# CHARACTERIZING MATHEMATICAL ACTIVITIES PROMOTED BY FERMI PROBLEMS

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## INTRODUCTION

Some questions that arise in everyday situations, such as *How long will it take to get to the airport?*, can be solved by making a quick estimate. In other cases, we can ask ourselves about situations that we have never asked ourselves before but that we still are interested in obtaining a first rough answer to. This latter would be the case if we consider what the number of planes that are flying all over the world at a specific time are, or the amount of $CO_2$ emissions we could avoid in a city if gasoline-powered cars were replaced by electric ones. These two questions are specific cases of a type of question known as *Fermi problems*, which owe their name to Enrico Fermi who used this particular type of problems both in his scientific work and as a university teacher. Fermi problems, being smaller, more well-defined and delimited contextualized problems and not real-world problems given in all their complexity, have been considered to be miniature-modeling problems (Robinson, 2008).

Fermi problems can be solved by decomposing of the original problem into simpler sub-problems to reach a solution of the original question by means of reasonable estimates. This way of solving realistic problems has been used in Mathematical Education to introduce mathematical modeling to Primary Education students (Peter-Koop, 2009), to use real-world knowledge to support mathematical learning with Secondary Education students (Albarracín & Gorgorió, 2014; Ärlebäck, 2009) and at university level to assess modeling processes, such as validating models (Czocher, 2018).

Although educational research on Fermi problems generally emphasise estimation it has been suggested that the activity of estimation can be replaced by other (classroom) activities to achieve the unknown numerical information needed to solve the problem at hand (Sriraman & Knott, 2009). In this article we identify different types of mathematical activities that can be derived from the use of Fermi problems in the existing published research, with the goal to develop a characterization framework that can be used both for designing research interventions and as an analytical tool in research.

## FERMI PROBLEM ACTIVITY TEMPLATES

In this section we present a characterisation tool called *Fermi problem Activity Template* ($F_pAT$) that focuses on the structure of the Fermi problem, which can be used to describe either (a) the result of an a priori analysis of the Fermi problem split into interconnected subproblems needed to be solved; or (b) a structure that describes the activities students actually engage in when solving a Fermi problem. Hence, the $F_pAT$ can be used both in the design of research interventions and as an analytical tool.

Part of our inspiration for how we have come to represent the $F_pAT$ categorisation comes from Anderson and Sherman (2010) who proposed a representation based on an a priori analysis of the problem at hand that structures the solving process by explicitly differentiating the sub-problems the students have to engage in to solve the problem. The example they discuss in great detail is about estimating the number of hotdogs consumed at the Major League Baseball (MLB) games each season in the US. Differentiating between the values needed to be estimated (such as the number of hotdogs consumed by an attendant and game) from given values that can be looked up (such as the number of games in a season), Anderson and Sherman (2010) presented the breakdown of the problem as represented in Figure 1.

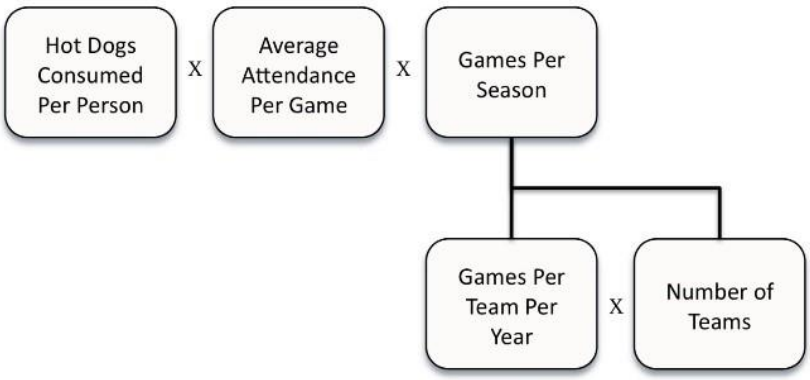


**Figure 1 –** The MBL hotdog Fermi problem as presented in Anderson and Sherman (2010, p. 37).

Here, one can consider engaging in different types if activities to achieve the unknown numerical values needed in solving the different sub-problems. For example, rather than using a standard way of estimating based on previous experiences, consulting the MLB website might provide the number of games each team play per year, and to find the number of spectators at a typical game one can turn to newspaper reports. In the case of finding the number of hotdogs consumed per person and game a survey that generates sufficient data to determine a reasonable estimate can be used. The decision and reason for using one activity rather than another can depend on the solver's understanding of the precision required in the solution of the problem, or be a conscious choice made in the design of the activity in order for the students to work toward given certain mathematical contents and curricula goals.

## MATHEMATICAL ACTIVITIES PROMOTED BY FERMI PROBLEMS

Some studies have used the so-called MAD framework (Ärlebäck, 2009) to analyse the activities students engage in when solving Fermi problem. This framework characterizes the process of solving a Fermi problem in term of the following sub-modelling activities: *reading*, *making model*, *estimating*, *calculating*, *validating* and *writing*. Here, the activity of estimating is the originally key feature of a Fermi problem, but in the following we concur with the suggestion by Sriraman and Knott (2009) and argue and illustrate that this activity can be changed and generalized to more broadly incorporate other ways for students to get the necessary numerical information they need to answer the problem or related sub-problems. The four types of mathematical activities that we propose to achieve the unknown numerical values needed of quantities to be able to provide a solution and answer to the problem in question are *guesstimation*, *experimentation*, *data search* and *statistical data collection*.

To explicitly visualize the structure of the problem and the different types of mathematical activities that can be used in the solving the problem discussed above, our $F_p$AT framework uses different graphical representations for of the four (sub-)activities; see Table 1. The purpose of using different geometrical shapes for the different activities is to enhance readability.

Table 1: The components of the Fermi problem Activity Template framework ($F_p$AT).

| **Activity / representation** | **Students obtain the quantity by engaging in…** |
|---|---|
|   | ...a mental process giving a rough solution through guessing and making comparisons based on previous experiences and intuition. |
|   | ...in-and-out-of-school experimentations and investigations, including making measurements. |

| | |
|---|---|
|  | ... searching for numerical information in external sources. |
|  | ...suitable ways of selecting, collecting and analysing statistical data. |

Figure 2 shows the $F_pAT$ framework applied to the structure of the Fermi problem discussed by Anderson and Sherman (2010) presented in Figure 1, illustrating that a statistical survey will be used to find the average number of hotdogs consumed per person in a game, that the number of how many people attend a game will be determined using an estimate, and that the rest of the needed numerical values will be searched for in appropriate sources. The different sub-problems identified in the problem are delimited using square brackets in order to clearly convey the structure of the problem.

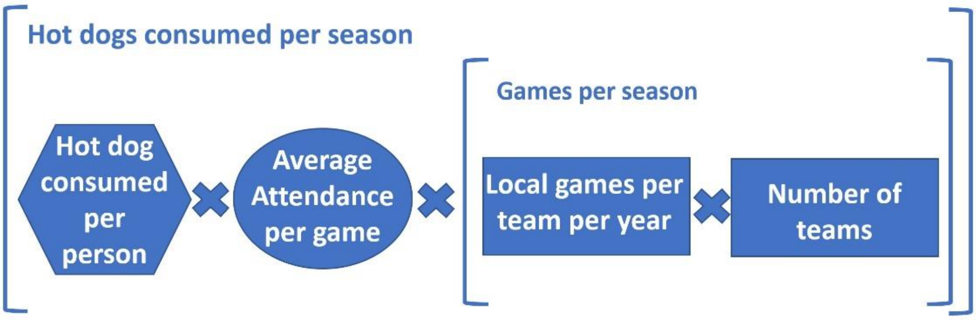


**Figure 2 –** Structure of Anderson and Sherman's (2010) MBL Hotdog problem characterized using $F_pAT$.

## USING $F_PAT$ AS AN ANALYTIC TOOL

In Albarracín and Gorgorió (2013) one of the problems studied asked individual students for an estimate of the total number of SMS sent in a day in Catalonia. Secondary students (12-16 years old) were asked to provide a detailed and concrete plan how to go about estimating the quantities asked for rather than solving the problem. The following excerpt presents one of proposed solutions given by a student (A1) in terms of the activities suggested would solve the problem in terms of a $F_pAT$ (see Figure 3).

> A1: I would make a survey to the people I know about how many messages they send per day, trying to take a balanced number of women and men, as well as for young people, adults and the elderly. Then, I would make an average and then multiply this by the number of inhabitants of Catalonia between the age ranges that might have a mobile phone.

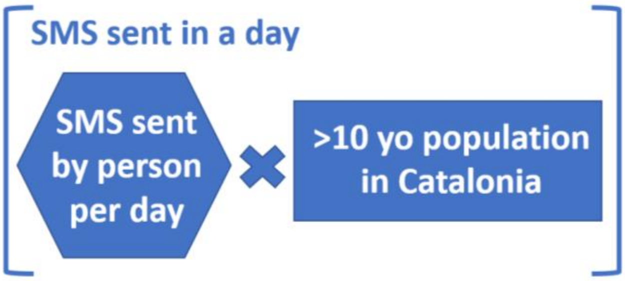


**Figure 3** – Structure of student A1's proposed solution to the SMS problem represented using $F_pAT$.

Student A1's proposal shows a strategy to solve the problem in which a survey is the suggested way to find out the number of SMS sent by the inhabitants of Catalonia in a day. The student considers that the participants surveyed must comply with some restrictions in order to the sample to be adequate but does not provide any details on how to realise this. Then, when the average value of SMS sent in a day per person is known, the student proposes to multiply this number by the number of mobile users in Catalonia.

Student A2's solution proposal was different and is outline below (Figure 4):

> A2: First, it [the survey] will take 10 teenagers, 10 adults, 10 senior citizens, from here I would ask them how many SMS they send per day to each group and I would find the average. Then I would calculate the total average respecting the percentage of people of each type and multiply it by the total number of people in Catalonia.

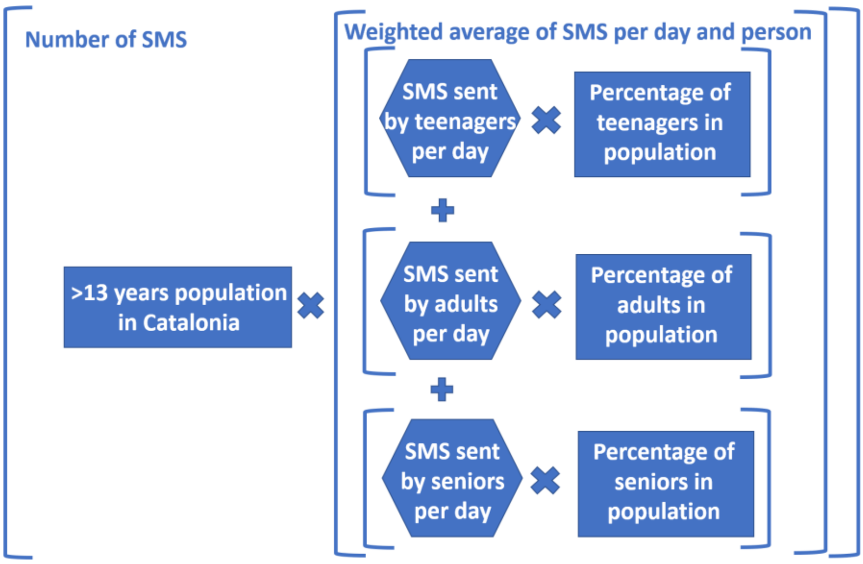


**Figure 4** – Structure of student A2's proposed solution to the SMS problem represented using $F_pAT$.

Student A2's proposal shows that this student also suggests using a survey to collect data as the basis for his solution, but in contrast to student A1's proposal suggests stratifying the sample according to his experiences and knowledge about the different age groups usage of SMS. As illustrated by the $F_pAT$ in Figure 4 however, student A2’s proposal not necessary use this stratification throughout in each population group, but rather suggests to use the collected data to derive a weighted average. These two examples illustrate that the $F_pAT$ as an analytical tool of students’ work facilitate a straightforward and compact representation of students’ (proposed) solutions as well as clearly can visualize subtle differentiating aspects of the processes (potentially) involved in solving the problem, such as in the case of student A2’s suggestion of using a weighted average instead of a generic average.

## USING $F_pAT$ AS A TOOL FOR TASK DESIGN

To illustrate the potential of the $F_pAT$ framework in designing research interventions, we apply the framework to do an a priori analysis of the activity proposed in Taggart et al. (2007) that asks students "How many semitrailer loads of bottled water [16.9 fluid ounces] were needed by New Orleans’ refugees in the week following Hurricane Katrina?" (p. 166). Taggart et al. discuss this question and list a number of sub-questions needed to be answered in order to answer the original question.

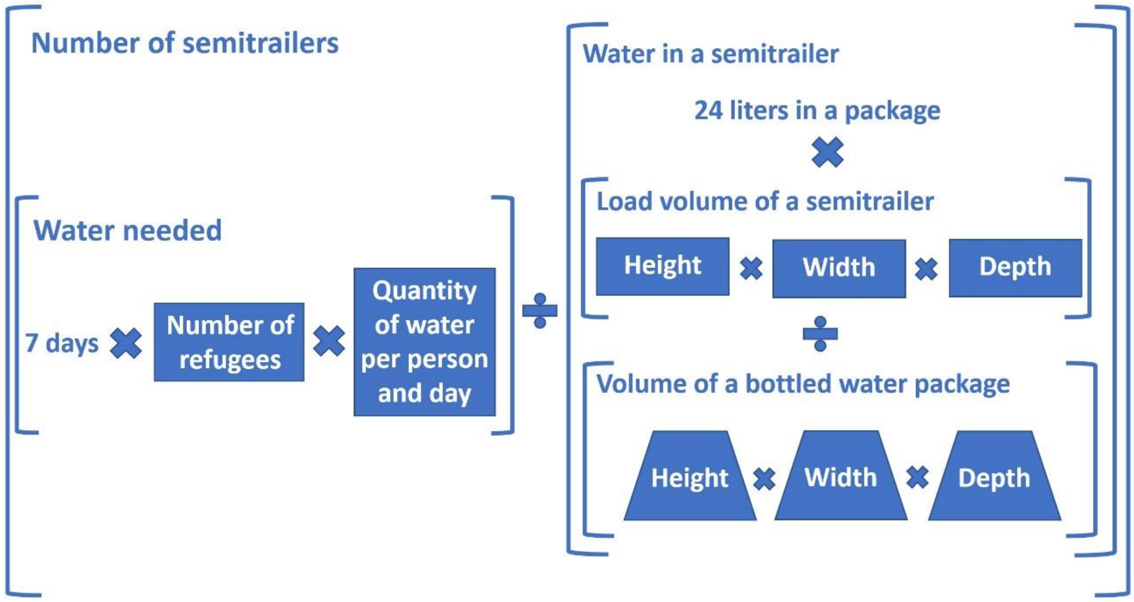


**Figure 5** – Structure of problem discussed in Taggart et al. (2007).

Figure 5 presents the $F_pAT$ based on the listed non-explicitly connected sub-questions in Taggart et al. (2007), and we can see that the problem is structured around determining two main quantities, namely the total amount of water needed and the amount of water that a semi-trailer can transport. However, these two quantities must be determined from a succession of estimates and calculations using more specific quantities. Given as a modelling problem in a research intervention, this a priori analysis highlights the different types of activities students productively can engage in have in tackling the problem and, depending on the focus of the research, hence can support the research design by preparing and propping up the research setting.

## FINAL THOUGHTS

Fermi problems have a long tradition in certain educational contexts. However, despite various recommendations in the mathematics education literature this type of problem has not yet to any larger degree been implemented in classrooms nor has its full potential been exploited or researched. As we argued for in this paper, Fermi problems in a modelling context promote multiple types of mathematical activities allowing for a broad range of solutions, classroom discussions and learning at all educational levels in multiple subjects. Indeed, given that the contexts of Fermi problems often come from the STEM disciplines, Fermi problems have the potential to function as integrating activities, allowing for the connection and transfer of strategies and applications of the different activities from other disciplines into mathematical classrooms (and vice versa). In addition, the different types of activities in the $F_pAT$ framework is an accessible entrance point to introduce and work with various types of technologies that can be used to obtain or process data, allowing Fermi problems to also be a facilitator of students' ICT literacy. With this in mind, we are hopeful that the presented $F_pAT$ framework can be used as both a research tool for designing interventions and analysis, especially focusing on what demands, roles, functions and effects different activities have on the problem-solving process, the solutions, and students' learning.

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