Alcántara, Vicent; Padilla, Emilio. «Key sectors in greenhouse gas emissions in Spain : an alternative input-output analysis». Journal of Industrial Ecology, Vol. 24 Núm. 3 (7 June 2020), p. 577-588. 12 pàg. DOI 10.1111/jiec.12948

This is the postprint version of the article:

Alcántara, V., Padilla, E. (2020) "Key sectors in greenhouse gas emissions in Spain: an alternative input–output analysis", *Journal of Industrial Ecology*, Vol. 24 (3), 577–588. https://doi.org/10.1111/jiec.12948

Key sectors in greenhouse gas emissions in Spain: an alternative input–output analysis

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Abstract

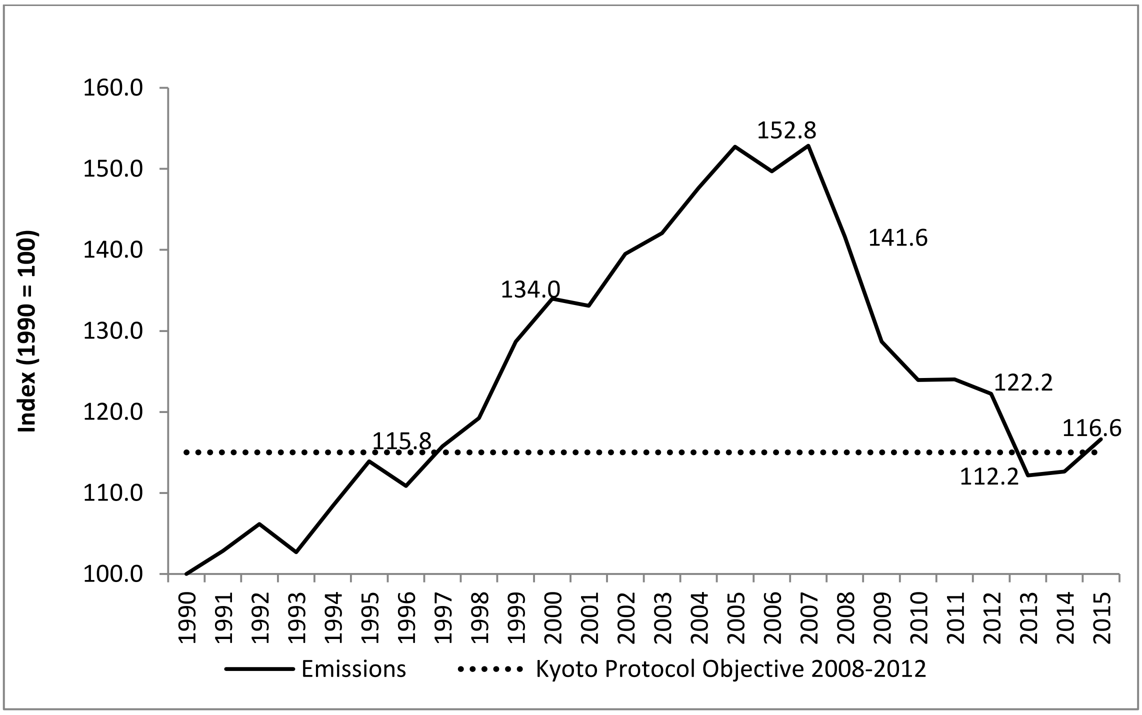
We develop an alternative input–output approach and apply it to the determination of key sectors in emissions. This methodology allows us to assess and classify the different productive sectors according to their greenhouse gas emissions and the role that they play in the productive structure, as well as the participation of their output in the total volume of production. In contrast with previous approaches, we do not focus on the responsibility of final demand, but on the responsibility of the total production of each sector. We apply our methodology to the 2014 input–output table for Spain provided by the World Input–Output Database (2016). The results show that the sectors that induce more emissions from other sectors are manufacture of food products, wholesale and retail trade, and construction. Those that are pulled to emit coincide with those that are relevant for their own final demand, being the most important electricity and gas provision, agriculture, and transportation. The classification obtained allows to orient the design of greenhouse gas emission mitigation policies for the different sectors.

Keywords: greenhouse gas emissions, input–output, key sectors, production structure, Spain.

1. Introduction

According to the Kyoto protocol and to the internal allocation of emission objectives within the European Union, the annual average of the greenhouse gas (GHG) emissions of Spain in the period 2008–2012 should have not been above a 15% increase of the 1990 emissions (taking into account the CO_2 equivalent aggregate of the six gases considered by the protocol: CO_2, N_2O, CH_4, HFCs, PFCs and SF_6) (Commission Decision 2006/944/EC, EC, 2006). The emissions of Spain for the base year of the protocol were 289.7 Mt, while the average annual emissions from 2008 to 2012 amounted to 360.0 Mt, a 24.5% increase over the base year. The emissions increased well above the objective and the protocol was only fulfilled thanks to the use of the flexibility mechanisms (EEA, 2017). Moreover, if it were not for the economic crisis, the mismatch would have been significantly worse.

Figure 1. GHG emissions in Spain, 1990–2016



Source: Prepared by the authors with EEA (2017) data.

The peak in emissions occurred in 2007, while the fall in economic activity since 2008 involved a quite drastic reduction in emissions (Figure 1). Emissions fell to their minimum in 2013, when they were even below the emission level objective of the protocol for 2008–2012. However, the economic recovery that began in 2014 led to an increase in emissions that continued in 2015, again exceeding the protocol objective. In 2015 emissions amounted to 335,661.5 kt of CO_2 equivalent and represented an increase of 3.5% over the previous year, while GDP grew by 3.2%. The key element for this expansion of emissions was the generation of electricity in thermal power stations (Observatorio de la Sostenibilidad, 2016).

The EU has proposed a plan to reduce emissions that involves a 20% reduction of 2005 levels for the year 2020. The plan shows two paths for reduction. There is a difference between the sectors subject to the emissions trading system (ETS) and the so-called diffuse sectors (non-ETS). The first group consists of fixed sources, which by 2020 would have to reduce their emissions by 21% with respect to the 2005 emissions. This is a global objective for the EU, without any explicit distribution of this effort among countries. On the other hand, the sectors not subject to the ETS must reduce their emissions by 10% with respect to 2005 levels for the EU as a whole. However, through Decision 406/2009/EC (Effort Sharing Decision), this global effort is distributed among the different countries. According to this Decision, Spain must reduce these emissions by 10%. To fulfill these objectives much more important mitigation efforts than those carried out so far must be implemented.

Although having different objectives, the two groups of sectors have clear structural relationships from a production perspective. In this research, we will show the sectoral interdependencies between the different productive sectors (including the relationships between the sectors included in the ETS directive and those not included) in order to inform the design of policies oriented to mitigate the GHG emissions of the Spanish economy. In contrast with previous methodological approaches for the determination of key sectors, which focus on the relevance of final demand, we will focus our analysis on the responsibility associated with the production of each sector in order to provide a more complete picture of the role played by different sectors in the productive structure regarding the implications for emission. From this point of view, our interest focuses on showing, first, the intersectoral linkages generated by the different productive processes, and second, the intrasectoral requirements to obtain the net output of the different sectors. This justifies our methodological proposal as an alternative to other proposals in the determination of key sectors. For example, in the conventional demand analysis, the production of any sector oriented to other sectors in direct and/or indirect relation with its final demand is computed as a feed-back in such sector. However, from a purely industrial perspective, these supplies must be computed in the sector that makes use of them. In a way, what we intend to show is, first, the output–output relations (Milana, 1985) derived from the productive structure of the country and, second, the own production directly linked to obtaining the final demand of each sector[1]. With this purpose, Section 2 provides an input–output methodological perspective based on the Leontief model and in the approach of Heimler (1991). Section 3 applies it and analyzes the results obtained. Finally, Section 4 presents our conclusions and orientations for the design of policies.

2. Methodological proposal

In an input–output framework, the relationship between polluting emissions and the behavior of the economic system is shown by linking the emission coefficients (the emission generated per unit of output) of each of the productive sectors of the economy in the Leontief model (1936)[2]. This allows an analysis of environmental impacts from a demand-side perspective. Alternatively, from the Ghosh (1958) model, similar relationships can be established but allowing an analysis from a supply-side perspective.

Let us assume an economy composed of *n* productive sectors; then, the emission coefficients would be given by the following expression:

(1) $$\mathbf{c} = \hat{\mathbf{x}}^{-1}\mathbf{e}$$

in which **c** is a column vector (*n x 1*) of emission coefficients, **x** is the vector (*n x 1*) of sectoral productions and **e** is a vector (*n x 1*) of sectoral emissions.[3] The emission coefficients obtained in this research are shown in the fourth column of Appendix I. Then,

(2) $$E = \mathbf{c}'\mathbf{x}$$

[1] We acknowledge the relevance of considering the impact on other countries of the Spanish productive behavior according to its international economic relations. In this sense, there are several interesting investigations from this perspective on Spain, such as Sánchez-Chóliz and Duarte (2004), Arto et al. (2014), López, et al. (2014) and Cadarso et al. (2015), among others. However, our intention was to provide a first approach of our alternative analysis from a purely territorial perspective.

[2] For a good review on the input–output methodology and its applications, see Miller and Blair (2009).

[3] In this work, (^) denotes the diagonalization or expression in form of diagonal matrix of a vector. Vectors are written in lowercase and bold. Matrixes are written in capital letters and bold. The mark (') expresses the transposition of both matrices and vectors. Scalars are written in italics.

expresses total emission, E, generated by the total productive activity of the economy.

Substituting **x** by its known value in the open Leontief model, we obtain:

(3) $$\mathbf{e} = \hat{\mathbf{c}}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \hat{\mathbf{c}}\mathbf{L}\mathbf{y}$$

where **A** is the matrix of technical coefficients or inputs and **y** the vector of sectoral net outputs. $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is the well-known Leontief inverse.

If we operate the same substitution but in the Ghosh model, we obtain:

(4) $$\mathbf{e}' = \mathbf{v}'(\mathbf{I} - \mathbf{B})^{-1}\hat{\mathbf{c}} = \mathbf{v}'\mathbf{G}\hat{\mathbf{c}}$$

where **B** is a (n x n) matrix of output coefficients defined as the production of i for sector j in relation to the total production of sector i. $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ is the so-called Ghosh inverse.[4] The characteristic element of the Ghosh inverse, G_{ij}, denotes the production of sector j linked to a unit of primary inputs used by sector i.

Taking into account the approaches of Rasmussen (1956) and Hirschman (1958), it is well known that it is possible to obtain the backward linkages (BLs) and forward linkages (FLs) from equation (3) in order to determine the so-called key sectors. This approach, however, was questioned by Skolka (1986).

Following the proposal of Jones (1976), the conventional analysis of environmental impacts complements the measure of BLs to determine the key sectors in the emission with the FLs obtained from the Ghosh model. While in equation (3) we obtained the vector of emissions of the economy from the input coefficients, in equation (4) we obtain it from the distribution or output coefficients.

[4] The Ghosh inverse is related to the Leontief inverse through the next similarity transformation: $\hat{\mathbf{x}}^{-1}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{x}} = (\mathbf{I} - \mathbf{B})^{-1}$.

The joint use of backward and forward linkages for the analysis of impacts is not exempt from criticisms, which we will not review again here. However, from an ex-post perspective, for a comparative sectorial study, they could be used as a descriptive tool, as proposed in Lenzen (2003).

Though this is an interesting approach from a final demand perspective, it is insufficient insofar as the productive sectors are not only producers of final goods. As Heimler (1991) states, the final demand of a sector can be zero. Therefore, according to the previous analysis, the final emission of this sector would be zero. If a sector exhibits a direct emission and a null final demand, its emissions will surely be located in its intermediate demand. The model does not allow for capturing that situation. Moreover, although the joint use of BLs and FLs allows us to determine the key sectors in the emission, as in Alcántara et al. (2010) or Piaggio et al. (2014), we will not develop an analysis of this type here. More than determining "key sectors" in the emission, the objective of our research is to show the role played by the relevant productive sectors, from the perspective of GHG emissions, within the framework of the Spanish productive structure. We will develop a method to show the total emissions of the system linked to the production of each sector considering the use of this production in the different sectors. Our aim is to determine the intersectoral linkages according to the direct and indirect requirements of the gross products, whether they are used to obtain the final demand or the intermediate demand. This approach was developed by Karunaratne (1976), in a semi-input–output framework, and was included in the careful exploration of Hewings (1982) on the identification of key sectors. Later, it was adopted with a rigorous mathematical description by Milana (1985) to determine the total use of the final outputs and the gross outputs of each industry in Italy in 1975. Heimler (1991) used the same method for the assessment of the key industries in the Chinese economy. The method that we propose, based on the previous research cited above, allows us to identify those sectors that are key regarding emissions in a precise way, in terms of their relationship with other sectors given the productive and technological structure of the economy. According to the research objectives, this approach could also be used to complement other types of analysis, including the conventional analysis of demand impacts, though it is not our objective in this study.

Our starting point is the basic equation of the Leontief model:[5]

(5) $\mathbf{Ax} + \mathbf{y} = \mathbf{x}$

Let **A** be equal to the sum of the next matrixes:

(6) $\mathbf{A} = \mathbf{A}^D + \mathbf{A}^O$

where $\mathbf{A}^D$ is a diagonal matrix built with the elements of the main diagonal of **A** and $\mathbf{A}^O$ the rest of the elements of **A** outside the diagonal.

The system can be rewritten:

(7) $\mathbf{A}^D\mathbf{x} + \mathbf{A}^O\mathbf{x} + \mathbf{y} = \mathbf{x}$

and then:

(8) $(\mathbf{I} - \mathbf{A}^D)^{-1}\mathbf{A}^O\mathbf{x} + (\mathbf{I} - \mathbf{A}^D)^{-1}\mathbf{y} = \mathbf{x}$

The first term on the left side shows the impact that the intermediate demand has on the different productive sectors. It can be easily verified that the main diagonal of the matrix resulting from the matrix product in this addend is composed of zeros. The second term shows the own impact of the final demand on its respective sector. Notice that this own impact does not include the feedback that occurs in the case of the Leontief inverse. This is now assigned to the sector to which the corresponding commodity has been sold. That is, if sector *i* sells to sector *j* a given quantity of output, the latter sector, from a production perspective, is responsible for the increase in production of *i* (hence the zeros in the mentioned main diagonal). However, from a final demand perspective, it would be responsible for the increase in its own production due to the need for inputs from other sectors, given that the Leontief inverse indicates the responsibility for final consumption.

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[5] To avoid making the methodological development tedious, we include the mathematical development of the methodology in Appendix III.

From an environmental perspective:

(9) $$\hat{\mathbf{c}}(\mathbf{I}-\mathbf{A}^{D})^{-1}\mathbf{A}^{O}\mathbf{x}+\hat{\mathbf{c}}(\mathbf{I}-\mathbf{A}^{D})^{-1}\mathbf{y}=\hat{\mathbf{c}}\mathbf{x}$$

We can now obtain the BL from the first component of the left side of equation (9):

(10) $$\boldsymbol{\mu}'_{B}=\mathbf{c}'(\mathbf{I}-\mathbf{A}^{D})^{-1}\mathbf{A}^{O}\hat{\mathbf{x}}$$

This vector expresses the pure BL (PBL) for each productive sector. It coincides with that proposed by Sonis et al. (1995).

With the aim of capturing the horizontal impact of all the sectors on a given sector for a specific moment of time, we compute the following expression:

(11) $$\boldsymbol{\mu}_{F}=\hat{\mathbf{c}}(\mathbf{I}-\mathbf{A}^{D})^{-1}\mathbf{A}^{O}\mathbf{x}$$

This expression should not be interpreted as a pure FL (PFL) in the sense of Sonis et al. (1995). It is, however, the pure impact of all sectors on each particular sector that interests us from the perspective of our subsequent analysis. Nevertheless, since it is defined as a FL in the perspective of conventional analyses that use the Rasmussen coefficients, we will also denote it with this term and the acronym PFL.

Last, the necessary production of each sector to meet its own demand, regardless of possible feedback, would be given by the expression:

(12) $$\boldsymbol{\mu}_{D}=\hat{\mathbf{c}}(\mathbf{I}-\mathbf{A}^{D})^{-1}\mathbf{x}$$

From the results obtained for the three former equations, we establish three indices that will allow us to characterize the different sectors according to their relative importance as regards their GHG emission orientation, taking into account their role in the framework of the economic structure.

Since the overall emissions are obviously equal from both BL and FL perspectives, we can establish the average as a reference quantity.[6]

Let $\bar{\mu}$ be the average reference. From equation (10), we can establish the following relationship:

$$(13) \qquad \mu^{*}_{B,j} = \frac{\mu_{B,j}}{\bar{\mu}}$$

which shows the relative importance of the BL of the productive sector *j*.

We proceed in the same way with expressions (11) and (12):

$$(14) \qquad \mu^{*}_{F,i} = \frac{\mu_{F,i}}{\bar{\mu}}$$

$$(15) \qquad \mu^{*}_{D,i} = \frac{\mu_{D,i}}{\bar{\mu}_D}$$

In equation (15), the average is obviously different.

From equations (13) and (14), we can establish a classification of the different sectors, as shown in Table 1.

Table 1. Sector classification

	$\mu^{*}_{B,j} < 1$	$\mu^{*}_{B,j} \geq 1$
$\mu^{*}_{F,j} \geq 1$	Sectors that are significantly induced to emit by other sectors	Key sectors
$\mu^{*}_{F,j} < 1$	All other sectors	Sectors that are relevant inductors of emission

[6] The reference magnitude to compare the indicators could be different, such as the median, as in Alcántara and Padilla (2003).

Equation (15) shows the relative importance of the emission generated by a sector for its own demand. As indicated above, there could be feedback as a consequence of the purchases that one sector makes from other sectors to obtain its final demand. From the consumption perspective, the responsibility should be allocated to this final demand. However, this is not our objective. The interest of the present research focuses on the emitting behavior of the different sectors in the framework of the productive structure. Our aim is to provide information in terms of responsibility from the point of view of the emissions required for the total production of each sector, so that the sectoral emissions from a production perspective are better reflected. This is the reason for the analytical separation between final and intermediate demand. We will see below that the indicator provided by equation (15) plays a very relevant role, combined with the other two, in order to guide environmental policies.

3. Data and results

We use the national input–output table (NIOT) for Spain prepared by the World Input–Output Database (WIOD, 2016) for 2014, the last year available[7]. The WIOD database includes homogeneous tables for 28 European countries and 15 other countries among the principal economies of the world for the period 2000–2014. The emission data have been obtained from INE (2016), which classifies them by productive sectors compatible with the classification used by the WIOD.[8] Appendix I shows the relevant variables of the economy and GHG emissions by sector.

We apply the methods proposed in the previous section to assess the role and importance of the different sectors in emissions according to the Spanish productive structure. From the computation of equations (10), (11) and (12), we obtain the results shown in Appendix II. Table 2 shows the main results obtained from the computations included in Appendix II.

[7] The use of the NIOT involves some limitations, such as not accounting the induced emissions in other countries. However, as stated above, in this work we wanted to provide a first application of our alternative methodology from a purely territorial perspective.
[8] Both classifications are based on the NACE Rev. 2 of Eurostat.

Table 2. GHG emissions by type of impact (kt) and % on the emissions of the productive sectors

		Backward emission	%	Forward emission	%	Own demand	%	Direct emission	%
				Key sectors					
20 H	Transportation and storage	4,318.5	1.7	18,302.2	7.2	16,117.5	6.4	34,419.7	13.6
9 CG	Manufacture of rubber and plastics products, and other non-metallic mineral products	4,670.3	1.8	14,838.2	5.8	13,390.8	5.3	28,229.0	11.1
10 CH	Manufacture of basic metals and fabricated metal products, except machinery and equipment	9,121.8	3.6	6,314.2	2.5	6,723.2	2.6	13,037.4	5.1
7 CE	Manufacture of chemicals and chemical products	7,179.9	2.8	4,302.7	1.7	6,535.9	2.6	10,838.6	4.3
	Total	25,290.6	10.0	43,757.3	17.2	42,767.4	16.9	86,524.7	34.1
				Sectors induced by other sectors (forward orientation)					
16 D	Electricity, gas, steam and air-conditioning supply	3,154.4	1.2	42,017.9	16.6	20,425.2	8.0	62,443.1	24.6
1A	Agriculture, forestry and fishing	2,088.9	0.8	23,499.0	9.3	17,424.2	6.9	40,923.2	16.1
17 EW	Water supply, sewerage, waste management and remediation	1,395.3	0.5	6,496.3	2.6	8,037.7	3.2	14,534.0	5.7
6 CD	Manufacture of coke, and refined petroleum products	1,171.4	0.5	4,856.8	1.9	13,527.7	5.3	18,384.5	7.2
	Total	7,809.9	3.1	76,869.9	30.3	59,414.9	23.4	136,284.8	53.7
				Sectors that induce emissions (backward orientation)					
3 CA	Manufacture of food products, beverages and tobacco products	28,025.0	11.0	1,967.2	0.8	3,231.2	1.3	5,198.4	2.0
19 G	Wholesale and retail trade, repair of motor vehicles and motorcycles	15,132.5	6.0	1,617.9	0.6	3,543.5	1.4	5,161.4	2.0
18 F	Construction	10,271.8	4.0	127.4	0.1	897.6	0.4	1,025.0	0.4
14 CL	Manufacture of transport equipment	6,148.7	2.4	126.3	0.0	1,438.3	0.6	1,564.6	0.6
21 I	Accommodation and food service activities	4,903.6	1.9	158.7	0.1	1,513.5	0.6	1,672.2	0.7
	Total	64,481.6	25.4	3,997.5	1.6	10,624.1	4.2	14,621.6	5.8
		25.4%	0.0	1.6%	0.0	4.2%	0.0	5.8%	0.0
				All other sectors					
All other sectors		32,295,9	12.7	5,253.3	2.1	11,111.8	4.4	16,365.1	6.4

Emissions of productive sectors: 253,796.2 kt

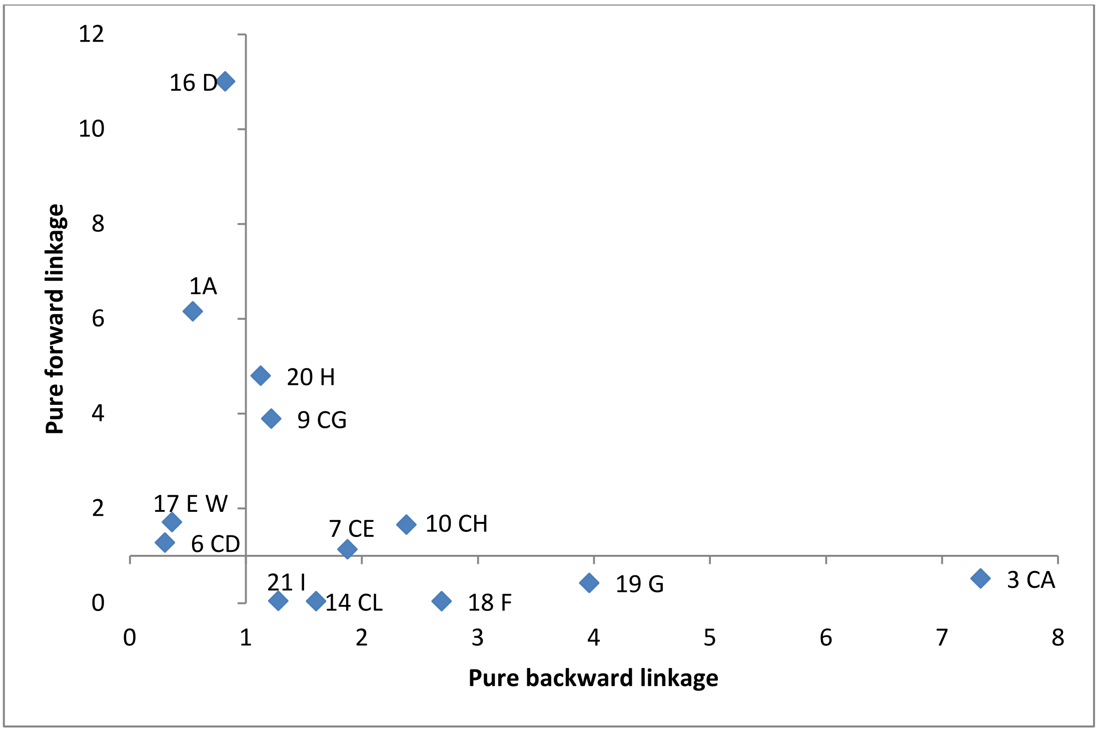
Households: 70,375.2 kt

Total emissions of the economy: 324,171.4 kt

Source: Prepared by the authors with WIOD (2016) and INE (2016) data.

In order to gauge the importance and the role played by the different productive sectors in the emissions within the framework of the Spanish productive structure, we also compute the indicators provided by equations (13), (14) and (15) based on the information provided in Appendix II. We can classify the sectors according to the classification criterion established in Table 1, taking into account the intersectoral relationships given by the PBLs and PFLs. Figure 2 shows this classification, where the top right quadrant shows the key sectors, the top left quadrant the sectors that are relevant from a perspective of PFLs and the bottom right quadrant shows the sectors that are relevant from a perspective of PBLs. The figure does not include the non-relevant sectors that would be placed in the bottom left quadrant (the "all other sectors" in Table 2). A value above 1 indicates that the linkage considered is above the average.

Figure 2. PBL and PFL



Four sectors are classified as "key sectors" in emission: Transport and storage (20H), Manufacture of rubber and plastics products (9CG), Manufacture of basic metals (10CH) and Manufacture of chemicals and chemical products (7CE). Table 2 shows that the emission generated by these sectors from an FL perspective represents 17.2% of the total emission of productive activities. However, from the perspective of the emissions that these sectors induce, they only would be responsible for 10% of emission. These sectors are defined as key because they have certain relative relevance from both a backward and a forward perspective. However, when only the FLs or BLs are considered, there are two other sectors in each case that are more important from these perspectives than these four key sectors.

As regards the sectors that induce other sectors to emit, besides the key sectors indicated, five sectors stand out above the rest. The most relevant is the Manufacture of food products sector (3CA), followed by the other four: Wholesale and retail trade (19G), Construction, Manufacture of transport equipment (14CL) and Accommodation and food service activities (21I). The total impact of this group over other sectors represents 25.4% of the emission of the system production. While in the other two groups the direct emission is significant, Table 1 shows that the direct emission of these sectors is only 5.8%. Thus, in terms of their direct emission, these sectors could be considered not very relevant for the emission of the system. In contrast, our analysis allows to detect that the total production of these sectors is responsible for one-fourth of the emissions of the system, due to the requirement of inputs from other sectors.

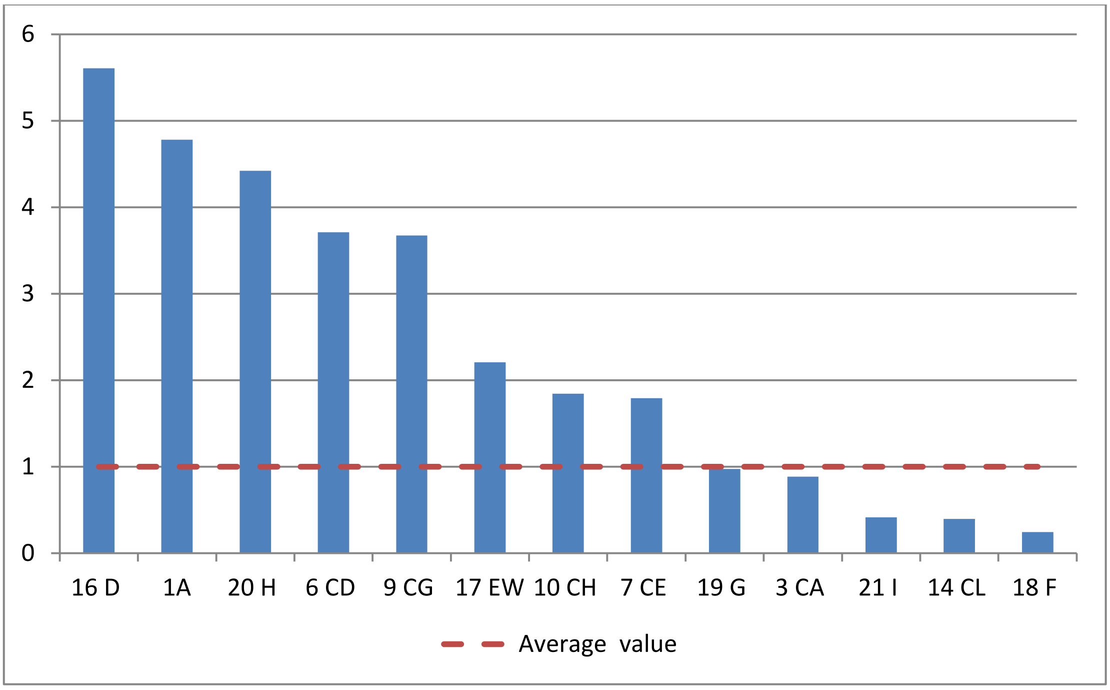
The sectors that are significantly induced to emit by other sectors, but that do not have significant BLs, are Electricity, gas, steam and air-conditioning supply (16D), Agriculture, forestry and fishing (1A), Water supply, sewerage, waste management and remediation (17EW) and Manufacture of coke, and refined petroleum products (6CD). Table 2 shows that the PFLs over these sectors amount to 30.35 of the total sectoral emission, while their impact on other sectors represents only 3.1%. The two first sectors mentioned stand out above the others, although the key sectors Transport and storage (20H) and Manufacture of rubber and plastics products (9CG) also have a very significant forward component.

In order to orient environmental policies, the previous analysis is not enough, even though it is necessary and fundamental from an intersectoral perspective. Environmental policies can be directed to the intersectoral network, with technological actions, but they can also be directed to the final demands of some sectors as the repercussion of the final demand of some sectors on their emissions can play a significant role.

In the case of the Transport and storage sector (20H), for example, its FL component represents 53.1% of its direct emission and the impact of its demand 46.8% of it. It seems reasonable to think that the policies to be applied should be aimed at replacing some modes of transport with others. But since the weight of the intermediate and final demands are very close, it would be necessary to study the possibilities of substitution in both cases. In addition, freight transport is not the same as the transport of passengers (note also that our research does not consider private transport).

From equation (15), we compute the sectoral relative impact of the own final demand, which can be higher, lower or equal to 1 depending on whether its impact on final demand is higher, lower or equal to the average. This indicator would tell where an action on the final demand of a sector can have a relevant incidence in the emissions associated with its production. Figure 3 shows the values of this indicator for the different sectors.

Figure 3. Emissions to satisfy the own final demand



Eight sectors are above the average (value above 1) as regards their impact on their final demand. Among these, we can highlight Electricity, gas, steam and air-conditioning supply (16D), Agriculture, forestry and fishing (1A), Transport and storage (20H), Manufacture of coke, and refined petroleum products (6CD), and Manufacture of rubber and plastics products (9CG). These sectors are also the most relevant for inducing emissions, according to the previous classification, since they all have important FLs. All sectors that were relevant for their FLs, including the key sectors, are also important from the point of view of their own demand. In contrast, most sectors that were relevant from a perspective of PBLs are not relevant from the point of view of the emissions generated by their final demand, with the indicated exception of the key sectors. That is to say, the sectors that are relevant because of their own final demand component are also relevant in terms of the emissions associated with their production for intermediate demand. The fact that their BLs are irrelevant means that changes to reduce their emissions depend to a great extent on their own technology, while actions that affect their own final and intermediate demand would also be effective. For example, in the case of energy, besides the actions that lead to reducing the demand for electricity via energy efficiency, it is fundamental to change the electric mix by increasing the share of renewables.

In most of these sectors, emissions due to their FLs and emissions for their own demand have a similar relevance, except in two cases in which the differences are substantial. The first is the Electricity, gas, steam and air-conditioning supply sector (16D), the main sector induced to emit by other sectors (from this perspective it would be responsible for 16.6% of all emissions, while its final demand is responsible for 8% of the emissions of the system). Conversely, in the case of Manufacture of coke, and refined petroleum products (6CD), it stands out much more from the point of view of its own final demand than for the missions linked to the intermediate demand from other sectors.

Comparing the results with existing studies on key sectors in GHG emissions for other countries is a delicate issue because, besides the different productive structure, the results are determined by the different approaches and the different sectoral aggregation levels employed. We can, in any case, highlight that some of the key sectors detected (or activities included in them), such as transport and food production appear also as relevant sectors in the studies for the Australian economy in 1995 (Lenzen, 2003), the Brazilian economy in 2004 (Imori and Guilhoto, 2010) or the Uruguayan economy in 2004 (Piaggio et al. 2014). It is also remarkable that all of these studies find various service activities among the most relevant sectors for GHG emissions, activities that would otherwise have been ignored given its low direct emissions.

4. Conclusions and policy recommendations

We have proposed an input–output methodology for the classification of the different productive sectors according to their role in GHG emission, taking into account the role they play in the productive structure and their participation in the total volume of production. With this aim, we have developed a methodological approach based on Heimler (1991) and adapted for the determination of key sectors in emissions, from both a supply and a demand perspective.

The analysis developed allows us to see which sectors are more important, both because they induce other sectors and because they provide, due to their FLs, inputs to other sectors. In contrast with other approaches, our research does not focus on the importance from the perspective of the responsibility of final demand, but from the perspective of the total production of each sector. In this sense, we complement the previous methodological contributions in the determination of key sectors, providing a clearer perspective on the responsibility associated with the production of each sector. Our methodology provides information that is more focused on the productive structure and the emission connections between the different sectors than in the importance of the demand structure. In addition, to complete the information provided by our methodology, we have computed an indicator that shows the importance of the own demand of each sector in its emissions.

Table 3 gathers the classification of the different sectors of the Spanish economy according to the methodology used and indicates the types of measures that could be more effective in each case.

Table 3. Classification of sectors according to their role in the emission of the production system and appropriate policy measures

	Sectors of the Spanish productive structure	Policy measures
Sectors induced to emit		
Sectors that are relevant because they pollute when supplying inputs for the (final and intermediate) production of other sectors.	Electricity, gas, steam and air-conditioning supply (16D) Agriculture, forestry and fishing (1A) Water supply, sewerage, waste management and remediation (17EW) Manufacture of coke, and refined petroleum products (6CD)	Sectors in which technological actions and the adoption of best practices may be more effective. These actions can significantly reduce the emissions in their production, which in large part is incorporated into other sectors. Electricity, gas, steam and air-conditioning supply and Agriculture, forestry and fishing are the most important sectors quantitatively. They amount to 16.6 and 9% of these emissions, respectively. In the case of the first sector, the actions oriented to change the energy mix used to generate electricity and, in short, to continue the trend to increase the share of renewable energies should be reinforced. Related to this, a negative point is that some policy actions have gone in the in an undesirable direction from the environmental point of view, such as the increased use of coal to generate electricity that has occurred in recent years (Observatorio de la Sostenibilidad, 2016). As regards agriculture, actions should be taken to facilitate more sustainable practices in this sector.
Sectors that induce emission		
Sectors that are relevant because their (final and intermediate) production requires inputs from other sectors that pollute during production.	Manufacture of food products, beverages and tobacco products (3CA) Wholesale and retail trade (19G) Construction (18F) Manufacture of transport equipment (14CL) Accommodation and food service activities (21I)	Measures aimed at the substitution of inputs would be appropriate, replacing inputs that incorporate more pollution with those whose production is cleaner. It would also be important to improve the efficiency in the use of polluting inputs, reducing their demand. Among these sectors, the most significant quantitatively are the first three. The presence of service sectors is remarkable; they are not usually paid much attention because they do not emit much directly, but their production in fact requires inputs whose production is highly polluting.
Key sectors		
Sectors with strong FLs and BLs. They are relevant both because their (final and intermediate) production requires inputs from polluting sectors and because their production is used in the production of other sectors, generating direct emissions.	Transport and storage (20H) Manufacture of rubber and plastics products (9CG) Manufacture of basic metals (10CH) Manufacture of chemicals and chemical products (7CE)	Being relevant from both perspectives, the types of measures discussed for the two previous groups could be effective. In any case, as shown in Figure 2, for the first two sectors (20H and 9CG) the forward component is more important, while the opposite is the case for the other two sectors, which indicates the types of policies that could be more effective in each case.

Sectors that are relevant due to their final demand		
Sectors with important direct emissions that orient a good part of their production to satisfy their own final demand.	Electricity, gas, steam and air-conditioning supply (16D) Agriculture, forestry and fishing (1A) Transport and storage (20H) Manufacture of coke, and refined petroleum products (6CD) Manufacture of rubber and plastics products (9CG) Water supply, sewerage, waste management and remediation (17EW) Manufacture of basic metals (10CH) Manufacture of chemicals and chemical products (7CE)	In the case of these sectors, policies aimed to impact the final demand of the sector would be effective. They can be policies aimed at discouraging the consumption of the most polluting sectors, either to reduce this type of consumption or to promote the consumption of less polluting alternatives when there are substitute goods. The relative importance of the first five sectors is much higher than the rest, being the sectors in which these types of measures should be done with more emphasis. It should be noted that these sectors coincide with those that are relevant due to their forward component, so in general it will be necessary to combine the actions on the final and intermediate demand. However, in the case of Electricity and gas the action on intermediate demand seems more relevant, while for Manufacture of rubber and plastic products, the action on its final demand seems the most relevant.
Other (non-relevant) sectors		
Sectors that are not quantitatively relevant in terms of either the emission that occurs in the production of the inputs they use from other sectors or their production of inputs for other sectors.	Other sectors	In these sectors, actions through specific policies would not be a priority, although they would be equally affected by systemic measures such as carbon pricing, achieving reductions wherever they were most efficient.

Note: Sectors are ordered from more to less relevant in each category.

The application of this methodology to the Spanish economy has allowed us to determine the sectors that, beyond their direct emissions, induce other sectors to pollute, producing their total output (to satisfy either their final or intermediate demand). The weight of the Manufacture of food products sector (3CA) stands out above the rest. It is the sector that requires more emissions from the other sectors to produce its output (11% of total emission). This is due in large part to the high direct emissions of the agriculture sector, from which it obtains a good part of its inputs, where there are methane emissions from intensive cattle-raising, nitrous oxide derived from chemical fertilization, and carbon dioxide emission from the fossil energy consumed. Another sector that pulls emission in a relevant way is the Construction sector (18F). Despite being far below the activity of the construction "boom" years, it pulls a significant amount of pollution from other sectors for its production, such as the manufacture of non-metallic minerals and others. Also important are two service sectors that are usually not considered among the relevant sectors: Wholesale and retail trade (19G) and

Accommodation and food service activities (21I). Concerning sectors that are relevant both for pulling emissions as well as for being pulled by other sectors to emit, two of the key sectors also stand out because of their pulling component (Manufacture of basic metals (10CH) and Manufacture of chemicals and chemical products (7CE)).

It is remarkable that the sectors that are relevant because they are pulled to emit coincide with those that are relevant for their own final demand. The most important sectors are Electricity, gas, steam and air-conditioning supply (16D), Manufacture of food products (3CA) and, in third place, Transport and storage (20H), which is also a key sector as it has an important pulling component. The measures aimed at the specific demands of these sectors and, especially, the technological measures would both be relevant in these cases.

The best systemic policy to mitigate emissions in an efficient way seems to be carbon pricing (see Baranzini et al. 2017), though many measures would also be appropriate to complement it. However, in the absence of a carbon price that affects all sectors and that is of sufficient magnitude to incentivize changes, and in the face of the difficulties for its implementation, it is even more necessary to act through different complementary measures on the sectors that, according to our analysis, have been shown to be more relevant in GHG emission.

Acknowledgements

We are grateful to three anonymous reviewers for their helpful comments. We acknowledge support from project RTI2018-095484-B-I00 (Spanish Ministry of Economics and Competitiveness and FEDER).

References

Alcántara, V., del Río, P., Hernández, F. (2010). "Structural analysis of electricity consumption by productive sectors. The Spanish case", *Energy*, 35 (5), 2088–2098.

Alcántara, V., Padilla, E. (2003). ""Key" sectors in final energy consumption: an input-output application to the Spanish case", *Energy Policy*, 31 (15), 1673–1678.

Arto, I., Roca, J., Serrano, M. (2014). "Measuring emissions avoided by international trade: Accounting for price differences", *Ecological Economics*, 97, 93–100.

Baranzini, A., van den Bergh, J., Carattini, S., Howarth, R., Padilla, E., Roca, J. (2017). "Carbon pricing in climate policy: Seven reasons, complementary instruments, and political-economy considerations", *WIREs Climate Change*, 8 (4), e462.

Cadarso, M.A., Gómez, N., López, L.A., Tobarra, M.A., Zafrilla, J.E. (2015). "Quantifying Spanish tourism's carbon footprint: the contributions of residents and visitors: a longitudinal study", *Journal of Sustainable Tourism*, 23 (6), 922–946.

European Commission (EC) (2006). Commission Decision 2006/944/EC of 14 December 2006 determining the respective emission levels allocated to the Community and each of its Member States under the Kyoto Protocol pursuant to Council Decision 2002/358/EC. Available at: https://www.eea.europa.eu/data-and-maps/data/external/commission-decision-2006-944-ec (accessed on May 2019).

European Environmental Agency (EEA) (2017). EEA greenhouse gas - data viewer. Availableat: https://www.eea.europa.eu/data-and-maps/data/data-viewers/greenhouse-gases-viewer (accessed on April 2017).

Ghosh, A. (1958). "Input–output approach in an allocation system", *Economica, 25, 58–64.*

Heimler, A. (1991). "Linkages and vertical integration in the Chinese economy", *Review of Economics and Statistics*, 73, 261–267

Hewings, G.J.D. (1982). “The empirical identification of key sectors in an economy: A regional perspective”, *Developing Economies*, 20 (2), 173–195.

Hirschman, A.O. (1958). *The Strategy of Economic Development*. Yale University Press, New Haven, CT.

Imori, D., J.J.M. Guilhoto (2010). Estrutura produtiva brasileira e emissão de CO2. In: J.E.Veiga (ed.) Economia Socioambiental. São Paulo, Editora Senac, 205–233.

Instituto Nacional de Estadística (INE) (2016). Cuentas de emisiones a la atmósfera. Base 2010.Serie 2008–2014. INE base, available at: www.ine.es (accessed on April 2017).

Jones, L.P. (1976). “The measurement of Hirschmanian linkages”, *Quarterly Journal of Economics*, 90, 323–333.

Karunaratne, N. D. (1976). “Quantificacion of Sectoral Developements Prospects in Papua New Guinea Using Tinbergen and Rasmussen Criteria”, *Developing Economies*, 14 (3), 280–305.

Lenzen, M. (2003). “Environmentally important paths, linkages and key sectors in the Australian economy”, *Structural Change and Economic Dynamics*, 14, 1–34.

Leontief, W. (1936). “Quantitative input–output relations in the economic system of the United States”, *The Review of Economics and Statistics*, 18, 105–125.

López, L. A., Arce, G., Zafrilla, J. (2014). “Financial crisis, virtual carbon in global value chains, and the importance of linkage effects. The Spain–China case”, *Environmental Science and Technology*, 48 (1), 36–44.

Milana, C. (1985). “Direct and indirectrequirementsfor gross output in input–output analysis”, *Metroeconomica*, 37, 283–292.

Miller, R., Blair, P. (2009). *Input–Output Analysis: Foundations and Extensions*. Cambridge University Press, Cambridge. 2nd edition.

Observatorio de la Sostenibilidad (2016). *Cambio climático: Evidencias, emisiones y políticas, CC16 (Por una política inteligente contra el cambio climático)*. Observatorio de la Sostenibilidad.

Piaggio, M., Alcántara, V., Padilla, E. (2014). "Greenhouse gas emissions and economic structure in Uruguay", *Economics System Research*, 26 (2), 155–176.

Rasmussen, P. (1956). *Studies in Intersectoral Relations*. North-Holland Publishing Company, Amsterdam.

Sánchez-Chóliz, J., Duarte, R. (2004): "CO_2 emissions embodied in international trade: evidence for Spain", *Energy Policy*, 32 (18), 1999–2005.

Skolka, J. (1986). "Input–output Multipliers and linkages". Eighth International Conference on Input–Output Techniques. Sapporo, Japan, July 28–August 2.

Sonis, M., Guilhoto, J., Hewings, G. J., Martins, E. B. (1995): "Linkages, key sectors, and structural change: some new perspectives", *The Developing Economies*, 33(3), 243–246.

World Input–Output Database (WIOD) (2016). "WIOD Data, 2016 release", available at: http://www.wiod.org/release16 (accessed on April 2017).

Appendix I. Relevant economic data and emissions by productive sector

		GHG emission (kt $CO_{2\text{-eq.}}$)	Emission coefficients (t/million US$)	Output (Million US$)	Value-added (Millions US$)	Full-time equivalent employment (Thousands of employees)
1A	Agriculture, forestry and fishing 01 to 03	40,923.2	679.75	60,203.6	31,755.1	691.3
2 B	Mining and quarrying 05 to 09	3,200.5	397.02	8,061.3	3,471.4	29.5
3 CA	Manufacture of food products, beverages and tobacco products 10 to 12	5,198.4	29.24	177,754.6	37,034.1	403.4
4 CB	Manufacture of textiles, apparel, leather and related products 13 to 15	647.7	24.39	26,560.5	8,012.7	136.4
5 CC	Manufacture of wood and paper products, and printing 16 to 18	3,464.4	100.24	34,560.3	10,895.7	153.3
6 CD	Manufacture of coke, and refined petroleum products 19	18,384.5	304.14	60,447.3	2,058.4	8.6
7 CE	Manufacture of chemicals and chemical products 20	10,838.6	166.97	64,911.7	13,360.3	91.0
8 CF	Manufacture of pharmaceuticals, medicinal chemical and botanical products 21	94.4	4.97	18,982.6	6,999.7	42.0
9 CG	Manufacture of rubber and plastics products, and other non-metallic mineral products 22 + 23	28,229.0	643.86	43,843.3	13,853.2	168.8
10 CH	Manufacture of basic metals and fabricated metal products, except machinery and equipment 24 + 25	13,037.4	145.48	89,614.0	20,659.3	279.3
11 CI	Manufacture of computer, electronic and optical products 26	132.5	14.49	9,141.9	4,157.3	39.4
12 CJ	Manufacture of electrical equipment 27	563.3	27.34	20,605.2	5,694.3	53.2
13 CK	Manufacture of machinery and equipment n.e.c. 28	527.2	16.82	31,336.4	11,039.3	123.8
14 CL	Manufacture of transport equipment 29 + 30	1,564.6	18.18	86,060.0	19,401.3	188.1
15 CM	Other manufacturing, and repair and installation of machinery and equipment 31 to 33	609.7	20.64	29,535.5	13,650.1	191.9
16 D	Electricity, gas, steam and air-conditioning supply 35	62,443.1	518.02	120,541.4	29,595.0	56.7
17 E W	Water supply, sewerage, waste management and remediation 36 to 39	14,534.0	430.29	33,777.1	14,205.7	140.9
18 F	Construction 41 to 43	1,025.0	6.27	163,380.3	68,243.7	924.4
19 G	Wholesale and retail trade, repair of motor vehicles and motorcycles 45 to 47	5,161.4	19.60	263,364.5	153,791.1	3,003.2
20 H	Transportation and storage 49 to 53	34,419.7	240.78	142,947.9	62,974.9	760.0
21 I	Accommodation and food service activities 55 + 56	1,672.2	11.30	148,042.7	87,056.6	1,160.4
22 JA	Publishing, audiovisual and broadcasting activities 58 to 60	267.7	10.62	25,197.2	9,677.5	122.5
23 JB	Telecommunications 61	113.6	2.52	45,123.2	23,148.3	66.3
24 JC	IT and other information services 62 +63	137.9	3.74	36,914.8	20,829.6	239.4
25 K	Financial and insurance activities 64 to 66	434.0	4.84	89,584.7	52,241.9	349.0
26 L	Real estate activities 68	61.6	0.36	171,763.1	151,576.5	172.9
27 MA	Legal, accounting, management, architecture, engineering, technical testing and analysis activities 69 to 71	287.2	4.29	66,966.0	39,217.4	630.9
28 MB	Scientific research and development 72	5.9	0.70	8,445.0	5,574.9	58.2
29 MC	Other professional, scientific and technical activities 73 to 75	107.3	5.52	19,454.4	9,744.0	195.2
30 N	Administrative and support service activities 77 to 82	1,270.8	18.91	67,195.5	38,450.8	996.8
31 O	Public administration and defence, compulsory social security 84	1,882.0	16.84	111,742.8	82,740.3	1,403.3
32 P	Education 85	932.3	10.77	86,600.9	74,961.9	1,083.8
Q	Human health and social work activities	1,332.3	10.78	123,642.2	79,254.3	1,203.3
R_S	Other service activities	292.8	4.21	69,493.8	42,392.4	863.5
All activity branches		253,796,2		2,555,796.2	1,247,719.2	16,030.8
h: Households		70,375.2				
Total emissions		324,171.4				

Source: Prepared by the authors with WIOD (2016) and INE (2016) data.

Appendix II. Results of the analysis

		Own demand μ_D	%	Backward emissions μ_B	%	Forward emissions μ_F	%	Direct emissions	%
1A	Agriculture, forestry and fishing 01 to 03	17,424.2	6.87	2,088.9	0.82	23,499.0	9.26	40,923.2	16.12
2 B	Mining and quarrying 05 to 09	2,993.0	1.18	1,021.1	0.40	207.5	0.08	3,200.5	1.26
3 CA	Manufacture of food products, beverages and tobacco products 10 to 12	3,231.2	1.27	28,025.0	11.04	1,967.2	0.78	5,198.4	2.05
4 CB	Manufacture of textiles, apparel, leather and related products 13 to 15	560.2	0.22	1,730.7	0.68	87.5	0.03	647.7	0.26
5 CC	Manufacture of wood and paper products, and printing 16 to 18	1,222.8	0.48	3,432.0	1.35	2,241.6	0.88	3,464.4	1.37
6 CD	Manufacture of coke, and refined petroleum products 19	13,527.7	5.33	1,171.4	0.46	4,856.8	1.91	18,384.5	7.24
7 CE	Manufacture of chemicals and chemical products 20	6,535.9	2.58	7,179.9	2.83	4,302.7	1.70	10,838.6	4.27
8 CF	Manufacture of pharmaceuticals, medicinal chemical and botanical products 21	74.9	0.03	1,324.9	0.52	19.5	0.01	94.4	0.04
9 CG	Manufacture of rubber and plastics products, and other non-metallic mineral products 22 + 23	13,390.8	5.28	4,670.3	1.84	14,838.2	5.85	28,229.0	11.12
10 CH	Manufacture of basic metals and fabricated metal products, except machinery and equipment 24 + 25	6,723.2	2.65	9,121.8	3.59	6,314.2	2.49	13,037.4	5.14
11 CI	Manufacture of computer, electronic and optical products 26	96.2	0.04	406.3	0.16	36.3	0.01	132.5	0.05
12 CJ	Manufacture of electrical equipment 27	354.3	0.14	1,707.0	0.67	209.0	0.08	563.3	0.22
13 CK	Manufacture of machinery and equipment n.e.c. 28	372.0	0.15	2,542.4	1.00	155.2	0.06	527.2	0.21
14 CL	Manufacture of transport equipment 29 + 30	1,438.3	0.57	6,148.7	2.42	126.3	0.05	1,564.6	0.62
15 CM	Other manufacturing, and repair and installation of machinery and equipment 31 to 33	308.5	0.12	1,580.1	0.62	301.2	0.12	609.7	0.24
16 D	Electricity, gas, steam and air-conditioning supply 35	20,425.2	8.05	3,154.4	1.24	42,017.9	16.56	62,443.1	24.60
17 E W	Water supply, sewerage, waste management and remediation 36 to 39	8,037.7	3.17	1,395.3	0.55	6,496.3	2.56	14,534.0	5.73
18 F	Construction 41 to 43	897.6	0.35	10,271.8	4.05	127.4	0.05	1,025.0	0.40
19 G	Wholesale and retail trade, repair of motor vehicles and motorcycles 45 to 47	3,543.5	1.40	15,132.5	5.96	1,617.9	0.64	5,161.4	2.03
20 H	Transportation and storage 49 to 53	16,117.5	6.35	4,318.5	1.70	18,302.2	7.21	34,419.7	13.56
21 I	Accommodation and food service activities 55 + 56	1,513.5	0.60	4,903.6	1.93	158.7	0.06	1,672.2	0.66
22 JA	Publishing, audiovisual and broadcasting activities 58 to 60	153.2	0.06	1,132.5	0.45	114.5	0.05	267.7	0.11
23 JB	Telecommunications 61	61.4	0.02	1,367.3	0.54	52.2	0.02	113.6	0.04
24 JC	IT and other information services 62 +63	110.6	0.04	762.9	0.30	27.3	0.01	137.9	0.05
25 K	Financial and insurance activities 64 to 66	197.8	0.08	1,029.0	0.41	236.2	0.09	434.0	0.17
26 L	Real estate activities 68	43.4	0.02	559.3	0.22	18.2	0.01	61.6	0.02
27 MA	Legal, accounting, management, architecture, engineering, technical testing and analysis activities 69 to 71	119.2	0.05	1,581.2	0.62	168.0	0.07	287.2	0.11
28 MB	Scientific research and development 72	5.9	0.00	146.9	0.06	0.0	0.00	5.9	0.00
29 MC	Other professional, scientific and technical activities 73 to 75	15.7	0.01	636.7	0.25	91.6	0.04	107.3	0.04
30 N	Administrative and support service activities 77 to 82	387.2	0.15	1,917.7	0.76	883.6	0.35	1,270.8	0.50
31 O	Public administration and defence, compulsory social security 84	1,651.8	0.65	3,288.2	1.30	230.2	0.09	1,882.0	0.74
32 P	Education 85	867.9	0.34	1,489.9	0.59	64.4	0.03	932.3	0.37
Q	Human health and social work activities	1,276.4	0.50	2,578.4	1.02	55.9	0.02	1,332.3	0.52
R_S	Other service activities	239.4	0.09	2,061.4	0.81	53.4	0.02	292.8	0.12
	Total	123,918.2	48.83	129,878.0	51.17	129,878.0	51.17	253,796.2	100.00

Source: Prepared by the authors with WIOD (2016) and INE (2016) data.

Appendix III. Mathematical development of the methodology

Starting from the solution to the Leontief system given by the expression:

(A.1) $$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$$

where **x** is the sectoral output vector, $(\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse and **y** the final demand (or net outputs) vector.

If we isolate sector *s* we can rewrite matrix **A** as:

(A.2) $$\mathbf{A} = \begin{pmatrix} a_{ss} & \mathbf{a}_{sr} \\ \mathbf{a}_{rs} & \mathbf{A}_{rr} \end{pmatrix}$$

where a_{ss} is a scalar, $\mathbf{a}_{sr}$ is a row verctor and $\mathbf{a}_{rs}$ is a column vector, que se corresponden con los coeficientes técnicos pertinentes.

We can transform (A.1) inverse in blocks. We can then write:

(A.3) $$\begin{pmatrix} x_s \\ \mathbf{x}_r \end{pmatrix} = \begin{pmatrix} (1-a_{ss}) & -\mathbf{a}_{sr} \\ -\mathbf{a}_{rs} & (\mathbf{I}-\mathbf{A}_{rr}) \end{pmatrix}^{-1} \begin{pmatrix} y_s \\ \mathbf{y}_r \end{pmatrix}$$

If we assume that $(\mathbf{I} - \mathbf{A}_{rr})$ can be inverted, the Schur complement of $(\mathbf{I} - \mathbf{A}_{rr})$ would be:

(A.4) $$\Theta_s = (1-a_{ss}) - \mathbf{a}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{a}_{rs}$$

and the inverse by parts in (A.3) would be given by the next expression:

(A.5) $$(\mathbf{I}-\mathbf{A})^{-1} = \begin{pmatrix} \Theta_s^{-1} & \Theta_s^{-1}\mathbf{a}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} \\ (\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{a}_{rs}\Theta_s^{-1} & (\mathbf{I}-\mathbf{A}_{rr})^{-1} + (\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{a}_{rs}\Theta_s^{-1}\mathbf{a}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} \end{pmatrix}$$

The solution to the system (A.3) would then be:

(A.6) $x_s = \Theta_s^{-1} y_s + \Theta_s^{-1} \mathbf{a}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r$

(A.7) $\mathbf{x}_r = (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{a}_{rs} \Theta_s^{-1} y_s + (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r + (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{a}_{rs} \Theta_s^{-1} \mathbf{a}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r$

Reordering terms in (A.7), we obtain:

(A.8) $\mathbf{x}_r = (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{a}_{rs} \underbrace{\left[\Theta_r^{-1} \mathbf{y}_s + \Theta_r^{-1} \mathbf{a}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r \right]}_{= \mathbf{x}_S} + (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r$

and thus:

(A.9) $\mathbf{x}_r = (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{a}_{rs} x_s + (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{y}_r$

The first vector on the right side expresses the output made by the rest of the sectors of the economy for sector *s*. The second vector expresses the output of the rest of the sectors for their own final demand. Then, we can express:

(A.10) $PBL_s = \mathbf{u}'(\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{a}_{rs} x_s$

as the PBL of sector *s*.

Let us now consider the output of sector *s* for the rest of the sectors.

The Schur complement of $(1 - a_{ss})$ is:

(A.11) $\Theta_r = (\mathbf{I} - \mathbf{A}_{rr}) - \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{A}_{sr}$

The Leontief inverse is then given by the expression:

(A.12) $$(\mathbf{I}-\mathbf{A})^{-1} = \begin{pmatrix} (1-a_{ss})^{-1} + (I-a_{ss})^{-1}\mathbf{a}_{sr}\Theta_r^{-1}\mathbf{a}_{rs}(1-a_{ss})^{-1} & (1-a_{ss})^{-1}\mathbf{a}_{sr}\Theta_r^{-1} \\ \Theta_r^{-1}\mathbf{a}_{rs}(1-a_{ss})^{-1} & \Theta_r^{-1} \end{pmatrix}$$

and the solution to (A.3) would now be:

(A.13) $$x_s = (1-a_{ss})^{-1} y_s + (I-a_{ss})^{-1}\mathbf{a}_{sr}\Theta_r^{-1}\mathbf{a}_{rs}(1-a_{ss})^{-1} y_s + (1-a_{ss})^{-1}\mathbf{a}_{sr}\Theta_r^{-1}\mathbf{y}_r$$

(A.14) $$\mathbf{x}_r = \Theta_r^{-1}\mathbf{a}_{rs}(1-a_{ss})^{-1} y_s + \Theta_r^{-1}\mathbf{y}_r$$

Proceeding in the same way as previously on expression (A.13) we have:

(A.15) $$x_s = (1-a_{ss})^{-1}\mathbf{a}_{sr}\mathbf{x}_r + (1-a_{ss})^{-1} y_s$$

in which the first term on the right side shows the output of sector *s* that this sector has to produce to make possible the production of the rest of the sectors. The second term shows the production that it makes to obtain its own final demand. Then,

(A.16) $$PFL_s = (1-a_{ss})^{-1}\mathbf{a}_{sr}\mathbf{x}_r$$

is the PFL of sector *s*, as it has been defined in the text.

This formulation allows us to show the intersectoral output–output linkages, on the basis of the productive structure of the economy analyzed given by the technical coefficients matrix. The results obtained can easily be generalized to the *n* sectors of the economic system, as shown in our methodological proposal.