

Replicator dynamics in value chains: explaining some puzzles of market selection*

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Abstract

The pure model of replicator dynamics provides important insights in the evolution of markets, but has not met with much empirical support. This paper extends the model to the case of firms vertically integrated into value chains. Through an extended analytical model and numerical simulations we show that i) by taking value chains into account, the replicator dynamics may reverse its effect. In these ‘regressive developments’ of market selection, firms with low fitness expand because of being integrated with highly fit partners, and the other way around; ii) allowing a partner’s switching re-introduces selection forces into the upper layers of value chains and iii) periods of instability in the early stage of the industry life-cycle may be the result of an ‘optimisation’ of partners within a value chain, thus providing a novel and simple explanation of the evidence discussed earlier in the literature.

Keywords: *innovation, replicator dynamics, returns to scale, value chain.*

JEL Classification: C63, D24, L14, O32.

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1 Introduction

This paper studies the influence of firms' integration into value chains (henceforth VC) on the pattern of competitive market selection. Integration into VC implies that the performance of a firm in its market is not solely dependent on its own factors of competitiveness (e.g. cost efficiency, productivity, or profitability), but also on that of its partners with whom it is vertically related to produce a final good for consumers.¹ VCs are characterized by a certain degree of stability of their vertical relationships, because trust and division of labor among supplier and user industries are usually well-developed, while flexibility in the choice of partners is lower compared to a pure arm-length market transaction, where firms can compete for the best suppliers.²

The idea of this study is that accounting for VC relations sheds new light on the pattern of competition dynamics in markets. This pattern might be at odds with the original stylized model of market selection developed by Metcalfe (1994) and known as 'replicator dynamics model'. In the replicator dynamics, a firm's market success entirely depends on its own competitiveness in the following sense: a firm with above-average fitness will increase its market share whereas a firm with below-average fitness will lose it.³ In this classical replicator dynamics model, vertical relationships among firms are implicitly considered only as long as their effect is homogeneous across firms acting in the focal market. The differential performance of firms is therefore only attributable to different idiosyncratic competences and abilities. Contrariwise, we argue that VC connections can have a decisive influence on the firms' success or failure in market selection, because value chains can be highly heterogeneous due to suppliers' different cost structures and product qualities and a certain degree of stickiness of the connections.

The importance of the firms' vertical relations is confirmed not only by marketing research reporting that the value of business-to-business (B2B) contracts in many in-

¹In the paper, we use the term fitness rather than performance to indicate the abstract 'goodness' of the unit of analysis with respect to its reference group (competitors) or environment (industry, market) to keep consistency with the literature on the replicator dynamics.

²For a recent literature overview on the relevance of contractual frictions (within VC) for the organization of production, see Antràs and Chor (2013).

³An alternative derivation of the replicator dynamics used by economics scholars can be found in Schuster and Sigmund (1983). See Silverberg (1997) for a discussion.

dustries exceeds that of business-to-consumer (B2C) contracts,⁴ but also by numerous studies indicating that in contemporary economy the degree of specialisation and division of labour increases constantly and instead of conducting the entire production cycle in-house, many stages are outsourced to firms specializing in certain tasks and phases of the production process.⁵ An important feature of those vertical relationships, however, is that firms collaborating on a long-term basis adjust their production processes to each other so that switching one's partner becomes a very (if not prohibitively) costly issue.⁶ As a result, a firm may get locked into cooperation with less fit partners over time, which has a direct impact on the firm's performance and market share development.

The principle of reallocation of market shares from less efficient firms to their more fit competitors is the key principle of selection-based theories (Friedman, 1953; Foster et al., 2008), which also play an important role in the evolutionary economics literature (Nelson and Winter, 1982). However, when it comes to empirical testing of the theory, evidence of that principle is at best mixed and at worst contradictory (Cantner, 2016; Dosi et al., 2015). A first set of explanations for this set of results ranges from the choice of inappropriate variables for firm performance (fitness) to not clearly demarcated units and populations under analysis (firms vs products, industries vs markets/sub-markets) (Cantner, 2016). Other explanations refer to neglected fitness-relevant components (such as sunk costs, see Hölzl, 2015) or — as suggested in this paper — to the exclusion of factors relevant to market share changes, such as a firm's integration into a VC.

⁴The major reason for this is that in a typical VC there are many B2B transactions and only one B2C transaction, namely, sale of the finished good to the end customer. For example, a computer manufacturer makes several B2B transactions, such as buying microchips, different cables, cooler. Producers in turn, buy e.g., nanometer transistors, rubber, plastics and metal.

⁵This is in line with the recent literature illustrating increasing interdependencies in economic systems, being it credit networks, trade systems or supply chains (Schweitzer et al., 2009; Poledna et al., 2015). Hence, new approaches dealing with that system complexity are required, and the current study presents a step towards this objective.

⁶In general, the issue of switching costs and their economic effect are widely covered in the economic literature (Farrell and Klemperer, 2007). More along the story discussed in this paper, the fact that partnerships and linkages based on economic transaction but also on trust may be 'sticky' is confirmed, for example, by a wide range of literature on innovation networks (Cantner and Graf, 2006; Egbetokun and Savin, 2014). Networks stand between pure hierarchies and pure market settings and are characterised by a mix of formal and informal drivers of tying. In the end, our model — a chain — may be considered as a very special case of networks, for which, however, the general considerations relating to partner switching hold. Even more related to the issue we deal with in this paper is the literature on strategic management of supply chains (see for example Fisher (1997)).

The topic of vertical relations and VCs is usually studied through the lens of transaction-cost theory to assess advantages and disadvantages of integration and complex contractual arrangements (Bresnahan and Levin, 2012). By linking the literature on vertical relations and market selection, this paper fills a relevant gap and is the first, to our knowledge, to explicitly model vertical relations as determinants of selection dynamics.

To simplify the analysis, we assume that the number of output units of the downstream firm entirely determines the number of output units of the upstream firm.⁷ The strict relation in production units between downstream and upstream layers (i.e. production stages each representing a separate market) of a VC implies that capacity expansion (or reduction) of the upstream firm is not so much dependent on this firm's success in its own market, but on the success of its vertically related partner in its downstream market. As a result, and in order to focus on the effects VC-structures produce on selection dynamics, the replicator dynamics is at work in the final good market only, while upstream firms just respond to downstream market shares reallocation. This simplification becomes useful later on when we assess the effects on selection due to downstream competition and innovation dynamics in all the VC layers. By applying simulation techniques, we explore different possibilities of matching firms in a VC and focus on those where the usually expected outcome of the replicator dynamics is not showing up or even reversed in its results. This assumption is also in line with the central purpose of the paper of identifying the conditions impairing the market selection. The representation of VC, in this sense, is subordinate to the study of selection, as our focus is on the latter issue.

The paper is organised as follows. The next section provides a literature review. Section 3 describes the model of the replicator dynamics adapted to the VC context. The main results of the computational exercise are summarised in Section 4. Section 5 discusses the model's implications and concludes.

⁷Otherwise, firms would have to be considered as actors recombining complementary and potentially discrete resources (Wernerfelt, 1984) from different upstream sources. Discrete adjustments and the presence of indivisibilities — for example, in capacity expansion investments — may also be sources of frictions for the smooth working of the replicator dynamics model. Though compatible with our approach, this would add a lot of complexity to our model, while we prefer to keep it simple for the sake of clarity. Furthermore, our modelling strategy fits with the proper definition of VC, as compared to the related — though different — concept of production network (Sturgeon, 2001).

2 Literature review

Below, the literature on market selection and the replicator dynamics model is reviewed. The theoretical prediction of the replicator dynamics model is in line with the Darwinian ‘survival of the fittest’ principle: a firm with a higher (lower) fitness than the (share-weighted) average of the population increases (decreases) its market share and drives the selection dynamics by affecting the level of the share-weighted average fitness in the following periods (see equation (1)), unless negative feedback dampen the dynamics.

The view of the replicator dynamics as a stylized representation of the selection mechanism at work in the market is, however, not a consensus one. The literature on market dynamics treats selection in different ways; given that, we arrange our review around a main fault line separating, on the one hand, theoretical and empirical contributions and, on the other hand, contributions pertaining to a Neoclassical or Neo-Schumpeterian tradition.

Theoretical contributions from the Neoclassical standpoint draw from the toolbox of industrial organization assumptions and building blocks to produce models of ‘equilibrium evolution’ (e.g. Jovanovic, 1982 and Hopenhayn, 1992) and Markov-Perfect Industry Dynamics (Ericson and Pakes, 1995; Doraszelski and Satterthwaite, 2010). Firms’ heterogeneity, entry and exit dynamics and idiosyncratic shocks are accounted for, but the distribution of primitives of the system lead nevertheless to ergodic outcomes, namely equilibrium distribution of firms’ growth rates or productivity. A more applied approach to market dynamics and selection is that of Foster et al. (2008), that discuss the impact of selection on productivity and profitability by building and estimating a model in which the selection dynamics is determined by ‘physical’ productivity, prices, and demand shocks.

Neo-Schumpeterian theoretical models depart from the stringent assumptions required by equilibrium models (e.g. rational expectations) and focus on the meso-level of analysis. The aim of these analytical frameworks is to outline the ‘evolutionary’ data-generating process that reproduces real-world statistical regularities and stylized facts (Dosi et al. (1995); Winter et al. (2003)). The focus here is shifted to the interweaving

and co-existence of outcomes of turbulence and persistence in the distribution of industries' characteristics. These structural patterns emerge from innovation and learning in different regimes, and are a function of the different stages of the industries' life cycles. Studies in this tradition rely more explicitly on the evolutionary assumptions of the replicator dynamics. The idea is to test Schumpeter's concept of competition *for the market*, rather than competition *in the market*.⁸

Empirical contributions on market shares reallocation are strongly related (and overlapping) to those on productivity dynamics, especially when focusing on firm level analysis. Studies on productivity at the micro level have been boosted by the recent availability of firm and establishment level data, which shed light on the determinants of a firm's heterogeneity and characteristics (e.g., productivity and profitability) dispersion (Bartelsman, 2010).

Also in the empirical field, we can distinguish between studies in Empirical Industrial Organization (EIO) that operate a 'dissection' of aggregate productivity (usually on the industry level) by decomposing it into more fundamental components, either in a static or in a dynamic way, and Neo-Schumpeterian studies that borrow the mentioned decomposition framework to test for the persistent, non-equilibrium nature of market dynamics.

Static decompositions in the EIO tradition operationalise the above 'equilibrium evolution' model, often producing evidence at odds with the models predictions. These studies follow the seminal exercise of Olley and Pakes (1996), that separates the first moment of the productivity distribution (the non-weighted average productivity) from a covariance term measuring the distortion caused by the reallocation of shares from less to more productive firms. Maliranta and Määttänen (2015) extend the static Olley-Pakes decomposition to account for different categories of firms (stayers, entrants, exiters, and visitors). Dynamic decompositions usually build upon the contribution of Baily and Campbell (1992) to explain changes in aggregate productivity rather than its level. Griliches and

⁸We aim to capture the essence of what is usually discussed as 'Schumpeterian competition' – meaning not the setting in which firms (theoretically) set quantities or prices given the structure of the market, but a setting in which, through innovation, firms act pro-actively to gain shares of the market (see Geroski (2003)).

Regev (1995) and Foster et al. (2001) extended the dynamic decomposition framework to account for entry and exit (the two methods differ only by the benchmark productivity used to calculate the change). Dynamic decompositions distinguish between two main sources of productivity change: a *within component*, standing for firm-specific learning, and a *between* (plus covariance) *component*, capturing the competition and reallocation (selection) dynamics. A combination of the static and dynamic decomposition is derived in Melitz and Polanec (2015), where a dynamic Olley–Pakes decomposition with entry and exit is considered in order to explain aggregate productivity changes while maintaining the distributional approach of the static methods. In those studies, the magnitude of market selection is assessed indirectly through the sign and level of the between effect.

The Neo-Schumpeterian empirical literature employs as well (mostly dynamic) decomposition techniques. They are re-framed in this context as ‘evolutionary accounting’ exercises, and a conceptual connection between the reallocation (between) effect and the replicator dynamics model is usually highlighted. Some examples are Cantner and Krüger (2008) and Krüger (2014), that found for German manufacturing firms from 1981–1998 a weak tendency that above-average productivity firms are favoured over below-average productivity firms — thus supporting a market selection process in line with the replicator dynamics. By splitting the sample into two periods (before and after German reunification), Cantner and Krüger (2008) were able to highlight the stronger effect of market reallocation in the period 1990–1998, which may be interpreted as a consequences of increased competition due to the reunification shock. In a follow-up study by Krüger (2014), however, these results could not be confirmed. Similar results were also obtained by Bottazzi et al. (2008) and Coad (2007), where it is the within component — that is, learning — that mainly drives productivity growth. Finally, Yu et al. (2015) applied decomposition techniques to explain the spectacular productivity growth in China in the last two decades. Also in this case, the ‘between component’ capturing market selection seemed to have only a weak explanatory power.

In general, Metcalfe and Ramlogan (2006) take a critical view to the decomposition exercises, considering them useful to uncover the dynamics behind the restless nature

of capitalism on the one hand, but sensitive to the assumptions on the ‘shapes’ of the within and between components on the other. They call for a sound theory of the interplay between innovation and market reallocation, which is to be constructed above these evolutionary accounting methods.

Where Neo-Schumpeterian studies fully depart from their Neoclassical empirical counterparts are the attempts at a direct operationalisation of the replicator dynamics. In particular, Metcalfe and Calderini (2000) measured the speed of selection, a specific parameter in the replicator equation, for a dataset of the Italian steel industry. They did not find any convincing evidence of the replicator dynamics being at work. More recently, Dosi et al. (2015) enriched the picture on the strength of selection by combining direct and indirect approaches. They conducted a decomposition exercise using firm-level productivity data for four countries (US, France, Germany, and the UK) and also estimated directly the speed of selection for different industries in each country. For both exercises, the results are rather mixed and do not support the standard replicator model. A major reason is that an industry is not a market, but a collection of markets, the firms are multi-product, and the fitness variable is entirely determined by the supply side, in particular in terms of unit costs of production. Cantner et al. (2012) analysed the rather narrowly defined market for compact cars in Germany using a relative quality-price ratio and aggregated information on four main product characteristics and prices as a proxy for a firm’s fitness. They found rather compelling evidence of the selection effect working in the expected direction.

To sum up, empirical studies on selection found the reallocation component (in decomposition analyses) and the direct operationalisation of the replicator equation to display a negligible explanatory power. This result, however, might be very sensitive to identification problems regarding the unit of analysis.

Indeed, the replicator dynamics typically⁹ requires a market with one price (and market clearing) and profits (or losses) used to invest into capacity expansion (or for disinvestment, respectively). With a firm’s integration into VCs, this mechanism may not

⁹For an example of a model avoiding such an assumption explicitly see Dosi et al. (2017).

work properly. A first issue is the law of one price: with long-term relationships, special prices can be negotiated, leading to heterogeneity in prices within the VC structure.¹⁰ A second issue, related to the former, is that vertically related firms may cooperate and invest into capacity extension together (one example is Intel sharing and jointly planning capacity extension with some of its main suppliers (Shamin and Kempf, 2006)). A third issue is related to the demand for an intermediate product: if the downstream firm performs successfully, it will increase its productive capacity and, hence, the demand for the intermediate product; as a result, the intermediate supplier, even when performing below average in its own market, will face this increased demand, provide for the necessary expansion in capacity, and experience a growing market share. Those issues, though perfectly realistic, are at odds with the selection mechanism and may lead to the observation that in a specific market a below-average-performing firm is able to increase market shares, because its very well-performing partners along the value chain are able and willing to pay higher intermediate product prices or are engaged in an investment cooperation with that firm or demand more intermediate products.

Leaving aside the features of differentiated prices and cooperation in investment and arguing in terms of the output relationships between firms, what we demonstrate in this paper is that if all firms of a value chain hold in their own market the *same* fitness rank, then market selection on each layer follows the principle of replicator dynamics; however, if downstream firms in their own market showing a *different* fitness rank than their upstream partners, the principle of replicator dynamics does not necessarily hold in the upstream markets.

3 Replicator dynamics in value chains: the model

In this section we provide the analytical derivation of the replicator dynamics model and its extension to multiple markets connected via VCs. We start by clarifying the variables and notation we use in this study.

¹⁰Hereinafter, we assume that the law of one price continues to hold in the final (consumption) market — so that consumers face the same price for a homogeneous product delivered by the competing VCs, while it may not hold in the upper layers of the VCs.

The setting we are analyzing consists of M markets, on each of which n firms compete for the market share producing a homogeneous good but with different costs. No firm can produce a finished good alone, but only in cooperation with firms from other layers. Thus, we leave out the possibility of vertical integration with one single firm present on more than one layer (market). Furthermore, we abstract from entry phenomena to isolate the effect of selection dynamics. For the sake of simplicity, we also ignore sources of uncertainty for value chains, such as demand (volume and product specification), process (e.g., machine downtime and transportation reliability) and supply (e.g., delivery reliability), described in detail in Strader et al. (1998). Instead, we assume perfect collection and sharing of information between VC members, which results in no inventory holdings and immediate order fulfillment cycle time. Also for simplicity, goods on each market (including the market of finished good M) are homogeneous: market dynamics is only driven by the firms' differential unit costs and VC relationships.

In order to analyze the competition taking place in the different markets we first need to choose a proxy for the competition variable — that is, for the fitness variable. As discussed in the literature review, any performance indicator may serve this purpose; usually, more or less elaborated versions of the (labor or total factor) productivity, value added per employee, product quality or unit costs are used. In order to facilitate comparison and to highlight our contribution with respect to other modeling exercises (e.g. Mazzucato, 1998), we focus on unit costs and label firm i 's total unit cost as C_m^i (for each firm $i \in n_m$ in market m). Once we consider innovative (see Section 4.2) activities, a firm's unit cost is allowed to change; otherwise it is considered fixed and independent of scale. We analyze this setting using a continuous-time version of the replicator model (Metcalfe, 1994); first, we describe the model for one market and, second, we extend it to the case of connected markets.

3.1 The model for a single market

The structural pattern and dynamics in a single market are analyzed tracking the development of firms' market shares. The market share of firm i on market m is given by

$s_m^i = \frac{y_m^i}{y_m}$, where y_m^i is firm i 's volume of output and $y_m = \sum_{j=1}^n y_m^j$ is the total volume of output of all firms in market m . To analyze market shares development we take the time derivative of s_m^i . With $g_m^i = \frac{\dot{y}_m^i}{y_m^i}$ as a firm's growth rate and \dot{y}_m^i being the time derivative $\frac{\partial y_m^i}{\partial t}$, the following relationship for the market share development can be derived:

$$\frac{\partial s_m^i}{\partial t} = \dot{s}_m^i = s_m^i (g_m^i - \bar{g}_m) \quad (1)$$

where $\bar{g}_m = \sum_{j=1}^n s_m^j g_m^j$ is the market share weighted average growth rate in market m . This dynamics is driven by differential growth in the sense that firms with above (below) average growth increase (decrease) their market shares. In a nutshell, market share growth is a function of capacity expansion (the growth rate g), which might also be negative indicating capacity contraction.

A firm's growth rate can be decomposed into several determinants, endogenous to market results and exogenously driven by factors outside the market. Concerning the endogenous mechanism for firm growth, for each market, assuming that the law of one price holds, there is a market clearing price p_m . Depending on unit cost of production C_m^i firms experience gains or losses which drive their investment or disinvestment into capacity with $g_m^i = \lambda(p_m - C_m^i)$. Here, the economic interpretation for the parameter λ is the (time and market-invariant) share of investment out of unit profits. By substituting $g_m^i = \lambda(p_m - C_m^i)$ into $\dot{s}_m^i = s_m^i (g_m^i - \bar{g})$ we obtain a unit cost based version of the market competition equation:

$$\dot{s}_m^i = s_m^i \lambda (\bar{C}_m - C_m^i) \quad (2)$$

Independent of the market price, a firm i with unit costs C_m^i below (above) the market share weighted average unit costs $\bar{C}_m (= \sum_{j=1}^n s_m^j C_m^j)$ in market m will increase (decrease) its market share. λ drives *ceteris paribus* the speed of competition among firms and, hence, the development of market shares.

Next to the endogenous mechanism, we now include an exogenous factor driving the growth of a firm, namely firm-specific demand growth. Formally, we posit that firms' rate of capacity expansion g_m^i is additionally affected — besides (dis)investments out of profits/losses — by firm-specific demand growth for each market m (that we label $g_{d,m}^i$). Weighting these factors with $\theta_{c,m}$, and $\theta_{d,m} = (1 - \theta_{c,m})$, the respective shares of the served demands in total capacity of firm i , the expression for growth dynamics becomes:

$$g_m^i = \theta_{c,m}\lambda(p_m - C_m^i) + \theta_{d,m}g_{d,m}^i \quad (3)$$

Substituting (3) into the selection equation and after some manipulation we obtain:

$$\dot{s}_m^i = s_m^i [\theta_{c,m}\lambda(\bar{C}_m - C_m^i) + \theta_{d,m}(g_{d,m}^i - \bar{g}_{d,m})] \quad (4)$$

where $\bar{g}_{d,m} (= \sum_{j=1}^n s_m^j g_{d,m}^j)$ is the market-share-weighted average exogenous firm-specific demand shift. In this new selection equation market share dynamics depends on two different competition variables: unit cost C_m^i and firm-specific demand growth $g_{d,m}^i$.

3.2 The model for connected markets

We now develop further the model derived in equation (4) to analyze a setting in which markets are vertically related through VCs. To simplify the model, we assume that each firm can have at most one upstream and one downstream partner — a VC firm in the final goods market delivers only to consumers and has only one upstream partner; a VC firm most upstream equivalently has only one downstream partner. VC partners coordinate themselves by contracts on how much to deliver and at what price. This relationship is based on the demand of the downstream firm for the upstream firm's output — an intermediate good. In our model, we take this into account through the firm-specific demand growth variable $g_{d,m}^i$ used in equations (3) and (4).

Being the result of a successive transfer of intermediate goods from layer to layer, the unit price of layer $m - 1$ becomes part of the total unit cost of layer m , to which its own *layer-specific unit cost* has to be added. In general, the total unit cost for each

layer can be decomposed as $C_m^i = p_{m-1}^i + c_m^i$, where p_{m-1}^i is the price of the intermediate good of the upstream layer $m - 1$ and c_m^i is the layer-specific unit cost. The price of the upstream intermediate good $m - 1$ can be expressed as $p_{m-1}^i = p_{m-2}^i + c_{m-1}^i$, with this relation holding for all the layers of a VC. For the sake of simplicity, we assume that each layer charges only its layer-specific cost on top of the cost of supplies it receives from upstream in order to determine the price of its product.

In sum, for any VC and for the first layer (layer one), C^i and c^i are necessarily identical ($C_1^i = c_1^i$), as the firm has only its own cost of extracting primary resources of production; for the second layer of the VC, the total unit cost becomes $C_2^j = c_2^j + p_1^j = c_2^j + c_1^j$ and so on until for the M th layer of production, $C_M^j = c_M^j + p_{M-1}^j = c_M^j + \sum_{m=1}^{M-1} c_m^j = \sum_{m=1}^M c_m^j$.

As stated earlier, we assume that on each layer m one unit of the intermediate output from $m - 1$ is used to produce one unit of output. Hence, the quantity of output units of firm i in the final layer must be equal to its supplier's one in each preceding layer $y_M^i = y_{M-1}^i = \dots = y_1^i$, while the output volume of the entire market M is equal to the preceding ones: $y_M = y_{M-1} = \dots = y_1$. As a consequence, the following equalities have to hold:

$$\dot{y}_M = \dot{y}_{M-1} = \dots = \dot{y}_1, \quad (5)$$

i.e. aggregate changes in outputs on all markets are equal. It further holds that

$$\dot{y}_M^j = \dot{y}_{M-1}^j = \dots = \dot{y}_1^j, \quad (6)$$

i.e. changes in outputs of all firms on different layers matched into a VC are also equal. As a result, the model yields that also changes in market share of all firms from different layers related to one VC are the same:

$$\dot{s}_M^j = \dot{s}_{M-1}^j = \dots = \dot{s}_1^j. \quad (7)$$

On the basis of these quantity and unit cost relations within VCs we can outline the

market selection model for a VC. For the final layer in a VC, layer M , differential growth is only driven by competition for the market with total unit cost C_M^i as competition variable; hence, with $\theta_{c,M} = 1$ and $\theta_{d,M} = 0$, we use:

$$\dot{s}_M^i = s_M^i \lambda (\bar{C}_M - C_M^i), \quad (8)$$

For all other layers, $m < M$, we assume that the (intermediate) goods are only traded within VCs. Hence, we set the parameters $\theta_{c,m} = 0$ and $\theta_{d,m} = 1$. This implies that all firms are integrated in a VC and conduct transactions only with their VC partners.

$$\dot{s}_m^i = s_m^i [g_{d,m}^i - \bar{g}_{d,m}], \quad \forall m < M \quad (9)$$

The rationale of this choice has to do with our aim to highlight the conditions under which the outcome of the replicator dynamics for firms integrated into one of the VCs and doing business *exclusively* with their VC partners deviates from its baseline one-market formulation. To prevent firms from dominating on their markets purely based on VC relation to highly fit partners, we later introduce the possibility for firms in any layer to switch VC according to a simple matching algorithm (Section 4.3), thus, re-introducing competition into the upper layers.¹¹

With (8) and (9) the pattern and dynamics of competition among firms and VCs in vertically related markets is fully described. Competition based on firm-specific total unit costs C_m^i takes place within markets. Competition among VCs takes place in the final market M . It is the last layer's total unit cost C_M^i that equals the sum of all chain elements' unit cost and which governs competition in M . The result of this competition is transmitted to upstream firms ($m < M$) via the direct specific demand growth $g_{d,m}^i$ within a VC. In this VC dimension the share dynamics in all the upstream markets follows

¹¹We thank a referee for pointing out that without competition among non-final layers, our VCs are equivalent to firms having fully internalized the VC structure. We consider the point well-taken; therefore we developed the more general version of the selection equation in Section 3.1.

the share development in the last — downstream — market M .

4 Numerical analysis for different model scenarios

4.1 Value chain matching with no innovation

We consider two contrasting scenarios: in the first one, firms integrated in a VC are matched according to their fitness (layer-specific unit cost) rank: the fittest firm in market M with the fittest ones in markets $M - 1$, $M - 2$ etc. and the other way around. We label this scenario ‘ordered matching.’ In the second scenario, firms are matched in an unordered manner — some less fit firms may either be matched with fitter ones (see Figure 1) or not. We label this scenario ‘unordered matching’ (i.e. firms being matched without being sorted according to their fitness). To focus on the selection dynamics driven by VC relations, we assign to all the firms the same initial market share. Furthermore, we consider for ordered matching a situation in which firms located on each market m have their layer-specific unit cost drawn in a way that each downstream firm surpasses the next one by the same amount (e.g., 1, 1.5, 2, ...).

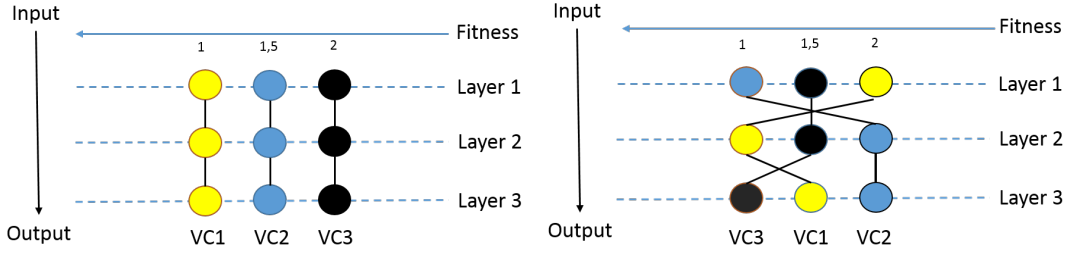


Figure 1: Firms’ ordered and unordered matching in value chains

Note: The left panel corresponds to ordered matching, while the right one to unordered matching.

Let us denote the fittest firm in each layer with index a , the second fittest with b , and (for the simplified case of three firms only) the least fit firm with c . Hence, in ordered matching we have all a firms linked together (having total unit cost C_M^a), while in unordered matching they are randomly assigned to different VCs. In the ordered matching scenario (benchmark) the fittest firm in each layer increases its market share

according to equation (1). In particular, for the final layer it holds:¹²

$$\Delta s_M^{a,t} = s^{a,t} - s^{a,t-1} = s_M^{a,t-1} \lambda (\bar{C}_M^{t-1} - C_M^{a,t-1}). \quad (10)$$

The difference between (10) and (1) is that due to accumulating costs along the VC, ‘monopolisation’ takes place faster:

$$C_M^a - C_M^b = c_M^a - c_M^b + \left(\sum_{m=1}^{M-1} c_m^a - \sum_{m=1}^{M-1} c_m^b \right) > c_M^a - c_M^b. \quad (11)$$

The inequality in (11) should not be misinterpreted with an attempt to ‘mixing up apples and oranges’ by comparing selection with and without VCs or, in other words, between a scenario of arm-length transactions and one of contracted VC ties. In fact, products produced within a single production step necessarily differ from products requiring more than this single step and the resulting production chains are different in nature and structure. The inequality in (11) rather points out that if VCs in the real world would all have been arranged according to ordered matching, reallocation of market shares towards the most fit firm in the respective market would have been easier to identify. This is simply because fitness gap between the two VCs in (11) is larger than the fitness gap between any two firms belonging to those VCs on any single layer. Now, given that former studies had enormous difficulty to find robust empirical evidence for the replicator dynamics at work (Section 2), one simple explanation may be the unordered matching of VCs resulting in more fit partners from one layer being integrated with less fit partners from other layer(s).

In the unordered matching scenario, thus, monopolisation takes place more slowly than under the ordered matching. Eventually, one of the VCs can dominate the other one (as long as its total unit cost is lower), but this has the side effect of a less fit firm in one (or more than one) layer dominating with its market share its counter-partners. To illustrate that, consider Figure 2.¹³ The leftmost charts in the upper and lower panels display the differences in the speed of market reallocation for ordered and unordered

¹²Here we outline the selection equation in discrete form, as this to be computed in the simulations.

¹³A case with 10 layers is exemplified in Figure 13 in the Appendix.

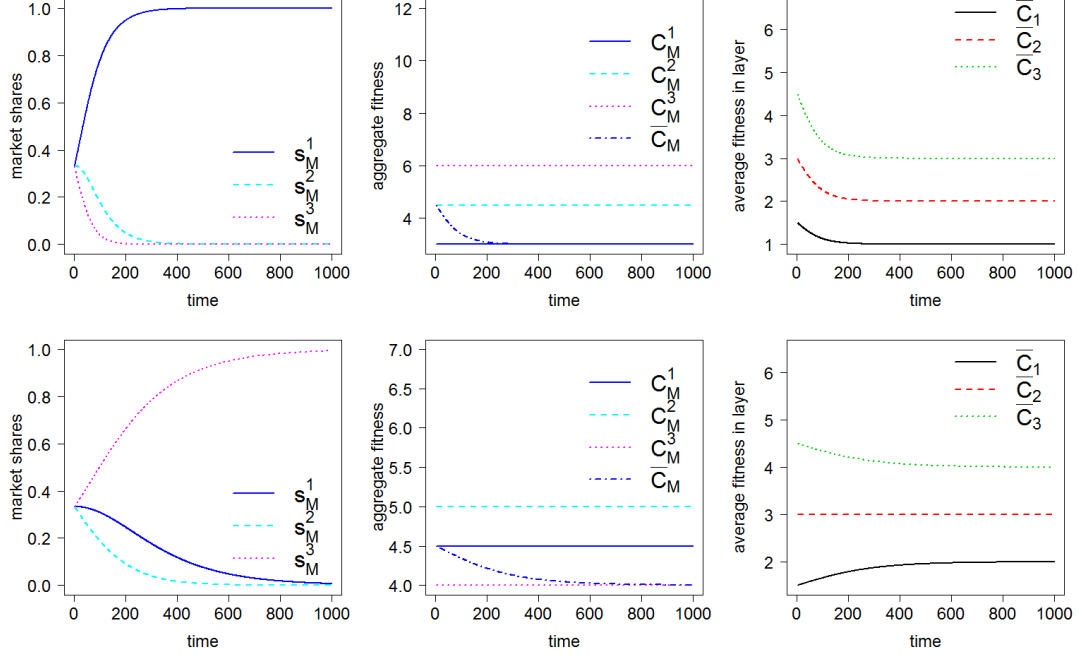


Figure 2: Dynamics with ordered and unordered matching

Note: The upper panel corresponds to ordered matching, while the lower one to unordered matching. $M = 3$ and $N = 3$.

matching, respectively; the overall selection dynamics of the three different VCs looks rather similar, except that the final winner is different. The middle charts show the corresponding dynamics with respect to the aggregate fitness of the VCs. Finally, the rightmost charts illustrate the development of the average total unit costs \bar{C}_m on each of the three markets in a layer-wise manner. In case of ordered matching, the first VC is built up by the dominating and, hence, best firms in *each* market (layer); these firms drive down the average \bar{C}_m in each layer to the level of VC1, just in line with the replicator dynamics. Also in the unordered matching case, the average \bar{C}_m in each layer approaches that of the prevailing VC — in this case VC3. However, the average fitness approached in each layer is not necessarily the ‘best’, as it should be for replicator dynamics. This is more evident when looking at the average total unit cost in layers 1 and 2: they do not decrease (one even increase) as in the corresponding chart for the ordered matching scenario. We name such violation a *regressive development*.

Furthermore, the famous *Fisher’s principle* — stating that the change in the average fitness and, hence, the speed of market shares reallocation in a population of competing firms is proportional to the variance in fitness — is also valid for this model of firms

matched into value chains.¹⁴ In particular, it is obvious from Figure 2 that the difference in aggregate fitness between value chains in case of ordered matching is higher (since all fittest firms are matched together against all least fit firms). As a result, average total unit cost improvement and market share reallocation take place much faster. A similar effect can be also obtained for larger variance in fitness between firms on any layer: increasing the variance in fitness between firms on any layer $(\sigma^m)^2 = \sum_{n=1}^N s^n (C_m^n - \bar{C}_m)$ in our model also increases the differences in expected total unit costs between the value chains, which automatically leads to a faster market reallocation process. This result holds for both, ordered and unordered matching scenarios (Figure 13 in the Appendix).

4.2 Value chain matching with innovation

Now, we further extend the model by allowing firms to endogenously improve their specific fitness (that is to reduce their layer-specific unit costs) through innovative activities. In this way, the selection dynamics is affected by another force: firms' innovation activities resulting in performance improvements on each specific layer (market). This choice is justified by the fact that it captures the real-world behaviour of firms. In fact, firms on each layer are subject to shifts in their market shares but, at the same time, take efforts to improve their own idiosyncratic processes. Those innovation activities result, first, in performance improvements on each specific layer/market (reduction of layer-specific unit cost c_m^i). Second, these improvements add up to a reduction of VC total unit costs showing up in the last layer/market.

More specifically, we adopt three alternative specifications of a cost-reducing innovation process: with *constant*, *decreasing*, and *increasing returns to scale* (henceforth, CRS,

¹⁴This strictly holds for the aggregate fitness (total unit cost) of VCs (since competition on the end consumer market defines market share reallocation, $\Delta \bar{C} \sim \sigma^2(C^j)$), but also — indirectly — for each market layer (since the variance in firm's fitness on each of the layers contributes to the respective variance of the value chains). For each single layer, however, this may not necessarily hold, since even though there is a low variance in fitness, e.g., on layer one, the market share reallocation may still be high and *equal in speed for all layers* due to high variance in fitness on other layers.

DRS and IRS, respectively). Following Mazzucato (1998), this is done by setting

$$\dot{c}_m^i = -c_m^i \gamma \quad \text{for constant returns} \quad (12)$$

$$\dot{c}_m^i = -c_m^i \gamma (1 - s_m^i) \quad \text{for decreasing returns} \quad (13)$$

$$\dot{c}_m^i = -c_m^i \gamma s_m^i \quad \text{for increasing returns,} \quad (14)$$

where γ is an exogenous rate of technical improvement (cost reduction) being reinforced, dampened, or neutrally affected by firm size (measured by the market share).

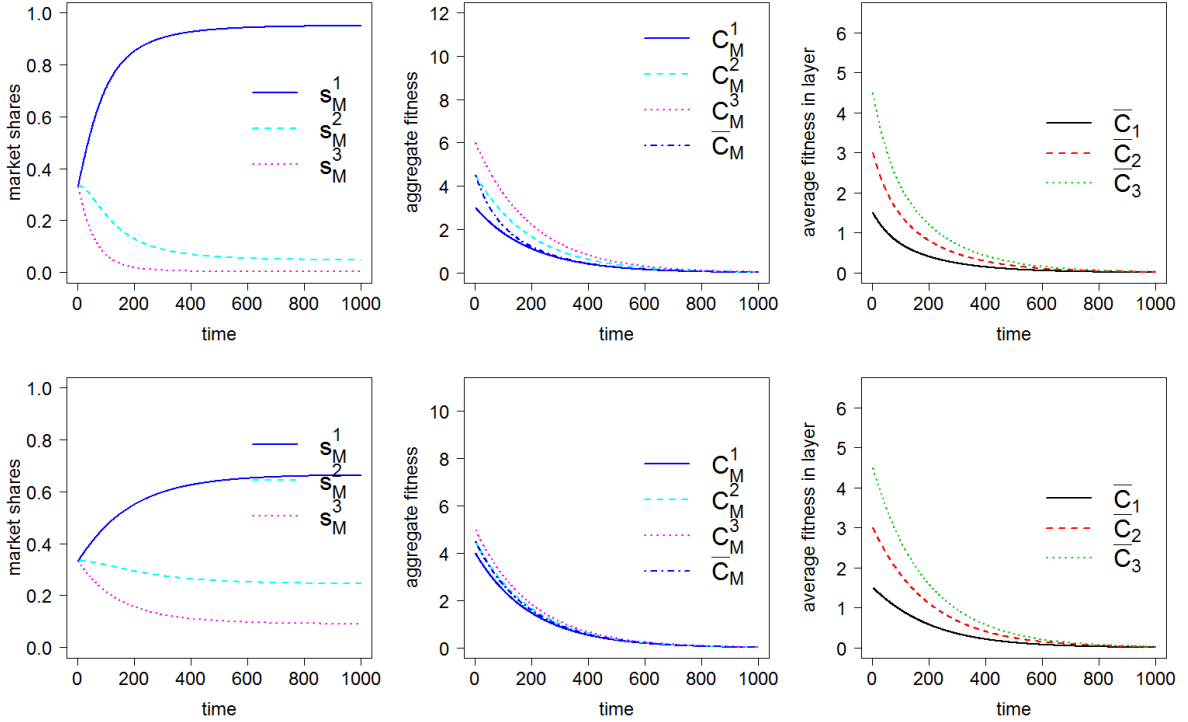


Figure 3: Dynamics with ordered and unordered matching and innovation with CRS

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. $M = 3$, $N = 3$, and $\gamma = 0.005$.

In accordance with the standard replicator model, the possibility of cost reduction with **constant returns to scale (CRS)** creates the possibility of more than one VC staying on the market (see leftmost charts in Figure 3). Since the difference in aggregated fitness between VCs is larger in ordered matching, the dominating value chain achieves a higher market share than in case of unordered matching (alternatively, it reaches the monopoly position faster). The fact that the less fit firm obtains an advantage through linkages

with strong partners in other layers can also be seen from Figure 3. The dynamics of the average fitness in each layer, as shown in the rightmost charts, looks similar in ordered and unordered matching, but its interpretation becomes less trivial. In fact, the reduction in average total unit costs in each layer can be driven by two forces. First, if the firm with initially lowest cost increases its market share, then average layer fitness decreases. Second, even if a firm with above-average cost linked to a more fit VC gains market share, the average layer's fitness may still decrease. This is because the magnitude of within-firm improvement compensates the regressive development.

To disentangle these two related forces, consider the following decomposition of the change in market-weighted average total unit cost in layer m ¹⁵:

$$\begin{aligned}
\Delta \bar{C}_m^t &= \bar{C}_m^t - \bar{C}_m^{t-1} = \sum_i s_m^{i,t} C_m^{i,t} - \sum_i s_m^{i,t-1} C_m^{i,t-1} = \\
&= \sum_i (s_m^{i,t} C_m^{i,t} - s_m^{i,t-1} C_m^{i,t-1}) = \sum_i ((s_m^{i,t-1} + \Delta s_m^{i,t})(C_m^{i,t-1} + \Delta C_m^{i,t}) - s_m^{i,t-1} C_m^{i,t-1}) = \\
&= \sum_i (s_m^{i,t-1} \Delta C_m^{i,t} + s_m^{i,t-1} \Delta C_m^{i,t} + \Delta s_m^{i,t} C_m^{i,t-1} + \Delta s_m^{i,t} \Delta C_m^{i,t} - s_m^{i,t-1} C_m^{i,t-1}) = \\
&= \sum_i s_m^{i,t-1} \Delta C_m^{i,t} + \sum_i \Delta s_m^{i,t} C_m^{i,t-1} + \sum_i \Delta s_m^{i,t} \Delta C_m^{i,t} - 0 = \\
&= \sum_i s_m^{i,t-1} \Delta C_m^{i,t} + \sum_i \Delta s_m^{i,t} C_m^{i,t-1} + \sum_i \Delta s_m^{i,t} \Delta C_m^{i,t} - \sum_i \Delta s_m^{i,t} \bar{C}_m^{t-1} = \\
&= \sum_i s_m^{i,t-1} \Delta C_m^{i,t} + \sum_i \Delta s_m^{i,t} (C_m^{i,t-1} - \bar{C}_m^{t-1}) + \sum_i \Delta s_m^{i,t} \Delta C_m^{i,t}, \tag{15}
\end{aligned}$$

where any $\Delta C_m^{i,t} = \Delta \sum_{m=1}^{M-1} c_m^j + \Delta c_m^{i,t}$, that is the total unit cost of a firm changes as a result of innovation in all the suppliers' layers and in its own specific production process. The first term in (15) captures the within effect (the sum over all the individual firms cost changes, with each multiplied by the market share before the change in fitness), the second term — the between effect (the sum of market share changes weighted by the deviation of a firm's cost level from the market-weighted average, that is basically **the replicator term** we are most interested in), while the third term is the so-called covariance effect (a negative/positive value indicating that the selection is faster/slower

¹⁵The derivation of the decomposition formula reads as follows when a 0 is added: $0 = \bar{C}_m^{t-1}(1 - 1)$; as any sum of shares must equal unity, we can rewrite the expression as $\bar{C}_m^{t-1}(\sum_i s_m^{i,t} - \sum_i s_m^{i,t-1}) = \bar{C}_m^{t-1} \sum_i (s_m^{i,t} - s_m^{i,t-1}) = \bar{C}_m^{t-1} \sum_i \Delta s_m^{i,t} = \sum_i \Delta s_m^{i,t} \bar{C}_m^{t-1} = 0$.

than predicted by the replicator mechanism alone (Cantner and Krüger, 2008)). The covariance component can be interpreted as the dynamics returns to scale introduced by innovative activities.

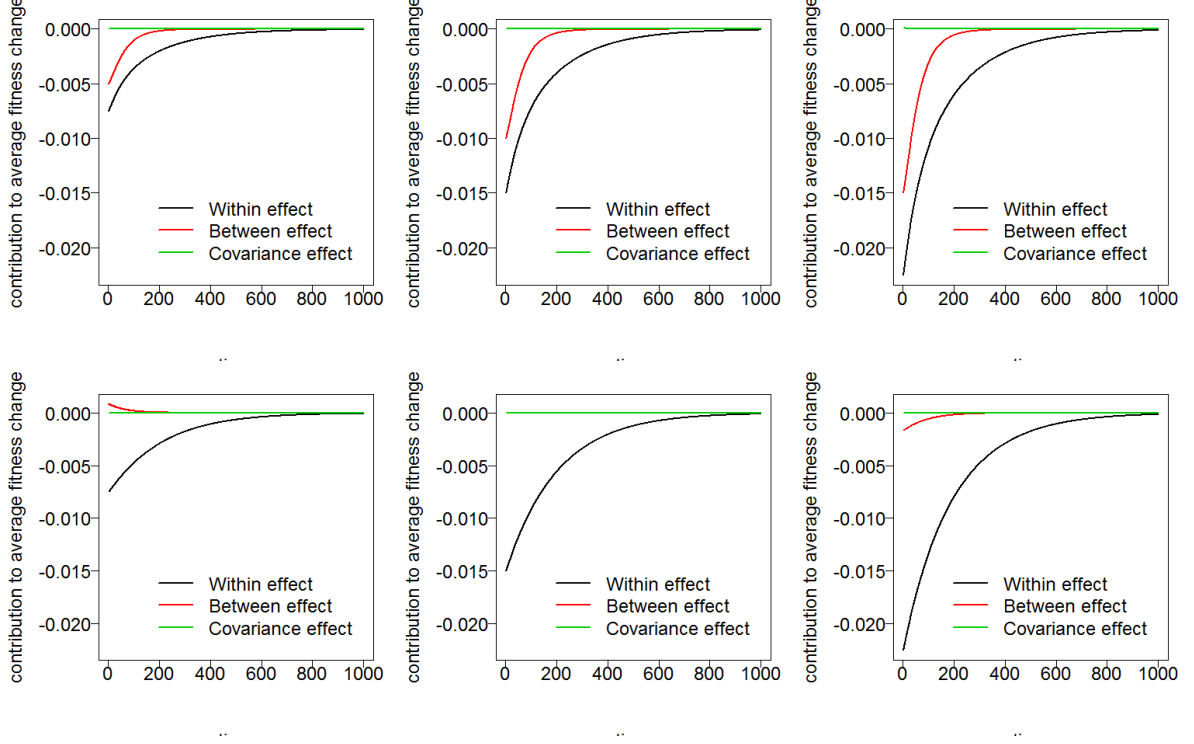


Figure 4: Decomposition of change in average unit cost with CRS

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. From the left to the right, the markets (layers) 1, 2, and 3 are shown.

For the standard replicator dynamics to hold, the between effect has to be negative, i.e. each firm displaying a cost higher (or a fitness lower) than the share-weighted market average should decrease its market share (the corresponding decompositions are reported in Figure 4). While in ordered matching the between effect is consistently negative and of a magnitude comparable with that of the within effect in all three layers, the pattern is very different for unordered matching. In particular, the between effect is much smaller in absolute terms and turns to be positive in layer one, indicating that in this market a firm integrated into a strong VC increased its market share, although its fitness was below the market average. Hence, from the decomposition exercise, it becomes clear that the replicator dynamics does not necessarily hold in markets that are vertically related but not the final good market (layer M).¹⁶

¹⁶Note that in the former exercise with no innovation the within and covariance effects are zero, as

For decreasing returns to scale (DRS): setting the rate of cost reduction to be inversely proportional to market share, a typical pattern of high volatility of market shares is obtained in the initial period. This volatility is higher in ordered matching, where the differences in fitness between the value chains are higher (Figure 5). The corresponding contribution of the within, between, and covariance effects to the change in market-weighted average fitness is presented in Figure 6. Again, the between effect is close to zero and occasionally turns positive in the unordered matching scenario, but not in the ordered one. Since firms with a smaller market share innovate here faster by definition, the most cost-efficient firm in the upper layer matched in the least cost-efficient VC soon becomes a part of the most cost-efficient VC. As a result, the positive between effect turns negative soon after (see leftmost chart in the bottom panel of Figure 6).

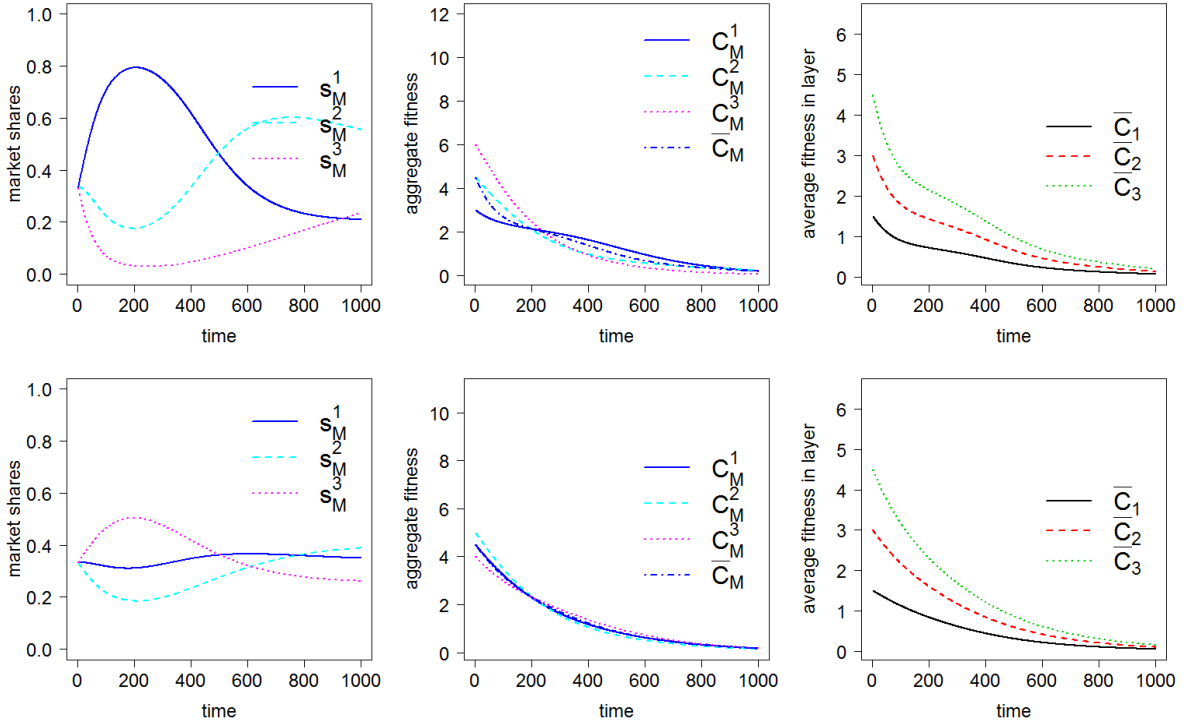


Figure 5: Dynamics with ordered and unordered matching and innovation with DRS

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. $M = 3$, $N = 3$, and $\gamma = 0.005$.

there is no change in layer-specific unit costs over time. The between effects, however, are present and also occasionally turn positive in one or the other layer in the unordered matching scenario.

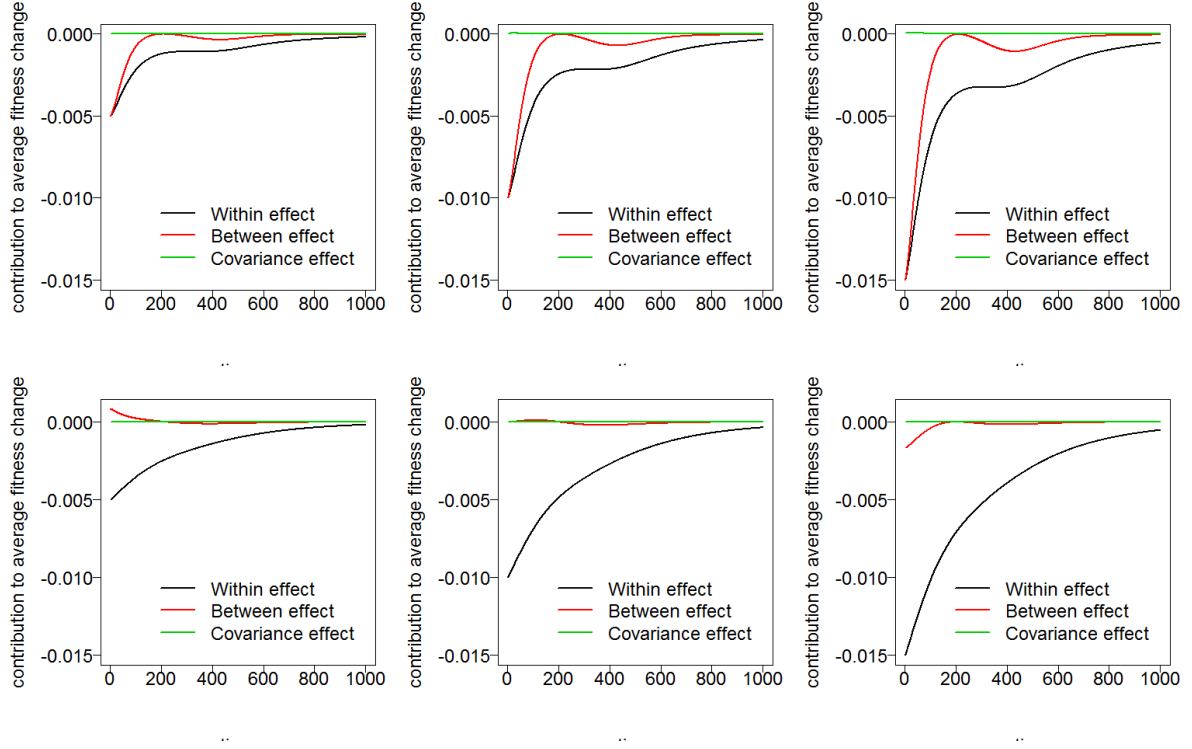


Figure 6: Decomposition of change in average unit cost with DRS

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. From the left to the right, the markets (layers) 1, 2, and 3 are shown.

For increasing returns to scale (IRS): as it is typical for IRS, once firms start innovating, the unit costs and market shares (at least for the leading VC) change much faster than in the scenario with constant returns to scale (Figure 7). The process of market monopolisation again takes place more quickly in ordered matching, as the initial advantage of the fittest VC over its counterparts is bigger. The decomposition into the between, within, and covariance effects demonstrates that for unordered matching in the first layer, the between effect deviates from the prediction. It is positive in the first three hundred periods, but turns negative afterwards (Figure 8). The reason for this is that the firm with unit costs below the average improves its fitness faster than market monopolisation takes place. Hence, before the VC this firm is integrated into dominates the market, this firm becomes the fittest one on its respective layer. It can therefore be concluded that under IRS, a less fit firm integrated into a superior VC is given an opportunity to improve its fitness rank to the level of the partners.

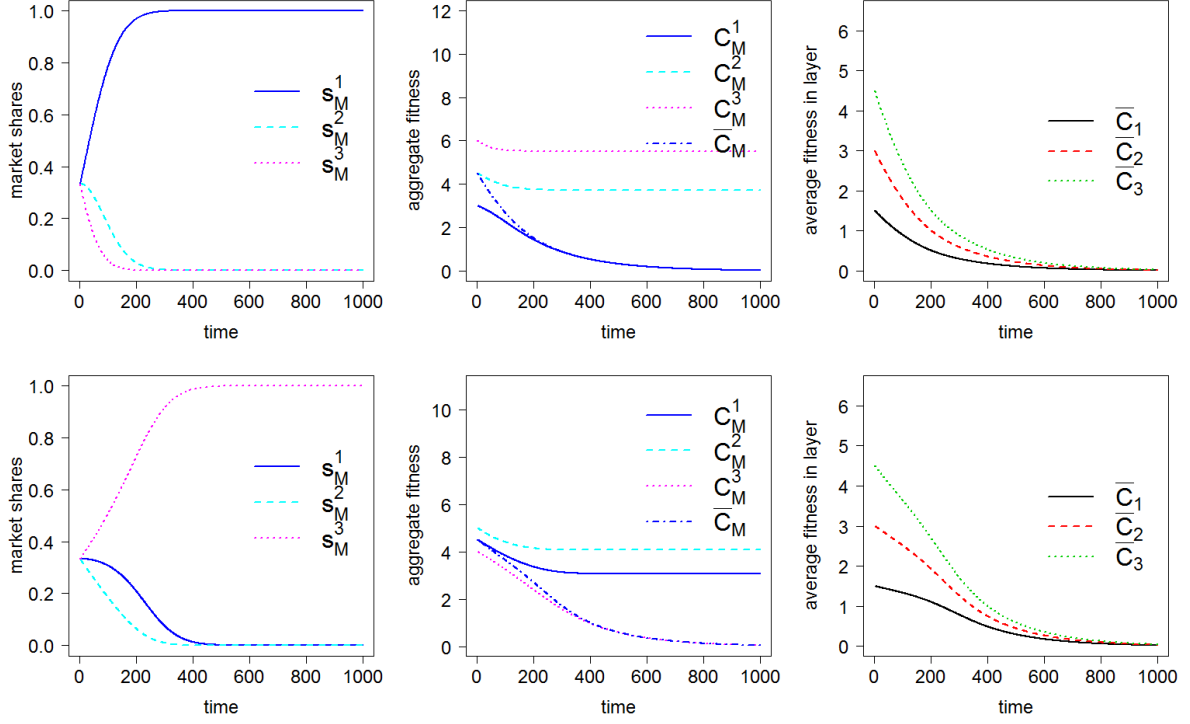


Figure 7: Dynamics with ordered and unordered matching and innovation with IRS

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. $M = 3$, $N = 3$, and $\gamma = 0.005$.

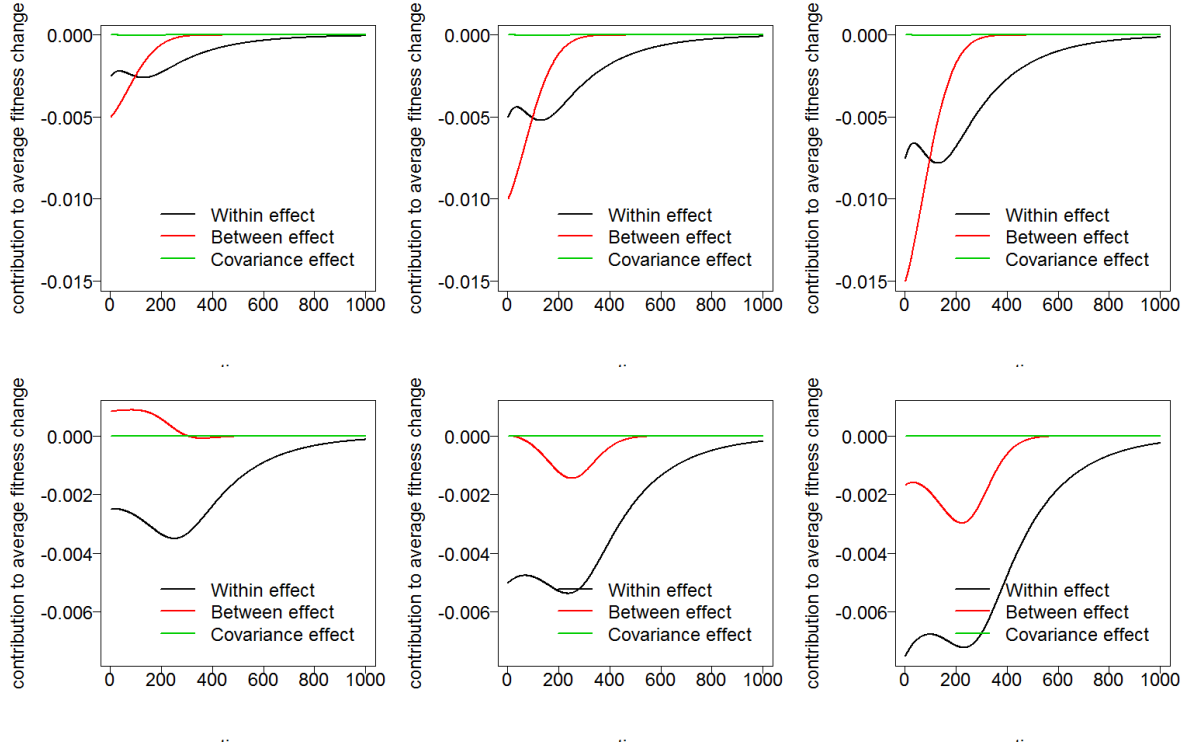


Figure 8: Decomposition of change in average unit cost with IRS

Note: The upper panel corresponds to ordered matching, while the lower — to unordered matching. From the left to the right, the markets (layers) 1, 2, and 3 are shown.

4.3 Possibility of partner switching

While in the previous two exercises the value chains were assumed to be fixed due to prohibitively high switching cost, this assumption might be relaxed and firms active in upper VC layers might be allowed to experience a pseudo-competition that we originally ruled out in the model to highlight the differences existing between selection in a single market and in a set of vertically-related markets.¹⁷ Switching costs may involve simply a fixed cost SC , and those firms which either compensate this cost by gaining a lower price of a new supplier multiplied by existing orders or by gaining more orders requested by new downstream partner at the current price, will be willing to switch. We propose to model SC as a percentage parameter: a firm is willing to switch a partner, if it improves fitness by at least a certain percentage compared to the current fitness level, e.g., if the new supplier has a lower price than the old one.

To make sure that a firm can switch only if there is reciprocity from the other side (the potential partner also finds it attractive to switch to that firm), we introduce a simple search and acceptance algorithm. In particular, if a firm j from a layer m_1 in a value chain x (VC^x) considers changing its current partner jj from a layer m_2 (which can be either $m_1 + 1$ or $m_1 - 1$) and takes (randomly) firm $jk \neq jj$ from a different value chain VC^y into consideration (which, in turn, currently has a partnership with firm kk from layer m_1), then those two firms, j and jk , will do the switching *iff* the fitness of the part of the value chain VC^y (into which firm jj from the layer m_2 is currently integrated) j is switching to is better than the fitness of the corresponding part of VC^x , while the opposite holds true for the remaining parts of those two VCs: the fitness of the remaining part of VC^y into which jk is integrated is worse than the corresponding part of the VC^x into which j is integrated (see Figure 9).¹⁸

¹⁷The possibility of switching partners in a value chain is less unrealistic than it may appear at a first glance; the whole worldwide structural re-organisation or production around global value chains is the most recent example of vertical relations between industries being neither completely frictionless nor totally rigid (Timmer et al., 2014).

¹⁸We also considered a simpler option of switching a partner when the randomly drawn candidate has a better fitness than our current partner, i.e. $c_{m_2}^{jj} - c_{m_2}^{jk} > SC \times c_{m_2}^{jj}$ and $c_{m_1}^{kk} - c_{m_1}^j > SC \times c_{m_1}^{kk}$. But given that this rule ignores the fitness of other partners integrated into VCs, such a rule is an oversimplification of reality and results in a much larger number of partner switches. The overall result (in terms of market reallocation and fitness improvement), however, is consistent with our preferred acceptance algorithm.

$$\begin{array}{c}
\downarrow \text{switching--point} \\
VC^x = \left(\overbrace{c_1^x \quad \dots \quad c_m^j} \quad \bigg| \quad c_{m+1}^{jj} \quad \dots \quad c_M^{xx} \right)_{1 \times M} \\
VC^y = \left(c_1^y \quad \dots \quad c_m^{kk} \quad \bigg| \quad \underbrace{c_{m+1}^{jk} \quad \dots \quad c_M^{yy}} \right)_{1 \times M}
\end{array}$$

Figure 9: Comparison of fitnesses for switching

Necessarily, the switching cost $SC \in [0, 1]$ becomes a key parameter, allowing situations from ‘fast and easy’ switching as if no sunk costs of partnership formation existed (close to frictionless markets on upstream layers) to no switching (and respectively, no competition) at all. Figure 10 is given for the simplified case ($M = 3$ and $N = 3$), while the case with more VCs and layers is presented in Figure 14 in the Appendix.

As in the ordered switching scenario fittest firms in the respective layers are matched, there is basically no room left for switching. In case of unordered value chain matching, by contrast, firms occasionally switch (no matter whether innovative activity is present and if yes, in which scenario of scale returns). The moment of switching can be captured by the ‘zig-zag’ evolution (abrupt shifts) of the total unit costs of the VCs (middle charts in all four panels of Figure 10) and the corresponding adjustments in the evolution of VC market shares (leftmost charts of the same figure).¹⁹

As a result, a period of volatility in the market share dynamics can be observed in early periods of simulation (which can be interpreted, for example, as early stage of an industry life-cycle). Except for the scenario with DRS, a dominating VC is identified relatively quickly. It drives other VCs out of the market and kills any volatility in market shares dynamics. The observation for the DRS scenario is not surprising, as by design DRS is meant to preserve competition between actors for a longer period of time. More interesting is that market share volatility in early periods is more universal and not so sensitive to scale returns, which contradicts the earlier argument by Mazzucato (1998) that high volatility in the early period of a life-cycle is found for DRS only.

¹⁹Since for ordered matching the possibility of switching is never exploited, we do not include the related charts.

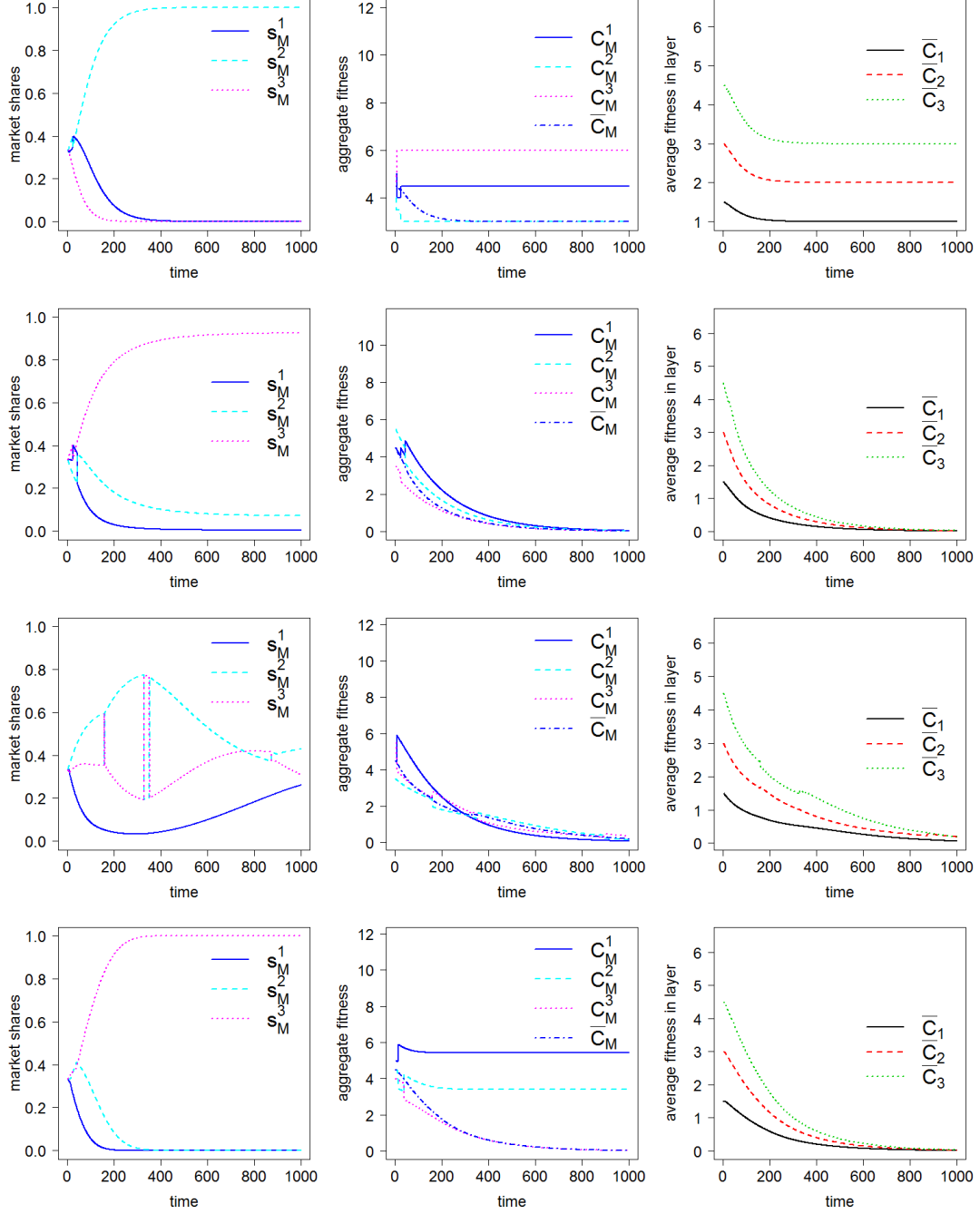


Figure 10: Dynamics with unordered matching, different innovation scenarios, and switching

Note: The upper panel corresponds to no innovation, the next to innovation with CRS, the third from the top to innovation with DRS, and the bottom one to innovation with IRS. $M = 3$, $N = 3$, $SC = 10\%$, and $\gamma = 0.005$.

Given that the firms in the VCs are connected via constant quantity relations (the firm in the last market competes for an output quantity (market share) and the upstream firms serve as suppliers of intermediary products for this final quantity), a switch of a VC partner implies a big change in the output quantity and produces an instantaneous

shock, which takes place synchronously on all M layers, but with different magnitudes.²⁰

Figure 11 presents the corresponding decompositions for the four scenarios with switching. Note that by construction, the covariance effect can gain value in our model when switching occurs, since once firms switch up- or downstream partners, they instantaneously experience an improvement in their own fitness (e.g., the price of their good can increase because of the higher fitness of new suppliers), while they still experience a market share increase/reduction because of the competition on the end consumer market. We suggest to disregard the covariance effect by virtually splitting it equally in the between and within effect while analysing our results.²¹

Thus, firms in the up- or downstream part of the VC that switched to a stronger group of partners experience a sudden increase in their market share. Given that by construction switching requires reciprocity and fitter firms tend to build stronger VCs, firms gaining additional market shares have a cost below their market average experiencing negative shocks in the between effects, while firms losing stronger counter-partners experience positive shocks. Furthermore, given that the firms' fitness accumulates over layers depending on the respective VC partners, these firms experience not only shocks in the between effect, but also shocks in the within effect, as fitness of own products can 'jump' and increase after a successful partner switch. Those shocks clearly correspond to the moments when switching takes place and are concentrated in the early periods of the simulation (see Figure 11). The main reason why the switches (and the corresponding shocks in the between effect) tend to take place so early is that the cost differences in the early phases are stronger (this is true for all scenarios with innovation) so that the term indicating the deviation from the cost average ($C_m^{i,t-1} - \bar{C}_m^{t-1}$) is greater.

²⁰This is due to the different deviation of each VC member's layer-specific cost level from the market-weighted mean layer-specific cost level, see (15).

²¹In fact, splitting the covariance effect between the two is not new to the literature and was done, among others, in Griliches and Regev (1995) and Dosi et al. (2015).

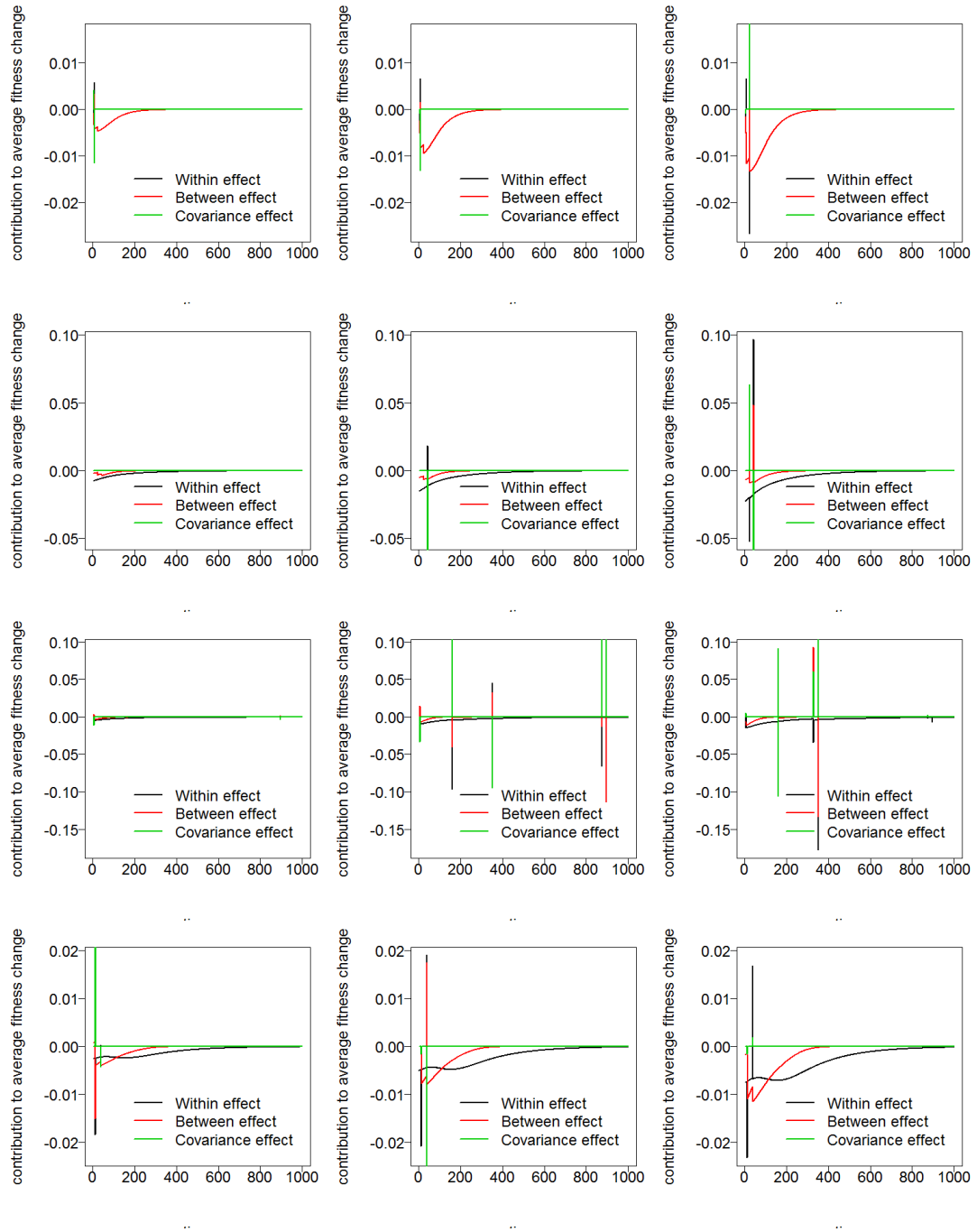


Figure 11: Decomposition of change in average unit cost for different innovation scenarios and switching

Note: The upper panel corresponds to no innovation, the next to innovation with CRS, the third from the top to innovation with DRS, and the bottom one to innovation with IRS.

4.4 Summary on the average unit cost decomposition

To summarise the differences between the above-mentioned scenarios in terms of the average layer-specific unit cost decomposition, consider Table 1 where the three effects

are averaged over all M (here $M = 10$ and $N = 10$ are taken as default) layers and all T (as before, equal to 1000) periods for 1000 restarts. Comparing the left and right hand sides of the table, it can be noticed immediately that the between effect in unordered matching is consistently smaller than in ordered one (even if the full covariance effect would have been attributed to the between effect), which is due to the fact that only in some markets the replicator dynamics works in the ‘right’ way, while in other markets regressive developments take place. In the unordered matching scenario the within effect clearly dominates over the between effect in all scenarios, except for the no innovation scenario.²² Such result generally supports our idea that the clear-cut expected results of market selection are made more ambiguous by the innovation process.

Table 1: Results for the average unit cost decomposition over different scenarios

		Ordered VC matching			Unordered VC matching		
		Between Effect	Within Effect	Covariance	Between Effect	Within Effect	Covariance
Without Switching	No innovation	−0.0124 (0.1017)	0 (0)	0 (0)	−0.0040 (0.0214)	0 (0)	0 (0)
	CRS	−0.0118 (0.0986)	−0.0061 (0.0113)	0.0001 (0.0005)	−0.0034 (0.0189)	−0.0144 (0.0226)	0.0000 (0.0001)
	DRS	−0.0139 (0.0986)	−0.0038 (0.0079)	0.0001 (0.0005)	−0.0044 (0.0190)	−0.0133 (0.0182)	0.0000 (0.0013)
	IRS	−0.0125 (0.1016)	−0.0053 (0.0073)	−0.0000 (0.0005)	−0.0046 (0.0222)	−0.0130 (0.0166)	−0.0002 (0.0009)
Switching, $SC = 10\%$	No innovation	−0.0124 (0.11017)	0 (0)	0 (0)	−0.0019 (0.2470)	0.0082 (0.2802)	−0.0152 (0.5041)
	CRS	−0.0118 (0.0986)	−0.0061 (0.0113)	0.0001 (0.0005)	−0.0030 (0.0926)	−0.0096 (0.1032)	−0.0052 (0.1835)
	DRS	−0.0139 (0.0986)	−0.0038 (0.0079)	0.0001 (0.0005)	−0.0069 (0.0956)	−0.0093 (0.1110)	−0.0013 (0.1939)
	IRS	−0.0125 (0.1016)	−0.0053 (0.0073)	−0.0000 (0.0000)	−0.0035 (0.1713)	−0.0072 (0.1835)	−0.0071 (0.3359)
Switching, $SC = 50\%$	No innovation	−0.0124 (0.1017)	0 (0)	0 (0)	−0.0039 (0.0710)	0.0004 (0.0797)	−0.0007 (0.1410)
	CRS	−0.0118 (0.0986)	−0.0061 (0.0113)	0.0001 (0.0005)	−0.0034 (0.0283)	−0.0142 (0.0332)	−0.0001 (0.0423)
	DRS	−0.0139 (0.0986)	−0.0038 (0.0080)	0.0001 (0.0005)	−0.0045 (0.0211)	−0.0131 (0.0220)	0.0001 (0.0205)
	IRS	−0.0125 (0.1016)	−0.0053 (0.0073)	−0.0000 (0.0001)	−0.0043 (0.0945)	−0.0120 (0.1032)	−0.0014 (0.1891)

Note: Results are averaged over 1000 restarts for all vertically integrated layers and time periods. Standard deviations are reported in parentheses.

Looking at the results for the possibility of switching partners within a VC (second and third panels in Table 1), one can notice that in all the scenarios considered, the

²²The within effect directed to firm-specific fitness here is certainly zero. However, since the firm’s total fitness includes costs of input, the within effect can deviate from zero due to switching. Note that comparing ordered and unordered matching in case of no innovation proves the between effects and an overall improvement that is always higher for the former case, which is consistent with our prediction.

possibility of inexpensive switching ($SC = 10\%$) contributes to the between effect by re-introducing the competition into the upper layers of the value chains. As it becomes more costly to switch ($SC = 50\%$), the role of the within effect dominates again and one has to rely more on internal improvements (R&D).

5 Conclusion

In this paper we generalise the pure replicator dynamics model to the case of firms vertically related in value chains with a view to highlight the differences existing between selection in a single market and in a set of vertically related markets. This is achieved by introducing a growth process driving the selection equation. Next to the firm fitness, we include firm-specific demand growth related to the change in market share of its downstream VC partner.

Using the extended model we conduct a series of exercises, starting from the simplest one without innovation and enriching the setting in a stepwise manner by introducing different cost-reducing innovation scenarios. Doing this, we distinguish two scenarios with firms matched according to their fitness rank (ordered matching scenario) and those matched randomly (unordered matching scenario). In addition, we introduce a simple rule of partner switching to ensure reciprocity from both sides. Using numerical techniques, we show how the two scenarios differ. In a nutshell, we first simulate the basic model to provide evidence of ‘failures’ of market selection due to VC relations. Second, we add innovation dynamics and partner switching to enrich our understanding of the interplay between VCs and selection. Third, we provide a decomposition analysis to disentangle the role played by within, between, and covariance effects under the different scenarios tested. Fourth, we look at average effects (over 1000 runs of the model), demonstrating that although the ‘regressive developments’ may be less pronounced over many replications, the ‘averaged’ results largely differ between ordered and unordered matching. All this provides a novel explanation why market selection may not work properly.

First of all, we demonstrate that firms related in a VC structure and dependent in

their output capacity on their downstream partners do not necessarily increase their market share, even though they exhibit the highest fitness among firms on the respective production layer. This is due to the limited competition on upstream markets (firms being locked into VCs) and to the fact that aggregate fitness of a VC is crucial to the success on the final consumer market. Thus, the very existence of VC relations may induce violations of the replicator dynamics and generate what we call regressive developments of market selection; in these situations the average fitness may decrease rather than increase over time.

Furthermore, we show that for firms in the unordered VC matching scenario the possibility of switching partners produces a period of high market share volatility in any innovation and returns to scale setting in the beginning, which provides a novel and simple explanation to the evidence discussed by Mazzucato (1998).

Our last result indicates that the possibility of partner switching re-introduces the competition into the upper layers of VCs. This demonstrates that the intensity of the replicator effect is crucially dependent on the cost of switching own VC partners. The latter may be taken into consideration to derive policy implications. Although policy makers generally have limited influence on the firms' strategic decisions with regard to partner selection, certain measures may be considered to facilitate the 'survival of the fittest' principle and to support the productivity improvement on a given market. Such measures may be increasing market transparency or financial support for firms at the early period of alliance formation.

Our results call for both, more differentiated analysis of the replicator dynamics on different stages of value creation²³ and application of different competition policy measures to different markets. In general, the idea of market selection 'biting' more in certain layers of a value chain gives rise to two broad sets of questions. First, how should policy interventions (targeting innovation and competition) focus more on upstream and downstream bottlenecks rather than just looking at a single layer's rate of innovation

²³Thus, while it may be easier to find evidence of the replicator model on the downstream market, such as stage of assembling and selling compact cars, it is more challenging for producers of intermediate parts, and this has to be taken into account.

and production? Second, how is the current reconfiguration of production into global value chains (Timmer et al., 2014) affected by (and how can it affect) the Schumpeterian competition for the market?

For further research we plan to explore at least the two following trajectories. First, we plan to deepen our understanding of the VC structures under which the replicator dynamics is violated and regressive developments take place. Second, we want to generalise the exercise and allow firms to partner more than one firm from the same layer at the same time. This will allow to address network properties of production chains and to explore in more detail the differences between VC and production networks (Sturgeon, 2001). Moreover, a very relevant issue is to provide an empirical test for our findings. As a first idea, the extents of market selection in the production layers might be compared to see whether downstream markets demonstrate higher within effects. Fortunately, novel metrics to measure industry ‘downstreamness’ were proposed recently (see Antràs and Chor (2013)). Finally, a viable empirical trajectory should explore multi-layer interconnectedness in productivity dynamics, especially in times of high academic and policy interest in productivity slowdown (Syverson, 2016).

By introducing value chains into the mechanism of market selection, our contribution sheds a light on the multi-dimensional nature of the replicator dynamics model. Instead of confining it among the theoretically elegant, but empirically irrelevant economic tools, we hope this will induce new attempts to enrich its framework and understand its validity and explanatory power.

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6 Appendix

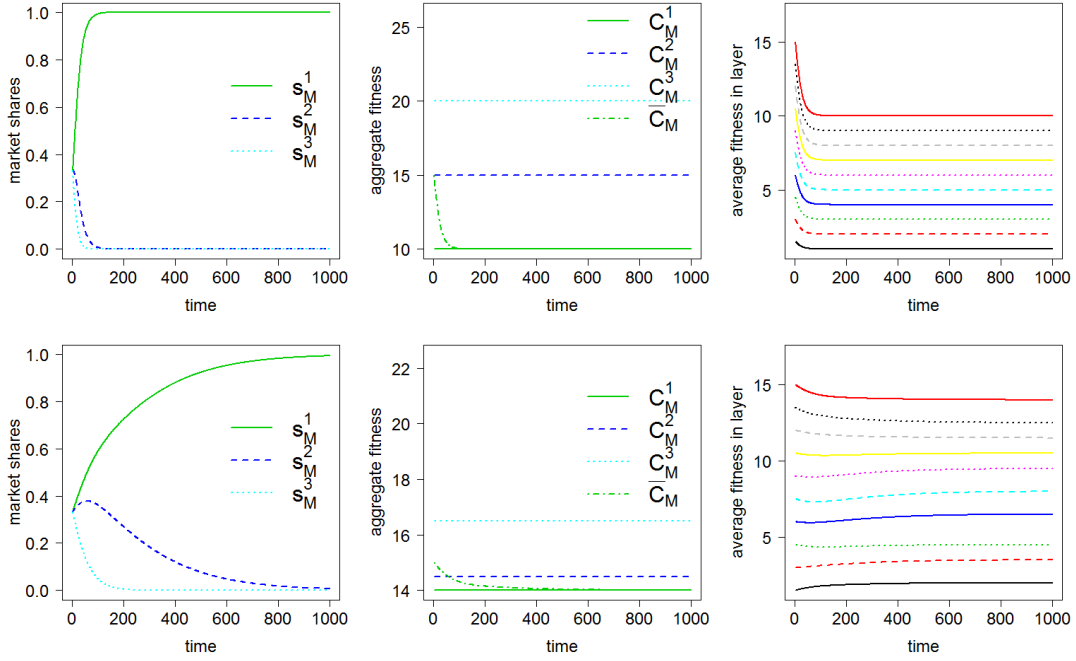


Figure 12: Dynamics with ordered and unordered matching with ten layers

Note: The upper panel corresponds to ordered matching, while the lower to unordered matching. $M = 10$ and $N = 3$.

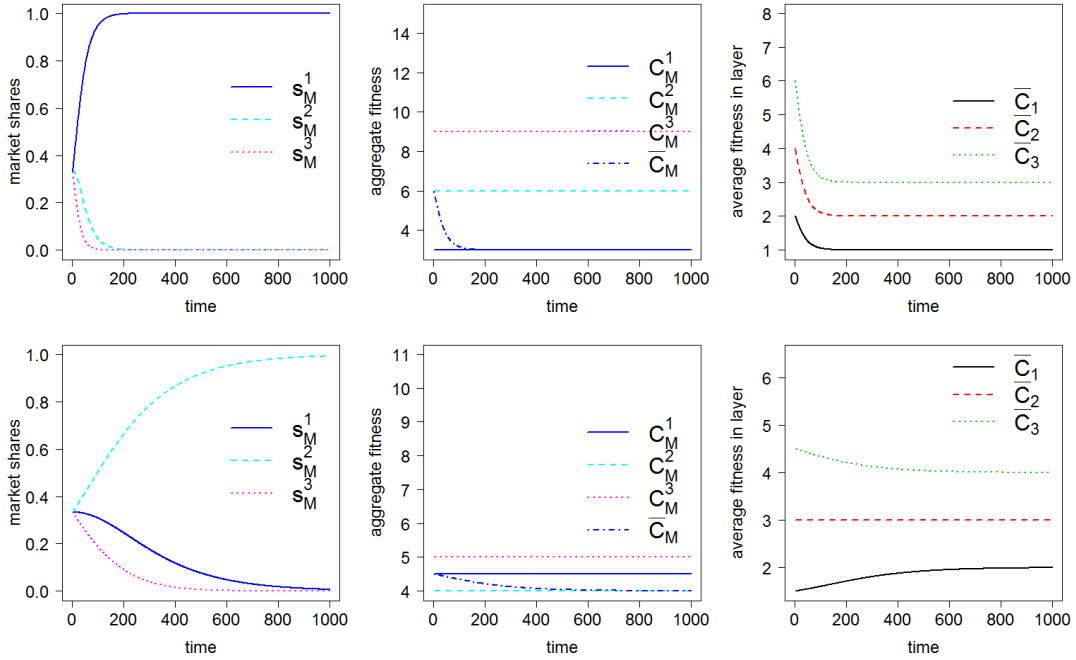


Figure 13: Dynamics with ordered and unordered matching with larger variance in fitness

Note: While in the default case, as it was mentioned earlier, the firms' productivity has been drawn in a way that each firm surpasses the next one by 0.5 (which was leading to $(\sigma_c^m)^2 \approx 0.167$), we now increase the step to 1 and, respectively, $(\sigma_c^m)^2$ to ≈ 0.67 . The upper panel corresponds to ordered matching, while the lower to unordered matching. $M = 3$ and $N = 3$.

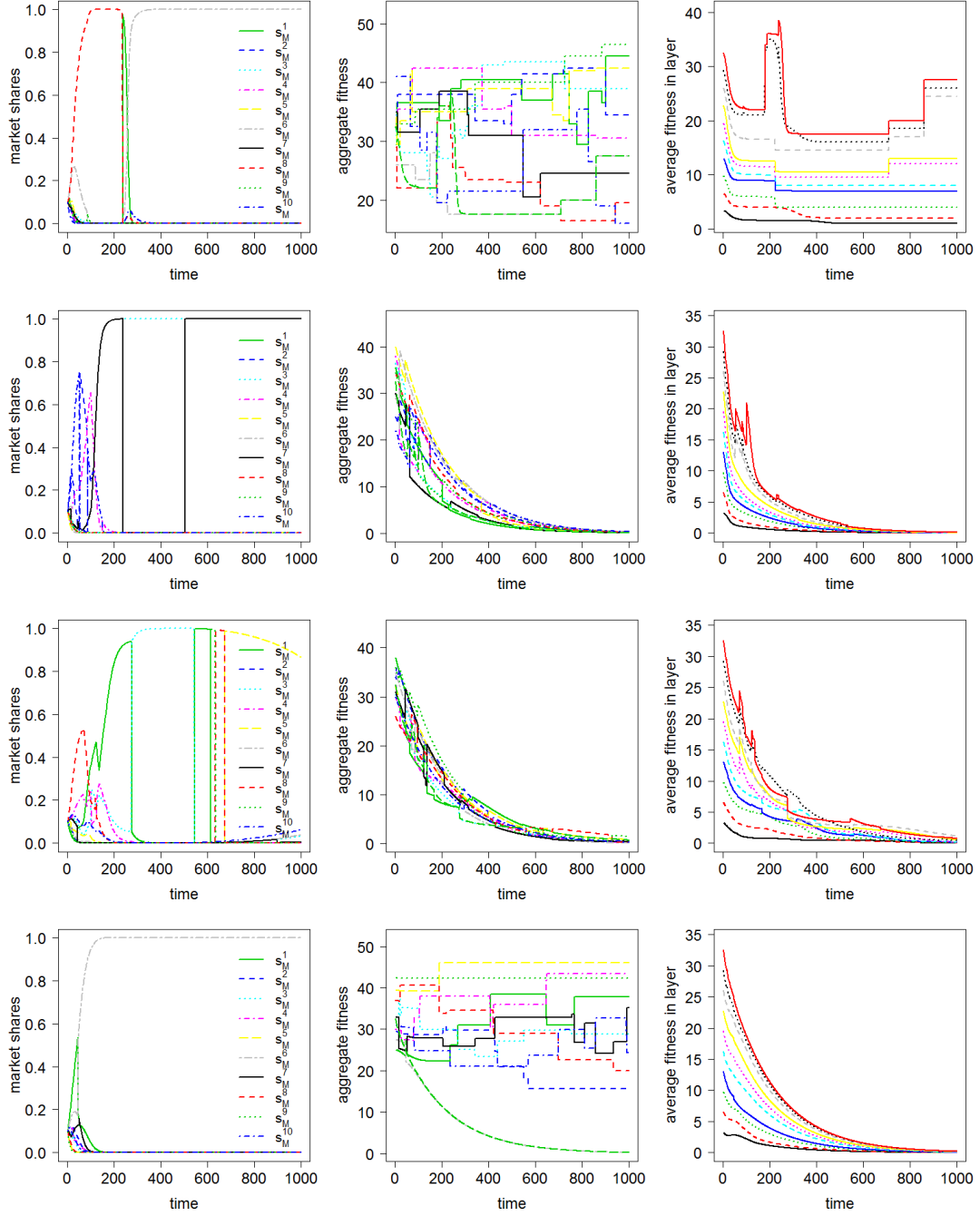


Figure 14: Dynamics with unordered matching and switching for different innovation scenarios

Note: The upper panel corresponds to no innovation, the next to innovation with CRS, the third from the top to innovation with DRS, and the bottom one— to innovation with IRS. $M = 10$, $N = 10$, and $SC = 10\%$.