The Use of Equity Financing in Debt Renegotiation

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Abstract

Debt renegotiation is often modeled as pure debt for equity or debt for debt swaps in the theoretical literature. However, the empirical evidence in the debt repurchase literature shows that a combination of debt and equity is used in renegotiation. In this paper we analyze the use of equity financing in addition to debt financing in debt repurchases. Firms with larger volatility, lower cash flow growth rates, or higher recovery rates are more likely to use equity financing in debt renegotiation. Flotation and renegotiation costs, the bargaining power of the creditors, and macroeconomic variables also influence this choice. When equity issuance is a possible source of financing in renegotiation, firms optimally choose smoother coupons and welfare increases as compared to pure debt for debt swaps. We provide closed-form solutions for the optimal use of funding and we derive novel testable empirical implications regarding the use of equity financing in debt repurchases.

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1. Introduction

Corporate debt renegotiation has been extensively studied in the literature. Different formulations of reorganization have been proposed starting from the well-known strategic debt service (Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997; Fan and Sundaresan, 2000), to the debt for equity swap (Fan and Sundaresan, 2000), and to the pure debt for debt swap (Mella-Barral, 1999; Lambrecht, 2001; Moraux and Silaghi, 2014).

Unlike bank debt that is relatively easy to renegotiate in a private workout, publicly traded debt is difficult or impossible to renegotiate outside of a formal bankruptcy procedure (Bolton and Scharfstein, 1996). In this context, Brandon (2013) argues that debt repurchases are a "market-based substitute for the renegotiation of corporate bonds". Debt tender and exchange offers have been extensively studied in the empirical literature (see Franks and Torous, 1994; Chatterjee et al., 1995; Lie et al., 2001, among others). The existing evidence shows that bond holders typically receive a bundle of securities (debt, equity and cash) in exchange for their debt (Franks and Torous, 1994). More recently, Altman and Karlin (2009) find that although some firms make pure debt for debt or debt for equity offers, others use a combination of both debt and equity. Moreover, Kruse et al. (2014) find that only 39.9% of the debt tender offers in their sample use as a source of funds public debt, the other sources of financing being asset sales (14.9%), bank debt (13.9%), and common equity (13.9%).

Therefore, the empirical literature shows that a combination of debt and equity is used in debt renegotiation. However, the use of equity financing along with debt in debt renegotiation has received very little attention in the theoretical literature.[1] In this paper, we attempt to fill this gap in the literature by providing a first theoretical analysis, to our best knowledge, of the use of equity financing in debt renegotiation. We therefore bridge the gap between two strands of the literature: the theoretical work on structural models of debt renegotiation and the empirical literature on debt reducing tender and exchange offers. On the one hand, we propose a structural model that incorporates taxes, bankruptcy and renegotiation costs. Renegotiation timing is optimally decided by the claimholders. Following Mella-Barral (1999), Lambrecht (2001), and Moraux and Silaghi (2014), renegotiation consists of

[1] Two notable exceptions analyzed below are Sarkar (2013) and Nishihara and Shibata (2016).

a permanent coupon reduction. Unlike these studies however, we do not restrain the choice of the optimal reduced coupon, by allowing for transfers among claimholders. This implies that renegotiation is not just a pure debt for debt swap, but that it can also involve the use of new equity financing. We provide closed-form solutions for the optimal reduced coupon in debt renegotiation with equity financing. Furthermore, since forced asset sales are common in practice,[2] we extend our benchmark model to account for asset sales as a third source of financing for debt renegotiation. On the other hand, we contribute to the literature on debt reducing tender and exchange offers by analyzing which firms are more likely to use equity financing in renegotiation, and how the use and amount of equity financing are influenced by the firm characteristics (volatility, cash flow growth rate, recovery rate), market variables (tax rate, interest rate), equity issuance costs and renegotiation costs. The novel empirical implications derived from the model could foster further empirical research on debt repurchase.

We find that firms that have lower cash flow growth rates, larger volatility, and larger recovery rates, which are forced to sell assets, or which operate on markets with low corporate tax rates and interest rates, are more likely to use equity financing in renegotiation. On the contrary, firms with a relative large bargaining power for the equity holder, relatively larger equity issuance costs, and lower renegotiation costs are less likely to issue equity to repurchase debt.

Comparing with a constrained situation where transfers between claimholders are not allowed, we find that allowing for transfers and thus for equity issuance in debt renegotiation leads to an increase in welfare. For our benchmark parameter values, the firm value at renegotiation increases by up to 2.81%. Moreover, the firm's choice of the optimal coupon is smoother. When the equity holder has a relatively high bargaining power, the optimal coupon is up to 14% higher compared to a case where transfers are not allowed. In the opposite case when creditors have a relatively high bargaining power, the optimal coupon is up to 19% lower compared to the constrained case.

Closely related papers in the theoretical literature are Sarkar (2013), Moraux and Silaghi (2014), and Nishihara and Shibata (2016). Moraux and Silaghi (2014) analyze the optimal number of debt renegotiations in a framework with multiple costly renegotiations, where transfers between

[2] Djankov et al. (2008) find that excessive forced asset sales of viable businesses make debt enforcement inefficient.

claimholders are not allowed. Renegotiation in their model is thus a pure debt for debt swap. On the contrary, we relax this assumption, by allowing for transfers between the equity holder and the creditors, and we consider a single renegotiation in order to keep the analysis tractable.[3] When transfers are allowed the firm can issue equity in renegotiation and debt can be repurchased using both types of financing sources. This leads to smoother coupon reductions and an increase in welfare.

Nishihara and Shibata (2016) focus on the choice between debt renegotiation using partial asset sales and direct full liquidation. They allow for equity financing to be used in renegotiation, however they do not investigate how the use of equity financing varies across firms, nor do they provide analytical solutions for the optimal reduced coupon. They note in their numerical analysis that equity financing always appears to be positive for reasonable parameter values. In this paper nevertheless, we provide a thorough analytical study of the use of equity financing in renegotiation, and show that, on the contrary, different costs can deter the use of equity financing in renegotiation.

Lastly, Sarkar (2013) proposes a general exchange offer, with both a permanent debt reduction and an equity stake for the creditors, modeled through a Nash bargaining game. When the firm is in distress, shareholders can threaten lenders with default and obtain concessions from them. As opposed to our model, cash and asset sales are excluded in their approach. Moreover, there are two other key differences with respect to our model. On the one hand, the shareholders make the exchange offer before they would declare bankruptcy, and not at the bankruptcy threshold as in our model. This raises the problem that the threat becomes non-credible given that equity holders threaten to liquidate the firm although it would be better for them to keep servicing the existing debt (Christensen et al., 2014). On the other hand, for the baseline parameter values considered, their results show that creditors receive a new debt with a reduced coupon and a *positive* equity stake in exchange for the initial debt claim. On the contrary, we show that the new debt claim can sometimes more than compensate the creditors for the initial one, which implies a "*negative* equity stake" in the firm. That is, there exists a transfer from the creditors to the equity holder in renegotia-

[3] Although multiple rounds are common, Godlewski (2015b) finds that more than 65% of loans in his sample are renegotiated only once. Therefore, even analyzing this simple case of one single renegotiation round can have relevant implications in practice.

tion, and not the other way around. We thus challenge Sarkar (2013)'s result that a combination of debt and equity dominates a pure debt swap.

Finally, our paper is also related to the literature on financial distress and equity issuance. On the one hand, according to the debt overhang theory, firms in distress would not issue equity since the benefits obtained from additional equity issuance would be entirely transferred from the equity holder to existing debt holders. However, the empirical evidence from the equity issuance literature is contrary to this prediction. A strong positive relationship between distress and equity issuance is documented for both public and private placements (DeAngelo et al., 2010; Brophy et al., 2009; Park, 2014). This is in line with our model's prediction that more distressed firms are more likely to use equity financing in renegotiation. We can therefore provide a theoretical explanation for this stylized fact. On the other hand, firms in financial distress might be near their debt capacity or might have debt covenants that legally restrict them from issuing more debt, leaving equity issuance as the only viable option. We show that this is not the case, firms in our model are far from their debt capacity (thus not forced to issue equity), and optimally choose to issue equity even in the absence of covenants. The rest of this paper is structured as follows: Section 2 describes our financial setup and valuation of financial claims. Section 3 presents our benchmark model regarding the use of equity financing in debt renegotiation, while section 4 illustrates it through numerical simulations. In section 5 we extend the benchmark model to allow for forced asset sales. Empirical implications are derived in section 6. Finally, section 7 concludes.

2. Financial setup and valuation

In this section we initially describe the continuous-time financial setup we use. We then introduce our model of debt renegotiation and present the valuation of financial claims. We consider a firm that is financed by equity and a consol debt only. The initial coupon value is denoted by c. The firms' EBIT (Earnings before interests and taxes), X, follows a geometric Brownian motion:[4]

$$dX_t = \mu X_t dt + \sigma X_t dW_t, X_0 = x, \tag{1}$$

[4]The EBIT we consider is net of any running costs, which implies that the equity holder of a fully-equity financed firm would perpetually operate the firm without liquidation.

where $W = (W_t)_t$ is a standard Brownian motion, $x > 0$, and μ and σ represent the drift and volatility terms, respectively.

The firm pays income taxes at a rate τ. The interest rate is denoted by $r > \mu$ (see Dixit and Pindyck, 1994). In case of liquidation, the proceeds are $\frac{\alpha X_t}{r-\mu}$, where $\alpha \in (0, 1)$ represents the recovery rate.[5]

For the purpose of valuation, we will consider first the case in which there is no renegotiation, which will serve as a benchmark for the case with renegotiation.

2.1. No renegotiation

Let us assume that there exists no renegotiation and that the firm is directly liquidated. We denote the equity, debt, and firm values by $E(x, c)$, $D(x, c)$, and $V(x, c)$, respectively. Following the standard literature (see Leland, 1994; Goldstein et al., 2001), we obtain:

$$E(x,c) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)c}{r} - \left(\frac{(1-\tau)x_B(c)}{r-\mu} - \frac{(1-\tau)c}{r}\right)\left(\frac{x}{x_B(c)}\right)^{\gamma}, \tag{2}$$

$$D(x,c) = \frac{c}{r}\left(1 - \left(\frac{x}{x_B(c)}\right)^{\gamma}\right) + \frac{\alpha x_B(c)}{r-\mu}\left(\frac{x}{x_B(c)}\right)^{\gamma}, \tag{3}$$

$$\begin{aligned} V(x,c) &= E(x,c) + D(x,c) \\ &= \frac{(1-\tau)x}{r-\mu} + \frac{\tau c}{r} - \left(\frac{(1-\tau)x_B(c)}{r-\mu} + \frac{\tau c}{r} - \frac{\alpha x_B(c)}{r-\mu}\right)\left(\frac{x}{x_B(c)}\right)^{\gamma}, \end{aligned} \tag{4}$$

with

$$x_B(c) = \frac{\gamma(r-\mu)c}{(\gamma-1)r}, \tag{5}$$

representing the default threshold. Here, the constant γ is given by $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} < 0$ and the EBIT value x is

[5] In the main model we assume that partial asset sales are not possible. As an extension we will consider forced partial asset sales in Section 5.

higher than the default threshold, which is endogenously chosen by the equity holder. Note that $x_B(c)$ is the optimal default threshold in the absence of renegotiation.

2.2. Debt renegotiation

Renegotiation consists of permanently reducing the initial coupon c to a lower payment of c_1. We thus have a lump-sum and permanent coupon reduction, following Mella-Barral (1999), Lambrecht (2001) and Moraux and Silaghi (2014). Indeed, continuous and infinitesimal coupon reductions as in Mella-Barral and Perraudin (1997) or Fan and Sundaresan (2000) are not likely to occur in practice due to renegotiation costs.[6] The renegotiation time is optimally chosen according to the bargaining power of the claimants (equity holder and creditors). We denote the renegotiation threshold by x_R. The claim values at renegotiation then become simple no-renegotiation values as expressed in the previous section, given by $E(x_R, c_1)$, $D(x_R, c_1)$, and $V(x_R, c_1)$. The final post-renegotiation liquidation time is endogenously chosen by the equity holder and denoted by $x_{B_1} \equiv x_B(c_1)$.

Debt renegotiation is costly and implies renegotiation costs proportional to the debt value just prior to renegotiation: $k_R D(x_R, c)$.[7] These renegotiation costs are suffered by the equity holder.[8]

The renegotiation surplus net of renegotiation costs is divided between the

[6]In practice, renegotiation can consist of amending one or several contractual terms, such as the amount, the maturity, the covenants, etc. In our framework however, since we consider a perpetual debt, modeling renegotiation through a maturity extension or face value reduction is not feasible. Nevertheless, our permanent coupon reduction for a perpetual debt is similar to an amount amendment for a finite debt, which seems to be quite relevant in practice. Indeed, Godlewski (2015a) finds that the amount is the most often amended term in his sample of European loans.

[7]We could also assume that the renegotiation costs are proportional to the firm value at restructuring like in Koziol (2010) or to the coupon reduction as in Hackbarth et al. (2007), or we could assume fixed renegotiation costs as in Moraux and Silaghi (2014). As long as the renegotiation costs are not proportional to the renegotiation surplus, we would obtain a finite number of renegotiations in a context of multiple renegotiations. The implications of the model are robust to this assumption.

[8]Nishihara and Shibata (2016) also assume that the equity holder suffers the renegotiation costs. Moraux and Silaghi (2014) allow for renegotiation costs to be suffered either by the party that has the bargaining power, or always by the equity holder. The former assumption has no impact on the main implications of our model, and brings no further insights.

claimants according to their bargaining power. In particular, the creditors get $\beta D(x_R, c)$, where $\beta \geq 1$ represents the creditors' premium, and is an indicator of the bargaining power of the creditors. If $\beta = 1$, then creditors are indifferent between renegotiation and liquidation, and the equity holder captures all the renegotiation surplus.

Unlike in previous studies, in this model debt renegotiation does not consist of a pure debt for debt swap. We allow for lump-sum transfers between the claimants. Although the creditors should obtain $\beta D(x_R, c)$ in renegotiation, the new debt value at renegotiation is $D(x_R, c_1)$, which could be different. Therefore, there is a lump-sum transfer of $\beta D(x_R, c) - D(x_R, c_1)$. Note that this transfer could be either positive or negative, depending on the bargaining power of the creditors, β, and the reduced coupon, c_1. The total transfer made by the equity holder at renegotiation is therefore:

$$EF(c_1) = (\beta + k_R)D(x_R, c) - D(x_R, c_1), \tag{6}$$

including the renegotiation costs and the transfer to the creditors.

If this amount is positive, the equity holder will need to issue equity in order to raise these funds. When the new debt value is lower than what the creditors should obtain in renegotiation, the firm issues equity and compensates the creditors using the funds of the equity issuance. The empirical evidence shows that using external equity financing is costly (Masulis and Korwar, 1986; Chacko et al., 2001; Dhillon et al., 2001; Hennessy and Whited, 2007). The main costs of external equity financing include tax costs, dilution losses from adverse selection or corporate control reasons, and flotation costs (underwriting, legal and registration fees). We thus assume that equity financing implies a proportional cost k_F.

Nevertheless, if the total amount is negative, then the new debt value $D(x_R, c_1)$ is larger than the debt value in the absence of renegotiation $D(x_R, c)$.[9] The firm refinances the old debt by issuing a larger new debt. This actually implies a negative transfer, i.e., a transfer from the creditors to the equity holder. The negative transfer simply represents the fact that after repurchasing the old debt using part of the funds from the new debt issue, the equity holder receives the extra proceeds from the new debt issue.

We now derive the equity, debt, and firm values of a firm that proceeds

[9]This is likely to happen since despite having a lower coupon, we also have a lower probability of default, which could increase debt value.

to a debt renegotiation, denoted by $E_R(x)$, $D_R(x)$, and $V_R(x)$, respectively.

The equity value with renegotiation is given by:

$$\begin{aligned} E_R(x) = {} & \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)c}{r} + \Big\{ V(x_R, c_1) - (\beta + k_R) D(x_R, c) \\ & - k_F \max\{EF(c_1), 0\} - \frac{(1-\tau)x_R}{r-\mu} + \frac{(1-\tau)c}{r} \Big\} \left(\frac{x}{x_R}\right)^{\gamma}, \end{aligned} \tag{7}$$

The equity holder initially has a claim on the EBIT net of taxes and coupons (accounting for the tax shield) until renegotiation. At renegotiation, she exchanges this claim for a new claim with a reduced coupon, net of the total transfer at renegotiation (transfer to creditors plus renegotiation costs and equity issuance costs).[10]

The debt and firm values in the case of renegotiation are:

$$D_R(x) = \frac{c}{r} - \left(\frac{c}{r} - \beta D(x_R, c)\right) \left(\frac{x}{x_R}\right)^{\gamma}, \tag{8}$$

$$\begin{aligned} V_R(x) &= E_R(x) + D_R(x) \\ &= \frac{(1-\tau)x}{r-\mu} + \frac{\tau c}{r} + \Big\{ V(x_R, c_1) - k_R D(x_R, c) \\ &\quad - k_F \max\{EF(c_1), 0\} - \frac{(1-\tau)x_R}{r-\mu} - \frac{\tau c}{r} \Big\} \left(\frac{x}{x_R}\right)^{\gamma}, \end{aligned} \tag{9}$$

The renegotiation threshold and the reduced coupon are optimally chosen by the claimholders. Consistent with the previous literature (Lambrecht, 2001; Moraux and Silaghi, 2014; Nishihara and Shibata, 2016), it is optimal for the claimholders to renegotiate as late as possible. In the case when the equity holder has all bargaining power, she wants to delay renegotiation, as a later renegotiation implies a larger coupon reduction. As far as the creditors are concerned, they prefer to receive the full original coupon as long as possible. Therefore, it can be shown that the optimal renegotiation threshold x_R is equal to the original bankruptcy threshold $x_B(c)$ without debt renegotiation.[11]

[10] Note that since $V(x_R, c_1) = E(x_R, c_1) + D(x_R, c_1)$, we have that $V(x_R, c_1) - (\beta + k_R)D(x_R, c) = E(x_R, c_1) - EF(c_1)$.

[11] On the contrary, in the model of Sarkar (2013), the firm proposes renegotiation before

3. Equity financing in debt renegotiation

3.1. When is renegotiation possible?

Before determining the optimal reduced coupon, we need to answer another important question: When is renegotiation possible? As we have seen above, renegotiation is designed such that the creditors accept it, since they receive at least as much as they had in case of liquidation. We need to check however, if the equity holder is willing to renegotiate or whether she prefers liquidation. Renegotiation is beneficial to her if the surplus that she receives in renegotiation covers the total costs associated to it: renegotiation costs, transfers to the creditors, as well of equity financing costs, if new equity is issued. Formally, renegotiation is preferred to bankruptcy if in equation (7) the following condition is satisfied:

$$V(x_R, c_1) - (\beta + k_R)D(x_R, c) - k_F \max\{EF(c_1), 0\} \geq 0 \tag{10}$$

This means that the new firm value net of the creditors' part, and of renegotiation and equity financing costs has to be positive. Alternatively, given that $V(x_R, c_1) = E(x_R, c_1) + D(x_R, c_1)$ and that $EF(c_1) = (\beta + k_R)D(x_R, c) - D(x_R, c_1)$, we can rewrite the previous condition as:

$$E(x_R, c_1) \geq EF(c_1) + k_F \max\{EF(c_1), 0\} \tag{11}$$

A necessary condition for the firm to be able to raise funds at renegotiation is that the equity value has to be larger than the amount of funds that needs to be raised. Moreover, if equity issuance is costly, then the equity value has to be large enough to cover those costs as well.

A particular case appears when the new debt value at renegotiation $D(x_R, c_1)$ is large enough to cover the creditors' premium and the renegotiation costs. In this case, the firm does not need to raise funds at renegotiation, we have $EF(c_1) < 0$. Therefore, the condition above is always satisfied in that case since $E(x_R, c_1) \geq 0 > EF(c_1)$, and renegotiation is possible.

In case the firm needs to raise funds, i.e., $EF(c_1) > 0$, renegotiation fails either when $E(x_R, c_1) < EF(c_1)$ because the renegotiation costs k_R and/or the creditors' premium β are too large, or when $EF(c_1) \leq E(x_R, c_1) <$

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it would declare bankruptcy. This poses the problem of a non-credible threat since equity holders threaten with liquidation even if it would be better for them to continue servicing the debt (Christensen et al., 2014).

$(1 + k_F)EF(c_1)$ because equity issuance costs k_F are too large. Of course, this is rather intuitive, the larger the costs (either k_R or k_F) and the larger the creditors' premium, the less likely it is that renegotiation will take place. This is also in line with Nishihara and Shibata (2016), who study the choice between renegotiation and liquidation by making a numerical comparative statics analysis with respect to these parameters.

The respective constraints for renegotiation to take place are expressed as a function of a general reduced coupon c_1. Of course, they should be evaluated at the optimal reduced coupon c_1^* that we will derive in the next subsection.

3.2. Optimal debt reduction

Regarding the optimal reduced coupon, since transfers between the two parties are allowed, there is no constraint regarding the choice of the reduced coupon,[12] and it is optimal to choose the coupon that maximizes the total firm value. Looking at equation (9), we can see that this reduces to choosing the coupon that maximizes the new firm value at renegotiation net of equity issuance costs. Formally, the new coupon solves:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F \max\{EF(c_1), 0\} \tag{12}$$

The following proposition then applies.

Proposition 1. *The optimal reduced coupon is given by:*

(i) *Negative transfers*
If $(\beta + k_R)\alpha\gamma/(\gamma - 1) < A'$ *then* $EF(c_1^*) < 0$ *and:*

$$c_1^* \equiv c_1^A = c_0 * A \tag{13}$$

(ii) *Positive equity financing*

[12] If transfers were not allowed, as it is the case in Moraux and Silaghi (2014), then there would be a lower boundary below which the new coupon could not descend, since creditors would refuse renegotiation. This boundary would be given by a coupon value that makes the creditors indifferent between renegotiating and liquidating, leading to a new debt value at least as large as the old debt value. When transfers are allowed, we can choose a reduced coupon that implies a lower debt value for the creditors, since we can compensate them by making them a lump-sum transfer.

If $(\beta + k_R)\alpha\gamma/(\gamma - 1) > B'$ *then* $EF(c_1^*) > 0$ *and:*

$$c_1^* \equiv c_1^B = c_0 * B \tag{14}$$

(iii) *No equity issuance*
If $A' \leq (\beta+k_R)\alpha\gamma/(\gamma-1) \leq B'$ *then* $c_1^* \equiv c_1^{EF}$ *is such that* $EF(c_1^*) = 0$ *and:*

$$c_1^* \in (c_1^A, c_1^B), \tag{15}$$

where

$$\begin{aligned} A &= \left(\frac{\tau - (1-\alpha)\gamma}{\tau}\right)^{1/\gamma} \\ B &= \left(\frac{\tau + k_F - (1+k_F)(1-\alpha)\gamma}{\tau + k_F}\right)^{1/\gamma} \\ A' &= A + A^{1-\gamma}\left(\frac{\alpha\gamma}{\gamma - 1} - 1\right) \\ B' &= B + B^{1-\gamma}\left(\frac{\alpha\gamma}{\gamma - 1} - 1\right), \end{aligned} \tag{16}$$

with $A \leq B$ and $A' \leq B'$, with equality $A = B$ and $A' = B'$ for $k_F = 0$.

Proof of Proposition 1. See Appendix.

We have three possible reduced coupons depending on the parameter values, which will be illustrated later on in the numerical section. If the renegotiation costs and the bargaining power of the creditors are relatively low, the firm does not need to issue equity in order to raise funds to renegotiate the debt. On the contrary, the firm refinances by issuing a larger new debt, and we have a negative transfer, that is, a transfer from the creditors to the equity holder. The equity holder receives the extra proceeds of the new debt issue, after having reimbursed the old debt. The optimal reduced coupon in this case is relatively low and does not depend on the renegotiation costs k_R, on the creditors' premium β, nor on the equity financing cost k_F, since there is no need for equity financing.[13]

When the renegotiation costs and the bargaining power of the creditors are relatively high, the equity holder does not have enough funds to cover the renegotiation costs as well as the transfer due to the creditors. In this case we have positive equity financing, and the reduced coupon is relatively large. This means that we have a smaller coupon reduction, which implies a smaller transfer from the equity holder to the creditors. Indeed, since obtaining funds to finance the transfer is costly, it is optimal to try to minimize the transfer. Moreover, the reduced coupon depends on the costs of equity financing, k_F. Higher costs of equity financing lead to a higher reduced coupon, thus the firm retires less debt, we have a lower coupon reduction in order to minimize the amount of costly equity issuance. However, the reduced coupon does not depend on the renegotiation costs k_R, nor on the bargaining power of the creditors, β.

Finally, for intermediate values of the renegotiation costs and the bargaining power of the creditors, the reduced coupon will be chosen such that there is no equity issuance and the firm has the exact required funds to cover the renegotiation costs and the transfer to the creditors. In this case, the intermediate reduced coupon depends on the renegotiation costs k_R and the bargaining power of the creditors β, but not on the equity financing costs k_F.

Since it will be useful in the following subsections, we derive simple expressions for the total transfer made by the equity holder to cover the renegotiation costs and the transfer to the creditors, $EF(c_1^*) = (\beta + k_R)D(x_R, c_0) - D(x_R, c_1^*)$, in the first two cases. If we denote $Q \equiv (\beta + k_R)\alpha\gamma/(\gamma - 1)$, then using equation (3) we have that $(\beta + k_R)D(x_R, c_0) = Qc_0/r$. Also, using equations (3), (13), (14), and (16) we have that $D(x_R, c_1^A) = A'c_0/r$ and $D(x_R, c_1^B) = B'c_0/r$. Then we can rewrite $EF(c_1^*)$ as:

$$\begin{aligned} EF(c_1^A) &= (Q - A')\, c_0/r < 0 \\ EF(c_1^B) &= (Q - B')\, c_0/r > 0 \end{aligned} \tag{17}$$

[13] As Altman and Karlin (2009) argue, although one of the most common distressed exchanges involves the substitution of equity for debt, distressed exchanges can also result in a reduction of the coupon of the debt. This situation is still called a distressed exchange even if the price of the existing debt increases after the announcement.

3.2.1. The impact of transfers

We would like to end this section by discussing the effect of allowing for transfers on the renegotiation process. First, we have seen that allowing for transfers between the claimholders has resulted in situations in which the firm proceeds to a debt renegotiation financed both by a new debt and by new equity issuance. Thus we no longer have a simple debt for debt swap. Secondly, regarding the optimal reduced coupon, we compare with the case in which transfers are not allowed. In that case, the optimal reduced coupon will range between two polar values, monotonically increasing with the bargaining power of the creditors. When the equity holder has all the bargaining power the coupon is set to its lowest value such that the creditors are indifferent between renegotiation and liquidation ($\beta = 1$). A coupon below this value is not possible since creditors would refuse renegotiation. When the creditors have all the bargaining power the coupon takes the highest possible value, being set such that it maximizes debt value at renegotiation (this corresponds to the debt capacity of the firm).

When transfers are allowed, the coupon choice is not constrained anymore. For low values of the creditors' premium, the optimal unconstrained coupon is larger than the constrained one as can be seen in panel a) of Figure 1. The firm is refinancing the debt by issuing a larger amount of debt. The initial creditors receive part of this issuance in exchange for their old debt claim. The rest of the debt issuance is subscribed by new debt holders. The equity holder will receive the proceeds of the debt from the new bondholders (what we called a "negative transfer", from the creditors to the equity holder). For relatively large values of the creditors' premium, the optimal unconstrained coupon is lower than the constrained one. This implies that the new debt value is inferior to the old debt claim. The creditors are then compensated with a transfer financed by equity issuance ("positive transfer"). It is important to notice that since the unconstrained coupon is much lower than the constrained one, the firm is not near its debt capacity.[14] This implies that equity issuance is not the only viable option. Therefore, the firm is optimally choosing to issue equity, it is not forced to issue equity. Finally, for intermediate values of the creditors' premium, the unconstrained coupon coincides with the constrained one and there are no transfers. The firm does not issue equity, nor new debt to new debt holders.

[14]The maximum constrained coupon corresponds to the debt capacity of the firm.

[Figure 1 about here.]

Allowing for transfers between claimholders gives the firm the flexibility to optimally adjust the coupon in order to maximize firm value. The firm's choice of the optimal unconstrained coupon is smoother with respect to the constrained case. Since the spectrum of the choice has widened, this implies that the total firm value increases (see panel b) of Figure 1). Given that the constrained coupon is still an available option for the firm, if the unconstrained coupon differs from it, it is because it leads to a higher firm value. Thus, eliminating restrictions on transfers leads to increased welfare.[15] Lastly, allowing for transfers increases the maximum premium that the creditors can obtain in renegotiation. While for a constrained coupon, the maximum creditors' premium is limited by the firm's debt capacity, when transfers are allowed, creditors can receive the proceeds of equity issuance increasing their part of the renegotiation surplus (in Figure 1 for high values of the creditors' premium, renegotiation is only possible under the unconstrained case, the firm is liquidated under the constrained case, therefore the reduced coupon and firm value for the constrained case are missing).

The change in the coupon and firm value at renegotiation are significant. For our benchmark parameter values (see section 4), allowing for transfers leads to an increase of 14.03% in the coupon in the polar case when the equity holder has all the bargaining power. At the other extreme, when the creditors have all bargaining power, allowing for transfers implies a decrease of 19.04% in the new coupon. The firm value at renegotiation increases up to 2.81%.

3.3. When is equity financing more likely in renegotiation?

We have seen that allowing for transfers between the claimholders can lead to equity issuance in debt renegotiation. We now analyze how likely it is for the firm to issue equity in debt renegotiation. Whether the firm issues equity or not depends on the size of the renegotiation costs and creditors'

[15]Note that allowing for transfers does not only mean allowing for equity financing (positive transfers), but also for new debt issuance (negative transfers). Moreover, the fact that allowing for transfers is welfare increasing does not imply that a combination of equity and debt financing would be a predominant form of financing used in renegotiation. The optimal form of financing depends on the firm, debt and market conditions, and it could be any of the three renegotiation outcomes.

premium relative to the equity issuance costs. We know from Proposition 1 that when $(\beta+k_R)\alpha\gamma/(\gamma-1) > B'$ the firm will have positive equity financing, $EF(c_1^B) > 0$. On the contrary, when renegotiation costs are relatively low, $(\beta + k_R)\alpha\gamma/(\gamma - 1) < A'$, the firm will not issue equity, the transfers from the firm to the creditors being negative, $EF(c_1^A) < 0$. For intermediate costs, $A' \leq (\beta + k_R)\alpha\gamma/(\gamma - 1) \leq B'$, the firm will set the reduced coupon such that the equity issuance is exactly equal to zero.

Therefore, we present comparative statics of these three quantities ($Q \equiv (\beta + k_R)\alpha\gamma/(\gamma - 1)$, A', and B') with respect to the parameters of the model (k_R, β, k_F, τ, α, and γ). These comparative statics will allow us to derive empirical implications regarding the use of equity financing in debt renegotiation, and to contrast them with the empirical evidence.

For intuitive purposes, note that Q is proportional to the debt value at the renegotiation threshold with the original coupon, $D(x_R, c_0)$, while A' and B' are proportional to the new debt value at renegotiation with the reduced coupon, $D(x_R, c_1^*)$.[16]

Renegotiation costs and creditors' premium, k_R and β. An increase in the renegotiation costs and the creditors' premium leads to an increase in the total transfer that the equity holder will have to make at renegotiation, by increasing Q and not affecting A' or B'. It is more likely then for a firm with larger renegotiation costs or larger bargaining power for the creditors to need to issue equity to finance these costs and transfers.

Equity issuance costs, k_F. A firm which faces larger equity issuance costs will adjust the reduced coupon such that it limits the amount of funds it needs to raise. Therefore, we have a larger reduced coupon c_1 (a smaller coupon reduction) which implies a larger debt value $D(x_R, c_1)$, which will reduce the transfers to the creditors. Formally, Q and A' are not affected by the issuance costs k_F, while B' increases with k_F. Thus, it is more likely that when facing higher issuance costs, the firm will be less likely to issue equity at renegotiation (it is more likely for Q to be lower than B').

Tax rate, τ . We can see that the renegotiation costs as well as the creditors' premium do not depend on the tax rate since renegotiation takes place at the

[16] We remind the reader that $Q = (\beta + k_R)D(x_R, c_0)/(c_0/r)$, $A' = D(x_R, c_1^A)/(c_0/r)$ and $B' = D(x_R, c_1^B)/(c_0/r)$.

optimal no-renegotiation bankruptcy threshold, $x_B(c_0)$. At this threshold, the initial debt value $D(x_R, c_0)$ is simply equal to the liquidation value of the firm, and does not depend on the tax rate. Thus, Q will not depend on τ. On the other hand, we can show that both A' and B' increase with the tax rate. Intuitively, a larger tax rate makes the firm choose a larger reduced coupon value to benefit from the tax advantage of debt, which will increase the value of the debt value just after renegotiation, $D(x_R, c_1)$.[17] Thus, it is more likely that the firm will not issue equity when the tax rate increases, since it will not need additional funds to make transfers to the creditors.

Recovery rate, α . We can show regarding the proportion recovered in liquidation α that an increase in this parameter leads to an increase in A', B' and Q. Intuitively, since these quantities are proportional to the debt values at x_R with or without renegotiation, it is logical that an increase in the recovery rate leads to an increase in the debt values. In order to know whether issuing equity is more likely for a firm with a higher recovery value (lower bankruptcy costs) we would need to know which of these quantities increases more. Although it is not possible to answer this question analytically, we can show numerically that Q increases more than A' and B' for low values of the recovery rate, i.e., $D(x_R, c_0)$ increases more with the recovery rate α than $D(x_R, c_1)$. This is due to the fact that the impact of the recovery rate is larger the closer the firm is to bankruptcy. Indeed, in the absence of renegotiation, we know that the firm would default at $x_R = x_B(c_0)$, this is why the impact is larger on $D(x_R, c_0)$. Therefore, it is more likely that $Q > B'$, i.e., the renegotiation costs together with the creditors' premium are larger than the debt value with the reduced coupon, and the firm needs to make a positive transfer. Hence, a firm with a larger recovery value will more likely issue equity in renegotiation to finance the required funds.

Drift, volatility, and interest rate, γ. We remind the reader that $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}$, thus it actually incorporates three parameters: μ, r and σ, the first two decreasing with γ and the last one increasing with γ. It is straightforward to show that Q decreases with γ, that is, it increases with μ and r and it decreases with σ. Intuitively, a firm with a larger

[17] In general we know that debt is a hump-shape value of the coupon. However, since we know that the reduced coupon is below the coupon that maximizes debt value, we can conclude that debt is increasing in the reduced coupon.

volatility will decide to renegotiate (or to default in case of no renegotiation) at a lower threshold. The old debt value of the firm, $D(x_R, c_0)$, will therefore be lower, and so will be the renegotiation costs and the creditors' premium. Unfortunately, we cannot show analytically how an increase in γ affects A' or B'. However, we can show numerically (for reasonable parameter values) that they also decrease with γ. Intuitively, when the volatility increases, the new debt value with the reduced coupon decreases for two reasons. First, the renegotiation threshold is lower, and second the optimal reduced coupon is lower. Thus, the new debt value at renegotiation decreases more than the old debt value as volatility increases. This implies that the larger the volatility (or the lower the drift or the interest rate), the more likely the firm is to issue equity at renegotiation.

4. Numerical analysis

We study the numerical implications of our benchmark model on the value of the optimal reduced coupon, and the use of equity financing in renegotiation, in the following two subsections.

As far as the parameter values are concerned, we choose orders of magnitude similar to those assumed by previous models of debt renegotiation, in order to facilitate comparison between models. For our baseline case, we set the riskless interest rate to 6% (as Leland, 1994, Mella-Barral and Perraudin, 1997, and Nishihara and Shibata, 2016 did),[18] the drift to 1% (as in Bruche and Naqvi, 2010), the tax rate to 35% (as in Leland, 1994), the volatility to 20% (Leland, 1994 and Fan and Sundaresan, 2000 set it to 25%, while Nishihara and Shibata, 2016 set it at 20%), the recovery rate to 60% (Mella-Barral and Perraudin, 1997 chose bankruptcy costs of 20%, while Leland, 1994 chose bankruptcy costs of 50%), the renegotiation costs to 5%, the equity issuance costs to 10%,[19] and the creditors' premium β to 1.05 (the

[18] Since a risk-free rate of 6% seems very high compared to current rates, we also analyze the numerical implications of the model for any value of r between 1% (the drift value) and 6%.

[19] Dhillon et al. (2001) obtain estimates of equity issuance costs that range from 2% (consistent with lower equity dilution costs documented for regulated utilities by Masulis and Korwar, 1986), to 15 and 18.4% (comparable in magnitude with the estimates of 16 and 20% used in Chacko et al., 2001). Hennessy and Whited (2007) use structural estimation and find that equity flotation costs start at 5% for large firms and 10.7% for small firms.

last three in line with Nishihara and Shibata, 2016). Finally, without loss of generality, we consider an initial coupon value of 2 and we set the initial cash flow value equal to 2.

[Table 1 about here.]

Our baseline case parameter values are presented in Table 1. These parameter values are used in all the tables and figures presented in this paper, unless specified otherwise. Nevertheless, we also present comparative statics for every parameter, thus we will let each variable vary along a quite large interval around the baseline values. This is done with a twofold aim: in order to be able to illustrate the different solutions we obtain for our model (renegotiation versus bankruptcy, equity financing in renegotiation or pure debt for debt swap) and for robustness purposes.

4.1. Optimal reduced coupon

In this subsection we illustrate the three cases for the optimal reduced coupon, we present its comparative statics, and we analyze the impact of allowing for transfers between the claimholders, thus having an unconstrained coupon choice.

We start by presenting three different set of parameters under which we obtain three different cases for the optimal reduced coupon: c_1^A, where we have no equity financing, c_1^{EF}, where the coupon is exactly set such that there is no need to issue equity financing, and c_1^B, where there is positive equity financing in renegotiation. Table 2 illustrates the variables of interest for three different values of the tax rate: 15%, 25% and 35%. We focus on the following variables: Q, A' and B', which will tell us whether equity financing occurs in renegotiation or not and what the optimal reduced coupon is, the reduced coupon c_1^*, the equity financing value $EF(c_1^*)$, the equity and debt value at renegotiation $E(x_R, c_1^*)$ and $D(x_R, c_1^*)$, the total equity, debt and firm value at time 0 with and without renegotiation, $E_R(x_0)$, $D_R(x_0)$, $V_R(x_0)$, and $E(x_0, c_0)$, $D(x_0, c_0)$, $V(x_0, c_0)$, respectively.

[Table 2 about here.]

In order to simplify the analysis, we set the renegotiation costs, $k_R = 0$ and the creditors' premium, $\beta = 1$. The rest of the parameters are the baseline parameters. In the first case in which the tax rate is equal to 35% (second column of Table 2), we see that $Q < A'$, since the creditors' share and renegotiation costs are relatively low, thus the firm does not need to issue equity financing, and the optimal coupon is c_1^A. Indeed, we observe that for this optimal reduced coupon, the new debt value obtained with the new coupon at the renegotiation threshold, $D(x_R, c_1^*)$ is larger than the debt value without renegotiation at the same threshold, $D(x_R, c_0)$. The firm issues a larger new debt, and the equity holder receives the proceeds of the new debt after having reimbursed the old debt: $D(x_R, c_1^*) - D(x_R, c_0) = 1.092$. This is the case of "negative transfers", transfers from the creditors to the equity holder. Comparing the debt value at time 0 with and without renegotiation, we can see that the creditors are indifferent between renegotiating or not, which is normal given that we set $\beta = 1$. The equity holder has all the bargaining power and takes all the renegotiation surplus, $E_R(x_0) - E(x_0, c_0) = 1.625$.

For a low tax rate of 15%, we have that $B' < Q$, which implies that the firm will issue equity at renegotiation in order to finance the transfer that the equity holder needs to make to the creditors. Since the tax rate is low, it is optimal for the firm to choose a lower reduced coupon, as the tax advantage of debt is reduced. This leads to a lower debt value at renegotiation, $D(x_R, c_1^*) = 11.610$. Given that the debt value without renegotiation at x_R is $D(x_R, c_0^*) = 12.000$, the creditors would only accept renegotiation if they receive a positive transfer from the equity holder of at least 0.390. This is precisely the amount of funds that the firm has to raise through equity issuance, as renegotiation costs are null. As before, the equity holder captures all the surplus from renegotiation, and the creditor is indifferent between renegotiation and liquidation.

For an intermediate tax rate of 25%, we have that $A' < Q < B'$, which means that the firm will optimally choose the reduced coupon such that it avoids equity issuance, $EF(c_1^*) = 0$. Given that there are no renegotiation costs, the new coupon is chosen such that the new debt value at renegotiation $D(x_R, c_1^*)$ is exactly equal to the debt value without renegotiation $D(x_R, c_0)$, and there are no transfers from the equity holder to the creditors. The optimal coupon in this case coincides with the coupon that Moraux and Silaghi (2014) find (the case of one costless renegotiation, full bargaining power to the equity holder). Indeed, they assumed that lump-sum transfers between the two claimholders were not possible.

The large coupon reductions obtained are consistent with very large fractions of debt retired in debt repurchases found by Kruse et al. (2014) and Brandon (2013). Kruse et al. (2014) find that in debt tender offers the average and median offers represent 80.2% and 40.5% respectively, of the debt book value in the year prior to the tender offer. Brandon (2013) finds that the repurchase retires 53% of the face value of the bond on average, and reduces the firm's leverage ratio by more than 16%. These are consistent with our large coupon reductions.

[Figure 2 about here.]

We now represent graphically the optimal reduced coupon and its comparative statics with respect to the parameters of the model. Figure 2 plots the optimal reduced coupon as a function of the equity issuance costs for different values of the renegotiation costs. In panel a), for a low tax rate of 15%, we have that for $k_F < 0.34$ the optimal reduced coupon is c_1^B. In this case there exists positive equity financing in renegotiation, which implies that the optimal coupon is increasing in the issuance costs. For higher issuance costs (above 0.34), it is optimal for the firm to set the coupon such that the equity issuance amount is exactly equal to zero, thus the optimal coupon is c_1^{EF}, and does not depend on the issuance costs.

In panel b), we illustrate how the reduced coupon varies with the bargaining power of the creditors for a tax rate of 35%. The optimal coupon obtained depends on the value of the creditors' premium. When the latter one is relatively large, the firm issues equity in renegotiation, the optimal coupon is c_1^B, and does not vary with β. For relatively low values of the creditors' premium, there are negative transfers (the firm issues new debt to new debt holders), the optimal coupon is c_1^A, and again, it does not vary with β. For intermediate values of the creditors' premium, the firm chooses the coupon such that there is no need to issue equity financing in renegotiation. The optimal coupon is c_1^{EF} and it increases in the creditors' premium. The larger the bargaining power of the creditors, the larger the optimal new coupon in order to reduce the need for a transfer to the creditors, and thus the need for equity financing.

We have thus provided three numerical examples of the three cases that we had previously characterized analytically. For reasonable parameter values, we can either have positive equity issuance in renegotiation, new debt issuance (negative transfers) or no equity issuance in renegotiation. At first sight, this evidence is in contrast with the findings of Sarkar (2013) and Nishihara and Shibata (2016). The former finds that the creditors always receive a new debt claim with a reduced coupon and a *positive* equity stake in the firm for a tax rate of 20%. The latter report that equity financing is always positive for a tax rate of 15%. One can verify however, that increasing the tax rate in both models would lead to opposite results. In particular, a *negative* equity stake is obtained for the model of Sarkar (2013) when using tax rates of 20-35% and relatively large bargaining power of the shareholders.[20] Similarly, no equity financing is obtained using higher interest rates for the model of Nishihara and Shibata (2016). This is consistent with our results. For higher tax rates, we have seen that the firm might not issue equity in renegotiation if the bargaining power of the creditor is relatively low. Unlike Nishihara and Shibata (2016) and Sarkar (2013), we characterize analytically the conditions under which the firm issues equity in renegotiation. We thus offer a complete characterization of equity financing in renegotiation.

4.2. Equity financing in renegotiation

Our goal in this subsection is to analyze numerically how the use and amount of equity financing in renegotiation depend on the parameters of the model. For this purpose, we will plot the equity value at renegotiation and the amount of equity financing with its corresponding cost. Comparing the equity value at renegotiation with the amount of equity financing and its cost does not only allow us to see under what conditions renegotiation is preferred to liquidation (as in inequality (11)), but also to analyze the evolution of equity financing with respect to the parameters of the model.

In Figure 3 we plot the equity value at renegotiation, $E(x_R, c_1)$, the equity financing amount, $EF(c_1)$, and the equity financing amount plus the cost that it involves, $EF(c_1) + \max(EF(c_1), 0)$. All these three quantities are plotted as a function of the equity financing costs k_F for different levels of the renegotiation costs, k_R.

[Figure 3 about here.]

In panel a), for low renegotiation costs, $k_R = 0.05$, we can see that renegotiation is possible irrespective of the value of the equity issuance costs.

[20]Indeed, the reduced coupon is chosen in order to maximize firm value, and the new debt claim with this reduced coupon more than compensates the creditors for their initial debt claim. Thus, they do not need to additionally receive a positive equity stake in the firm. On the contrary, there will be a transfer from the creditors to the equity holder (a "negative equity stake").

Additionally, we can see that the amount of equity financing decreases with the costs of issuing new equity, as discussed in section 3.3. When equity issuance costs are large the firm will optimally adjust the level of the coupon in order to reduce the costs of equity issuance. The firm therefore increases the level of the reduced coupon (we have a lower coupon reduction) to be able to issue a lower amount of equity (see panel a) of Figure 2). Furthermore, we observe that as issuance costs increase, the firm is less likely to use equity financing in renegotiation. For issuance costs larger than $k_F = 0.34$ the firm renegotiates its debt through a pure debt to debt swap, without issuing equity.

In panel b) of Figure 3 we notice that renegotiation is only possible for $k_F < 0.45$, since renegotiation costs are larger, $k_R = 0.3$. Although the equity value at renegotiation exceeds the amount of equity financing needed, the presence of large equity issuance costs ($k_F > 0.45$) prevents renegotiation: $EF(c_1) < E(x_R, c_1) < EF(c_1) + k_F \max(EF(c_1), 0)$. Finally, in panel c) it is the presence of large renegotiation costs, $k_R = 0.5$, that prevents renegotiation. The firm is not able to renegotiate its debt for any value of the equity issuance costs, k_F (we have $E(x_R, c_1) < EF(c_1)$). Fixing $k_F = 0.4$ and comparing the first two panels (across renegotiation costs), we notice that equity financing is more likely to occur in renegotiation as renegotiation costs increase. The firm needs more funding to finance the larger costs and the transfers to the creditors as renegotiation costs increase.

[Figure 4 about here.]

Regarding the use of equity financing as a function of the creditors' premium we plot the same variables as before as a function of β for different values of the tax rate. For a low tax rate of 15% we can see in panel a) of Figure 4 that renegotiation is possible whenever $\beta < 1.4$ and that for any value of the creditors' premium, there exists positive equity financing in renegotiation. A low tax rate leads to a low optimal coupon, and therefore the need to issue equity to compensate the creditors. The amount of equity financing increases in the creditors' premium as expected. In panel b) of Figure 4, similarly to panel b) of Figure 2, we obtain the three cases of: negative transfers (for $\beta < 1.04$), no equity issuance (for $\beta \in [1.04, 1.08]$), and positive equity issuance (for $\beta > 1.08$). Whenever there exists equity financing in renegotiation, it will be increasing in the creditors' premium as shown analytically. Comparing the two panels (across tax rates), we observe that as the tax rate decreases it is more likely that the firm issues equity in renegotiation.

[Figure 5 about here.]

Whether the firm will use equity financing to renegotiate its debt also depends on the recovery rate, α. Figure 5 plots the evolution of the amount of equity financing with changes in the recovery rate for different values of the tax rate. Renegotiation is only possible for a recovery rate below 0.83 at a tax rate of 15%, and below 0.79 for a tax rate of 35%. For the values of the recovery rate for which renegotiation is preferred to liquidation, and there exists positive equity financing in renegotiation, the amount of equity financing is either increasing in the recovery rate or hump-shaped. Firms with larger recovery rates are thus more likely to issue equity in renegotiation, however, they do not necessarily issue a higher amount of equity.

Finally, we illustrate graphically how the use and amount of equity financing in renegotiation relate to the three parameters contained in γ: the volatility σ, the drift μ, and the interest rate r.

[Figure 6 about here.]

The effect of volatility on the use of equity financing in renegotiation is plotted in Figure 6. Equity financing is used in renegotiation for volatility values above 12% in the case of a low tax rate (panel a)), and above 24% for a high tax rate (panel b)). Firms with larger volatility are more likely to use equity financing in renegotiation. This is due to the fact that a larger volatility decreases the new debt value at renegotiation, $D(x_R, c_1)$ not only by lowering the renegotiation threshold, but also by decreasing the coupon. Therefore, the firm needs to issue equity to be able to make the transfers to the creditors to compensate for the reduction in the debt value and to cover the renegotiation costs.[21] Similarly to the comparative statics with respect to the recovery rate, the amount of equity financing is either increasing or hump-shaped in the volatility.

[21]The value of the initial debt at renegotiation, $D(x_R, c_0)$, is only affected by a higher volatility through the impact on the renegotiation threshold, but not on the coupon. Consequently, this debt value and the creditors' premium and renegotiation costs proportional to it decrease less when volatility increases, as compared to $D(x_R, c_1)$.

[Figure 7 about here.]

Unlike the volatility, the drift is negatively related to the parameter γ. We observe in Figure 7 that as the drift increases it is less likely for the firm to use equity financing in renegotiation. In panel a), the firm issues positive equity financing as long as the drift is below 0.025. For larger drifts the firm has a pure debt for debt swap, without issuing equity in renegotiation. The amount of equity financing used in renegotiation can be either decreasing, hump-shaped on increasing in the drift, depending on the parameter values.

[Figure 8 about here.]

Regarding the last parameter, the interest rate, this is also negatively related with the parameter γ. We observe in Figure 8 that the higher the interest rate the less likely it is that the firm will issue equity in renegotiation. In panel b), whenever the interest rate is above 0.037 the firm will not issue equity to renegotiate the debt. The amount of equity financing decreases in the interest rate. This is due to the fact that besides the indirect effect r has on the equity financing amount through γ, r also directly and negatively affects the amount of equity financing as shown in equation (17).[22]

5. Extension:Forced asset sales

We now extend the previous framework in order to account for forced asset sales. Following other papers in the literature, Mella-Barral (1999), and Nishihara and Shibata (2016), we assume economies of scale, which implies that partial liquidation is inefficient, i.e., assets sold piecemeal are less valuable than the same assets sold as a going concern. Therefore, it is optimal for the firm not to partially sell assets. Nevertheless, *forced* asset sales of viable businesses do occur in practice, and, as documented by Djankov et al.

[22]The comparative statics analysis regarding the decision to renegotiate or to liquidate and the use of equity financing in renegotiation was conducted keeping the initial coupon fixed to our baseline parameter. However, we also made the same analysis for the optimal initial coupon that maximizes firm value at time zero, and we obtain the same comparative statics. For the sake of space they are omitted here. Furthermore, in terms of the optimal capital structure we obtain that the optimal leverage ratio increases with renegotiation since renegotiation reduces the bankruptcy costs, confirming the evidence from the previous studies, such as Christensen et al. (2014) and Nishihara and Shibata (2016).

(2008), they make debt enforcement inefficient. Moreover, according to the evidence of Kruse et al. (2014), 14.9% of debt tender offers use asset sales as a financing source.

Formally, we assume that by selling a fraction $\phi \in (0,1)$ of the assets at time t, the equity holder receives the proceeds $P(X(t), \phi)$ after taxes. We let $P(x, \phi) = F(\phi)x$, where F is a non-decreasing convex function with $F(0) = 0$. The convexity implies that full liquidation will always be preferred to partial liquidation. Therefore, if the firm could optimally choose the fraction of assets to liquidate at renegotiation, it would choose not to sell assets, i.e., $\phi = 0$, which is the case of our baseline framework.

We therefore adjust our initial equity, debt, and firm value under no renegotiation, to account for asset sales. Consider a firm that is operating with the asset size ϕ. Its equity, debt and firm value are given by the following equations:

$$E(x,\phi,c) = \frac{(1-\tau)\phi x}{r-\mu} - \frac{(1-\tau)c}{r} - \left(\frac{(1-\tau)\phi x_B(\phi,c)}{r-\mu} - \frac{(1-\tau)c}{r}\right)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma} \tag{18}$$

$$D(x,\phi,c) = \frac{c}{r}\left(1 - \left(\frac{x}{x_B(\phi,c)}\right)^{\gamma}\right) + P(x_B(\phi,c),\phi)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma}, \tag{19}$$

$$\begin{aligned} V(x,\phi,c) &= E(x,\phi,c) + D(x,\phi,c) \\ &= \frac{(1-\tau)\phi x}{r-\mu} + \frac{\tau c}{r} - \left(\frac{(1-\tau)\phi x_B(\phi,c)}{r-\mu} + \frac{\tau c}{r} - P(x_B(\phi,c),\phi)\right)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma}, \end{aligned} \tag{20}$$

with

$$x_B(\phi,c) = \frac{\gamma(r-\mu)c}{(\gamma-1)r\phi}, \tag{21}$$

representing the default threshold.

If the firm sells a fraction ϕ of its assets at renegotiation, then the amount of equity financing needed in renegotiation will depend on the proceeds from the asset sales:

$$EF(c_1,\phi) = (\beta + k_R)D(x_R, 1, c) - D(x_R, 1-\phi, c_1) - P(x_R, \phi), \tag{22}$$

where $x_R = x_B(1, c)$.

If the new debt value of a firm operating with an asset size $1-\phi$ plus the proceeds from selling a fraction ϕ of the assets are not enough to cover the creditors' premium and the renegotiation costs, then the firm will have to issue equity financing.

Renegotiation is possible only if:

$$V(x_R, 1-\phi, c_1) + P(x_R, \phi) - (\beta + k_R)D(x_R, 1, c) - k_F \max\{EF(c_1,\phi), 0\} \geq 0 \tag{23}$$

The optimal coupon is chosen in order to maximize the new firm value at renegotiation after the asset sale, net of equity issuance costs:

$$c_1^*(\phi) = \arg\max_{c_1} V(x_R, 1-\phi, c_1) - k_F \max\{EF(c_1,\phi), 0\} \tag{24}$$

In order to obtain closed-form solutions for the optimal reduced coupon, we take an explicit function of the proceeds obtained through asset sales:[23]

$$P(x_R, \phi) = \frac{\alpha \phi^{1.01} x_R}{r - \mu}, \tag{25}$$

which is a convex function. Note that for $\phi = 1$, the full liquidation value is given by $\frac{\alpha x_R}{r-\mu}$, which is the same as in the benchmark model.

The following proposition then applies.

Proposition 2. *The optimal reduced coupon in the general case in which we allow for asset sales in renegotiation is given by:*

(i) *Negative transfers*
If $(\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma - 1) < A'(\phi)$ *then* $EF(c_1^*(\phi), \phi) < 0$ *and:*

$$c_1^*(\phi) \equiv c_1^A(\phi) = c_0 * A(\phi) \tag{26}$$

(ii) *Positive equity financing*
If $(\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma - 1) > B'(\phi)$ *then* $EF(c_1^*(\phi), \phi) > 0$ *and:*

$$c_1^*(\phi) \equiv c_1^B(\phi) = c_0 * B(\phi) \tag{27}$$

[23]This liquidation function is also used in Nishihara and Shibata (2016).

(iii) *No equity issuance*
If $A'(\phi) \leq (\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma-1) \leq B'(\phi)$ *then* $c_1^*(\phi) \equiv c_1^{EF}(\phi)$ *is such that* $EF(c_1^*(\phi), \phi) = 0$ *and:*

$$c_1^*(\phi) \in (c_1^A(\phi), c_1^B(\phi)), \tag{28}$$

where

$$\begin{aligned} A(\phi) &= (1-\phi)\left(\frac{(\tau-\gamma)(1-\phi)+\alpha\gamma(1-\phi)^{1.01}}{\tau(1-\phi)}\right)^{1/\gamma} \\ B(\phi) &= (1-\phi)\left(\frac{\tau+k_F-(1+k_F)(\gamma-\alpha\gamma(1-\phi)^{0.01})}{\tau+k_F}\right)^{1/\gamma} \\ A'(\phi) &= A(\phi)+A(\phi)\left(\frac{\alpha\gamma(1-\phi)^{0.01}}{\gamma-1}-1\right)\left(\frac{1-\phi}{A(\phi)}\right)^{\gamma} \\ B'(\phi) &= B(\phi)+B(\phi)\left(\frac{\alpha\gamma(1-\phi)^{0.01}}{\gamma-1}-1\right)\left(\frac{1-\phi}{B(\phi)}\right)^{\gamma}, \end{aligned} \tag{29}$$

with $A(\phi) \leq B(\phi)$ and $A'(\phi) \leq B'(\phi)$, with equality $A(\phi) = B(\phi)$ and $A'(\phi) = B'(\phi)$ for $k_F = 0$. Note that for $\phi = 0$, i.e., no asset sales in renegotiation, we obtain the same results as in Proposition 1 in our benchmark model.

PROOF OF PROPOSITION 2. See Appendix.

We now illustrate numerically the impact of allowing for forced asset sales on the debt renegotiation process. In line with the findings of Nishihara and Shibata (2016), we obtain that firms with larger fractions of asset sales are more likely to proceed to direct liquidation rather than to renegotiate their debt due to the inefficiency of partial asset sales in renegotiation. Moreover, the optimal reduced coupon decreases with the fraction of asset sales since a firm with a lower operating size chooses a lower optimal coupon.[24]

[Figure 9 about here.]

[24] In the interest of space these results are not reported.

Regarding the use of equity financing, we find that firms which do not issue equity when they are not forced to sell assets ($\phi = 0$), are likely to start using equity financing as they are forced to sell more assets (see panel b) of Figure 9). This happens because an increase in the fraction of asset sales has two opposite effects on the amount of equity financing (see equation 22). On the one hand, when the fraction of asset sales increases, the proceeds from asset sales will be larger, so the firm needs to issue less equity (negative effect). On the other hand, a higher fraction of asset sales also implies that the firm's scale is reduced and the value of the new debt with the reduced coupon is smaller, $D(x_R, 1 - \phi, c_1)$. So the firm needs to issue more equity (positive effect). For these firms, the second effect dominates, i.e., selling assets reduces the new debt value more than it increases the proceeds from asset sales. Thus, they are more likely to use equity financing as the fraction of asset sales increases. For firms which optimally decide to use equity financing even when they are not forced to sell assets, we find that they will also use equity financing when forced to sell assets. The amount of equity issued can however either increase or decrease when they sell assets, depending on which of the two opposite effects dominate (see panel a) of Figure 9 for an example of decreasing equity issuance). This result is different from the finding of Nishihara and Shibata (2016), who argue that selling assets increases the amount of equity financing in renegotiation. In our view, this is because for the parameter values that they consider, the second effect dominates, i.e., selling assets reduces the new debt value more than it increases the proceeds from asset sales.

6. Empirical implications

Our model has a number of testable implications for debt renegotiation, several of which are novel with respect to the existent literature. Given that the main contribution of the paper is to account for the use of equity financing, we will focus on the implications regarding equity financing.

Our analysis implies first of all that firms which have more intangible assets (firms from the technological sector for example that have low recovery rates) are less likely to use equity financing in renegotiation. Second, smaller/younger firms (which tend to have larger cash flows volatility) are more likely to issue equity financing to renegotiate their debt (if the volatility is not so high as to push them into direct liquidation). Third, firms with higher cash flow growth rates are less likely to issue equity financing in order to repurchase debt.

A simple way to empirically test these implications would be to perform an event study to analyze the stock price reaction to different forms of financing for debt repurchase. According to the model, the use of equity financing in debt repurchases would be a signal of lower firm quality or financial distress. Thus, we would expect to see a more negative price reaction when equity financing is used than when only debt is used.

These implications are consistent with the empirical evidence of Kruse et al. (2014) who study the cumulative abnormal returns (CARs) of debt tender offers. They find that when debt is used to repurchase debt, but equity is not, the mean CAR is positive and significant. That is, the market values more firms which do not use equity financing to repurchase debt. Our implications are also consistent with the evidence of Brennan and Kraus (1987) who shows that using equity proceeds to retire debt has a more negative share price reaction than other uses.

The model's prediction that firms in distress are more likely to use equity financing in renegotiation is also in line with the empirical evidence on equity issuance in financial distress. Indeed, this literature documents a strong positive relationship between distress and equity issuance for both public and private placements (DeAngelo et al., 2010; Brophy et al., 2009; Park, 2014). Park (2014) argued that this empirical evidence was in line with the agency theory, but not with the debt-overhang problem. We thus provide an alternative theoretical explanation for this stylized fact.

The model also predicts that small firms, which have a concentrated group of creditors who closely monitor them (in which creditors' premium is higher), are more likely to use equity financing compared to large firms with a high number of institutional investor (in which the creditors' premium is lower). At the same time, firms are less likely to use equity financing in renegotiation on markets where equity financing is more expensive (higher flotation costs) or the corporate tax rate and interest rate are higher. Finally, firms which are forced to sell assets in renegotiation are more likely to use equity financing.

7. Conclusions

In this paper we have analyzed the use of equity financing in debt renegotiation. To the best of our knowledge, this is the first paper to provide a thorough analysis of equity issuance in renegotiation, both analytically and numerically. We have studied the conditions under which a firm repurchases debt using a combination of debt financing and equity financing (along with forced asset sales) rather than a pure debt for debt swap. Depending on the firm characteristics, the market variables or different costs involved in the renegotiation process, some firms will choose to issue equity in order to finance part of the proceeds of the debt repurchase, while others will prefer not to use equity financing in renegotiation. If new equity is issued, the optimal reduced coupon will be adjusted in order to minimize equity issuance costs.

When transfers between the equity holder and the creditors are allowed, and equity financing can be used in renegotiation, the firm optimally chooses a smoother pattern of coupon reductions than previously documented in the literature. Allowing for transfers, and thus for equity financing is welfare increasing. Indeed, an unconstrained choice of the optimal reduced coupon increases the net surplus from renegotiation.

Several novel empirical implications regarding the use of equity financing in debt renegotiation are also derived. These implications could motivate further empirical evidence on the use of equity financing in debt repurchases. Indeed, despite a few recent empirical studies on debt repurchases, the literature remains quite scarce.

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Appendix A.

Proof of Proposition 1. Our maximization problem is the following:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F \max\{EF(c_1), 0\} \tag{A.1}$$

This can be rewritten as two maximization problems:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) \tag{A.2}$$

s.t.

$$EF(c_1) \leq 0$$

and

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F EF(c_1) \tag{A.3}$$

s.t.

$$EF(c_1) \geq 0$$

We know that $EF(c_1) = (\beta + k_R)D(x_R, c) - D(x_R, c_1)$. Since the first term does not depend on c_1, the second maximization problem reduces to:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) + k_F D(x_R, c_1) \tag{A.4}$$

s.t.

$$EF(c_1) \geq 0$$

We can compute the two derivatives that interest us $\frac{\partial V(x_R,c_1)}{\partial c_1}$ and $\frac{\partial D(x_R,c_1)}{\partial c_1}$.

$$\begin{aligned}
\frac{\partial V(x_R, c_1)}{\partial c_1} &= \frac{\tau}{r} - \left(\frac{(1-\tau)\partial x_B(c_1)/\partial c_1}{r-\mu} + \frac{\tau}{r} - \frac{\alpha \partial x_B(c_1)/\partial c_1}{r-\mu}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&+ \left(\frac{(1-\tau)x_B(c_1)}{r-\mu} + \frac{\tau c_1}{r} - \frac{\alpha x_B(c_1)}{r-\mu}\right)\frac{\gamma}{x_B(c_1)}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \frac{\partial x_B(c_1)}{\partial c_1} \\
&= \frac{\tau}{r} - \frac{1-\tau}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}(1-\gamma)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\tau}{r}\left(1 - \frac{c_1\gamma}{x_B(c_1)}\frac{\gamma(r-\mu)}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&+ \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}(1-\gamma)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&= \frac{\tau}{r} + \frac{(1-\tau)\gamma}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\tau(1-\gamma)}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\alpha\gamma}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&= \frac{\tau}{r} + \left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \frac{\gamma(1-\alpha)-\tau}{r} \\
&= \frac{1}{r}\left(\tau + \left(\frac{c}{c_1}\right)^\gamma (\gamma(1-\alpha)-\tau)\right)
\end{aligned} \tag{A.5}$$

$$
\begin{aligned}
\frac{\partial D(x_R, c_1)}{\partial c_1} &= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha}{r-\mu}\frac{\partial x_B(c_1)}{\partial c_1}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&+ \left(\frac{c_1}{r} - \frac{\alpha x_B(c_1)}{r-\mu}\right)\frac{\gamma}{x_B(c_1)}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\frac{\partial x_B(c_1)}{\partial c_1} \\
&= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&+ \left(\frac{c_1}{r} - \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)c_1}{(\gamma-1)r}\right)\frac{\gamma(\gamma-1)r}{\gamma(r-\mu)c_1}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\frac{\gamma(r-\mu)}{(\gamma-1)r} \\
&= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha\gamma}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} + \left(\frac{c_1}{r} - \frac{\alpha\gamma c_1}{(\gamma-1)r}\right)\frac{\gamma}{c_1}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&= \frac{1}{r} - \left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\left(\frac{1}{r} - \frac{\alpha\gamma}{(\gamma-1)r} - \frac{\gamma}{r} + \frac{\alpha\gamma^2}{(\gamma-1)r}\right) \\
&= \frac{1}{r} - \left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\left(\frac{1}{r} - \frac{\gamma}{r} + \frac{\alpha\gamma}{r}\right) \\
&= \frac{1}{r}\left(1 - \left(\frac{c}{c_1}\right)^{\gamma}(1-(1-\alpha)\gamma)\right)
\end{aligned} \tag{A.6}
$$

We start by solving the first maximization problem. Assume the constraint is not binding, i.e. $EF(c_1) < 0$. Then the optimal coupon is such that $\frac{\partial V(x_R,c_1)}{\partial c_1} = 0$. Letting the derivative from equation (A.5) be equal to zero and solving for c_1 we obtain the optimal coupon c_1^A. Since we assumed that the constraint is not binding, we are in the case in which $EF(c_1^A) < 0$, which is equivalent to $Q < A'$.

We solve the second maximization problem assuming again that the constraint is not binding, i.e., $EF(c_1) > 0$. Then the optimal coupon is such that $\frac{\partial (V(x_R,c_1)+k_F D(x_R,c_1))}{\partial c_1} = 0$. Solving for c_1 we obtain the optimal coupon c_1^B. As before, the constraint is not binding, and we are in the case in which $EF(c_1^B) > 0$, which is equivalent to $Q > B'$.

Finally, when the constraint is binding, $EF(c_1) = 0$, the optimal coupon is such that this condition is satisfied, and we have that $c_1^{EF} \in (c_1^A, c_1^B)$. □

Proof of Proposition 2. The proof is similar to the proof of Proposition 1, there exists just an extra parameter, ϕ, for the asset sales.

References

Altman, E., Karlin, B., 2009. The re-emergence of distressed exchanges in corporate restructurings. NYU Working Paper.

Anderson, R.W., Sundaresan, S., 1996. Design and valuation of debt contracts. Review of Financial Studies 9, 37–68.

Bolton, P., Scharfstein, D., 1996. Optimal debt structure and the number of creditors. Journal of Political Economy 104, 1–25.

Brandon, J., 2013. Corporate investment and the option to repurchase debt. Working paper London Business School.

Brennan, M., Kraus, A., 1987. Efficient financing under asymmetric information. Journal of Finance 42, 1225–1243.

Brophy, D., Ouimet, P., Sialm, C., 2009. Hedge funds as investors of last resort? Review of Financial Studies 22, 541–574.

Bruche, M., Naqvi, H., 2010. A structural model of debt pricing with creditor-determined liquidation. Journal of Economic Dynamics and Control 34, 951–967.

Chacko, G., Tufano, P., Verter, C., 2001. Cephalon, Inc. Taking risk management seriously. Journal of Financial Economics 60, 449–485.

Chatterjee, S., Dhillon, U., Ramirez, G., 1995. Coercive tender and exchange offers in distressed high-yield debt restructurings: An empirical analysis. Journal of Financial Economics 38, 333–360.

Christensen, P., Flor, C., Lando, D., Miltersen, K., 2014. Dynamic capital structure with callable debt and debt renegotiations. Journal of Corporate Finance 29, 644–661.

DeAngelo, H., DeAngelo, L., Stulz, R., 2010. Seasoned equity offerings, market timing, and the corporate lifecycle. Journal of Financial Economics 95, 275–295.

Dhillon, U., Noe, T., Ramirez, G., 2001. Bond calls, credible commitment, and equity dilution: A theoretical and clinical analysis of simultaneous tender and call (STAC) offers. Journal of Financial Economics 60, 573–611.

Dixit, A., Pindyck, R., 1994. Investment Under Uncertainty. Princeton University Press: Princeton.

Djankov, S., Hart, O., McLiesh, C., Shleifer, A., 2008. Debt enforcement around the world. Journal of Political Economy 116, 1105–1149.

Fan, H., Sundaresan, S.M., 2000. Debt valuation, renegotiation, and optimal dividend policy. Review of Financial Studies 13, 1057–1099.

Franks, J.R., Torous, W., 1994. A comparison of financial recontracting in distressed exchanges andchapter 11 reorganizations. Journal of Financial Economics 35, 349–370.

Godlewski, C., 2015a. Debt renegotiation and the design of financial contracts. Working Paper.

Godlewski, C.J., 2015b. The certification value of private debt renegotiation and the design of financial contracts: Empirical evidence from Europe. Journal of Banking and Finance 53, 1–17.

Goldstein, R., Ju, N., Leland, H., 2001. An EBIT-based model of dynamic capital structure. The Journal of Business 74, 483–512.

Hackbarth, D., Hennessy, C.A., Leland, H.E., 2007. Can the trade-off theory explain debt structure? Review of Financial Studies 20, 1389–1428.

Hennessy, C., Whited, T., 2007. How costly is external financing? Evidence from a structural estimation. Journal of Finance 62, 1705–1747.

Koziol, C., 2010. Why adding firm value with a put feature in debt contracts is better than renegotiation. Review of Managerial Science 4, 53–90.

Kruse, T., Nohel, T., Todd, S., 2014. The decision to repurchase debt. Journal of Applied Corporate Finance 26, 85–94.

Lambrecht, B.M., 2001. The impact of debt financing on entry and exit in a duopoly. Review of Financial Studies 14, 765–804.

Landier, A., Ueda, K., 2012. The economics of bank restructuring: Understanding the options. IMF Staff Position Note.

Leland, H.E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49, 1213–1252.

Lie, E., Lie, H., McConnell, J., 2001. Debt-reducing exchange offers. Journal of Corporate Finance 7, 179–207.

Masulis, R., Korwar, A., 1986. Seasoned equity offerings: An empirical investigation. Journal of Financial Economics 15, 91–118.

Mella-Barral, P., 1999. The dynamics of default and debt reorganization. Review of Financial Studies 12, 535–578.

Mella-Barral, P., Perraudin, W., 1997. Strategic debt service. Journal of Finance 52, 531–556.

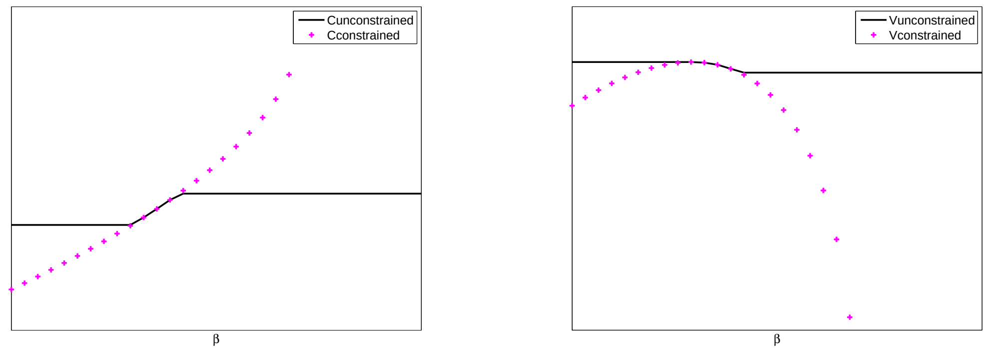
Moraux, F., Silaghi, F., 2014. Inside debt renegotiation: Optimal debt reduction, timing, and the number of rounds. Journal of Corporate Finance 27, 269–295.

Nishihara, M., Shibata, T., 2016. Asset sale, debt restructuring, and liquidation. Journal of Economic Dynamics and Control 67, 73–92.

Park, J., 2014. Equity issuance of distressed firms. Working Paper Korea University.

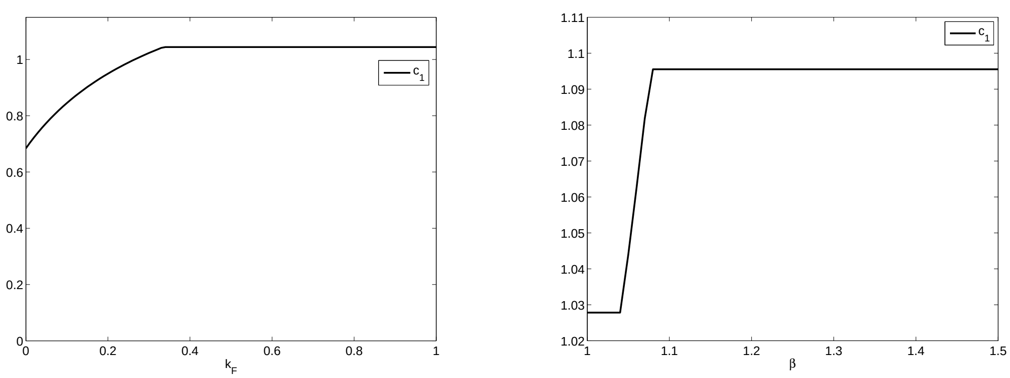
Sarkar, S., 2013. Distressed exchange, bargaining power, and prior capital structure. Journal of Economic Dynamics and Control 37, 2695–2709.

Figures



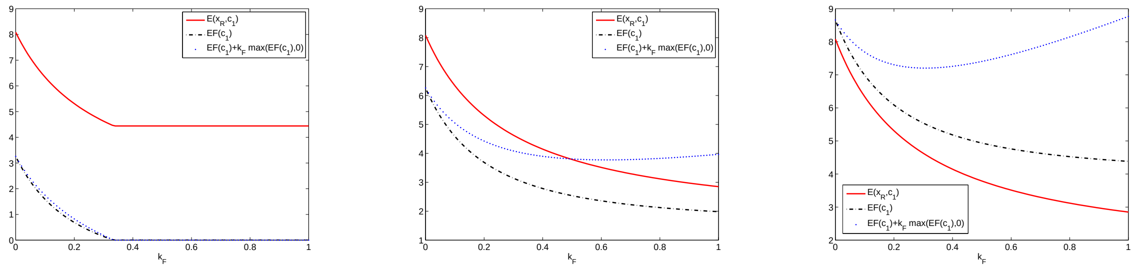
a) Coupon b) Firm value

Figure 1: The impact of transfers. Panel a) plots the optimal reduced coupon, while panel b) shows the firm value at renegotiation as a function of the creditors' premium, β. The constrained case corresponds to a situation where transfers between claimholders are not possible. In the unconstrained case transfers are possible.



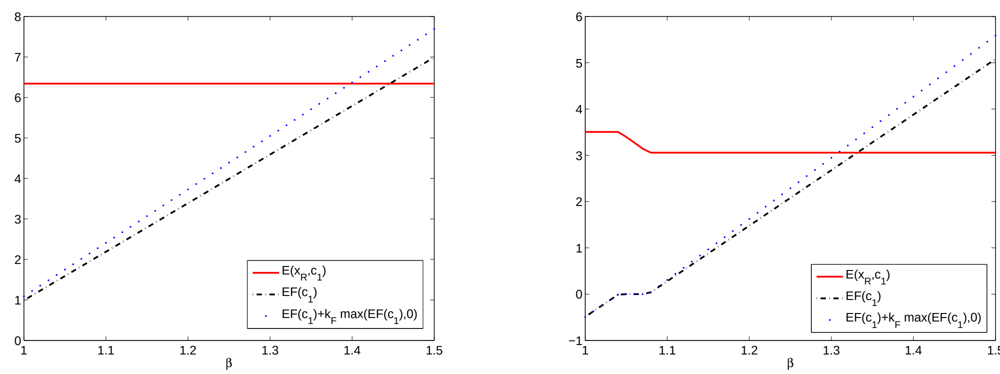
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 2: Comparative statics of the optimal reduced coupon. The figure plots the optimal reduced coupon c_1 as a function of the equity issuance costs parameter, k_F, for a tax rate equal to $\tau = 0.15$ in panel a). The comparative statics with respect to the creditors' premium, β, is plotted in panel b) for a tax rate of 35%.



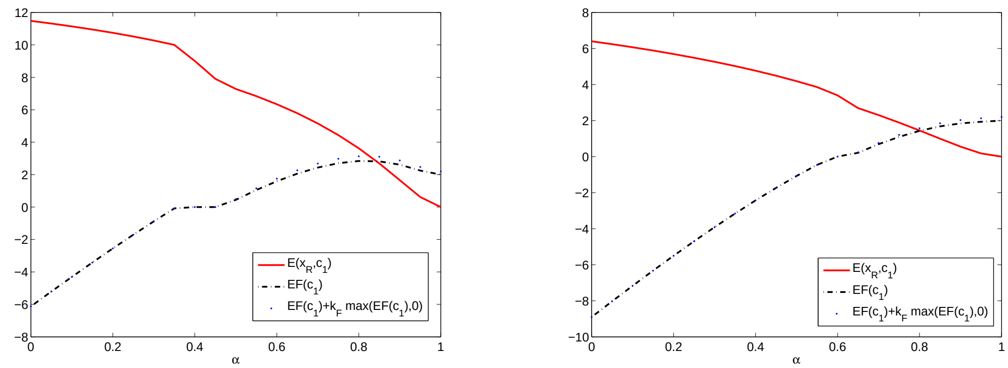
a) $k_R = 0.05$ b) $k_R = 0.3$ c) $k_R = 0.5$

Figure 3: Equity financing and issuance costs. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the equity issuance costs parameter, k_F, for different values of the renegotiation costs, k_R. The tax rate is equal to $\tau = 0.15$.



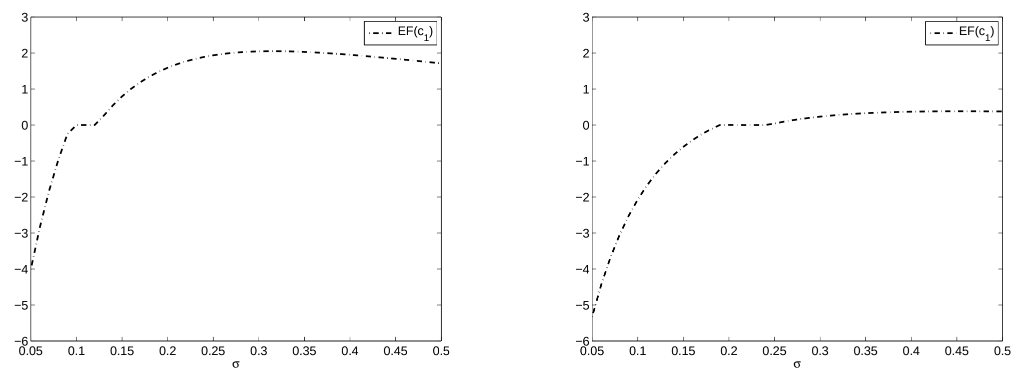
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 4: Equity financing and creditors' premium. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the creditors' premium, β, for different values of the tax rate.



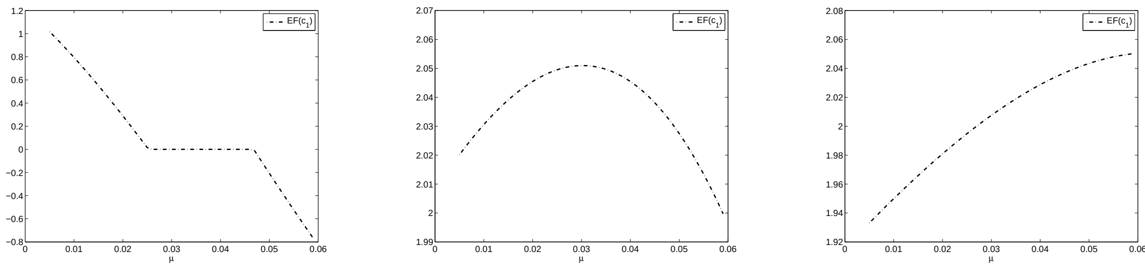
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 5: Equity financing and the recovery rate. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the recovery rate, α, for different values of the tax rate.



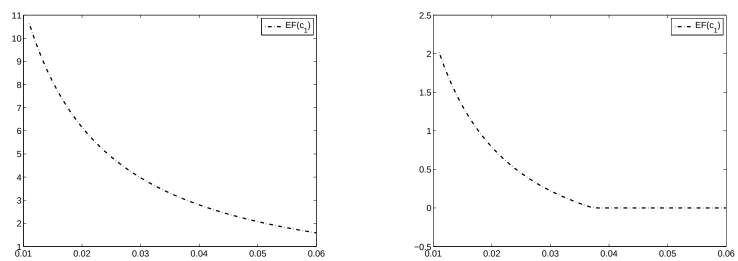
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 6: Equity financing and the volatility. The figure plots the amount of equity financing, $EF(C_1)$, as a function of the volatility, σ, for different values of the tax rate.



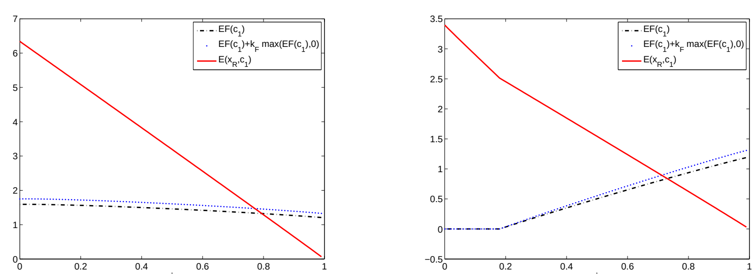
a) $\sigma = 0.15$ b) $\sigma = 0.35$ c) $\sigma = 0.40$

Figure 7: Equity financing and the drift. The figure plots the amount of equity financing, $EF(C_1)$, as a function of the drift, μ, for different levels of volatility. The tax rate is set to $\tau = 0.15$.



a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 8: Equity financing and the interest rate. The figure plots the amount of equity financing, $EF(C_1)$, as a function of the interest rate, r, for different values of the tax rate.



a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 9: Equity financing and the fraction of asset sales. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the fraction of asset sales, ϕ, for different values of the tax rate.

Tables

Table 1: Parameter values for the baseline case

Parameter	Value	Parameter	Value
x_0	2	r	6%
μ	1%	τ	35%
σ	20%	α	60%
c_0	2	β	1.05
k_R	5%	k_F	10%

Table 2: Numerical results of the baseline case ($k_R = 0$ and $\beta = 1$)

Results	$\tau = 0.35\%$	$\tau = 0.25\%$	$\tau = 0.15\%$
Q	0.360	0.360	0.360
A'	0.393	0.359	0.298
B'	0.406	0.384	0.348
c_1^*	c_1^A =1.028	c_1^{EF} =0.888	c_1^B =0.845
$EF(c_1^*)$	-1.092	0.000	0.390
$E(x_R, c_1^*)$	3.506	5.211	6.342
$D(x_R, c_1^*)$	13.092	12.000	11.610
$D(x_R, c_1^*)$	12.000	12.000	12.000
$E_R(x_0)$	9.023	10.378	11.764
$D_R(x_0)$	25.791	25.791	25.791
$V_R(x_0)$	34.814	36.169	37.555
$E(x_0, c_0)$	7.398	8.536	9.674
$D(x_0, c_0)$	25.791	25.791	25.791
$V(x_0, c_0)$	33.189	34.327	35.465