- What theories and concepts are visible in the papers compiled in this issue of ZDM? (Sect. 3)
- What are the works of some of the authors and terms that seem to be interpreted differently? What does this complexity imply for research in mathematics education? (Sect. 4)

Since the publication of the paper by Austin and Howson (1979), the increased number of survey papers reveals the exponential growth of the domain. Our current update is unique and timely for at least three reasons. First, it is common that researchers appeal to the same terms but use them in different ways. To address this different usage, brief evaluations are provided in the following sections to review theoretical similarities and differences among the papers in this issue that we situate in the same group, as well as among groups. Second, it is not always clear how much the newest approaches build on previous research even if this building is justified in terms of contrast, criticism or rupture. The change in the domain is evident, but it is not evident whether the present trends are a reordering of the same basic theoretical options, meaning there are not as many novelties as claimed. One of the unique aspects of this special issue is that authors have been asked to draw connections to existing theoretical-methodological approaches in the research of mathematics and language and to ensure that the respective theoretical-methodological scaffolding is explicit. All papers have been required to differentiate between aspects that are new and aspects that are reordering already-existing approaches with continued significance for research on mathematics and language. Third, it is important to trace some of the arguments raised by authors in order to examine specific contributions of particular theories and concepts. Although progress is being made in addressing these arguments and the underlying frameworks, we expect that they will remain significant for several years to come.

2 Domain of research, domain of knowledge

Domains of research tend to be perceived, interpreted, evaluated and valued in terms of what is taken as factually known and thus represented as knowledge in the research community. Research on mathematics education and language is invariably established as a distinct domain of research and domain of knowledge, or alternatively as an "area of study" (Moschkovich and Wagner 2018, p. 3). Today, these descriptions are not questioned and justifications of the work on mathematics education and language appear unnecessary. Which are the research problems (Sierpinska and Kilpatrick 1998) and what is taken as known (Ernest 2012) are issues aside from the qualification of the domain.

We use the term ontology to express what is taken as existing and knowable. In particular, we consider the set of institutionalized kinds of objects for description and recognition of features and (lack of) boundaries (Morgan 2013, 2014) within and about the domain. In a philosophical sense, the notion of ontology refers to the study of the objects constructed as existing, understandable and knowable under some sort of reasoning or logic of inquiry. Ontologies therefore consist of theories and concepts that are worthy of study in their own right and constitutive of domains of research and knowledge. As formulated by Ernest (2012) in the singular: "Ontology inquires into the kinds of objects we take for granted [either overtly or covertly] as populating the universe we study, and live and work in, as well as the worldviews associated with these objects" (p. 12). In this issue of ZDM, ontology in the singular stands for the entire set of theories and concepts whose being and existence are a focus of mathematics education researchers across the world in their study of language. Even if researchers do not discuss theories and concepts in ontological terms, they all form the conceptual basis for identifying, organising and evaluating research in the domain. This basis should be accessible to new researchers in the domain, so that they may make use of it in the anticipation of future research problems. We could talk about ontologies in the plural to refer to the fact that different researchers show different orientations and representations of the work, knowledge and notions in the domain. That option could be misinterpreted as an absence of definitions and principles. The productive focus is to look at the richness of meanings that strive to sustain their relevance as they interact with each other. We thus choose ontology in the singular to imply and be consistent with the idea of one diverse domain with many relevant frameworks contributing to "a continuum of complexity" (Planas et al. 2018, p. 197). This continuum admittedly emerges from discrepant uses of concepts in research in the domain, but as we discuss, it also provides foundations for potential results in the future.

Any ontology is problematic for a number of reasons, specifically because what the related domain is about tends to remain under-discussed and communicated as implicit content in the activity of research (Ernest 2012). Mathematics education research on language is not an exception. Various terms have become 'mainstream' not without some apparent vagueness and ambiguity in their meaning. Conceptualizations of language as a synonym for discourse or as reducible to specific kinds of text, for example, are not always addressed by authors in explicit ways. There seems

to be an interchangeable use of different terms for the same concepts, as well as the use of the same terms applied to different concepts, which suggests a certain weakness of ontological commitment. A similar point can be made with the apparently interchangeable use of different meanings for the same theories, as well as the mentioning of the same theory in competing interpretations of the same phenomenon. The Bourdieuean approach to language (see the paper by Jorgensen) takes the words 'structuralism' and 'structuralist', for example, in a sense very different and even antagonistic to the Saussurean theory, to mean that language exists within the social world, and not only within symbolic systems of linguistic forms, and independently of specific languages. We still find the expression of British structuralism to convey the legacy of Halliday (1985) and his systemic functional grammar. We could also refer to German structuralism, American structuralism… It is not such diversity of meanings in itself that weakens the building up of the domain, but rather the under-discussion of precise meanings for theories and concepts in use. We come back to this point later to argue that the domain would benefit from reflecting on what the existence of diverse theories and concepts (Sect. 3) as well as contended authors and terms (Sect. 4) may imply for research in mathematics education.

3 What theories and concepts are visible in this issue of ZDM?

Our method for unveiling an account of the diversity of ontology in our ways of researching and talking about language in the domain is primarily restricted to the papers selected out of a range of papers presented for this issue of ZDM. High-quality proposals were inevitably left out. Rather than looking for representative papers to cover preestablished subdomains and timely topics, we chose the papers with more explicit reflection on the theories adopted, and more clearly oriented to address theoretical concerns with some justified novelty in them. Many of the features and qualities of language highlighted in some of the works compiled are present in previously published works, but there are still important differences in how researchers conceive and write about language. The variety of approaches in the collection of papers offers a picture of some of the objects constructed as existing, understandable and knowable in today's mathematics education research on language. We identify a number of similarities and differences in the papers, which reveal three major groups around theories and concepts in the domain. These groups are as follows: (1) the politics of language and language diversity, (2) the modes of communication and representation in language, and (3) the interactionist dimension of language in classroom discussion. The extent to which features from one group either compete or coexist with those from another, varies for each paper. Thus, these groups themselves do not exclude each other but rather some of their features coexist in some of the papers. Especially in the account of interactionist papers, features of the other groups can be seen in, for example, the understanding of modes of representation and social relationships involved in the environments of teaching and learning. Overlaps are mentioned at various points of the next subsections and illustrated by cross references among papers and among claims situated in different groups. For each subsection, we first use more general references to introduce the discussion for the group of papers.

3.1 The politics of language and language diversity

A large body of mathematics education research on language considers that mathematics education is biased (Planas and Valero 2016) and language is political (Barwell 2016; Planas 2018). The notion of the political is revised to include all societal relationships and mechanisms of power of which language is a salient representation. Accordingly, the politics of language involves relationships of power embedded in language, as concerned with inequalities such as those based in race, social class, gender and differences of capital. The focus is therefore on theories of power. From this view, researchers object to the apolitical stances taken by theorists in the field who see language as internal, bounded and/or epistemologically neutral. These latter stances are criticized with the argument that language is never expressed in a social vacuum but subjected to macro societal structures while shaping thinking and practice of all human beings. Any use of language is thus produced within a configuration of social, cultural, economic and political circumstances, which it reflects and helps to fabricate. Words, phrases and grammars about language diversity, home and school cultures, second-language learners, native speakers or indigenous languages are naturalized as innocent and fixed in the exercise of power.

A subgroup of these works addresses socio-cultural-political features of the macro context for the understanding of language in mathematics education in line with the politics of language and language diversity in education. In contrast to works centred on processes on the micro level (often, but not always, classroom interaction), this subgroup relates the processes on the micro level to phenomena on a macro societal level. As shown in the references of the papers by Jorgensen and by Chronaki and Planas, and illustrated by these papers themselves, this subgroup identifies Bourdieuean, Foucauldian, Deleuzian and other theories of power and concepts being used in the domain for the examination of ideologies involved in the production of language during mathematics teaching and learning. These theories contribute to tackling the problem of the prevalence and

dominance of particular types of the language of mathematics and of the learner. The concept of discourse is associated with language and text, but specifically referred to unveil the ideologies underlying categories such as "competent/poor learner of mathematics", as applied to those who are represented as close to or far from the widely recognized language of mathematics of the teacher in the language of instruction. The paper of Chronaki and Planas is particularly focused on the politics of representation of language diversity as a set of variations on a few languages produced as legitimate. Together with the interrogation of the construction of language diversity in mathematics education research, this area of the domain interrogates claims that see language as a pedagogic resource for mathematics teaching and learning. The pedagogic potential of language is not denied but considered in relation to the discursive structuring practices at school and within the mathematics classroom that support spaces of both marginalization and resistance. This is the substantive focus of the paper by Jorgensen, working with mathematics teachers and indigenous learners in remote rural Australian schools.

A second subgroup is concerned with issues of positioning and identity. While in the papers of Jorgensen and of Chronaki and Planas, one pole is discursive and the other is sociopolitical, in the papers of Langer Osuna and of Tatsis et al., one pole is discursive and the other is psychological and sociocultural. In these accounts of the domain, subjectivity theories open up a discursive view of the sociocultural psychology of interpersonal encounters and provide explanations for how and why learners either accept or resist 'othering' positions in the interaction with peers and the teacher. Power is thought to shape these encounters and the positions that people occupy in settings of mathematics teaching and learning. From this perspective, any position and positioning is a dynamic manifestation of the many dialectical discourses materialized in school mathematics. This manifestation can be seen, for example, in the paper of Tatsis and colleagues, which relates the concepts of authority, positioning and politeness in the analyses of two episodes of group work that differ in the conditions of the teacher mediation. These authors are concerned with the fact that politeness and authority are expressed differently with different languages and among different cultures. The paper of Langer Osuna brings together studies of authority and of learner positioning to highlight the concepts of social and intellectual authority in the analyses of student-led activity in collaborative lessons. She gives an example of a boy she calls Terrance, who resists an identity of disengaged learner of mathematics, and storylines about historically marginalized social identity groups, by negotiating his position in peer discussion and producing a newer positioning of participation in the resolution of the task. Generally in this subgroup, analyses involve a methodology of detecting linguistic marks such as personal pronouns like 'you' or 'we', which can reveal positions and agency and patterns of positioning at particular moments.

3.2 Modes of communication and representation in language

There is another group of works in the domain and in the collection of papers, which also distinguishes language as a social phenomenon integral to culture. Similarly to political frameworks, this group criticizes the theoretical separation between social and individual aspects in the study of language. For this group, however, the substantial claim is that language is understandable, researchable and knowable through the numerous overlapping semiotic modes of communication and representation that always develop in the context of historical processes and cultures (Roth and Radford 2011). In mathematics learning, teaching and doing, this line of research traces language across modes of communication and representation and often in relation to epistemological effects in meaning making. Morgan and Kynigos (2014), in line with social semiotics (Halliday 1978, 1985), show how the algebraic expression ax^2+bx+c may orient learners towards practices of symbolic manipulation like factorization, whereas the drawing of a parabola may orient them towards specific visual features like intersection points with the axes. These effects in individual meaning making are not separable from school cultures and historical traditions such as the teaching of quadratic functions.

A subgroup of these works is particularly interested in the revision and refinement of a concept of representation for the domain. In the papers of Alshwaikh and Morgan and of Roth, this is clearly addressed but with different proposals for conceptualization. On the one hand, Alshwaikh and Morgan recognize representations as communicational means with the ontological role of bringing the mathematical objects of teaching and learning into existence. Discourse and activity are two key terms in this understanding since representations—specialized notations, diagrams, graphs…—are produced, framed and developed multimodally in discursive activity. Foucauldian approaches are also present in the paper of Alshwaikh and Morgan, but now in relation to social semiotics and critical discourse analyses applied to the ways in which mathematics is communicated and represented in Palestinian schools. On the other hand, Roth points to the materiality of representations and its role in providing material presence, with signs and words, to either real or ideal objects so that they can be made accessible to the senses. In this way, Roth draws on the representational function of language and signs, and destabilises Platonic views that see school mathematics as dealing with abstract objects and learners as dealing with their minds to capture these objects. The problem of how mathematics

learners and teachers make meaning of and with language becomes the problem of how they deal with and communicate representations that are found in contexts of mathematical activity. The emphasis is on meaning being produced in language and context, in comparison to emphases indicated in other frameworks (e.g., meaning primarily produced in discourse, in the papers of Moschkovich and Zahner, and of Tatsis et al.).

The paper by Robertson and Graven illustrates the integration of frameworks similar to those used by Alshwaikh and Morgan and by Roth in their papers. Robertson and Graven take the three theorists Bernstein, Halliday, and Vygotsky to emphasize the role of language and socio-cultural-historical factors in mathematics teaching and learning. They propose a multifaceted "transdisciplinary" framework for the study of classroom talk, which provides a triple focus on society, language and semiotic mediation. Bernstein is interpreted to inform the recognition and realisation of legitimate mathematical texts in the wider macro socio-cultural-historical context; Halliday is interpreted to examine the potential of language for mathematical meaning making in the micro context of the classroom; and Vygotsky is interpreted to address dialectic shifts between everyday and scientific mathematical meanings under the related influence of macro and micro contexts. Showing similarities to the interest in the politics of language of Chronaki and Planas, Robertson and Graven see their work as having particular cogency for analyses of postcolonial educational circumstances, where most learners learn mathematics in a colonial language not spoken at home. While these two authors claim the need to complement the attention to Vygotsky with issues of structure and power, in his paper Roth cautions against specific readings of Vygotsky. We actually find very different readings of Vygotsky across the papers in this group, from the idea that Vygotsky did not consider power in language but development only, to the idea that he developed a coherent social theory of historical materialism. In Sect. 4, we more generally discuss the implications of different interpretations of terms and authors in the domain.

3.3 The interactionist dimension of language in classroom discussion

A third group of research papers in the domain situates the study and view of language in strict association with classroom interaction, particularly interactionism and how people use language—their utterances, words, gestures and signs—to achieve different goals in the mathematics classroom. Underlying the understanding of language is the idea that by talking (or signing in the languages of the deaf) and writing in different ways, people engage in action and practice to achieve different goals. All ways of talking and writing are thus constituted in relation to each other (Krummheuer 1995). Differently to what happens in other frameworks, here the macro context is not a focus. For researchers in this line of study, the micro level is taken to develop detailed, microscopic description and examination of situated practices and meaning making processes in situations where people can encounter one another and interact in either physical or virtual spaces. Conversation analysis and discursive psychology together with ethnomethodological principles are key, stating that social reality is ordered, context-bound and a product of the experience of those acting and living in it (Schütz 1962). This being said, we see a crucial move towards the inclusion of sociocultural standpoints (Moschkovich 2002). Even though one might expect a sort of pure subjectivism in this group of works, the analyses of the mathematics classroom integrate the interpretation of subjective meanings of learners and teachers with the attention to meanings that lie outside the individual.

In this issue of ZDM there are four papers that can be located among the works that embrace subjectivist and sociocultural visions in the analysis of classroom interaction. Two of the papers are closer to subjectivism. These are the papers by Ingram and by Jung and Schütte, which might be seen as typically situated at the intersection between the interactionist and ethnomethodological traditions. However, in contrast to works of these traditions that come from earlier periods and precede the views of these authors, Ingram, Jung and Schütte transcend the classical opposition between individual and collective representation of meaning by seeing individual meanings as relative to systems of social meaning. Linguistic processes in the mathematics classroom are thus related to social environments that intervene in how learners and teachers tacitly agree on a basic common/taken-as-shared understanding of the situation. This view is visible in Ingram's paper, focused on the reconceptualization of some key concepts, such as actions and practices, in the ethnomethodological approaches linked to interactionism, conversational analysis and discursive psychology. The further consideration of how social structures and patterns of turn-taking are co-constructed in classroom interaction illustrates well the move beyond strict subjectivism and individual experience. In the paper by Jung and Schütte, this move is also visible. In their paper, the authors address the discourses entering the mathematics classroom together with the subjective meanings manifested in the interaction among the participants in mixed-ability groups of children. Although the emphasis is not on the modes of communication, some of the oral features of classroom discussion considered are present in the paper of Alshwaikh and Morgan.

While Ingram and Jung and Schütte base their analyses on interactionism and ethnomethodology, adopting a subjectivist perspective that can enrich sociocultural approaches, two other papers argue in favour of enriching subjectivist visions by means of sociocultural approaches. These are the

papers of Wessel and Erath and of Moschkovich and Zahner, where the understanding of any lesson is seen to require access to more than what is available in the spoken text of the visible interaction. Wessel and Erath bring to the fore the scaffolding role of the language of the teacher in mathematics classrooms with learners whose home language is not the language of instruction, as someone who occupies the elevated social position required to modify discursive practices and develop strategies of lexical support. The relevance of instructional practice is indicated in the attempt to compare micro scaffolding moves of different teachers who focus on lexical work and discursive activation. As in the paper of Tatsis and colleagues, the attention to the teacher mediation in the analysis is thus crucial. Moschkovich and Zahner add to the discussion the analysis of the mathematical languages and discursive practices of learners in peer communication in the multilingual classroom, specifically in their processes of appropriation of academic mathematical language through participation in practice. The two papers expand subjectivist visions by highlighting a notion of language as socioculturally situated in mathematical discursive practices and constitutive of classroom interaction by means of multiple modes and registers. Once more, we see similarities with papers in other groups. As in the paper of Robertson and Graven, registers in plural mainly consist of the everyday and the academic types. Modes of communication and of representation are also considered but they are seen as instrumental to the ultimate goal aimed at understanding the mathematical practices in the classroom.

4 What are the works of some of the authors and terms that seem to be interpreted differently? What does this complexity imply for research in mathematics education?

Section 3 reveals widespread thinking of language as social in mathematics education research on language. The move away from views of language as logical and biological is not surprising if we consider the continuing influence of linguistics and psychology in the domain together with changes in these fields. In both these latter fields progressive attention has been paid to social aspects such as gender, ethnicity, poverty and social class, together with more sophisticated understandings of the social layers intervening in the construction of identity and ability. The interest in mixed ability grouping arrangements as presented in the paper of Jung and Schütte, for example, is nowadays a major line of concern in the psychological research in education that questions the explanatory power of the idea of disability. Above, we have identified three major groups——politics, communication and representation, and interactionism—involved in the view of language as social across the papers in this issue. In this section, we address some of the constructs of authors, and terms used, that seem to be differently interpreted, the discussion of which could be strengthened in the coming years. There are a number of umbrella terms with different contended meanings for different researchers, which we see related to interpretations of the writings of fundamental authors and common terms used, and importantly to the theoretical development of the domain as a continuum of complexity (Planas et al. 2018).

In view of the differences between and within diverse groups of work in the domain (three of which can be traced in the papers compiled), some fundamental publications and common terms are the subject of divergent interpretations and uses. It is important to emphasize that what is contended is not the relevance of the terms and publications themselves, but the specific meanings and narratives attached to them. Whilst contended terms and authors' writings challenge theoretical clarity and sometimes even create confusion, they also reflect a more profound contention between frameworks. The recognition of contention of terms and authors as an asset can create opportunities to strengthen the building of the domain, and consequently should not be resisted or undermined. When we find terms that are used loosely in the literature, this is not necessarily due to diversity of meanings or lack of consensus. It is the presentation of such terms as certain and taken-as-shared, as if there were no other possible readings, that weakens the theoretical discussion of the research undertaken. That contended terms are of value is not new. The inception of the view of "culture" in Skovsmose's (2005) book is inspiring: "Culture is changing and developing, it includes new elements, good and problematic ones, in a complex mix" (p. 6). The complex mix of elements of changing culture was implicit already and acknowledged in Bishop's (1988) book. Today, culture is a strong notion subjected to diverse theorizations, which have allowed for a multiplicity of meanings. One meaning of culture shared by all mathematics education researchers would be symptomatic of only one framework, which would be detrimental to the variety of research interests and goals to be achieved. This same argument applies to other terms and the risks of building the domain around one framework only.

Attaching a single meaning to language, for example, would be detrimental and difficult to imagine since meanings and narratives continue to develop in the context of the domain, and different researchers tend to think exclusively within their frameworks. The challenge is not to set limits to meanings. Rather, we need to improve our capacity to view the diversity of interpretations of authors and terms as an asset. The 'problem' is not the existence of contended terms, writings and frameworks, etc., but the difficulty of capturing and exploiting the opportunities that emerge from

the complexity of meaning. The difficulty of deciding which meanings promulgated by an author are to be presented in each particular study is no less important. It is unclear what can be taken as shared with other researchers involved in the conversation and which are the other uses of the words at hand. This is indeed a renewed version of the problem of the meaning. There is no uniformity of meaning between what we say and what others understand but, be that as it may, uniformity is not a quality to be searched for in complex theoretical discussion.

4.1 Differing interpretations of the writings of fundamental authors

We started this paper with reference to the challenge of specifying and re-situating knowledge constituted as well-established in other domains of research. Jablonka and Bergsten (2010) point to the many attempts of mathematics education researchers to develop comprehensive readings of out-of-the-field theories and concepts. Some of these attempts are clear in the ways numerous authors align their work with Lev S. Vygotsky to emphasize, for example, the connection between the social and the individual, the relevance of mediated action and tools, or the role of scaffolding in human learning. It would be difficult to entirely agree on a list of fundamental authors. Nonetheless, it would be even more difficult not to agree in principle on the relevance of Vygotsky and his role (see Lerman 2000, for a discussion of the beginnings in the late 1980s of Vygotskian influences in mathematics education research). Seven of the papers in this issue refer to Vygotsky through either direct citation (the papers of Robertson and Graven, and Roth) or commentators like Wertsch (the paper of Moschkovich and Zahner), Bruner (the papers of Wessel and Erath, and Ingram), Bauersfeld (the paper of Jung and Schütte) and Lave and Wenger (the paper of Chronaki and Planas). These are papers that we have identified as implying different groups regarding the set of related theories and concepts. All these authors draw from Vygotsky, but their focuses take them in different directions.

The uptake of Vygotsky seems clear with respect to the representation of everyday languages to access scientific languages (the paper of Robertson and Graven) or the activity of scaffolding in the interaction (the paper of Wessel and Erath). Indeed, in his paper Roth reminds us that the use of the work of Vygotsky in the domain is inherently complex and diverse. There are theoretical subtleties of relevance, particularly regarding Vygotsky's later work, which is much less widely familiar. The notion of language as the external expression of something operating within the individual is produced as the most distinctive interpretation attributed to Vygotsky, also in the papers in this issue that refer to the mediating role of language. But this is not distinctive of the later Vygotsky. Roth questions the uptake of Vygotsky in educational research on language that focuses on the study of the dyad, the small group or the class speech as a context for learning. In such research, the moves from and towards the outside/speech to the inside/mind are generally used to indicate learning. By omitting the later Vygotsky, he argues, the conceptualization of the social is reduced to the plane of the individuals interacting. Importantly, the inseparability of the individual and the social is neglected as manifested in studies that differentiate between mental processes (e.g., thinking what to write in a personal diary, thinking what the activity in the small group is about) and material processes (e.g., writing a personal diary, speaking with peers in the small group about the task to be done).

The interrogation of some readings of Vygotsky confronts us with the larger question of the use of other fundamental theorists. In four of the papers in this issue, there are numerous mentions of Michael A. K. Halliday and his work in linguistics that contributes to the coordination of syntactics and semantics. Are there meanings taken as shared that are imposing a particular interpretation of Halliday in the domain? Is something new emerging from the readings of Halliday? There is evidence of Halliday's work starting to be used in mathematics education by the end of the last century by authors such as Morgan (1996), who refers to the notion of mathematical register—the meanings that belong to the language of mathematics—and to the idea of what is meant by the language of mathematics itself. Regarding the Hallidayan term "mathematical register", it is interesting how the paper of Langer-Osuna presents it as a "specialized form of discourse", meaning specific discursive practices in the mathematics classroom. If we take the paper of Alshwaikh and Morgan, citing Halliday and developing Hallidayan methods, we find the focus on the use of a specialized discursive form of language in the materiality of "specialized activities of school mathematics classrooms." Indeed, Morgan is a must-read interpreter of Halliday (e.g., Morgan 2006), for other researchers in the domain who wish to trace applications of Halliday to the analysis of school mathematical activity. Still citing Halliday, Moschkovich and Zahner suggest a substantially different use of the term 'register' in their distinction of three major registers in the mathematics classroom, namely, school mathematical language, home languages and the everyday register. In addition to the different interpretations of 'register' in these papers, in their paper Robertson and Graven propose a deconstruction of Halliday's linguistics in combination with Vygotsky's psychology and Bernstein's sociology for a multifaceted linguistic-psychological-sociological analysis of talk in the mathematics classroom.

What are then the implications of interpreting the constructs of authors differently, for research in mathematics education? Is it a 'problem' for the field that different researchers interpret and use the writings of the same author

in different ways? If so, is this a kind of problem that can be addressed, avoided and/or solved by the research community deciding, for example, who should be the ultimate interpreter? To us, it is reasonable and valuable that the continuous developments of domains of research promote new (readings of) theories and concepts, which in turn provide new frameworks in the sense of models of interpretation for the use of constructs of fundamental authors. There is not the correct representation of an author's original intended meaning in a text, but conventional meanings and representations that have lasted for years or decades. Therefore, authors and their writings have always to be considered and interpreted in relation to their time with its specific social, cultural, and political situation. What we actually see as a problem is the fact that already-established readings of an author in our domain are not always challenged and connected in dialogue with newer readings produced within newer frameworks. For example, the reinterpretations of Vygotsky cannot be outdated and separated from the context of development of the particular domain. Regarding the papers compiled, we see all them contributing to contemporary research in mathematics education and one of its present research problems, the rethinking of the individual and the social in more nuanced ways, in line with recent developments and different frameworks. Thus, the diversity of interpretations of authors' writings becomes a problem when these interpretations are not produced in strong relation to the present research problems in the domain. We can still argue about the difficulty of determining the precise research problems in the domain of mathematics education at present. However, this is a concern at a different meta-level.

4.2 Differing interpretations of common terms

To finish this section, we expand the idea of fundamental authors to common terms that have become fundamental in mathematics education research on language. We refer to terms which have a high value in the papers of this issue and beyond in the entire domain whose meanings cannot be described by simple connection to one major group. The term 'discourse' is a peculiar example of a term that has widely been produced and accepted as fundamental in the domain. This is not, however, without conflict between meanings and narratives attached to this term. Newer researchers or researchers from other domains encountering the word "discourse" for the first time cannot merely search for the definition in a dictionary. Indeed such definitions (e.g., "*any unit of connected speech or writing longer than a sentence*") may be detrimental to understanding the many complementary uses of the term in mathematics education research on language. As when for example an adjective is associated with a noun which specifies the meaning of it, it is in the combination of the word "discourse" with other words within particular frameworks that the concept emerges.

The presentation of different interpretations of the term 'discourse' is evident across the papers compiled, and to some extent is 'symptomatic' of differences in the understanding of the social in language. The papers that we group around the politics of language and language diversity use 'discourse' to mean that material written, spoken and signed texts are not enough to understand the social situations and actions of people in relation to mathematics teaching and learning, and to mathematics education research. In the paper of Langer-Osuna, discourse becomes a tool for claims and imputations of social identity to learners in the mathematics classroom. In their paper, Chronaki and Planas focus on "the discourses on language and on language diversity" in mathematics education research and their circulation in the academia. They present a Foucauldian type of discourse analysis of what other mathematics education researchers say and write about language (sometimes to mean only 'natural' languages like Catalan, Spanish or German) and language diversity in order to interrogate remedying discourses based on ideas of ownership encoding dichotomies of high/low language proficiency/deficiency. Another hegemonic discourse identified by these authors is that of language as a tool of communication in which lesson texts are seen or examined as autonomous of their social and political contexts. In this understanding, discourses are statements that structure knowledge and practice about a given topic, with the power to organise how that topic is talked about, conceptualised and researched. The discourses on language thus shape the possibilities for researchers to challenge and develop the domain in ways that can be more or less easily recognised as a legitimate option in the research community.

In many of the papers, there is a common sociocultural view of discourse as a concept that contributes to expand views of language as text. The paper of Moschkovich and Zahner, for example, considers mathematical discourse to include oral and written text in combination with academic and everyday languages, but also specific building practices. Here, the emphasis is on the mediation of 'language' (e.g., the language of instruction) and 'discourse practices' (e.g., generalising, abstracting, and searching for certainty) in bilingual classrooms settings. There is a similar distinction in the paper of Wessel and Erath, where the emphasis is on the articulation of 'the discursive level' and 'the lexical level' in the design of teaching and learning arrangements for multilingual mathematical communication. Examples of discourse practices in this paper are explaining, arguing, reporting and describing, while examples of language practices are revoicing, reformulating, repeating utterances and initiating students' self-repair of utterances. In accordance with these interpretations, the questions that have arisen with respect to discourse are researched through the analysis of

practice. The focus is not discourse but the whole intersection of social practices of which discourse is a part. In the papers grouped around the interactionist dimension of language in classroom discussion, 'discourse' is also sociocultural. Here, we find the strong link between discourse and discourse analysis. Thus, types of discourse analysis become the main source of meaning for discourse, despite the focus not being discourse per se. In the paper of Jung and Schütte, discourse is analysed through interactional analysis attached to ethnomethodology, discursive psychology, and symbolic interactionism. In the paper of Ingram, discourse is conceptualised as an instrument of communication whose structures and patterns can be brought to the surface and featured through conversational discourse analysis. In this paper, discourse is strongly equated with written and spoken discourse, and perceived as a cultural tool for the representation and making of meaning in classroom interaction. We come in this way to the paper of Alshwaikh and Morgan and the Hallidayan argument that all texts in a lesson come with cultural meanings that shape the kinds of actions and practices that can be done with them and those that cannot. But also to the paper of Jorgensen and the Bourdieuean argument that learners of mathematics need to learn how to appropriate certain uses of texts into their habitus and deploy them in situations of teaching and learning.

We could continue with the discussion of the diverse uses of the term 'discourse' in the papers compiled. As with the diversity of interpretations of the writings of authors, however, we intend to reach some kind of conclusion that allows us to consider the implications of using fundamental terms differently. At first sight, it is unclear whether the potential value argued for the existence of contended authors also applies in the case of terms such as discourse. The different authors of the papers in this issue develop different frameworks and, accordingly, work with different uses of the constructs of fundamental authors and terms. A conflict like the one between uses of the term 'discourse' is in the end a conflict between the development of different theoretical frameworks. It is not reasonable to see the diversity of interpretations of the same term as just a problem of unreliability, imprecision or rigor. The complexity of a dynamic domain with diverse theoretical frameworks at play grounds the view on the contention of authors' work and terms as unavoidable and valuable. To us, the problem lies in the poor dialogue between uses of the same term, and hence between frameworks in the domain. One serious effect of this poor dialogue is that researchers contributing to the same area of research may not understand each other. The aim of dialogue is not to correct the 'other' interpretations, but to understand them and learn from their contributions to specific research problems, as well as to identify differences and similarities in these in order to advance the field. Indeed, progress in mathematics education research on language has often originated in the dialogue and contrast between frameworks (Bikner-Ahsbahs and Prediger 2006).

5 Conclusion

With the aid of guiding questions, the study of the papers in this issue of ZDM has provided an overview of the diversity and complexity of theories and concepts of today's mathematics education research on language, as well as an evaluation of the contention of fundamental authors' work and common terms. To answer *what theories and concepts are visible in the papers compiled*, we have revealed the complexity of diverse thinking about language as social. In these papers, three prevalent groups can be identified around: 'the politics of language and language diversity', 'the modes of communication and representation in language', and 'the interactionistic dimension of language in classroom discussion'. To answer the questions *what are the works of some of the authors and terms that seem to be differently interpreted and what does this complexity imply for the domain*, we have reflected on the phenomenon of contention. The theoretical discussions that originate in the contention of meanings for authors like Vygotsky are, from our perspective, a potential beneficial source for the domain. Regarding terms, 'discourse' serves as a fundamental term whose use and understanding varies tremendously in line with the frameworks taken by researchers.

Although much research has been done, we have only just begun to explore the potential value of the related phenomena of complexity, diversity and contention that research in mathematics education and language has produced. The time has come to structure, evaluate and contrast different frameworks and detect—hitherto hidden—similarities and differences that have evolved along the way. The goal here is not to generate unified terms that may be taken as shared, but rather to draw out a network of uses and interpretations around terms and authors that are presumed to be clear-cut. This will lead to a better understanding of the possibilities of the domain and a richer foundation for numerous researchers, both those newly entering the domain and those more experienced already working within it. We strongly believe that this issue will provide an essential contribution to the building of the domain, together with the understanding of the strengths involved in its diversity, complexity and contention. The domain will certainly benefit from reinforcing the debates of the many research problems from multiple frameworks, as well as from fully accepting the diversity, complexity and contention inherent in the debates, as topics of research and debate themselves.

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