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Combining Finite and Infinite Elements.

Why do we use infinite idealizations in engineering?

Abstract

This contribution sheds light on the role of infinite idealizations in structural analysis, by exploring how infinite elements and finite element methods are combined in civil engineering models. This combination, I claim, should be read in terms of a 'complementarity function' through which the representational ideal of completeness is reached in engineering model-building. Taking a cue from Weisberg's definition of multiple-model idealization, I highlight how infinite idealizations are primarily meant to contribute to the prediction of structural behavior in Multiphysics approaches.

Keywords: infinite idealization, multiple-model idealization, engineering, finite elements method, infinite element method

1. Introduction: Idealization and Model-building

Idealization is an essential component in model-building. Especially after the works by Giere (1999), and Jones & Cartwright (2005), among others, there is now increasing awareness among philosophers and practitioners that there can be different types of idealization, which embody different functions and respond to different goals in the sciences (Potochnik 2017). Weisberg (2007), for instance, sheds light on three different types of idealization that can be identified in scientific model construction, i.e. Galilean idealization, minimalist idealization and multiple-models idealization (MMI). More recently, Strevens (2017) accurately distinguished between asymptotic and simple idealizations.[1] Connected, albeit indirectly, with the question of justifying the activity of idealization, is the problem of distinguishing it from approximation (see Norton 2012) and that of clarifying the use of a specific kind of idealization, i.e. infinite idealization in the sciences (Narens and Luce 1990). Thus, philosophers of science and mathematics focus on the epistemological and/or ontological consequences that can be derived from the application of infinite idealization, and whether a unified picture of its use in model-building can be supplied. This question is shaped with reference to various fields, depending on the role played by the infinite domains or systems used therein. With reference to physics, for instance, one of the major concerns regards phase transitions and their interpretation as irreducible emergent phenomena (Liu 1999; Morrison 2012), as well as the paradoxical consequences[2] that derive from assuming phase transitions in classical thermodynamics are a result

[1] To Strevens, in the case of a simple idealization a simple operation, namely, a *change in the parameters* of the veridical model, creates a new and partly fictional model that continues to model and to explain the target phenomenon. On the contrary, in asymptotic idealization, "a fiction is introduced by taking some sort of limit. Rather than setting a parameter to zero, for example, a limit is taken as that parameter approaches zero" (Strevens 2017, 3).

[2] A recent survey of this debate has been offered by Schech (2013). For different interpretations of PT and approximation, see Menon & Callender (2013) and Norton (2014). The latter in particular shows that approximations of arbitrarily large

of representing a system as infinite (Batterman 2005; Bangu 2009; Butterfield 2011; Bangu 2015).[3] Furthermore, there have been attempts to show that the infinite limit is an approximation for phase transition and not idealization (see Ardourel 2017). In this sense, the distinction proposed by Norton (2012) between idealization and approximation is extremely useful and appropriate, but it is still partial if one wants to reach a more unified picture of the functions to be attributed to infinite idealizations in the sciences.[4]

A comprehensive definition of infinite idealization and its functions across the sciences is still a desideratum. In the domain of biological sciences, for instance, emphasis is put on the use of infinite population size in population ecology models (see Strevens 2008; Rice 2012, 2015), or in economics emphasis is put on the use of an infinite number of agents consuming an infinite number of goods (Ross 2016; Albert and Kliemt 2017).

In this paper, I shall address the question of whether we are justified in using idealization within the framework of civil structural engineering. The latter represents a very interesting field that can enrich our view of idealization, by including what I call the 'complementarity function' of infinite idealization. In building up their models, engineers use a combination of the finite element method and infinite idealization to predict the behavior of structures which are interacting with the environment. This combination embodies the strive for the representational ideal of completeness, which cannot ever be achieved,[5] and other representational goals typical of structural engineering models, including robustness, safety and reliability, and so forth. Weisberg (2007) suggests that the ideal of completeness can be associated with two components: inclusion rules and fidelity rules.

"The inclusion rules state that each property of the target phenomenon must be included in the model. Additionally, anything external to the phenomenon that gives rise to its properties must also be included in the model. Finally, structural and causal relationships within the target phenomenon must be reflected in the structure of the model. Completeness's fidelity rules tell the theorist that the best model is one that represents every aspect of the target system and its exogenous causes with an arbitrarily high degree of precision and accuracy" (Weisberg 2007, 649-650).

but finite systems are often mistaken for infinite idealizations in statistical and thermal physics. For a general discussion on the use of infinite idealization in statistical mechanics, see Batterman (2011). For the distinction between approximation and idealization, see Norton (2012).

[3] More recently, Tegmark (2015) addressed the fundamental question of the use of infinity in physics and suggested dropping this notion from the realm of physics itself.

[4] According to Norton (2012), "an idealization is a real or fictitious system, distinct from the target system, some of whose properties provide an inexact description of some aspects of the target system".

[5] Weisberg (2007, 649-650) considers completeness as a unique representational ideal because it directs theorists towards a goal and makes them strive to include every aspect in their representations even if this ideal can never be achieved. Therefore, he suggested that it was reminiscent of a regulative function in Kantian terms.

How can we apply this notion of completeness to civil engineering? In order to represent every aspect of the behavior of the target system and fulfill the fidelity rules, engineers use infinite idealization in combination with the finite element method. When dealing with unbounded domain problems, e.g. in dam-foundation interaction or wave propagation studies, engineers aim at performing high-precision calculation of stress, strains, and displacement at points far away from the point of load application, Before the development of infinite elements (IE), engineering analysis proceeded by truncating the domain of analysis at a finite distance from the point of application of load. However, such an analysis required a huge number of finite elements (FE), thereby generating the need to handle a large number of simultaneous equations. This implied a huge computational effort, particularly when the non-linear behavior of the system was considered. Historically, the development of infinite elements helped engineers in modeling in the physics of the far field behavior and in reducing the number of elements and the computational time (see Godbole, Viladkar and Nodrzaei 1990). It is worth noting that nowadays, when studying the interaction between a structure and its environment, engineers model the structure or the artifact by using discrete numerical methods and assume a representation of the soil as a continuum and as homogeneous, then proceed in analyzing the interaction with the structure at different discrete contact points through specific numerical computational methods. This procedure represents a building block of Multiphysics approaches, because Multiphysics as a computational discipline deals with closely coupled interactions amongst separate continua phenomena. Areas of Multiphysics include magnetohydrodynamics, electrokinesis flow, physiochemical hydrodynamics, just to mention some of them.[6]

In section 2, I analyze which kind of idealization is present in engineering models and in Multiphysics models, by relating it to representational goals, such as completeness. In section 3, I identify the proper function of infinite idealization in engineering models: this is to allow the combination of finite and infinite elements that are analyzed in soil–structure interaction (SSI) models in subsection 4.1, and in the case study discussed by Dong and Selvadurai (2009) in subsection 4.2. Finally, in Section 5, I offer an account of the goal-oriented functions of infinite idealization in engineering models.

2. Structural Engineering Models: Idealization and Multiphysics

To begin with we have to offer a cogent definition of idealization that can account for practices within engineering. The approach of model-building in civil structural engineering responds nowadays to new concepts and challenges. On the one hand, one is tempted to say that engineers just use physics

[6] Zimmermann (2006, 11ff.) defined Multiphysics as any complete coupled system of differential equations that has more than one independent variable of different physical dimensions.

models and apply them to concrete problems. However, in the last two decades a new concept has emerged. This is the concept of Multiphysics, which is crucial for engineering and other sciences, including physics. This new concept should generate further reflection. Engineering, indeed, is transforming concepts used by experimental physics and biomedical sciences, by connecting different fields and models used therein. This represents the present and future challenges for modeling within engineering (Brown et al. 2008, 11).

Therefore, our approach to the study of engineering models should include new procedures introduced in engineering modeling in order to connect multiscale systems. In particular, structural engineering models must accomplish representational pragmatic goals, such as safety, robustness and reliability that can be derived from the representational ideal of completeness.[7] The means by which engineering reaches this representational ideal is Multiphysics; that is to say, engineering analysis as a computational discipline produces simulations that involve multiple physical models or multiple simultaneous physical phenomena. We should profit from the reflections that engineers developed on numerical methods and the foundations of Multiphysics, given that they largely debated the problem of infinite idealization in practice (see Keyes et al. 2012, 6). Indeed, civil engineers must give an account of complexity at a multiscale level and produce models that consider the largest number of variables, in order to obtain reliable results.

According to Weisberg (2007, 645), multiple-models idealization (MMI) plays a fundamental role in engineering. MMI is the practice of building multiple related but incompatible models, each of which makes distinct claims about the nature and causal structure which give rise to a phenomenon. According to Weisberg (2007, 648) engineering modelling uses MMI to find the set of idealized models that is maximally useful for creating new structures. Now, the question that is not addressed in Weisberg's paper, and that I explore herein, is when and why engineering appeals to MMI and specifically when and why it combines infinite idealization and approximation, thereby realizing a complementarity function in order to reach the representational ideal of completeness.[8] I can already anticipate here that precisely the use of infinite idealization is what makes it possible to harmonize

[7] Safety and reliability are representational goals in civil engineering, because its models must embody them as goals. In representing the structural behavior, engineers are not simply satisfied by simulating the collapse of a bridge, rather they aim at preventing it by finding out the model that accounts for the safety and reliability of the structure. This in turn influences the kind of formalism that is used and the way in which the Multiphysics approach is employed. For instance, when designing a building and trying to meet the safety criteria, engineers have to simulate a fire and will use a Multiphysics approach considering heat flow and mass transfer, but to reach completeness they will also use other means to simulate the psychology of the people escaping from the fire, in order to find the exact points in which the fire exit signs must be put. Civil engineering models do not simply describe reality but must anticipate it. In this respect it is very different from other sciences. See Van Lamsweerde (2001), Linde (2005). On goal-oriented approaches in engineering see Kavakli, E., & Loucopoulos, P. (2004).

[8] Multiple-models idealization assumes that there is no single idealization that can fully capture all the observed behavior relevant to a phenomenon. Therefore, multiple-models idealization can imply the existence of a combination of different methods and specifically the complementarity functions of methods such as FEM and IEM.

incompatible models in engineering when specific goals must be fulfilled. I follow here Norton (2012) in believing that only idealization and not approximation introduces the reference to a novel system and we shall see in the case studies under analysis that precisely infinite idealization allows the connection among models that are incompatible with each other. However, in order to implement Norton's definition of the distinction between approximation and idealization, I shall emphasize the role played by the function of completeness, and its consequences. Indeed, starting from this function one can 1) justify the complementarity that is typical in engineering models using infinite idealization and that characterizes MMI in Multiphysics (section 4), 2) identify the goal-oriented function of idealization in engineering models (section 5).

To reach this aim, I shall consider here infinite idealization as a qualitative idealization, namely it consists in representing a finite domain or system (already subject to either idealization or approximation) *as if it were* an infinite domain,[9] e.g. a homogenous non-denumerable domain in order to capture properties or behaviors of the systems under analysis that cannot be adequately simulated by the finite models at hand.

In structural engineering, a number of infinite idealizations are used to solve both simple and complex problems. Most of these idealizations arise from the need to represent a structure interacting with the environment. In particular, in order to represent interaction with soil, engineers appeal to an idealized system representing space as a continuous, homogeneous and isotropic medium.[10] This representation is used to solve problems in representing a building and its interaction with the environment under particular load and stress conditions, e.g. earthquakes, or in designing specific structures, such as dams, and in representing their interaction with soil. In order to consider viscosity or vibrations, infinite idealization is introduced. This move produces a transition from the already idealized or approximate system, e.g. soil as a continuous, isotropic, homogeneous medium, to a new one which includes new effects and makes the model(s) strive for completeness.

Methods	Characteristics and main use

[9] It is possible to think of infinite idealization in terms of a meta-idealization if the system to which it applies is a new one with respect to the original target system. This concept of meta-idealization could help in unifying different types of idealization. To deal with this topic would be beyond the scope of this paper, but it is worth mentioning it here.

[10] They include the infinite slope method, the infinite cylinder idealization and many others, see Na 1980; Das and Soboham 2013.

FEM	Finite Element Method (FEM) solves boundary value problems for PDEs by formulating the problem in a system of algebraic equations. In FEM the degree-of-freedom vector represents displacement, temperature, pressure, velocity, electric or magnetic potential, depending on the physical problem. Conjugate vector *f* can represent mechanical force, heat flux, particle velocity, charge density or magnetic density depending on the physical problem. Elements are modeled as disjointed and non-overlapping in space. USE: All kinds of structural analysis, heat transfer, chemical engineering, electromagnetics, Multiphysics and computational fluid dynamics (CFD).
FDM	Finite Difference Method (FDM) solves differential equations by approximating them with difference equations, in which finite differences approximate the derivatives. FDM is the dominant approach to numerical solutions of partial differential equations (Grossmann and Roos 2007, p. 23), but is more difficult to use for handling material discontinuities, given that elements are represented as connected in space. USE: Weather calculations, astrophysics, seismology.
FVM	Finite Volume Method (FVM) represents partial differential equations in the form of algebraic equations. Like in FEM and FDM, values are calculated at discrete places on a meshed geometry, but in FVM what counts is the small volume surrounding each node point on the mesh. USE: computational fluid dynamics.

Table 1 - Among the numerical methods used in computation, engineers appeal to the infinite element method (FEM), the finite difference method (FDM) and the finite volume method (FVM). However, the boundary element method (BEM) is often preferred these methods. The latter is a numerical computational method used in solving linear partial differential equations which have been formulated as integral equations, i.e. in boundary integral form. BEM can be applied in many areas of engineering and physics, including fluid mechanics, acoustics, electromagnetics, and fracture mechanics (see Briaud (2013), 304-305). All these methods share a common assumption: the description of the laws of physics for space- and time-dependent problems are expressed in terms of partial differential equations (PDEs).

3. How do Engineers Discretize? Finite and Infinite Elements

In many physics and engineering studies, PDEs cannot be solved with analytical methods. On the contrary, an approximation of the equations is constructed, by using discretization methods that approximate the PDEs with numerical model equations, whose solutions in turn approximate the real solution to the PDEs. As mentioned in Section 1, given the limit of FEM to deal with external boundary problems, the infinite element method has been introduced into engineering practice.

The main difficulty in solving exterior domain problems arises from their unboundedness, and therefore they must be discretized. Indeed, an exterior domain cannot be completely discretized with the standard finite element method based on polynomial shape functions. For this reason, in the last two decades, engineers have improved the treatment of exterior domains by means of the infinite element method (IEM).

, IEM was introduced by Bettess & Bettess (1991a; 1991b) about two decades ago, and since then has been refined and applied to a variety of problems. But what is meant by an infinite element? According to Agrawal and Hora (2009), it is “an element in which one or more dimensions extend to infinity”. Infinite elements find applications in all unbounded continual problems. According to Gerdes (2000, 44), one can represent infinite elements as extending to infinity in one direction with a corresponding shape function integrable over the infinite element or as constituting themselves as local absorbing boundary conditions, which is to say that they can be used to discretize the whole exterior domain. This means that infinite elements, just like finite elements, are local. However, the location of a truncation boundary (the common junction between the finite and infinite common layer) between finite and infinite elements is the most important and difficult aspect to solve and, as shown in section 4.2, it is the decay function that ensures that the behavior of the element at infinity is a reasonable reflection of the physics.

The incompleteness of models using FEM to solve fluid mechanics problems and in non-linear dynamical analysis, created the condition for its integration with infinite or semi-infinite idealization (Chen and Wang 2010). In other words, to be successfully applied in engineering studies of structure-soil interaction or in any other model representing how an artifact is affected by the environment, FEM, which is used to model the structure, must assume a counterpart. That is, it must assume, for instance, the representation of soil as a *continuum*, then calibrate the boundary conditions and through infinite elements create a truncation on the ground of such assumptions. This move, I claim, embodies the complementary function of finite and infinite idealization in engineering models. At first, the fact that engineers appeal to infinite idealizations might seem very far from the idea that engineering models must describe physical reality and must do so in a refined way giving the implications of structural failure or collapse that can derive from design errors. However, at a deeper level, we notice that structural engineers opt in many cases for models including infinite idealizations in order to implement FEM mesh refinement and to produce more effective predictions of structural behavior, thereby ensuring completeness in their models. Civil engineering models strive for completeness but never reach it, because the contingency of events make it impossible for them to be perfect (complete) in prediction. However, they reach relative completeness by fulfilling goals, such as structural reliability and safety, by simulating complex phenomena. To this purpose engineers appeal to Multiphysics and in representing two or more separate continua interacting, one can also appeal to methods that include infinite idealization but only in combination with FEM. Therefore, in order to respect the ideal of completeness one has to appeal to the complementarity function of IEM and FEM.

4. Combining Finite and Infinite Elements: The Complementarity Function in Practice

Let us now see how engineers combine finite and infinite idealization when dealing with structure-soil interaction or with structure-space interaction in aerospace and civil aerodynamics. These are all fields in which errors in the simulation must be kept to a minimum for obvious reasons and the analogies with problems of fluid dynamics and crack propagation naturally lead us to approach such problems using BEM and other infinite element methods.

4.1 Finite and infinite methods in SSI models

When dealing with the problem of structure-soil interaction, FEM and FDM model the continuum by discretizing the entire body of the soil mass (Briaud 2013, 304). If the soil mass extends to infinity, FEM requires a boundary at some distance from the imposed loading or deformation. On the contrary, no artificial boundaries are needed in BEM, because it models the continuum by discretizing only its boundaries in a very efficient way.[11] Now, since soil is a material showing nonlinear behavior,[12] the combination of BEM and FEM is ideal in modeling the structural soil interaction (SSI), in studying various effects of simulated earthquakes, and in designing nuclear power plants built on deep soft soil (Chuhan et al. 1999). However, in order to obtain reliable results through a complete model, it is first necessary to couple FEM with IEM and BEM with infinite boundary element (IBE). In building up their model, as if it were a hierarchical construction, Chuhan et al. (1999) obtained a complete model of structure-soil-bed rock system, but infinite idealization played a role only at certain stages and always in combination with FEM and BEM. The relevance of infinite idealization in building up this hierarchical model is clear in procedures aiming at solving exterior domain problems and open boundary field problems. This emerges, for instance, in Tang et al. (2010). They used IEM to solve open boundary field problems by converting an infinite field domain into a bounded field domain, without an arbitrary cut. For open boundary problems, indeed, FEM proceeds in a different way and must place an artificial boundary either far away or close to the near field domain. When the boundary is far away, a Dirichelet or Neumann boundary condition is applied to the artificial boundary. This method however needs a huge number of nodes to reach accuracy. Thus, engineers devised various strategies to process open boundary problems, by using IEM. The first strategy defines a kind of infinite element for the outer unbounded domain and then applies new shape functions for the infinite elements. This method ensures an easier coupling with FEM.[13] A second possible strategy proposed

[11] The mathematical techniques for BEM consist in replacing the governing differential equations valid over the entire soil mass by integral equations that consider only the boundary values. However, BEM is suitable only if the boundary surface is small compared to the volume of soil to be simulated. In other words, there must be a small boundary-to-volume ratio and this also makes the choice of BEM unsuitable in a number of cases (Bobet 2010).

[12] For an excellent overview of the application of infinite elements to dynamical problems, see Bettess and Bettess (1991a; 1991b).

[13] However, it cannot process exterior domains which contain more than one medium, especially when the interfaces of different mediums extend to infinity (this is a typical problem solved in Multiphysics approaches).

by Tang et al. (2010) does not have these limitations. This second type of IEM first divides the whole domain into bounded interior domain and unbounded exterior domain and then transforms the exterior domain to a bounded domain and applies similar FEM to the transformed domain. The total field, therefore, is the sum of these two parts: the interior domain which is close to and includes the source, and the exterior domain is the rest of domain. Thus, the final equations are formed by coupling the nodes of interior and exterior domains which lie in the common boundary of these two domains.[14]

As Chuhan et al. (1999) and Tang et al. (2010) clearly show, the finite element method alone is not able to produce a proper approximation of the soil medium and they also suggest modelling the soil media by means of BEM, and the finite structure using FEM. BEM and FEM thus can be matched by adding compatibility conditions and by restricting the number of boundary conditions. The latter can be limited via IB (infinite boundary) and IE (infinite element), respectively, in order to include vibrations in the dynamic soil-structure interaction (Chuhan et al. 1999; West and Pavlović 1999; Agrawal and Hora 2009).

Therefore, the combination of finite and infinite elements with the proper location of truncation boundary provides accurate and computationally economic solutions, but also reveals the complementary function of infinite idealization and the attempt to achieve the representational ideal of completeness for the models at hand.

Furthermore, infinite idealization occupies a place in the hierarchy of the model construction, but it is never the ultimate one. In this sense infinite idealization contributes to the effectiveness of explanation in engineering, but it does not appear to be explanatory *per se*.[15] In other words, the use of infinite elements is fundamental in acquiring accuracy, in predicting the behavior of the system under analysis. However, this only represents a stage in view of a refined approximation provided by other methods, such as FEM.[16] From this consideration it follows that the justification of the use of infinite idealization in engineering models is similar to that highlighted by Butterfield (2011) in the field of physics, which is to say that engineers use infinite idealization to attain mathematical convenience and empirical correctness. As shown in the next sections, indeed, infinite idealization is used to give meaning to the choice of the truncation boundary which allows us to account for physical effects that the use of FEM alone would disregard.

4.2 Infinite Idealization in SSI Models: A Case Study

[14] Following Silvester and Ferrari (1996), Tang et al. (2010) apply the Kelvin transformation to satisfy the need for finite element analysis and thereby enable the merging of FEM and IEM.

[15] Here I mean that infinite idealization is not explanatory if taken by itself and not in connection with FEM. See also De Bianchi (2016) for the general case of idealization as being not explanatory *per se*.

[16] For a review of the applications of infinite element methods, see Gerdes (2000).

In order to see how infinite idealization supplies such fundamental information, let us consider the following example. Dong and Selvadurai (2009) discussed the problem of the truncated boundary in an approach combining finite and infinite elements.[17] This approach includes Multiphysics couplings and is quite useful in geosciences and geo-environmental engineering, where it can be used to model structure-soil interaction by modelling the surface flow of a porous medium of infinite extent. Indeed, whereas FEM can be usefully applied to solve bounded domain problems, elements to be modeled, such as subsurface fluid flow, are generally considered as unbounded domain problems. This happens because the flow potential at significant distances (or *as if it were* at infinity) should be taken into consideration given that the physics of the problem in the remote region can influence the behavior of the solution in the near-field. FEM is able to provide accurate solutions to such problems involving unbounded domains, if and only if appropriate artificial boundary conditions are applied at truncated boundaries (Xia and Zhang 2006). There are various options which can be applied in order to extend FEM to the unbounded flow problems:

a) Truncate the far-field boundary at the location remote from the near-field region of interest.

b) To consider only a small region of interest of the structure and assume a simple state in the remote region such that can be described by analytical solutions.

In both a) and b), FEM can be combined with BEM, provided that a boundary integral equation can be developed for the problem in the exterior domain. However, this is not always possible, as is the case in nonlinear problems and for anisotropic media.

Another approach mixes FEM with IEM. The latter employs a shape function to describe the basic far-field characteristic of the problem in the exterior region. Such an infinite element shape function can be obtained by using a mapping to transform the global infinite region into a local finite domain through polynomial interpolation. This option, let us call it c), can lead to accurate results but requires a huge computational effort.[18]

After analyzing the limits of the approaches a), b) and c), the option that Dong and Selvadurai (2009) propose is to use the Dirichelet-to-Neumann operator and absorbing boundary condition that are used in wave propagation problems in the unbounded domain. In doing so, they obtained the appropriate expression of the truncated boundary condition in terms of a local differential operator. It is within

[17] Xia and Zhang (2006) had shown the superiority of merging the finite/infinite methods with respect to the use of finite element method in phenomena involving fluid flow in porous media. In numerical analysis of fluid flow problems the most difficult task is to find the proper approach to deal with an unbounded exterior domain. In most cases, the simplest solution to such difficulties is to truncate the mesh at some large but finite distance, which results in a relative approximation to infinity. Unfortunately, this method is inaccurate and computationally inefficient. Moreover, this method cannot satisfy the real boundary conditions pertaining to the infinite boundary since displacements and pore pressures are fixed only at infinity.

[18] According to Dong and Selvadurai (2009) this is due to the fact that the finite-infinite element coupling procedure is presented at the matrix level.

this context that the concept of Multiphysics plays a fundamental role. They proposed to implement the integro-differential equation in coding by using a Multiphysics software and its embedded PDE weak form formulation. In more detail, in the proposed finite-infinite method coupling they introduced a Neumann boundary condition at the truncated boundary to satisfy the C^1-continuity of the solution at the interface between the bounded interior domain and unbounded exterior domain. In this way the infinite element method is portrayed as complementary to the finite element method. Therefore, FEM is used to model the flow problem described by the weak form

$$\int_a^\infty w \frac{\partial \phi}{\partial t} R^2 dR + \int_a^\infty D \frac{dw}{dR} \frac{\partial \phi}{R} R^2 dR = 0 \quad (1)$$

that is derived from the classical piezo-conduction equation,

$$D \nabla^2 \phi = \frac{\partial \phi}{\partial t} \quad (2)$$

where ϕ represent the flow potential, $\nabla^2$is the Laplace operator and D is the diffusion coefficient of the flow potential, and the one-dimensional diffusion equation:

$$\frac{D}{R^2} \frac{\partial}{\partial R}\left(R^2 \frac{\partial \phi}{\partial R}\right) = \frac{\partial \phi}{\partial t} \quad (3)$$

FEM is used to model the flow problem described by Eq. (1) in a truncated domain, with $a \leq R \leq b$. Therefore, an appropriate artificial boundary condition should be applied on the truncated boundary $R = b$ in order to consider the regularity condition of the flow potential at infinity. This infinite idealization refers to a new system, different from the target system and from the system reproduced through FEM only. When constructing an infinite element of the transient flow transport problem in an unbounded region, it is necessary to choose an appropriate hydraulic potential distribution function that can be used as the shape function, in order to describe the flow potential distribution in the infinite element.

This feature is not present in the target system, but is rather constructed and attributed to the other system, i.e. the infinite one. Therefore, this is an idealization in Norton's (2012) terms, namely idealizations should be portrayed to be new systems or parts of new systems that inexactly describe the target system, but that contain reference to some novel object or property. Furthermore they must be carefully distinguished from approximations.

Indeed, in the present case study, after obtaining the shape function in the infinite element and its derivative, both can be substituted in the weak form (Eq. 1) of the diffusion equation (Eq. 3), thereby obtaining an integro-differential equation of the flow potential ϕ_{b^+} at the left-hand side of the infinite element. This move makes it possible to account for the influence of the flow potential in the exterior unbounded region at the interior computational domain of interest. In order to obtain the finite-infinite element coupling in addition to the governing PDE, two boundary conditions are imposed at each end of the truncated region. A cavity flow potential ϕ is used as a Dirichelet boundary condition on the left-hand side of the finite computational domain. A second condition is to be imposed on the right-hand side of the computational domain $R = b$, that is the interface between the finite and infinite elements.[19] This condition must include the influence of the flow potential in the exterior unbounded region on that in the finite computational domain. For this purpose the integro-differential equation for the flow potential ϕ_{b^+} is coupled to the finite element model through the Multiphysics software, and such a coupling procedure can be implemented using a Neumann coupling boundary condition $R = b$, leading to a C^1-continuity of the solution at the interface between the finite and infinite element.[20] This ensures that no iteration loop[21] is necessary in the finite-infinite element coupling, since only one flow potential is involved in the linear computation.

Thus, engineering analysis as Multiphysics embodies a multiple-models idealization and the use of infinite idealization not only serves for computational purposes and problem solving, but also for unifying different scales and models for the fulfillment of engineering goals. These goals could be, for instance, those of prediction and control that serve as guidelines in choosing appropriate numerical methods and idealization strategies. In this sense, when using infinite idealization, engineers do not simply describe a real system – that is to say, their goal is not to describe a structure as it is. On the contrary, they aim at predicting the behavior of a structure in its interaction with other structures and/or the environment. Therefore, they use infinite idealization in order to intervene on, predict, and control structural behavior, since they are not satisfied with the approximation that FEM provides. Engineers also need infinite idealizations in order to reveal and predict possible effects that can modify or damage their structure, effects that are not present or originally contained in the structure design which was created using FEM, but that might still manifest in concrete situations, such as that

[19] This is due to the fact that the truncated boundary $R = b$ is a connection point between the finite computational domain and the exterior of an unbounded region. The flow potential at $R = b$ should include the contribution of the flow potential from these two regions. For a comparison of FEM and other techniques used for the external boundary problems, see Chernysheva and Rozin (2016).

[20] This is due to the fact that the Dirichelet condition at the truncated boundary $R = b$ only ensures C^0-continuity of the flow potential at this point. The fact that the point $R = b$ is a physically internal boundary within the entire physical domain implies that both the flow potential and its first spatial derivative in the normal direction should be continuous across the point which leads to the set of C^1-continuous conditions.

[21] No infinite loop is generated in computation because even if the series is taken to be as infinite, still one does not count it.

of the flow potential distribution defining the hydraulic potential. All this in turn points to the fact that in the hierarchy of model-building, engineers can look for the complementarity of finite and infinite methods, but only under specific conditions and to fulfill goals that pertain to their field.

5. On the Goal-oriented Functions of Infinite Idealization

In engineering models, the use of infinite idealization helps in solving problems and in gaining accuracy in order to predict the structural interaction with the environment. Both explanatory and prediction functions can be found in those engineering models which appeal to infinite or semi-infinite idealization. But what is the element which enables infinite idealization to contribute to the effectiveness of mathematics in capturing features of the physical world? In other words, how could infinite idealization guarantee such effectiveness? The achievement of this effectiveness is based on mathematical considerations. Independently of the idealization, be it semi-infinite or infinite, it is their complementary use in conjunction with the finite element method that enhances the application of numerical methods such as FEM, indirectly contributing to the reduction of the number of DOF. By representing the continuum or two or more continua with certain properties that are not necessarily pertaining to the target system and to its numerical approximation, infinite idealization also contributes to the inclusion of crucial effects to be considered in interacting structures. Whereas the effectiveness of infinite idealization is based on its integration with numerical methods, such as FEM, its meaning is indissolubly associated with a mathematical representation of space, i.e. with a representation of DOF that is purely mathematical. It can assume a physical meaning if and only if it is coupled with other appropriate numerical methods. Thus, in order to achieve the representational ideal of completeness, infinite idealization must ensure a complementarity function in model-building. Now, the question is to establish whether such a mathematical representation of space and its properties is also explanatory and predictive. The answer is yes, but only under particular assumptions. First, one should consider that infinite idealization as examined so far does not directly correspond to any physical system. On the contrary, by simulating some relevant properties expressed by quantities, e.g. flow potential and distance of the source, the finite-infinite element coupling can finally represent the target system behavior and some of its properties. In engineering, the target system includes an artifact, be it a building, a bridge, or an airplane, as well as how it is affected by stress conditions and so forth. Therefore, an explanatory description (be it inexact or refined) of the target system or of its properties is given by the coupling of finite and infinite elements. Therefore, what enables infinite idealization to contribute to the capture the structural (dynamical) behavior is the choice of the boundary conditions and the choice of the truncation boundary. In other words, it is how engineers take control over the shape function that dictates the effectiveness of both finite element method and infinite idealization. Indeed, boundary conditions, which are needed to truncate the computational domain, cannot be properties that directly represent the target system, because in structural engineering they represent communication with the surrounding world, which is what engineers want to exclude from the final simulation.[22]

Second, it is only by using infinite idealization, e.g. semi-infinite cylinder, infinite slope method and so forth, that FEM can be applied to structures experiencing non-linear dynamics. Indeed, only by using infinite idealization, can FEM produce reliable results about non-linear dynamical systems and simulate the structural behavior in the case of earthquakes, for instance. Given the unsuitability of FEM to be applied to fluid mechanical problems, the best way to represent the structure's interaction with fluid or viscous soil is to appeal to infinite idealization of a medium or media. Thus, it is true that engineers define the realm of the finite method by using infinite idealization, namely infinite idealization can be the condition of applicability of the finite method to dynamic systems, because without it, Multiphysics analysis would be impossible in many cases and a unified simulation could not be attained. Thus, even if infinite idealization *taken alone* does not fulfill the functions of prediction and explanation,[23] it still represents a fundamental step towards obtaining both of these goals which orient engineering models. Finally, it is worth noting that infinite idealization plays a fundamental role in expressing certain boundary conditions and helps the computation and prediction of structural behavior on the grounds of both a mathematical fact, i.e. the solution of differential equations require computation and a truncation boundary, and the physical problem at hand, e.g. where we locate the source of radiation or the flow that interacts with the structure.

The upshot of all this is that the use of infinite idealization combined with finite element method makes it an essential ingredient of the explanatory function of a model that includes the truncation boundary problem solution. Therefore, our ability to predict structural behavior also depends on the use of infinite idealization, because it can account for the non-linearity of dynamical systems (e.g. soil viscosity affecting the structural behavior). More importantly, this means that infinite idealization offers the possibility of including "hidden physics" in models that would be otherwise unable to offer

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[22] For an interesting discussion of the need for idealization and the relevance of idealized cut in defining boundary conditions in physics and their relation to the notion of "effacement", see Wilson (1992, 570ff.) and Damour (1987).

[23] I cannot deal here with this topic for reasons of space, but the analysis of idealization and infinite idealization, particularly in relationship to explanation in engineering models is fascinating. In particular, to simulate different outcomes and structural behaviors, such as crack propagation depending on the use or otherwise of infinite idealization, sheds light on many engineering models. I shall propose elsewhere this analysis but for the time being it is important to mention the potential of the application of Woodward's (2003) approach to civil engineering models and the quest for explanation of its models. Consider the question 'what if things had been different?', for instance, by using or not using infinite idealization in modeling a bridge structure as it interacts with the combined action of water and wind. Which kind of model explaining vertical and horizontal vibrations would one obtain? This question relates to the reflection upon the ideal of completeness. Indeed, the latter requires that engineering models specify both (external) causal factors that affect the phenomena, as well as the difference-making properties with respect to whether or not a phenomenon occurs (for a discussion on these topics, see also van Eck 2016, 5).

accurate predictions. For instance, the use of infinite idealization in our case study (Section 4.2) embodies the need to represent the flow potential as continuous, namely the hydrodynamics of the soil at the point of contact with the structure is included in model-building, so that the structure can be designed to meet the required safety and stability standards. Therefore, engineering models embody goals, such as structural stability and safety that are not included in pure mathematical or physical models.

6. Closing Remarks

In this contribution, I explored the use of infinite idealization in structural engineering models. As shown, this use depends on both mathematical considerations and specific functions associated with engineering models. Without infinite idealization, fundamental physical effects would be neglected, and it would be harder for engineers to take control over the structural behavior, because fundamental boundary conditions would be neglected. In order to identify these boundary conditions, engineers consider how a certain portion of the world interacts with its surroundings along their mutual boundary. To reach this goal, engineers cannot rely on finite elements only. In the complex architecture of engineering model-building, the FEM/IEM coupling is fundamental to identifying the procedure that defines appropriate boundary conditions in view of the representational ideal of completeness and of the unity of Multiphysics simulations. Therefore, the use of infinite idealization in structural engineering discloses the strengths and weaknesses of the mathematical methods devised therein, their effectiveness, and the consequent reliability of Multiphysics models. Thus, infinite idealizations tell us when and why BEM, FEM, FVM and FDM pose severe limits to modeling, but at the same time their use contributes to successfully determining the correct boundary conditions and therefore the applicability of engineering methods and models.

Finally, in engineering, infinite idealization plays a fundamental role in building up goal-oriented models. It contributes to the representational ideal of completeness when discretizing methods, such as FEM, necessarily need a counterpart to solve PDEs. I call this necessary coupling the "complementarity function" of infinite idealization. This function is certainly to be detected in the multiple-model idealization that characterizes structural engineering and to Multiphysics in general.

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