Agency problems in public-private partnerships investment projects

This version: June 2020

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Abstract

This paper examines concession contracts between a private firm and a government in the presence of moral hazard within a real-options framework. The design of optimal contracts to provide incentives to the private firm to exert effort is analyzed. We show that although first-best investment timing can be implemented, contracts often do not provide firms with proper incentives to exert effort, resulting in high-cost projects being undertaken. This problem can be alleviated through the use of a monitoring technology that imposes a penalty on the shirking firm. Although monitoring distorts the investment timing leading to a delayed investment, it increases the government's profits at the expense of the firm, so that the government finds it optimal to induce effort exertion, increasing the likelihood of low-cost projects. Considering jointly incentives and an exit option, we show that the regular compensation of firms and their compensation upon termination act as substitutes in providing incentives. Governments should set these remunerations jointly in order to minimize the cost of a bailout option for the society.

Keywords: Finance, Investment analysis, Agency problem, Public and private partnership, Real options
JEL: G11, G13, G30, G31, G38, D82

Declarations of interest: none

[☆] We thank Domenico De Giovanni, Paulo Pereira, Jan Schneemeier, as well as conference participants at the IFABS Conference 2018, Porto, Portugal, the French Finance Association Conference 2018, Paris, France, the Finance Forum 2018, Santander, Spain and the Real Options Conference 2018, Dusseldorf, Germany and seminar participants at University of Calabria, Italy and NEOMA Business School, France for valuable comments and suggestions. Florina Silaghi, Serra Húnter Fellow, gratefully acknowledges financial support through the Juan de la Cierva Incorporation Grant IJCI-2017-31641 cofinanced by the Spanish National Research Agency and through Project ECO2017-86054-C3-1-R from the Spanish Ministry of Economics and Competitiveness.

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1. Introduction

1.1. General introduction

Investments in public-private-partnerships (PPP) have grown over the last decades, being widely used in both developed and emerging economies. Nevertheless, there has been considerable disatisfaction with such projects since incorrect contract terms can be very expensive to the government and taxpayers, e.g., the governments of Mexico and Spain recently had to pay 8.9 billion and 2.5 billion respectively to their private partners because of inappropriate contract terms (**?**). Moreover, projects with negative social value that suffer from severe demand overestimation and cost underestimation, commonly denoted as white elephants, are widespread around the world (**?**). To minimize the costs of such projects for society, the design of PPP contracts needs to be improved.

This paper analyzes the optimal contract design of a PPP project within a real-options framework, taking into account incentive mechanisms to ensure effort exertion and optimal investment timing. We show that contracts often do not provide incentives for firms to exert cost-reducing or quality-enhancing effort, thus contractors exert too little effort and as a consequence low-quality/high-cost infrastructures such as white elephants end up being built. Furthermore, we address the question of how concession contracts can be adjusted to provide proper incentives to contractors and increase social welfare.

Our results can be useful in guiding policy makers in designing PPP agreements that maximize social welfare. First, since PPPs generally supply a vital good such as transport, electricity and water, the government usually bails out the private party to avoid disruptions to supply. In this case, the government should explicitly take into account the bailout option in PPP design. In particular, we stress the importance of analyzing the interaction between two critical contract parameters in order to minimize social losses: the private firm's fraction of revenues (the firm's regular remuneration) and the bailout price (the firm's compensation upon termination). These two parameters should be simultaneously optimized, which implies that the firm's fraction of revenues should be inversely related to the buyout price: the larger the fraction of revenues, the lower the bailout price. Our findings are in line with recent evidence by **?** who highlight the importance of jointly optimizing the concession period and the level of the minimum revenue guarantee. Similar to us, they find these two parameters to be inversely related: the longer the concession period, the lower the level of the minimum revenue guarantee. Second, regarding governmental use of a monitoring technology, we show that it is not necessary for the government to monitor all projects. The government's choice to monitor a project should depend on the cost of effort of the firm. For relatively simple projects in which the firm can exert cost-reducing or quality-improving effort at a low cost, the government should not monitor. Monitoring should only be used for complex projects, involving high effort cost from the firm.

1.2. Summary of the model and results

Building on **?**, we propose a principal-agent model between the government (the principal) and the private firm (the agent) in a real-options framework. To our knowledge, this is the first model to incorporate a moral hazard problem into a real-options PPP framework, bridging the rich contract theory literature on PPPs with the real-options literature on PPPs.[1] The government has an option

[1]Most papers in the real-options literature assume perfect, symmetric information. A notable exception is the paper by **?**. They compare two forms of government support, loan guarantee and direct investment through PPPs

to invest in a project and it delegates the exercise decision to a private firm. The government's problem is to design an optimal contract under moral hazard (hidden action by the private firm). The true cost of the underlying project can be low or high. The hidden action problem is that the firm can influence the likelihood that the cost is low by exerting an unobservable effort. An optimal contract will induce the firm to exert costly (but unverifiable) effort. The assumption of unobservable or non-verifiable effort is very reasonable in the case of many PPP contracts. Although several aspects of service quality and cost in different types of concessions are observable and verifiable by third parties such as water quality, train punctuality, highway congestion rates, they are only indicative of the effort incurred by the firm in building the infrastructure. The intrinsic quality of the infrastructure remains by and large not observable (e.g. quantity of leakage, quality of the tubes, etc. in a water network as argued by **?**). In order to induce the private firm to exert effort and invest at the first-best trigger, the government designs a contract to provide the firm with the appropriate incentives. A contract consists of a pair of investment triggers and a remuneration for the private firm. We model the remuneration of the private firm as a fraction of net revenues that the firm receives starting from the time of the investment, consistent with the wide use of revenue-sharing contracts between the two parties in PPP projects such as transport projects (e.g., toll highways) and energy projects. Revenue-sharing contracts have been previously used in the literature in **?**, **?**, **?**, **?**, among others.

We derive the optimal concession contracts, and show that although the government can induce the firm to exercise at the first-best investment trigger, for a sufficiently high cost of effort, giving incentives to exert effort as in first-best is too costly, since the firm captures some informational rents. Consequently, the government finds it optimal not to induce cost-reducing effort, leading to high-cost infrastructures being built. We thus provide an incentives-based rationale for why PPP projects may exhibit higher costs and may be more likely to incur cost overruns. Our model's implications are supported by recent empirical evidence that challenges the conventional wisdom that the private sector can reduce the large cost escalations typically associated with public sector projects (**?**). **?** and **?** provide a comparative analysis of the incidence of project overruns in PPP and non-PPP road projects in India, and show that the use of PPP increased cost overruns.[2] For the health sector, there is evidence that the private finance initiative (a form of PPP) substantially increases the cost of hospital building (**?**; **?**). **?** reported that the £2.7 billion Scottish private finance initiative programme would cost, at a conservative estimate, £2 billion more than if the Treasury had acquired the assets directly. Moreover, we find that the problem of high-cost projects being built is particularly severe for projects characterized by huge investment costs and large demand risk. This is consistent with empirical evidence on cost overruns and project renegotiation in PPPs from across the world. According to **?**, **?** and **?** large-sized projects and works with high costs or high implementation risk are more likely to result in cost overruns.[3] Additionally,

in a hidden information framework: the government knows less about project quality than do private partners (the so-called plum problem).

[2]Our incentives-based explanation is in line with one of the possible reasons for higher cost overruns in PPP projects put forward by **?**, i.e., that the PPP contract either provided an incentive for incurring cost overruns or did not discourage it adequately in terms of penalties. Other possible reasons are uncertainty and imperfect estimation techniques, optimism bias or purposeful underestimation by politicians in the context of competition for scarce funding. **?** and **?** rule out some of these reasons since underestimation is not as likely as overestimation (as imperfect estimation techniques would indicate), cost overruns did not show a declining trend as more projects were implemented and project managers learned from past mistakes, and the government agency was not competing for funds with other ministries.

[3]**?** analyze 601 PPP contracts from countries in Africa, Asia, Europe, North and South America and the Pacific.

empirical evidence on project renegotiation in Latin America (?) shows that sectors characterized by huge sunk investments and demand risk such as transportation and water and sanitation exhibit significantly higher renegotiation rates (54.7% and 74.4% renegotiation rates respectively versus a 30% rate overall for all sectors).

The problems posed by the opportunistic behavior of contractors could be alleviated by monitoring (?). By designing a proper monitoring technology that detects the firm's effort exertion and imposes some penalty when the firm shirks, the government could increase its value and induce the private firm to exert effort. Most governments set out institutional mechanisms and procedures that guarantee close monitoring of the PPP firm's performance and general compliance with the agreed contract. ? describe the project monitoring process in Spain, where a team is appointed by the public authority to inspect, verify and monitor the works as many times as it deems necessary and request the information it deems appropriate for the correct control of the works. However, monitoring is costly. In the case of the U.S., ? report that the Office of Management and Budget recommends that monitoring costs of 10% of the contract value be budgeted in this type of contracts. We thus incorporate a costly monitoring technology into our benchmark model. We show that monitoring increases the incentives of the firm to exert effort by decreasing its payoff in case of shirking. Nevertheless, since monitoring is costly, delay in the low-cost project investment timing is needed to provide enough incentives to the firm to exert effort, while minimizing monitoring costs. This allows the government to increase its profits by reducing the informational rents paid to the firm for relatively low monitoring costs. For a sufficiently high cost of effort for which inducing effort exertion was too expensive in the second-best case, the government using a monitoring technology now finds it optimal to induce effort. Therefore, monitoring partially alleviates the distortion in the government's decision, resulting in fewer high-cost projects being undertaken. Governments should thus invest in designing efficient monitoring technologies, especially in the case of complex projects involving a high effort cost, to increase welfare.

Finally, we extend the basic model to incorporate a government guarantee prevalent in many PPP projects, an exit option (?; ?). We assume that the firm has the possibility of transferring the ownership of the project to the government in exchange for a bailout price when demand falls sufficiently. By jointly considering incentives and embedded real options, we observe that the firm's regular compensation (a fraction of net revenues) and its compensation upon termination (the bailout price) act as substitutes in providing it with incentives to exert effort. An important policy recommendation resulting from our analysis is that when setting the regular payment to the private partner, the government should take into account the possibility of a future bailout. The two payments should be set jointly in order to minimize the cost of a bailout for taxpayers, the regular payment being lower the higher the bailout price. Our recommendation complements previous recommendations regarding the design of the compensation upon termination. ? and institutions such as EPEC (European PPP Expertise Center), PPIAF (The Public-Private Infrastructure Advisory Facility) recommend compensating the bankrupt concessionaire according to the value of the project uncovered by auctioning the failed concession (market value method), not the cost incurred (book value method) in order to increase social value and minimize payments to the private partner. However, this might not be feasible in emerging market economies where the PPP market might not be sufficiently liquid for the project to be retendered to potentially interested purchasers. In this case, taking into account the probability of bailout when setting the regular

? study 258 projects located in 20 nations on five continents. Finally, ? provides evidence from India.

compensation of the private partner could mitigate social losses, as this is an alternative way of linking the compensation upon termination to performance, as in the market value method.

The rest of the paper is organized as follows. Section 2 presents the literature review and highlights our contributions. Section 3 describes our benchmark model including its setup, the first-best solution and the principal-agent setting. In Section 4 we simplify the optimization program of the government and we derive the optimal contracts, i.e., the second-best solution. Section 5 extends the benchmark model to include a monitoring technology. The exit or bailout option for the private firm is introduced in Section 6. Section 7 provides a numerical sensitivity analysis and implications. Finally, Section 8 concludes.

2. Literature review and contributions

The existing PPP literature has two strands – focusing on "real options" and "contract design". The former values real options embedded in the contract (revenue guarantee, revenue sharing, buyout clause, etc.), and determines the optimal investment policy in a real-option setting.[4] However, this literature does not address contract design and performance incentives.[5] The "contract design" literature examines the optimality of the PPP organizational form versus the traditional contract (building, operating and maintaining are bundled in the former and unbundled in the latter).[6] This literature focuses on asymmetric information, principal-agent problems, and the theory of incomplete contracts (?), but ignores the effect of the embedded real options which have a significant impact on project valuation and risk allocation (?; ?; ?). This paper aims to combine the two strands of the literature mentioned above.

The setting of our paper is most similar to that of ? and ?. While these papers focus on agency problems between managers and shareholders within a decentralized firm, we adapt the model of ? to a PPP framework where the agency problem is between the private firm and the government.[7] Instead of considering a one-time payment at the moment of investment, we propose a linear revenue-sharing contract in line with the use of these contracts in PPP (?). Moreover, to

[4]? and ? examine revenue guarantees, ? minimum traffic guarantees, ? revenue and income guarantees, and ? and ? fee deferral and buyout options. ? value the deferral, abandonment and expansion options, ? value revenue guarantees, revenue sharing and buyout options, ? value investment subsidies, revenue guarantees and termination options, ? study investment and revenue subsidies, together with a minimum demand guarantee and a rescue option, ? analyze investment subsidies under debt financing, while ? and ? show how to use option combinations (collars) to manage revenue risk.

[5]? models a high-speed rail PPP project identifying the optimal investment trigger. Although he does not consider a proper mechanism design with the corresponding optimization problem for the government, his study is the only real-option study to address the issue of giving incentives to the operator to achieve an exogenous quality level.

[6]? argues that the choice between PPP and traditional contract turns on whether it is easier to write contracts on service provision or on construction quality. For the former, PPP is optimal; for the latter, the traditional (unbundled) contract is better. ? show that bundling (unbundling) is optimal with positive (negative) externalities, and ? show that bundling of tasks is generally the optimal organizational form. They also show that, if the productivity shock is common knowledge and verifiable, the optimal contract would (i) ensure that the private party is not exposed to negative cash flows, and (ii) require a revenue-sharing arrangement independent of the productivity shock. ? show that bundling (or PPP) is optimal, while ? show that PPP is suitable for traditional infrastructure projects such as transportation and water, but less so for others (e.g., schools). In a framework where the firm can exert innovation effort and gather private information about future costs, ? show that the choice between PPP and traditional procurement depends on the information-gathering costs, the costs of innovation efforts, and the degree to which effort is contractible.

[7]For a review of the use of real options in Operations Research, including principal-agent settings, see ?.

make the model specific to a PPP framework, we include spillover benefits and consumer surplus, as well as a bailout option. While **?** consider both hidden information and hidden action, all the papers building on this model (**?**; **?**; **?**, **?**) focus only on hidden information (the manager privately knows the value of the project and can lie about it in order to gain some private benefits). On the contrary, we focus on a hidden action problem, as the contract theory literature on PPPs argues that a hidden action problem is more likely to affect PPPs (**?**).[8] **?** extends the framework of **?** by considering not only a bonus incentive (carrot), but also an auditing mechanism (stick). Auditing is appropriate for hidden information problems. However, in hidden action problems as ours, monitoring is adequate. To our knowledge, monitoring has never been modeled before in the literature within a real options framework accounting for agency problems.

While our benchmark results for the second-best case are not new, but consistent with previous findings in the literature (**?**), all the results that we derive under monitoring are novel. Appearing to be counterintuitive at first glance, they are quite different from findings in the previous literature. In particular, the most notable result is that monitoring only occurs with a positive probability for the low-cost project, not for the high-cost project. This is strongly contrasted with the well-known result that costly state verification should only be used in bad states of the world (**?**), or with the result of **?** that only the high-cost project is audited. Since the latter models a hidden information problem, there is no incentive for the manager of a high-cost project to pretend to have a low-cost project. On the contrary, the manager of a low-cost project has incentives to report a high-cost project. Therefore, only high-cost projects are audited, and when a lie is detected a punishment is applied to the manager of a low-cost project that has falsely reported a high-cost one. In contrast, in our model, the type of project is observable. What the government cannot observe is effort exertion, so a low-cost project could be the result of effort exertion or just of luck. Moreover, due to limited liability the maximum punishment that can be applied to a high-cost project is zero, which implies that it is inefficient to monitor such projects. Thus, only low-cost projects are monitored. Another novel result is that under monitoring the investment threshold of the low-cost project is distorted away from first-best. This is in contrast to the well-known result that hidden action does not distort investment timing (**?**), and also opposite to the hidden information model under auditing, where it is less costly for the owner to distort the investment threshold of the high-cost project (**?**). Furthermore, while in **?** auditing mitigated the distortion in investment timing (earlier investment compared to the case without auditing, closer to first-best), in our model monitoring leads to later investment compared to both the case without monitoring and to first-best. Thus, while auditing always reduces inefficiency in the investment trigger, monitoring creates such an inefficiency. This also implies that while auditing could lead to a larger total value of the project provided that the reduction in investment timing inefficiency compensates the auditing cost (**?**), monitoring always decreases the total value of the project (assuming effort exertion both with and without monitoring).

By including spillover benefits and consumer surplus, we can also derive novel results on how they are affected by monitoring. We show that by delaying investment timing for the low-cost project, monitoring reduces spillover benefits and consumer surplus. Nevertheless, monitoring allows the government to increase its value by reducing the information rents of the private firm,

[8]The solution in **?** includes three different regions depending on the cost-benefit ratio of effort: the hidden information only region, the joint hidden information/hidden action region and the hidden action only region. Assuming a relatively large cost-benefit ratio of effort would be sufficient for the solution of a model with both hidden information and hidden action to collapse to the solution of a model with hidden action only.

which is consistent with previous findings that auditing always leads to an increase in the owner's value and a decrease in the manager's value (**?**). Since the increase in the government's value more than compensates the decreases in spillover benefits and consumer surplus, monitoring leads to an increase in social welfare. Finally, while **?** and **?** only analyze an investment option, we extend this framework by including a bailout option, where we do not only consider the regular remuneration, but also a remuneration upon termination. This allows us to derive novel results regarding the substitution effect between these two types of remuneration.

Adapting and extending the framework of **?** to PPPs not only allows us to 1) exploit monitoring and the bailout option and derive several novel results in terms of modeling, but also to 2) bridge the contract theory literature on PPPs with the real options literature on PPPs, by treating incentives and real options within a unified framework and not separately, and 3) to obtain important empirical and policy implications. Our theoretical model can explain documented empirical facts such as cost overruns in PPPs and are in line with empirical data on PPP renegotiation, as previously mentioned. Moreover, we derive several implications for policy makers regarding monitoring and the design of a bailout option, with the aim of minimizing costs for society.

3. Model

We now start by describing the model setup. We will then derive the first-best benchmark solutions assuming full symmetric information. Finally, we will discuss the principal-agent maximization problem under asymmetric information.

3.1. Setup

The principal (government) owns an option to invest in a single project. We assume that the principal delegates the exercise decision to an agent (private firm). Once investment takes place, the firm produces q units of the output, which are sold at a price of \$$p$ per unit, both observable and contractible to both the government and the private firm. The price is given by the inverse demand function:

$$p = xq^{\eta} \tag{1}$$

where the state variable x represents a stochastic demand parameter and η is a measure of price sensitivity ($-1 \leq \eta \leq 0$).[9] The price elasticity of this demand function is given by $1/|\eta|$. The state variable x introduces uncertainty in the model and can be interpreted as the relative strength of the demand, since changes in this variable will be reflected in parallel shifts to the demand curve. Conditions affecting this variable include the target population size, disposable income, changing tastes, prices of substitutes, the level of industrial production, etc. (see, for example, **?** and **?**). When x is higher, demand is stronger, hence the market will pay a higher price per unit for the same output level, or will buy a larger amount at the same price. That is, when x is higher, price or quantity or both will be higher. Following **?**, **?**, **?**, **?** and **?**, we assume x follows a geometric Brownian motion:

$$dx(t) = \mu x(t)dt + \sigma x(t)dz(t), \tag{2}$$

[9]This iso-elastic demand function was used in **?**, **?**, **?**, **?** and **?**. A parameter value $\eta = 0$ implies perfect competition or zero market power, and the larger $|\eta|$ the larger the market power of the firm.

where μ and σ represent the drift and standard deviation per unit time respectively, and dz is the increment of a standard Wiener process.[10] Let x_0 denote the value of the state variable at time zero. There is a fixed cost of c per unit time, but no variable costs.[11] The production capacity is Q units per unit time. Since there are no variable costs, the firm will always operate at full capacity (**?**), i.e., $q = Q$.[12]

The cost expenditure to undertake the investment is given by $I - \theta$. Here I can be interpreted as the "gross" investment cost and θ as the amount by which the gross investment cost can be reduced depending on the private firm's unobservable effort. The variable θ can take on two possible values: θ_1 or θ_2, with $\theta_1 > \theta_2$, and $\Delta\theta = \theta_1 - \theta_2 > 0$. We can interpret a draw of θ_1 as a "lower cost" project, and a draw of θ_2 as a "higher cost" project. The effort that the private firm exerts influences the likelihood of obtaining a lower cost project. In particular, if the private firm exerts a one-time effort at time zero, the probability of drawing a lower cost project θ_1 is q_H. If no effort is exerted this probability is only $q_L < q_H$.[13] Effort is costly, when exerting effort the private firm incurs a cost $\xi > 0$.

Both the government and the private firm are risk-neutral, with the risk-free rate denoted by r. For convergence, we assume that $r > \mu$. Since the government can run a competitive auction to attract potential service providers, we assume following **?** and **?** that it has all bargaining power ex ante. The government is thus in the natural position of a Stackelberg leader, and the private firm acts as a follower.[14] We also assume that the firm is protected by limited liability and other public guarantees. Following **?**, we rule out the upfront sale of the project by the government to the private firm, which would achieve first-best. This can be justified by legal constraints or by liquidity constraints for the private firm.

Since effort is unobservable, the government cannot contract on the effort exerted by the private firm. However, it can contract on the observable cash flow of the project, $pq = xQ^{\eta+1}$. The firm's compensation will be contingent on the level of $x(t)$ at exercise. In **?**, **?**, **?**, and **??** the agent that is a manager of a firm receives a one-time payment at the time of exercise. However, in PPP contracts, revenue-sharing agreements between the two parties are widely employed according to **?** (p. 447). We therefore assume that the net revenue from the PPP project is shared, with the private party getting a fraction b and the public party getting the rest (with $0 \leq b \leq 1$), as in **?**, **?**, and **?**.[15] This is in general the case for infrastructure projects such as highways, for example. Moreover, as it is common in PPP projects for the government to offer subsidies to help covering

[10]This is a very standard and well-established way of modeling investment decisions with capacity choice in real-option models. Some examples with a stochastic demand parameter following a geometric Brownian motion and linear demand function include **?**, **?**, **?**, **?**, **?**, and **?**.

[11]Similarly, **?**, **?** and **?** assumed zero operating costs.

[12]Assuming positive variable costs would imply that the firm does not always operate at full capacity, i.e., $q < Q$ under certain conditions. We present the derivation of the results under this assumption in Appendix E. As we show in the appendix, the results are very similar to our benchmark case with zero variable costs.

[13]Exerting effort thus reduces the cost of the project (for the same given quality). The problem could be equivalently formulated as one in which the total revenue from the project is $pq + \theta$ and the cost of exercising the option is I (see **?**). In this case, exerting effort improves the quality of the project, that is, it increases the revenue for the same given cost. We nevertheless prefer the former formulation in line with the contract theory literature on PPPs that also models cost-reducing noncontractible effort (**?** and **?**).

[14]**?** examine possible negotiating conditions of a concession contract between a public and a private party ranging from a Nash bargaining game to a noncooperative Stackelberg equilibrium.

[15]While in **?** the revenue-sharing coefficient is independent of the state variable, in our case, this coefficient is contingent on the state variable's value at exercise.

the initial investment (?), we assume that the government offers a subsidy of $1-\alpha$ of the total investment cost, with $0 \leq \alpha \leq 1$ denoting the subsidy or cost sharing ratio. The firm's initial investment is thus $\alpha(I-\theta)$.[16]

The benefit generated by the infrastructure includes not only the profits, but also a benefit for the society as a whole (?, ?). This social benefit includes on the one hand spillover benefits (SP) generated by the project for the local economy in terms of employment or boosting economic activity, and on the other hand consumer surplus (CS). The government thus maximizes a social welfare function, defined as the discounted sum of the project's profits and the social benefit of the project net of the incurred investment cost and of the payment made to the firm.[17] The profits from the project are given by $pq - c = xQ^{\eta+1} - c$ per unit time. Spillover benefits are included in the objective function as γxq per unit time, following **?** who model spillover benefits in public ports. While they model spillover benefits as a percentage of the costs to process cargo capacity, we adapt the modeling of spillover benefits to our framework with stochastic demand parameter x and no variable costs, such that the spillover benefit parameter γ is expressed as percentage of economic activity in line with **?**. For example, in the case of a port, γ would be interpreted as the employment generated by the port actors as a percentage of overall economic activity directly recorded in the port, estimated by **?** to around 41%. The instantaneous consumer surplus stream is given by $\int_0^{q^*}[p-p^*]dq$, where p^* and q^* denote the actual price and quantity per unit time (?). Since $q^* = Q$ and $p = xq^\eta$, the instantaneous consumer surplus is given by $\int_0^Q [xq^\eta - xQ^\eta]dq = -\frac{xQ^{\eta+1}}{1+1/\eta}$ per unit time. To account for the fact that some governments might only consider in their objective function the consumer surplus relevant for the region they govern, let $\omega \in [0,1]$ denote the share of total consumer surplus considered by the government. Before analyzing the optimal contract, we first present the first-best benchmark, the no-agency solution.

3.2. First-best benchmark

Suppose that there is no delegation of the exercise decision.[18] Let $W(x_0, \theta_i)$ denote the discounted sum of the project's profits and social benefit net of the investment cost for $\theta = \theta_i$ for each $i \in \{1,2\}$, and x_0 denote the value at time zero of $x(t)$. Using standard arguments in real-options theory (?), we obtain the following value at time zero and the coresponding investment trigger:

$$W(x_0,\theta_i) = \left(\frac{x_0}{x_i^*}\right)^{\beta_1}\left[\left(\frac{x_i^* Q^{\eta+1}}{r-\mu} - \frac{c}{r}\right) - (I-\theta_i) + \frac{\gamma x_i^* Q}{r-\mu} + \omega\left(-\frac{x_i^* Q^{\eta+1}}{(1+1/\eta)(r-\mu)}\right)\right], \tag{3}$$

[16]Note that although the private effort is unobservable, the type of the project, θ, is observable. At the time of investment, the government can observe if the project is of type θ_1 or of type θ_2. Therefore, the government can subsidize a fraction of the net investment cost $I-\theta$. The fact that effort is unobservable implies that the government does not know if a realization of θ_2 (a high-cost project) is the result of shirking (no effort exertion) from the part of the agent, or simply of bad luck. Indeed, even if the agent exerts effort, there is still a positive probability that the project is of high cost, although this probability is smaller than when the agent shirks.

[17]As in **?** and **?**, the agent's profit has no weight in the social welfare function. Having a redistributive objective function for the government with a weight less than one for the firm's profit would lead to the same insights at the cost of an increased complexity in the modeling.

[18]This is equivalent to the case in which there is delegation but effort is observable and contractible. Since the government has all bargaining power ex ante, it sets the remuneration for the private firm such that the firm is just indifferent between providing the service or getting its outside option normalized at zero.

and

$$x_i^* = \frac{\beta_1}{\beta_1 - 1} \frac{(\frac{c}{r} + I - \theta_i)(r - \mu)}{\gamma Q + Q^{\eta+1}(1 - \frac{\omega}{1+1/\eta})}, \tag{4}$$

where β_1 is the positive root of the fundamental quadratic equation (see ?) $\frac{\beta(\beta-1)\sigma^2}{2} + \beta\mu = r$ and is given by

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{5}$$

and where the superscript "*" refers to the first-best (symmetric information) optimum.

The first term in between squared brackets of equation (3) represents the discounted value of the project's profits, the second term represents the investment cost of the project, while the last two terms correspond to the social benefit, that is, the discounted value of spillover benefits and consumer surplus streams respectively.

Throughout the paper we assume that the current value of the state variable x_0 is sufficiently low so that investment is not undertaken immediately, that is, it is lower than the lower investment trigger, x_1^*, to ensure some positive option value inherent in the project.

It is straightforward to show that $\partial x_i^*/\partial\gamma < 0$ and $\partial x_i^*/\partial\omega < 0$. That is, the investment trigger decreases with the spillover benefits and the considered share of consumer surplus. Thus, the larger the spillovers and the more consumer surplus is taken into account, the earlier the investment is realized.

Depending on whether effort is exerted or not, we get the following ex-ante value of the gross social welfare (before effort cost):

$$SW_i(x_0) = q_i W(x_0, \theta_1) + (1 - q_i) W(x_0, \theta_2), \tag{6}$$

where $i \in \{H, L\}$.

Given the components of $W(x_0, \theta_i)$, the social welfare value can be decomposed into the value of the project net of the investment cost, Π, the spillover benefits, SP, and the consumer surplus, CS:

$$SW_i(x_0) = \Pi_i(x_0) + SP_i(x_0) + CS_i(x_0), \tag{7}$$

with the expressions for $\Pi_i(x_0)$, $SP_i(x_0)$ and $CS_i(x_0)$ given in Appendix A.

Therefore, the ex-ante value of the net social welfare conditional on exerting effort is given by:[19]

$$SW^*(x_0) = SW_H(x_0) - \xi = (\Pi_H(x_0) - \xi) + SP_H(x_0) + CS_H(x_0) \tag{8}$$

Note that it is optimal for the government to exert effort if $SW^* \geq SW_L(x_0)$. That is, the government exerts effort if $\xi \leq SW_H(x_0) - SW_L(x_0)$. Substituting the expressions for $W(x_0, \theta_1)$

[19] If there is no delegation, the government exerts effort itself and incurrs the effort cost, ξ. In the equivalent case of delegation with observable effort, since effort is observable the government could make a payment to the firm such that the firm's option value is exactly equal to ξ, just sufficient to cover the effort cost.

and $W(x_0,\theta_2)$ and simplifying, we get that effort exertion is optimal if:

$$\begin{aligned}\frac{\xi}{\Delta q} \leq & \left(\frac{x_0}{x_1^*}\right)^{\beta_1}\left(\frac{x_1^* Q^{\eta+1}}{r-\mu}-\frac{c}{r}-(I-\theta_1)+\frac{\gamma x_1^* Q}{r-\mu}-\frac{\omega x_1^* Q^{\eta+1}}{(1+1/\eta)(r-\mu)}\right) \\ & -\left(\frac{x_0}{x_2^*}\right)^{\beta_1}\left(\frac{x_2^* Q^{\eta+1}}{r-\mu}-\frac{c}{r}-(I-\theta_2)+\frac{\gamma x_2^* Q}{r-\mu}-\frac{\omega x_2^* Q^{\eta+1}}{(1+1/\eta)(r-\mu)}\right) \qquad (9)\end{aligned}$$

Denoting by A the right hand side of the inequality, we have that effort exertion is optimal if $\frac{\xi}{\Delta q} \leq A$.[20] Thus, it is worth exerting effort only when the cost-benefit ratio $\frac{\xi}{\Delta q}$ is sufficiently low. To focus on the interesting case we assume that indeed $\frac{\xi}{\Delta q} \leq A$. We will see in the next section that when we have a moral hazard problem, this decision is distorted. We will find situations where even though it would be optimal for the government to exert effort in the first-best case, the government will optimally choose not to give incentives to the private firm to exert effort because it is too costly to do so.

3.3. A principal-agent setting

In order to give the private firm incentives to exert effort, the government offers a contract at time zero that commits the government to pay the firm a fraction b of the net cash flows of the project continuously starting with the time of exercise (investment).[21] As in **?**, this type of contract is contingent on the value of the cash flows at the time of exercise, i.e., b is a function of the realized value of $x(t)$ at the time of exercise. However, it differs from the contracts in **?**, **?**, **?**, and **??** since it is not a one-time payment at the time of exercise, but a linear revenue-sharing contract. This is in line with the contract theory literature on PPPs. Moreover, given the subsidy provided by the government, the firm pays a fraction α of the investment cost $I-\theta$.

Given a certain contract, the private firm chooses the exercise time in order to maximize the value of its investment option. The government will thus select the contract parameters such that it induces the private firm to exert effort and choose an exercise policy that maximizes social welfare.

Since θ can only take two values, the private firm will choose at most two exercise triggers. Therefore, the contract offered by the government will be (b_1,x_1), the firm receives a fraction b_1 of the cash flows if it exercises at x_1, and (b_2,x_2), receiving a fraction b_2 when exercising at x_2.

The government's payoff is: $(1-b_1)(xQ^{\eta+1}-c)$ if $\theta=\theta_1$, and $(1-b_2)(xQ^{\eta+1}-c)$ if $\theta=\theta_2$. Then, conditional on the private firm exerting effort, the government's option value, $\Pi_H^G(x_0,b_1,b_2,x_1,x_2)$, is given by:

$$\begin{aligned}\Pi_H^G(x_0,b_1,b_2,x_1,x_2) &= q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}\left(G(x_1,b_1)-(1-\alpha)(I-\theta_1)\right) \\ &+ (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}\left(G(x_2,b_2)-(1-\alpha)(I-\theta_2)\right), \qquad (10)\end{aligned}$$

where

$$G(x_i,b_i)=(1-b_i)\left(\frac{x_iQ^{\eta+1}}{r-\mu}-\frac{c}{r}\right) \qquad (11)$$

[20] Note that $A=\frac{SW_H(x_0)-SW_L(x_0)}{\Delta q}$.

[21] Following **?** we assume that renegotiation is not allowed. Indeed, although commitment might be ex-post inefficient, it leads to a higher ex-ante value.

Similarly, the private firm's payoff is: $b_1(xQ^{\eta+1}-c)$, if $\theta=\theta_1$ and $b_2(xQ^{\eta+1}-c)$ if $\theta=\theta_2$. Then, conditional on the private firm exerting effort, the private firm's option value, $\Pi_H^F(x_0,b_1,b_2,x_1,x_2)$, is given by:

$$\begin{aligned}\Pi_H^F(x_0,b_1,b_2,x_1,x_2) &= q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}\left(F(x_1,b_1)-\alpha(I-\theta_1)\right)\\ &+ (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}\left(F(x_2,b_2)-\alpha(I-\theta_2)\right), \qquad (12)\end{aligned}$$

where

$$F(x_i,b_i)=b_i\left(\frac{x_iQ^{\eta+1}}{r-\mu}-\frac{c}{r}\right) \qquad (13)$$

The sum of the two investment option values simply equals the total value of the project (the expected discounted sum of the project's profits) net of the investment cost, $\Pi_H=\Pi_H^G+\Pi_H^F$.

The government wants to set the contract parameters b_1,b_2,x_1,x_2 in order to maximize its objective function, i.e., the sum of the project's value and social benefit net of the investment cost and of the firm's option value,[22] subject to several constraints. First, the government needs to give incentives to the firm to exert effort (ex-ante incentive compatibility constraint, IC). Second, the contract has to ensure that the total value to the firm of accepting the contract is non-negative (ex-ante participation constraint, PC). Finally, we need to ensure that the firm has incentives to invest ex-post (ex-post limited liability constraints, LL). Therefore, the government's maximization problem is the following:

$$\begin{aligned}&\max_{b_1,b_2,x_1,x_2} q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}\left[G(x_1,b_1)-(1-\alpha)(I-\theta_1)+\frac{\gamma x_1Q}{r-\mu}-\frac{\omega x_1Q^{\eta+1}}{(1+1/\eta)(r-\mu)}\right]\\ &+(1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}\left[G(x_2,b_2)-(1-\alpha)(I-\theta_2)+\frac{\gamma x_2Q}{r-\mu}-\frac{\omega x_2Q^{\eta+1}}{(1+1/\eta)(r-\mu)}\right] \qquad (14)\end{aligned}$$

$$s.t.\left(\frac{x_0}{x_1}\right)^{\beta_1}\left(F(x_1,b_1)-\alpha(I-\theta_1)\right)-\left(\frac{x_0}{x_2}\right)^{\beta_1}\left(F(x_2,b_2)-\alpha(I-\theta_2)\right)\geq\frac{\xi}{\Delta q}\ \ (IC) \qquad (15)$$

$$q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}\left(F(x_1,b_1)-\alpha(I-\theta_1)\right)+(1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}\left(F(x_2,b_2)-\alpha(I-\theta_2)\right)-\xi\geq 0\ \ (PC) \qquad (16)$$

$$F(x_1,b_1)-\alpha(I-\theta_1)\geq 0\ \ (LL1),\ \ F(x_2,b_2)-\alpha(I-\theta_2)\geq 0\ \ (LL2)^{23} \qquad (17)$$

The social welfare represented in the objective function can be decomposed similarly to the

[22] Note that the project's value net of the investment cost and of the firm's option value actually represents the government's option value.

[23] Additionally, we should have the restrictions $0\leq b_i\leq 1$, with $i\in\{1,2\}$. Since we are dealing with revenue-sharing contracts, we focus on the interior solutions of this problem, assuming parameter values are such that these constraints are not binding, i.e., $0<b_i<1$.

first-best benchmark into:

$$SW^{**}(x_0) = \Pi_H^G(x_0) + SP_H(x_0) + CS_H(x_0), \tag{18}$$

Regarding the constraints, the first constraint is the ex-ante IC constraint which ensures that the firm exerts effort. To see this, note that it is just a simplified version of the following constraint:

$$\begin{aligned} &q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1,b_1) - \alpha(I-\theta_1)) + (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2,b_2) - \alpha(I-\theta_2)) - \xi \geq \\ &\geq q_L \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1,b_1) - \alpha(I-\theta_1)) + (1-q_L)\left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2,b_2) - \alpha(I-\theta_2)) \end{aligned} \tag{19}$$

The left-hand side of this inequality represents the value of the firm's investment option if it exerts costly effort minus the cost of effort, while the right-hand side represents the value of the firm's option if it does not exert effort.

The second constraint represents the ex-ante PC constraint which ensures that the total value to the firm of accepting the contract at time zero is non-negative. Finally, the last two constraints are the LL constraints which ensure a non-negative value of the firm at the time of investment, to provide an incentive for the firm to implement the exercise of the project (otherwise it would rather walk away from the contract).

4. Model solution: optimal contracts

We now solve the government's maximization problem. We show in Appendix B that only two of these constraints are binding in equilibrium, i.e., the limited liability constraint for a θ_2 type project, $F(x_2,b_2) = \alpha(I-\theta_2)$ and the IC constraint, $\left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1,b_1) - \alpha(I-\theta_1)) \geq \frac{\xi}{\Delta q}$.

The following proposition describes the solution of our model. The proof is provided in Appendix B.

Proposition 1. *The optimal contract under second-best (x_i, b_i), $i \in \{1,2\}$ is given by:*

$$x_1 = x_1^*, \quad x_2 = x_2^* \tag{20}$$

and

$$b_1 = \left(\frac{\xi}{\Delta q}\left(\frac{x_1^*}{x_0}\right)^{\beta_1} + \alpha(I-\theta_1)\right)\left(\frac{x_1^* Q^{\eta+1}}{r-\mu} - \frac{c}{r}\right)^{-1}, \quad b_2 = \alpha(I-\theta_2)\left(\frac{x_2^* Q^{\eta+1}}{r-\mu} - \frac{c}{r}\right)^{-1}. \tag{21}$$

Proposition 1 implies that the investment triggers equal the first-best outcome, thus the private firm invests at the same thresholds as the government would. Nevertheless, in order to give incentives to the firm to invest at the optimal triggers, the government needs to pay the firm some rents, which will eventually distort the decision of the government, as we show in the next section.

Corollary 1. *Comparing second-best with first-best we obtain:*

$$\Pi_H^G(x_0) = \Pi_H(x_0) - q_H \frac{\xi}{\Delta q} < \Pi_H(x_0) - \xi \tag{22}$$

and

$$\Pi_H^F(x_0) = q_H \frac{\xi}{\Delta q} > \xi \tag{23}$$

Corollary 1 implies that compared to first-best where the government only pays the effort cost, under second-best it has to pay the firm some informational rents above this effort cost. We can see that the firm is not only paid for the cost of effort, but also captures some rents to induce it to exert effort. The rents of the private firm are given by the difference between its option value and the effort cost: $\Pi_H^F(x_0) - \xi = q_L \frac{\xi}{\Delta q} > 0$.

4.1. Inefficiency of agency problems

The existence of rents distorts the government's choice of effort. To see this, we first need to compute the government's value in case of no effort. Since for the private firm not exerting effort is costless, the government can offer a contract to the firm at no cost, as follows:

$$x_1 = x_1^*, \quad x_2 = x_2^* \tag{24}$$

and

$$b_1 = \alpha(I - \theta_1)\left(\frac{x_1^* Q^{\eta+1}}{r-\mu} - \frac{c}{r}\right)^{-1}, \quad b_2 = \alpha(I - \theta_2)\left(\frac{x_2^* Q^{\eta+1}}{r-\mu} - \frac{c}{r}\right)^{-1}, \tag{25}$$

where b_1 and b_2 are derived from the binding limited liability constraints.

The government's value under no effort is then simply given by $\Pi_L^G(x_0) = \Pi_L(x_0)$, and the social welfare is $SW_L(x_0)$, as defined in equation (7).[24] Therefore, under the second-best case with moral hazard, the government will induce the firm to exert effort if $SW^{**} \geq SW_L$, where the superscript "**" refers to the second-best optimum. Using equations (18), (22), and (7), we have that the government induces effort exertion if:

$$SW_H(x_0) - q_H \frac{\xi}{\Delta q} \geq SW_L(x_0) \tag{26}$$

Thus, the government gives incentives to the firm to exert effort if:

$$\frac{\xi}{\Delta q} \leq \frac{SW_H(x_0) - SW_L(x_0)}{q_H} < \frac{SW_H(x_0) - SW_L(x_0)}{\Delta q} = A \tag{27}$$

This implies that for a cost-benefit ratio such that $\frac{\xi}{\Delta q} > \frac{SW_H(x_0) - SW_L(x_0)}{q_H}$, the government's decision is distorted. While the government would exert effort under first-best, it does not induce the private firm to exert effort because it is too costly to give incentives. It is thus more likely for the project to be of high cost, hence the ex-ante expected total cost of the project is higher. Moreover, it is more likely for the project to incur cost overruns since the probability of the actual cost exceeding the expected value of the cost is higher. In the numerical section we will analyze how the likelihood of having more high-cost projects depends on the parameters of the model.

[24] The reason for this is that the firm's value is zero since effort is costless and the participation constraint is binding: $\Pi_L^F(x_0) = q_L \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - (1-\alpha)(I - \theta_1)) + (1 - q_L)\left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - (1-\alpha)(I-\theta_2)) = 0$.

5. Monitoring

? argue that in the case of complex PPP projects "auditing and monitoring could alleviate some of the problems posed by opportunistic behavior of contractors". Auditing on the one hand is a possible solution in the case of hidden information. **?** shows that auditing reduces the ineficiency cost of hidden information in the context of a descentralized firm. Monitoring on the other hand is appropriate for problems of hidden action. Nevertheless, both auditing and monitoring are costly. According to empirical evidence reported by **?** on PPP projects in the United States, monitoring the performance of the private firm in PPP projects entails a cost between 3 and 25 percent of the contract value. In this section, we extend the base case model considering that the government has access to a monitoring technology which allows her, at a cost, to detect if effort has been exerted.[25]

We assume that the technology detects with probability p whether effort was exerted if the government incurs a cost $s(p)$ with $s(0) = 0$, $s' > 0$, $s'' > 0$ and $\lim_{p\to 1} s(p) = +\infty$. These are standard assumptions in microeconomics. The first one implies that the government does not incur any cost if it does not use the technology. The cost function is strictly increasing and convex according to the second and third assumptions. The final assumption implies that the government cannot completely monitor the firm since it would incur a huge cost that it cannot pay.

The possibility of monitoring enlarges the feasible contracts. In particular, the government does not only set the investment trigger x and the remuneration of the firm b, but also a probability of monitoring p and a penalty P in case the firm is found shirking.

Then the agency problem with monitoring is to maximize sum of the government's value plus the social benefit through the choice of these variables. For simplicity, we normalize $Q = 1$. The maximization problem is as follows:

$$\max_{b_1,b_2,x_1,x_2,p_1,p_2,P_1,P_2} q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} \left(G(x_1,b_1) - (1-\alpha)(I-\theta_1) + \frac{\gamma x_1}{r-\mu} - \frac{\omega x_1}{(1+1/\eta)(r-\mu)} - s(p_1)\right)$$
$$+(1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1} \left(G(x_2,b_2) - (1-\alpha)(I-\theta_2) + \frac{\gamma x_2}{r-\mu} - \frac{\omega x_2}{(1+1/\eta)(r-\mu)} - s(p_2)\right) \tag{28}$$

$$\begin{aligned} s.t. \quad & q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1,b_1) - \alpha(I-\theta_1)) + (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2,b_2) - \alpha(I-\theta_2)) - \xi \geq \\ & \geq q_L \left(\frac{x_0}{x_1}\right)^{\beta_1} \left[(1-p_1)\left(F(x_1,b_1) - \alpha(I-\theta_1)\right) + p_1\left(F(x_1,b_1) - \alpha(I-\theta_1) - P_1\right)\right] \\ & +(1-q_L)\left(\frac{x_0}{x_2}\right)^{\beta_1} \left[(1-p_2)\left(F(x_2,b_2) - \alpha(I-\theta_2)\right) + p_2\left(F(x_2,b_2) - \alpha(I-\theta_2) - P_2\right)\right] \ (IC) \end{aligned} \tag{29}$$

$$q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1,b_1) - \alpha(I-\theta_1)) + (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2,b_2) - \alpha(I-\theta_2)) - \xi \geq 0 \ (PC) \tag{30}$$

[25] **?** also argue that costly monitoring through periodic inspections of the infrastructure's performance is an essential part of the game.

$$F(x_1,b_1)-\alpha(I-\theta_1)\geq 0\ (LL1),\quad F(x_2,b_2)-\alpha(I-\theta_2)\geq 0\ (LL2) \tag{31}$$

$$F(x_1,b_1)-\alpha(I-\theta_1)-P_1\geq 0\ (LL1new),\quad F(x_2,b_2)-\alpha(I-\theta_2)-P_2\geq 0\ (LL2new) \tag{32}$$

$$0\leq p_1\leq 1,\quad 0\leq p_2\leq 1 \tag{33}$$

with $G(x_i,b_i)$ and $F(x_i,b_i)$ given by equations (11) and (13) respectively.

Since the penalties P_1 and P_2 appear only on the right-hand side of the incentive compatibility constraint we can see that it is optimal for the government to increase them as much as possible. Indeed, the larger the penalties, the more incentives the firm has to exert effort. Thus, given the limited liability constraints, it is optimal for the government to set the penalties as high as possible, that is, so that the new limited liability constraints bind: $P_1=F(x_1,b_1)-\alpha(I-\theta_1)$ and $P_2=F(x_2,b_2)-\alpha(I-\theta_2)$. Whenever the firm is found shirking, it is optimal to give it the lowest possible compensation. Substituting these values into the IC constraint and rearranging we obtain:

$$\left(\frac{x_0}{x_1}\right)^{\beta_1}(F(x_1,b_1)-\alpha(I-\theta_1))-\left(\frac{x_0}{x_2}\right)^{\beta_1}(F(x_2,b_2)-\alpha(I-\theta_2))\,\frac{\Delta q-(1-q_L)p_2}{\Delta q+q_Lp_1}\geq\frac{\xi}{\Delta q+q_Lp_1} \tag{34}$$

Similar to the benchmark model, we show in Appendix C that the LL1 and PC constraints do not bind and that the LL2 and IC constraints bind at the optimum. Let superscript "M" denote the optimum in the agency problem with monitoring. The optimal contract with monitoring is given in the following proposition. The proofs are provided in Appendix C.

Proposition 2. *The optimal contract with monitoring* (x_i^M,b_i^M,p_i,P_i), $i\in\{1,2\}$ *is obtained as follows:*

If $s'(0)\left(\frac{x_0}{x_1^*}\right)^{\beta_1}>\frac{q_L\xi}{(\Delta q)^2}$, *the optimal contract is given by*

$$\begin{aligned}(x_1^M,b_1^M,p_1,P_1)&=(x_1^*,b_1,0,F(x_1^*,b_1)-\alpha(I-\theta_1)),\\(x_2^M,b_2^M,p_2,P_2)&=(x_2^*,b_2,0,0).\end{aligned}$$

If $s'(0)\left(\frac{x_0}{x_1^*}\right)^{\beta_1}\leq\frac{q_L\xi}{(\Delta q)^2}$, *the optimal contract is given by*

$$(x_1^M,b_1^M,p_1,P_1)$$

with

$$x_1^M=\frac{\beta_1}{\beta_1-1}\frac{\left(\frac{c}{r}+I-\theta_1+s(p_1)\right)(r-\mu)}{\gamma+1-\frac{\omega}{1+1/\eta}}, \tag{35}$$

$$b_1^M=\left(\frac{\xi}{\Delta q+q_Lp_1}\left(\frac{x_1^M}{x_0}\right)^{\beta_1}+\alpha(I-\theta_1)\right)\left(\frac{x_1^M}{r-\mu}-\frac{c}{r}\right)^{-1}, \tag{36}$$

$$s'(p_1)=\frac{q_L\xi}{(\Delta q+q_Lp_1)^2}\left(\frac{x_1^M}{x_0}\right)^{\beta_1}, \tag{37}$$

$$P_1 = F(x_1^M, b_1^M) - \alpha(I - \theta_1). \tag{38}$$

and

$$(x_2^M, b_2^M, p_2, P_2) = (x_2^*, b_2, 0, 0).$$

Proposition 2 implies that depending on parameter values we obtain two cases. In the first case, intuitively, if the marginal cost of monitoring at $p_1 = 0$ is sufficiently large, the government optimally chooses not to monitor. Then we obtain $p_1 = p_2 = 0$, and the problem collapses back to our benchmark model (the investment thresholds are the first-best ones and b_1, b_2 are the ones obtained in the second-best model given by equation (21)). Otherwise, if the cost of monitoring is relatively low, the government monitors the low-cost project with a positive probability $p_1 > 0$ implicitly defined by equation (37). However, it does not monitor the high-cost project, $p_2 = 0$. The fact that $p_2 = 0$ has a very intuitive explanation. Since LL2 binds at the optimum, the firm gets zero in case of a high-cost project. The maximum penalty that the government could impose would thus be zero. Therefore, since monitoring is costly, there is no point of monitoring in this case since the firm already has the maximum punishment possible, it does not earn anything.

The following corollary presents the properties of the monitoring solution. The proof is provided in Appendix C.

Corollary 2. *The optimal contract with monitoring has the following properties:*

$$x_1^M \geq x_1^*, \; p_1 \geq 0, \; P_1 \geq 0$$

$$x_2^M = x_2^*, \; p_2 = 0, \; P_2 = 0$$

In particular, we obtain $x_1^M > x_1^*$ *when* $p_1 > 0$, *while* $x_1^M = x_1^*$ *when* $p_1 = 0$.

Whenever $p_1 > 0$ we have that $x_1^M > x_1^*$, that is, we have delayed investment for the low-cost project. It is less costly for the government to distort x_1^M away from x_1^* than to distort x_2^M away from x_2^* at the optimum. This is in contrast to previous findings: **?** finds that it is less costly for the owner to distort the investment threshold of the low-quality project in a shareholder-manager agency problem with auditing, **?** find that in order to minimize verification costs, costly state verification should only be used in bad states of the world (when cash flows are insufficient to repay the debt). This is due to the fact that in our case verification for the high-cost project is inefficient since given limited liability the maximum punishment is zero.

Since $x_2^M = x_2^*$, we also have that $b_2^M = b_2$ given that LL2 is binding. Therefore, the total value for the high-cost project is the same (since $x_2^M = x_2^*$ and $p_2 = 0$). However, since $x_1^M > x_1^*$, the total value of the low-cost project is lower under monitoring. Indeed, it can be shown that:

$$\left(\frac{x_0}{x_1^M}\right)^{\beta_1} \left(\frac{x_1^M}{r-\mu} - \frac{c}{r} - (I - \theta_1) - s(p_1)\right) < \left(\frac{x_0}{x_1^*}\right)^{\beta_1} \left(\frac{x_1^*}{r-\mu} - \frac{c}{r} - (I - \theta_1)\right) \tag{39}$$

by simply substituting the expressions for x_1^M and x_1^*.

Hence, assuming that the government finds it optimal to induce effort both with and without monitoring, monitoring leads to a decrease in the total value of the project $\Pi_H^M \leq \Pi_H$. This is due to the fact that not only the government pays a monitoring cost, but it also has to delay the investment timing of the low-cost project. On the contrary, in **?** and **?** auditing mitigated the distortion in investment timing (earlier investment compared to the case without auditing, closer to first best), which could sometimes lead to a larger total value of the project if the cost of auditing was relatively low.

Similarly, by the same reasoning it can be shown that the spillover benefits and consumer surplus decrease under monitoring. However, the government prefers to use monitoring when its cost is relatively low since in this way it can reduce the information rents of the firm. The following corollary formalizes this argument.

Corollary 3. *Comparing monitoring with second-best we obtain:*

$$\Pi_H^{GM} \geq \Pi_H^G,\ \Pi_H^{FM} \leq \Pi_H^F \tag{40}$$

In particular, we have $\Pi_H^{GM} > \Pi_H^G$, $\Pi_H^{FM} < \Pi_H^F$ *when* $p_1 > 0$.

By reducing the information rents the government increases its own option value, $\Pi_H^{GM} \geq \Pi_H^G$. This increase in the government's value more than compensates for the decrease in the social benefits, thus increasing social welfare, i.e., $SW^M \geq SW^{**}$.[26]

Assuming that the parameter values are such that the government induces effort exertion both with and without monitoring, we have seen above that monitoring leads to a decrease in total project value. Nevertheless, since monitoring increases social welfare, it will also have an impact on the government's decision to induce effort. In particular, under monitoring the government induces effort exertion when $SW^M > SW_L$, while without monitoring the government induces effort if $SW^{**} > SW_L$. Since $SW^M \geq SW^{**}$, the threshold of the cost-benefit ratio below which effort exertion is optimal will be higher with monitoring than without monitoring.[27] Therefore, for relatively large cost-benefit ratios of effort, monitoring can alleviate the distortion in the government's decision to induce effort, reducing the loss in the total value of the project and increasing social welfare. Indeed, while the government would not find it optimal to incentivize effort without monitoring, it would find it optimal to do so under monitoring, increasing the likelihood of obtaining a low-cost project. Our model thus implies that investing in efficient monitoring technologies is particularly important for complex projects characterized by a high cost of effort.

6. Bailout option

We now extend the benchmark model and incorporate a government guarantee prevalent in many PPP projects, an exit option (? and ?). In particular, assume that if the net revenue deteriorates, the firm has the right to transfer the project's ownership to the government in exchange for an amount K. The optimal transfer occurs as soon as the demand parameter falls to the exit threshold x_g.[28]

To obtain the optimal contracts we first need to adjust the expressions of $G(x,b)$ and $F(x,b)$ in order to account for the exit option. As before, we normalize $Q = 1$.

[26] This follows directly from the maximization problem. Since not monitoring is always an option, whenever the government optimally chooses $p_1 > 0$ it is because social welfare (the sum of government's value, spillover benefits and consumer surplus) under monitoring is larger.

[27] Since $SW^M = SW_H^M - q_H\xi/(\Delta q + q_L p_1) >$, it can be shown after some manipulations that the government induces effort exertion under monitoring if $\frac{\xi}{\Delta q} < \frac{SW_H^M - SW_L}{q_H} + \frac{\xi q_L p_1}{\Delta q(\Delta q + q_L p_1)}$. Similarly, since $SW^{**} = SW_H - q_H\xi/\Delta q$, in the second-best case without monitoring the government induces effort if $\frac{\xi}{\Delta q} < \frac{SW_H - SW_L}{q_H}$. Comparing the two cases, it can be shown that $\frac{SW_H - SW_L}{q_H} \leq \frac{SW_H^M - SW_L}{q_H} + \frac{\xi q_L p_1}{\Delta q(\Delta q + q_L p_1)}$.

[28] As standard in real options, and in particular in other analytical studies modeling the bailout option (? and ?), we have assumed that x follows a geometric Brownian motion. We have verified that there is a positive (and sometimes quite large) probability for x to hit the lower threshold within this setting, especially for larger K and within a longer period of time.

Following standard arguments in real-options theory, we show in Appendix D that the discounted value of the government's and private firm's profits at the time of investment in the case of a bailout option are given by:

$$G(x,b) = (1-b)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) - \frac{1}{1-\beta_2}\left(K + \frac{bc}{r}\right)\left(\frac{x}{x_g}\right)^{\beta_2} \tag{41}$$

and

$$F(x,b) = b\left(\frac{x}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{bc}{r}\right)\left(\frac{x}{x_g}\right)^{\beta_2} \tag{42}$$

and the threshold is

$$x_g \equiv x_g(b) = \frac{\beta_2}{\beta_2 - 1}\frac{\left(K + \frac{bc}{r}\right)(r-\mu)}{b}, \tag{43}$$

where β_2 is the negative root of the fundamental quadratic equation (see **?**) $\frac{\beta(\beta-1)\sigma^2}{2} + \beta\mu = r$ and is given by

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0 \tag{44}$$

The maximization problem of the government is expressed in equations (14-17), with $G(x,b)$ and $F(x,b)$ given by equations (41) and (42). The optimal contract is characterized in the following proposition, whose proof is similar to the one of Proposition 1.

Proposition 3. *The optimal contract with a bailout option* (x_i, b_i), $i \in \{1,2\}$ *is given by:*

$$x_1 = x_1^*, \;\; x_2 = x_2^*, \tag{45}$$

and b_1, b_2 *satisfying the following implicit equations:*

$$b_1\left(\frac{x_1^*}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{b_1 c}{r}\right)\left(\frac{x_1^*}{x_g(b_1)}\right)^{\beta_2} = \frac{\xi}{\Delta q}\left(\frac{x_1^*}{x_0}\right)^{\beta_1} + \alpha(I-\theta_1) \tag{46}$$

$$b_2\left(\frac{x_2^*}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{b_2 c}{r}\right)\left(\frac{x_2^*}{x_g(b_2)}\right)^{\beta_2} = \alpha(I-\theta_2) \tag{47}$$

Equations (46) and (47) must be solved numerically for the firm's fractions of cash flows b_1 and b_2. As in the base case model without an exit option, the investment triggers equal the first-best outcome, thus the private firm invests at the same thresholds as the government would. However, giving incentives to the firm is costly, thus the government's decision might be distorted by the rents paid to the firm. This distortion and the resulting inefficiency (higher likelihood of obtaining high-cost projects) are identical to those in the base case model without an exit option.[29] Although this might seem counterintuitive at first, the existence of a bailout option does not increase the inefficiency as long as the government takes into account the possibility of bailout when setting the remuneration of the firm. Indeed, it is straightforward to show that $\partial b_1/\partial K < 0$ and $\partial b_2/\partial K < 0$. This implies that the larger the buyout price the lower the fraction of net revenues that the firm receives. The regular compensation of the firm and the compensation upon termination act

[29]The same IC and LL1 constraints are binding, which implies that the firm obtains the same option value and the same rents as before.

as substitutes and the increase in value that the firm receives through the compensation upon termination (the exit option and thus K) is completely offset by the reduction in the regular compensation (b_1).[30] Failing to set both types of remuneration jointly however, as in the case of a "myopic" goyernment, leads to an excessive regular remuneration of the firm and losses in the government's profits, as we illustrate in the numerical section. Our results are consistent with recent evidence by **?** who find that the length of the concession period is inversely proportional to the level of the minimum revenue guarantee. Similar to us, they stress the importance of simultaneously optimizing these two parameters.

In a related asymmetric information model, **?** show that institutional arrangements that limit the losses that the firms can bear discourage firms from acquiring information about the future profitability of a project, which increases social losses as negative net present-value projects are undertaken. They thus support institutional reforms aimed to increase the potential losses that the firm incurs (or decrease the buyout price). Their policy recommendation is to compensate the bankrupt concessionaire according to the value of the project (uncovered by auctioning the failed concession), rather than according to the cost incurred, in order to increase social value and minimize payments to the private partner. However, this might not be feasible in countries where the PPP market is not sufficiently liquid for the project to be retendered to potentially interested purchasers. In these situations, our model suggests that it is possible to give incentives to the firm to exert cost-reducing effort and thus minimize social losses by making the regular compensation of the private partner a (an inversely related) function of the probability of bailout and the buyout price. This could mitigate social losses for emerging market economies and civil law countries that are more likely to compensate the concessionaire on termination based on the cost incurred (book value method).

7. Numerical analysis and implications

In this section we illustrate the dependence of our results on the main parameters of the model and discuss the implications of our model. We first analyze the benchmark model focusing on the optimal contract (investment thresholds and private firm's fraction of the net revenue) and the inefficiency of the second-best case. We then compare the benchmark model with the model including monitoring. Third, we analyze the effects of social benefits, including spillovers and consumer surplus. Finally, we look at how the bailout option affects the optimal contract.

For numerical examples regarding monitoring, we will use the following monitoring cost function:

$$s(p) = a\frac{p}{1-p}, \tag{48}$$

where a is a positive constant.[31]

The base case parameters used in this analysis are set similar to existing real-options PPP models (**?**; **?**). The demand volatility and growth rate are set to $\sigma = 0.2$ and $\mu = 0$, and the risk-free interest rate is $r = 0.04$. Following **?**, the price sensitivity parameter is set to $\eta = -0.75$

[30] Here we are ignoring any additional costs that the government might incur due to the bailout such as reputational costs or a cost of public funds to rescue the firm due to distortionary taxation. Such costs would imply a delay in the investment timing and an increase in inefficiency compared to the case without exit option.

[31] A similar cost function for audit is used in **?**. Note that this function satisfies $s(0) = 0$, $s' > 0$, $s'' > 0$ and $\lim_{p\to 1} s(p) = +\infty$.

(which implies a price elasticity of 1.33). The fixed cost is $c = 0.10$, the gross investment cost is $I = 10$, while $\theta_1 = 9$ and $\theta_2 = 3$. The probability of having a low-cost project when exerting effort is $q_H = 0.7$, while if effort is not exerted this probability is $q_L = 0.5$. The cost of effort is set to $\xi = 0.05$. Regarding the subsidy for the net investment cost, according to a recent note from the PPP Group of the World Bank and the PPI Database, about a quarter of the financing of PPP projects is from the public sector in low-to-middle-income countries. Moreover, public sector financing of transport projects (41%) tends to be higher than that of energy projects (16%). We therefore set the financing provided by the private party to $\alpha = 0.7$. The capacity of the firm is normalized to $Q = 1$. The bailout price is set to $K = 0.1$. The spillover parameter and the weight of the consumer surplus are set to $\gamma = \omega = 0$ in the base case, as we will analyze social benefits in a separate subsection. The monitoring cost function parameter is set to $a = 0.5$. Finally, following **?**, the curent value of the state variable is set to $x = 0.2$. We use these base-case parameter values in all tables and figures, unless specified otherwise. We also provide comparative statics with a wide range of parameter values to ensure the robustness of the results.

7.1. Optimal contract

Table 1 shows the numerical results for all the problems (full-information problem corresponding to the first-best case (FB), the agency problem without monitoring corresponding to the second-best (SB), and the agency problem with monitoring (M)). We focus here on the FB and SB cases, and analyze monitoring in the next subsection. At the current value $x = 0.2$ investment should not be undertaken now for all the cases. To be able to analyze the inefficiency generated by the agency problem we consider two cases for the cost of effort illustrated in two different panels. Panel A provides the results for a cost of effort $\xi = 0.05$, while panel B provides the results for a higher cost of effort $\xi = 0.07$. The investment triggers are equal in the first-best and second-best cases ($x_1 = 0.2800$ and $x_2 = 0.7600$), thus the agency problems do not lead to a delayed nor early investment. For a low cost-benefit ratio as in panel A it is optimal for the government to induce effort both in the first-best case and under the second-best case (no inefficiency and thus zero loss in the total value of the project). In the agency-problem without monitoring, the private firm receives a fraction $b_1 = 0.2644$ of the net revenues for a low-cost project and $b_2 = 0.2970$ for a high-cost project. For a high cost-benefit ratio as in panel B however, while the government exerts effort under first-best, it does not find it optimal to induce effort exertion under second-best. Since giving information rents to the firm to exert effort is costly, the government prefers not to induce effort and the remuneration of the firm for a low-cost project decreases to $b_1 = 0.1556$, to simply cover the investment cost. This leads to a decrease in the total value of the project compared to first-best, since the likelihood of obtaining a high-cost project is now higher under no effort exertion, which implies a loss of 16% (or in absolute value 0.2256).

We plot in Figures 1, 2 and 3 the comparative statics of the optimal contract (b_1, b_2, x_1, x_2) with respect to the parameters of the model. We use a blue dotted line for the variables corresponding to the low-cost project, and a red dash-dotted line for the high-cost project. Panels a) of Figures 1 and 3 depict the comparative statics of the optimal remuneration and investment thresholds with respect to the fixed operating cost, c. In panel a) of Figure 3 we observe that both investment triggers increase with the operating cost. A higher cost thus implies a delayed investment. In Figure 1, we note a jump in the fraction of net revenues obtained by the private firm for a low-cost project at $c = 0.13$. For $c < 0.13$ the government induces effort exertion and thus the firm has a relatively high remuneration. However, for $c > 0.13$ the government does not find it optimal to induce effort and the firm's remuneration decreases. Thus, the larger the operating cost the more

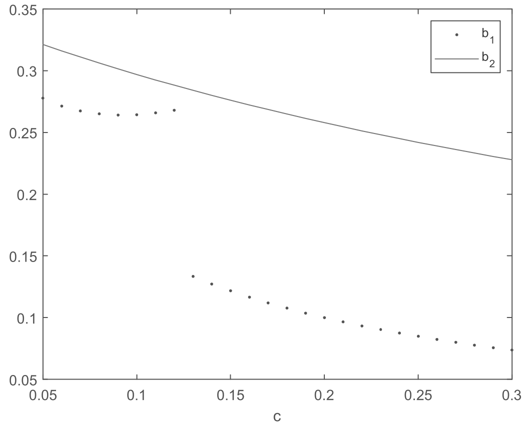
Table 1: Optimal contract, values and loss for the first-best case, second-best case without monitoring, and second-best case with monitoring.

	Panel A: $\xi = 0.05$			Panel B: $\xi = 0.07$		
	FB	SB	M	FB	SB	M
x_1	0.2800	0.2800	0.2869	0.2800	0.2800	0.2905
x_2	0.7600	0.7600	0.7600	0.7600	0.7600	0.7600
b_1	-	0.2644	0.2303	-	0.1556	0.2489
b_2	-	0.2970	0.2970	-	0.2970	0.2970
p_1	-	-	0.1471	-	-	0.2084
p_2	-	-	0	-	-	0
P_1	-	-	0.3761	-	-	0.4856
P_2	-	-	0	-	-	0
Π^G	-	1.2724	1.2894	-	1.2218	1.2410
Π^F	-	0.1750	0.1279	-	0	0.1611
Π	1.4474	1.4474	1.4173	1.4474	1.2218	1.4020
Loss	-	0	0.0301	-	0.2256	0.0454

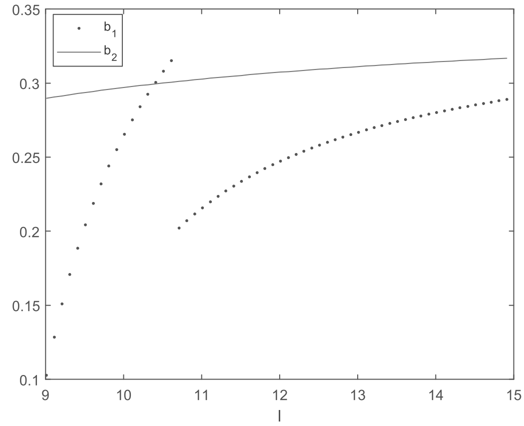
likely it is that the project will be of high cost. In this region b_1 decreases with the operating cost. This is due to the fact that the discounted value of future net revenues at the time of the investment increases with the operating cost. Therefore, a lower fraction of the net revenues is needed to compensate the firm for the share of investment cost that it covered. In the effort region b_1 is U-shaped in c. This is because in this case the firm has to be compensated for effort as well. The higher the operating cost the later the firm invests, and the larger the remuneration that the firm has to receive.

Panels b) of the same figures depict the comparative statics with respect to the gross investment cost I. The fraction of net revenues obtained by the private firm increases with the investment cost since the private firm shares this investment cost with the government in a proportion equal to α. Therefore, a larger investment cost implies that the firm requires a higher fraction of the net revenues to participate in the project. As before, both investment triggers increase with the investment cost. Thus, an increased investment cost delays investment. Again, for a relatively high investment cost the government will not give incentives to the firm to exert effort, hence the drop in b_1. The larger the investment cost the more likely it is that the project will be of high cost. Note that both for an investment cost of $I = 9.61$ and $I = 11.11$ the resulting optimal remuneration for a low-cost project is the same $b_1 = 0.22$, while the investment triggers are different, $x_1 = 0.25$ and $x_1 = 0.37$ respectively. For a relatively low investment cost of $I = 9.61$, the firm invests relatively early at $x_1 = 0.25$ and a fraction of $b_1 = 0.22$ of the net revenues not only compensates the firm for the share of the investment cost that it covered, but it also includes information rents that more than cover the cost of effort. For a relatively high investment cost of $I = 11.11$, the firm invests later at $x_1 = 0.37$ and the same fraction of $b_1 = 0.22$ barely compensates the firm for its share of the investment cost, resulting in no effort exertion.

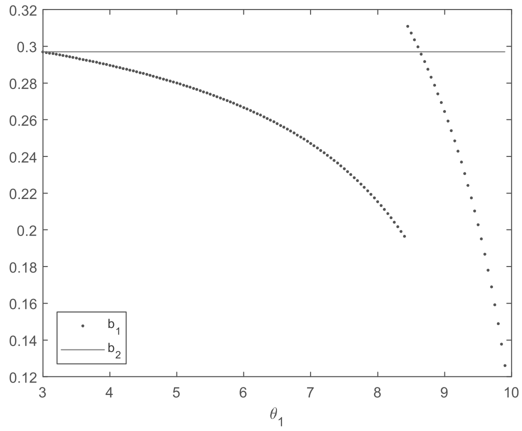
Panels c) and d) present the comparative statics with respect to the θ_1 and θ_2. In general the optimal contract of a θ_i-project depends only on θ_i, where $i \in \{1, 2\}$. Both the fraction of net



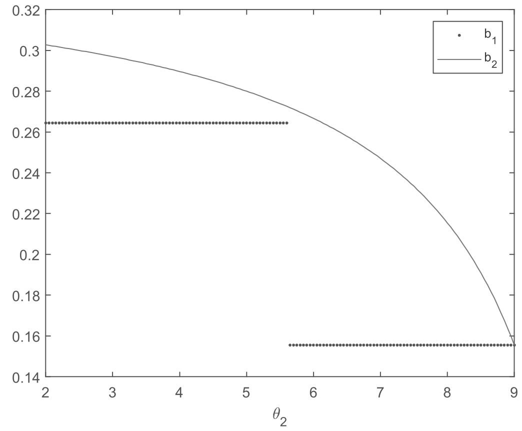
a) c



b) I



c) θ_1



d) θ_2

Figure 1: Optimal remuneration as a function of the cost parameter values: c, I, θ_1, θ_2.

revenues obtained by the private firm and the investment trigger decrease with θ. This is because θ represents the amount by which the gross investment cost can be reduced. However, we can see that b_1 depends indirectly on θ_2 since when the projects are very similar (θ_2 gets closer to θ_1, and the difference $\Delta\theta$ diminishes) the government will decide not to induce effort.

Panel a) of Figure 2 shows that the fraction of net revenues received by the private firm of a θ_1 project increases with the effort cost ξ in order to provide the firm with incentives to exert effort. Moreover, as expected, for a large effort cost the government no longer wants to induce the private firm to exert effort. The larger the effort cost the more likely it is for the project to be of high cost.

In panel b) we can see that when the benefit of exerting effort, Δq, is relatively small, the government will not induce effort. For a relatively large benefit of effort, when the government induces effort, the fraction of net revenues obtained by the private firm decreases with Δq. When the benefit of exerting effort increases, it is less costly for the government to provide incentives since the firm will naturally have more incentives to exert effort. As expected, we note in panel c) that the fraction of net revenues obtained by the private firm increases with the investment cost sharing ratio α (decreases with the subsidy $1 - \alpha$). However, the investment trigger does not

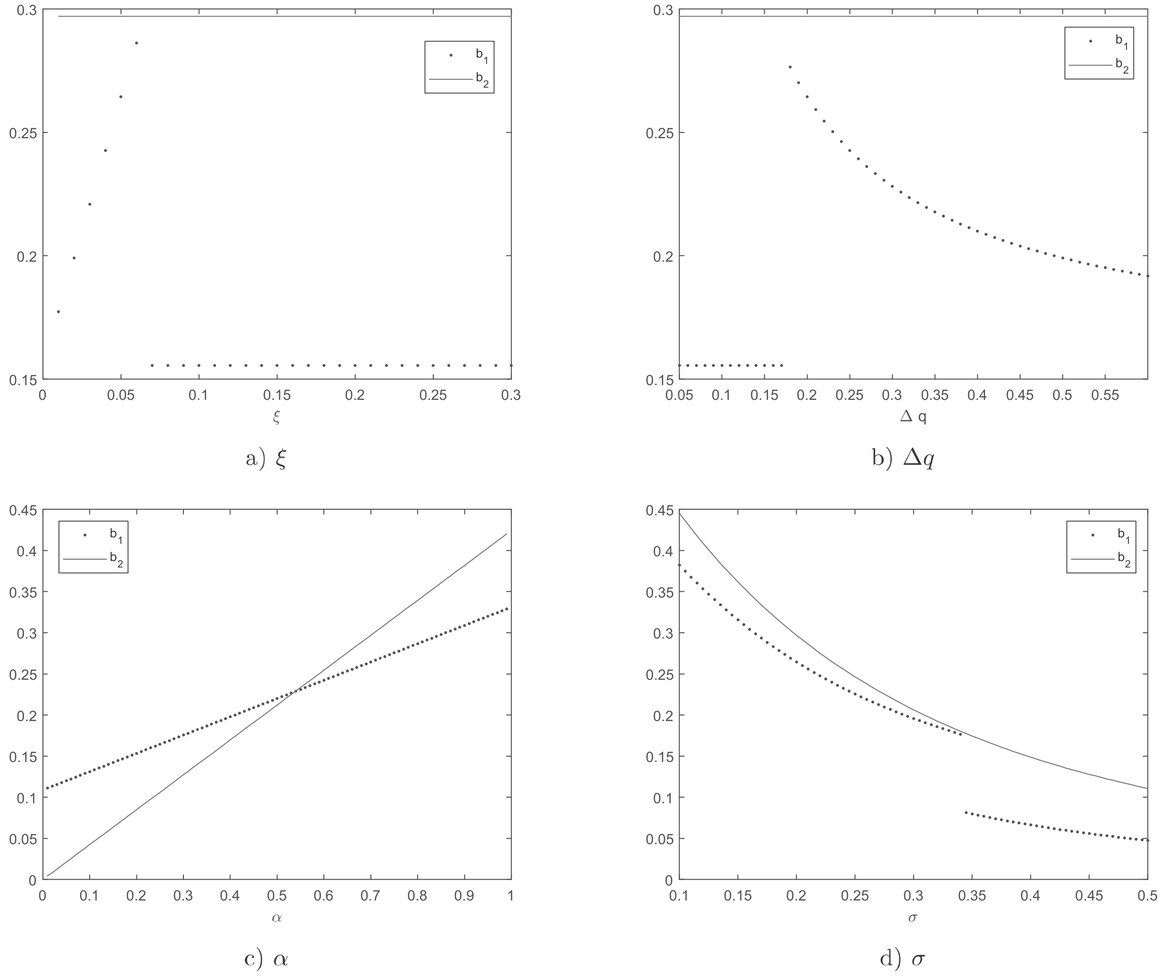


Figure 2: Optimal remuneration as a function of the cost-benefit ratio of effort parameters ξ and Δq, subsidy parameter α, and volatility σ.

depend on this parameter, nor does the government's effort choice. Finally, the remuneration of the firm decreases with the volatility of the demand as we observe in panel d). This is due to the fact that, as standard in real-options theory, the larger the volatility the larger the investment triggers, which are inversely related to the remuneration of the firm. The higher the investment trigger the lower the fraction of revenues needed to compensate the private firm. Moreover, for relatively large volatility levels, it is not optimal for the government to induce effort, hence the drop in b_1. Thus the likelihood of obtaining high-cost projects increases with the volatility of the demand.

We have thus shown that the higher the investment cost, the fixed operating cost, the cost of effort or the volatility of the demand, the more likely it is for the government not to find it optimal to give incentives to the firm to exert effort, resulting in higher cost projects, which are then more prone to renegotiation. These implications are in line with empirical evidence on cost overruns and renegotiation of PPP contracts. **?** show that large-sized projects, with high costs or a high level of implementation risk are more likely to result in cost overruns in a sample of 601

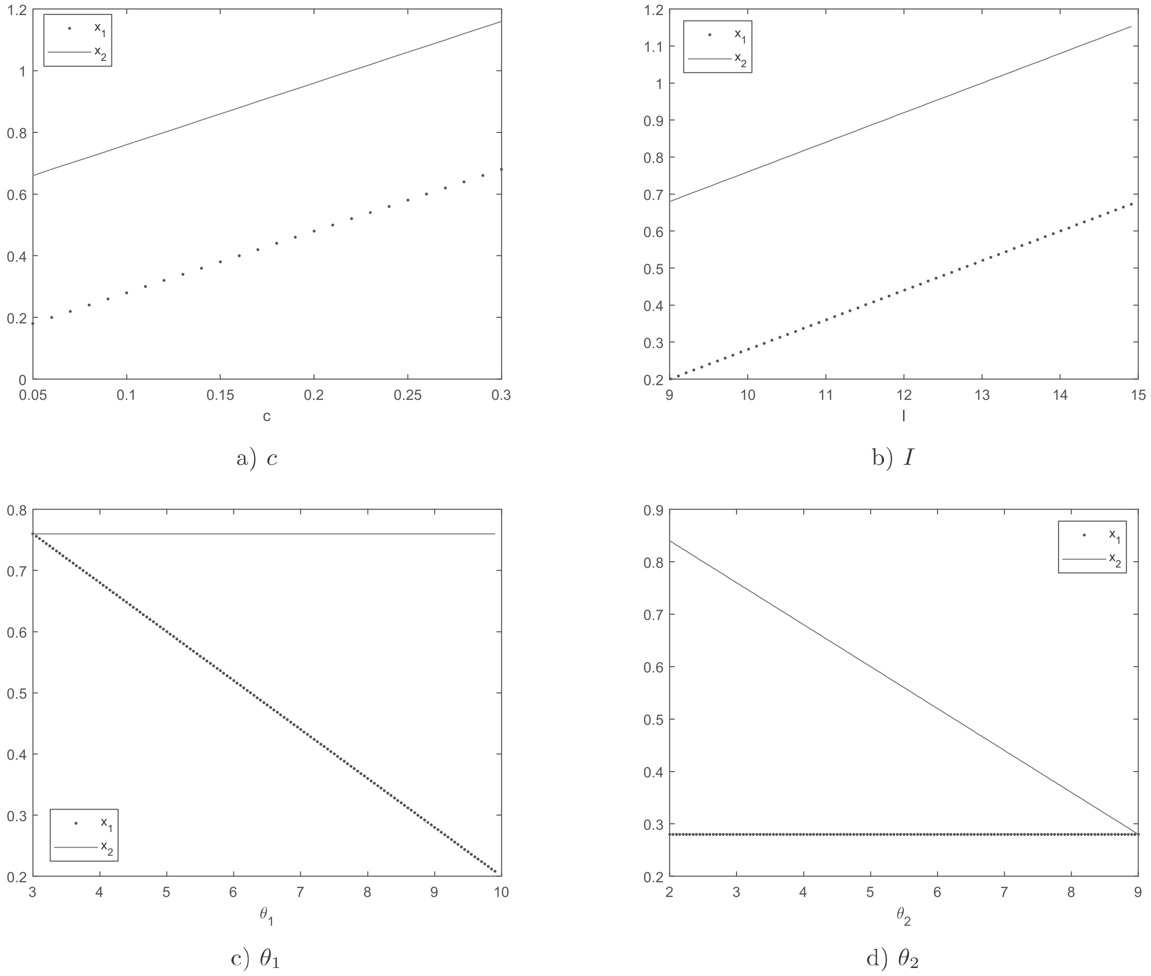
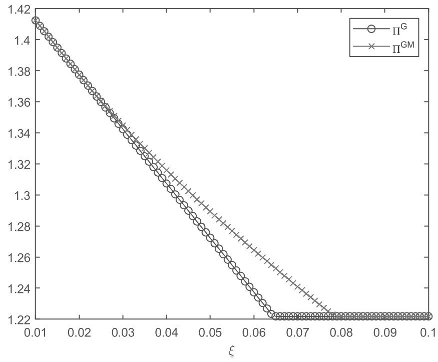


Figure 3: Optimal investment thresholds as a function of the parameter values.

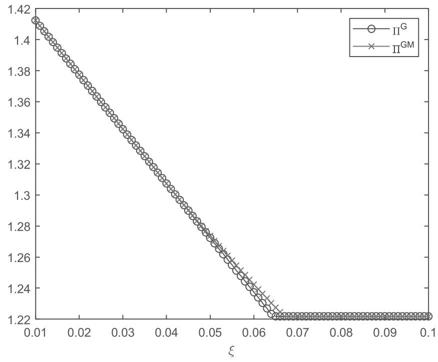
PPP highway projects. For roads and railways projects in India, **?** finds that relatively big projects experience much higher cost overruns compared to smaller ones. **?** also find that for bridges and tunnels larger projects have larger percentage cost escalations. Regarding evidence on renegotiation of concession contracts, **?** shows that in Latin America over 30% of concession contracts are renegotiated (the incidence of renegotiation raises to 41.5% not including the telecommunications sector, since practically all projects in this sector were privatized rather than concessioned). This proportion raises to 54.7% in the transportation sector and to 74.4% in the water and sanitation sector, where concessions are characterized by huge sunk investments and demand risk.

7.2. Monitoring

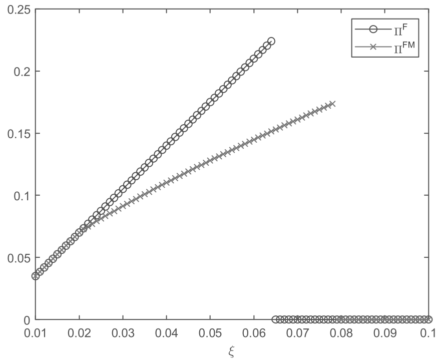
We now analyze the agency problem with monitoring. Columns 3 and 6 of Table 1 reports the variables of interest for the monitoring model. As we can see in panel A, since monitoring is costly, it leads to a delayed investment for the low-cost project, i.e., $x_1^M = 0.2869 > x_1 = 0.2800$. Effort exertion is detected with a probability of $p_1 = 0.1471$ for a low-cost project, while the government does not monitor high-cost projects, $p_2 = 0$. By monitoring the government can increase its profits $\Pi_H^{GM} = 1.2894 > \Pi_H^G = 1.2724$. However, the firm's profits decrease by more than that, and the total value of the project decreases, $\Pi_H^M = 1.4173 < \Pi_H = 1.4474$. In this case of a low effort cost, both in the second-best and monitoring problems the government induces effort. Nevertheless, we note in panel B that for a relatively high effort cost, while the government does not induce effort in the second-best case, it does induce effort with monitoring, since its profits increase under monitoring. The total value of the project increases with respect to the second-best case, $\Pi_H^M = 1.4020 > \Pi_L = 1.2218$, thus the loss is reduced to 0.0454 (around 3%).



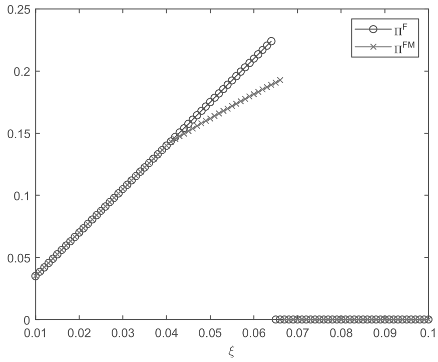
a) Π^G for $a = 0.5$



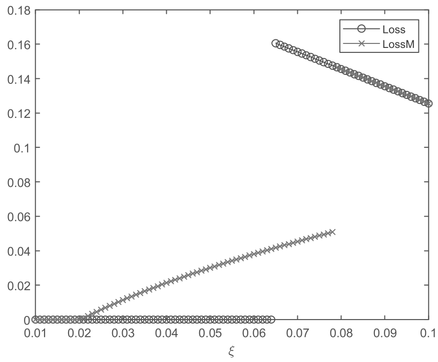
b) Π^G for $a = 1$



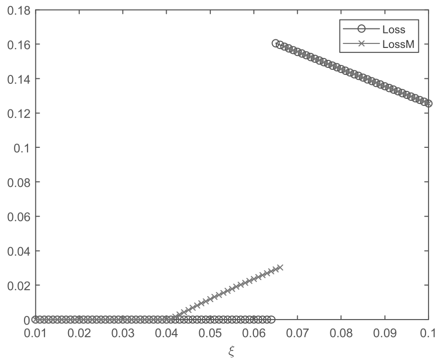
c) Π^F for $a = 0.5$



d) Π^F for $a = 1$



e) Loss for $a = 0.5$



f) Loss for $a = 1$

Figure 4: The effects of monitoring on the government's and firm's profits, as well as on the loss in total project value.

The effects of monitoring on the government's and firm's profits, as well as on the loss in total project value (compared to first-best) are depicted in Figure 4. Panels a), c) and e) plot the results for a monitoring cost parameter $a = 0.5$, while in panels b), d) and f) we have $a = 1$. In panel a) we observe that for an effort cost $\xi < 0.022$, it is optimal for the government to induce effort (no inefficiency, zero loss), but it is not optimal to use monitoring. For $0.022 < \xi < 0.064$, it is optimal for the firm to use monitoring and to exert effort both with and without monitoring. Since monitoring is costly, it implies a delayed investment and leads to a loss in the total value of the project in this region (see panel e)). For a relatively larger cost of effort, $0.064 < \xi < 0.079$, it is no longer optimal for the government to induce effort in the second-best case without monitoring, although it is optimal to do so with monitoring (since the government's profits increase under monitoring). Therefore, the use of monitoring leads to a smaller distortion in the government's decision to induce effort, and therefore a smaller loss. Finally, for $\xi > 0.079$, it is not optimal for the government to induce effort with monitoring either, the threshold for effort exertion under monitoring being larger than the second-best threshold, but lower than the first-best one. Comparing with panels b), d) and f) where the monitoring cost is larger, we note that monitoring is more efficient in alleviating inefficiencies when its cost is relatively low.

7.3. Spillovers

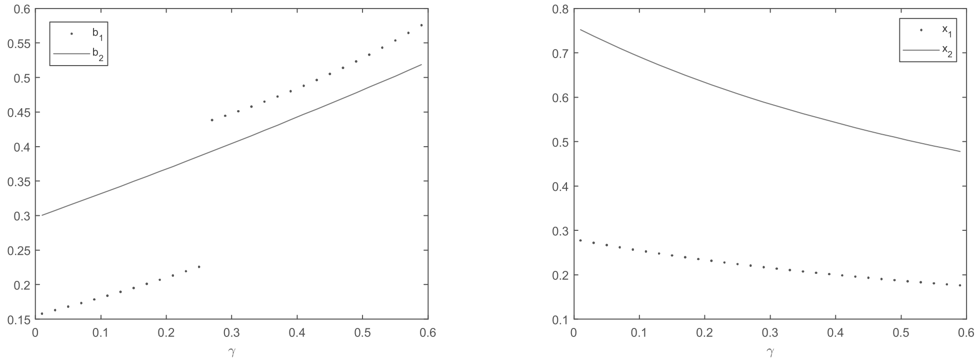
So far we have normalized the social benefits parameters $\gamma = \omega = 0$. We now analyze their effects on the optimal contract. We let γ vary from 0 to 0.6 in line with spillover parameters used by **?** and spillover benefits reported by **?**. According to **?**, in the case of a port infrastructure spillover effects range between 20 and 60 percent of the cost to process one unit of cargo capacity, depending on the method used and on the level of aggregation of the local government's jurisdiction. **?** report spillover benefits in a port as percentage of total economic activity between 36 and 41 percent. Figure 5 shows that the larger the spillover benefits the earlier investment takes place (panel b)) and the larger the fraction of revenues obtained by the private firm (panel a)).[32] Moreover, for spillover benefits $\gamma > 0.23$, it is optimal for the government to give incentives to the firm to exert effort. The higher the spillover benefits the more likely it is for the project to be of low cost.

The weight of the consumer surplus in the welfare function, ω, could vary from 0 to 1. To ensure that the optimal remunerations of the firm, b_1 and b_2, are between $(0, 1)$, we plot in Figure 6 the optimal contract for a weight ω varying from 0 to 0.3. The results are similar to those for the spillover benefits. The larger the weight of consumer surplus the earlier the firm invests, the lower its remuneration, and the more likely it is for the firm to exert effort.

7.4. The bailout option

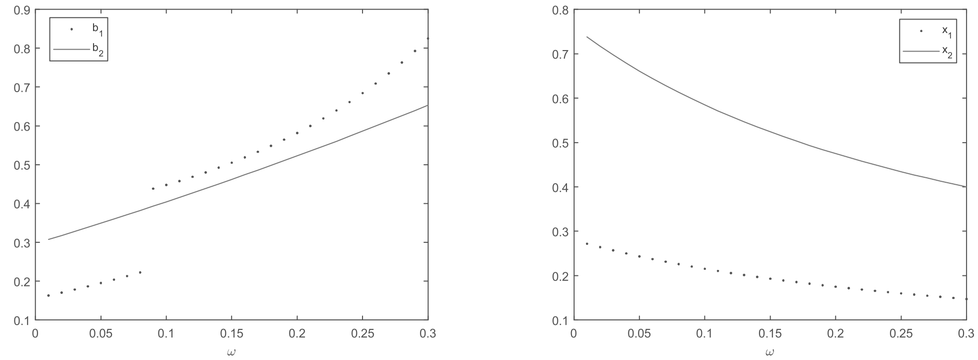
We finally analyze how the optimal contract changes when we add a bailout option. As expected, the bailout triggers increase with the bailout price. Thus a higher bailout price will lead to an earlier bailout (panel b) of Figure 7). Note that a firm with a high-cost project will ask for a bailout later than a firm with a low-cost project ($x_g(b_2) < x_g(b_1)$). When the government takes into account the bailout option in setting the remuneration of the firm, the fraction of net revenues received by the private firm decreases with the bailout price. Naturally, since now the firm has an exit option, the government does not need to offer such a high fraction of revenues to induce

[32]Since the investment triggers decrease with the spillover parameter, to ensure that the current value of the state variable is always lower than the investment triggers, we set $x_0 = 0.14$.



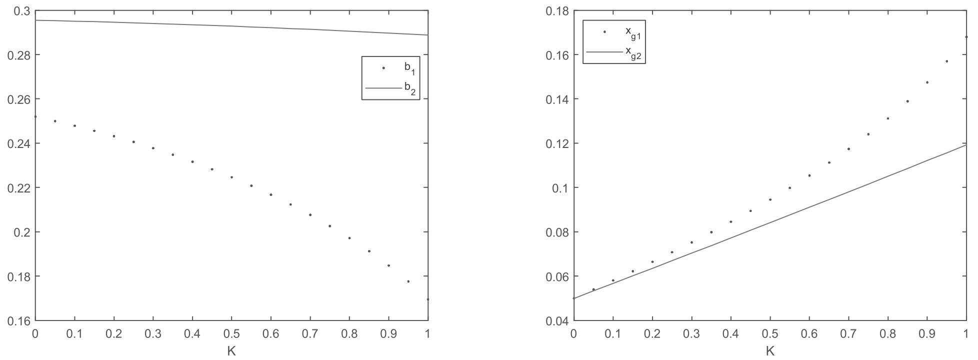
a) b_1 and b_2 b) x_1 and x_2

Figure 5: Optimal contract as a function of the spillover parameter γ ($x_0 = 0.14$).



a) b_1 and b_2 b) x_1 and x_2

Figure 6: Optimal contract as a function of the consumer surplus weight ω ($x_0 = 0.14$).



a) b_1 and b_2 b) x_{g1} and x_{g2}

Figure 7: Optimal remuneration and buyout thresholds as a function of the buyout price K.

effort. The regular compensation and the compensation upon termination are substitutes. Thus the higher the compensation upon termination the lower the regular compensation of the firm.

To see the importance of setting the regular compensation and the compensation upon termination jointly, consider the case of a "myopic" government, that sets the regular compensation irrespectively of the compensation upon termination. The regular compensation in this case is $b_1 = 0.2644$ and $b_2 = 0.2970$. This leads to a government's profit of $\Pi^G = 1.2101$, while the firm receives $\Pi^F = 0.2373$. On the contrary, if the government is not myopic, it will take into account the bailout option when setting the regular remuneration leading to lower regular remuneration, $b_1 = 0.2479$ and $b_2 = 0.2951$ for $K = 0.1$. This in turn leads to a higher government's profit $\Pi^G = 1.2724$ at the expense of the firm, $\Pi^F = 0.1750$.

8. Conclusions

In this paper, we incorporate a moral hazard problem into a real-options PPP framework, focusing on contract design and performance incentives. We show that although first-best investment timing is achievable, for a sufficiently high cost of effort, the government finds it too expensive to induce effort, resulting in high-cost projects being built. This is particularly the case for contracts characterized by high investment costs and demand risk, in line with empirical evidence on concession contracts renegotiation and cost overruns. This inefficiency can be alleviated through the use of a monitoring technology. Although monitoring distorts investment timing leading to a delayed investment, it also provides the firm with appropriate incentives by charging a penalty in case the firm shirks, thus increasing the government's profits. Finally, by jointly considering incentives and an exit option, we show that the regular compensation of the firm and its compensation upon termination act as substitutes.

We make several contributions to the literature. First, we contribute to the real options and contract theory literatures on PPPs by analyzing PPPs within a framework that combines real options with a principal-agent model, taking into account both the incentives of the private firm and the real options embedded in the contract. Second, we contribute to the growing literature on agency and information issues in a real options framework initiated by Grenadier and Wang (2005) by analyzing monitoring as a solution to a hidden action problem, as opposed to auditing as a solution to a hidden information problem. To our knowledge, these issues have not been studied before. While our benchmark results for the second-best case are not new, but consistent with previous literature, the novel results that we obtain for monitoring are in contrast with well-established results regarding auditing. Third, by extending the framework to include elements specific to PPP projects such as spillover benefits, consumer surplus and a bailout option, we derive novel results regarding the effect of monitoring on spillover benefits and consumer surplus, as well as the substitution effect between regular compensation and compensation upon termination for the bailout option. Fourth, although from a technical point of view our results for the benchmark case are not new, by adjusting the model to a PPP framework, we are able to provide a rationale for cost overruns in PPPs. This leads to empirical implications in line with empirical findings on cost overruns and PPP renegotiation. Finally, we provide important policy implications. On the one hand, we highlight that two key contract parameters, the firm's fraction of revenues and the bailout price, should be simultaneously optimized to minimize the costs of the bailout option for the society. On the other hand, it is not necessary to monitor all projects. Governmental use of low-cost monitoring technologies is important for complex projects characterized by high effort cost. Overall, both in terms of modeling and deriving novel results, as well as in terms of empirical and policy implications, our paper makes an important contribution to the existing literature.

Our model abstracts from several important aspects and our analytical framework relies on several restrictive assumptions. First, we consider a perpetual series of cash flows. In practice, although in some exceptional cases in the railways and roads sector we find projects of over 100 years, most PPP projects have a length between 20 and 30 years. Thus one should reformulate the cash flows and option valuation as finite annuities or as perpetuals less forward start options, especially for short-term contracts and low discount rates. For long-term contracts and high discount rates this is not a significant problem though. Second, we have assumed that the private firm is as efficient as the government in reducing costs. However, it might be the case that the firm is specialized in this kind of projects and has a competitive advantage over the government in reducing costs. In this case, whether the PPP project results in larger costs depends on which of the two effects dominates: the competitive advantage of the firm or the inefficiency caused by the hidden action problem. Third, we have abstracted from an important aspect of PPP projects, external debt financing. The government's task of monitoring the project might be easier in the presence of external finance since investors have the stake to incur monitoring costs, and might have the expertise and reputation for being credible monitors.