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Economic and distributional effects of different fare schemes: Evidence from the Metropolitan Region of Barcelona

Anna Matas^{a,c}, Josep-Lluís Raymond^{b,c} & Adriana Ruiz^a

^aDepartament Economia Aplicada, Universitat Autònoma de Barcelona

^bDepartament d'Economia i Història Econòmica, Universitat Autònoma de Barcelona

^cInstitut d'Economia de Barcelona

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Abstract: This paper assesses the ability of alternative fare system designs to change the modal-share of public transport use, as well as their corresponding impact on company revenues. Specifically, we provide evidence for how switching from flat to distance fares or from integrated to non-integrated tickets affects both the ridership and the financial situation of public transport companies. Secondly, because distributional concerns are at the heart of the policy debate, we evaluate the impact of the alternative fare schemes on equity. We distinguish between commuting and personal travel purposes. Focusing on the Metropolitan Region of Barcelona, our analysis shows that different pricing structures have only a moderate effect on ridership, while the potential for revenue changes is higher. Regarding equity, our results reveal that the distributional profiles of alternative pricing strategies are quite homogeneous. However, there appears to be a mild regressive effect when an integrated fare system is removed. Our results may help to guide policy decisions related to public transport pricing strategies.

Keywords: Public transport, fare schemes, integrated fares, subsidies, elasticities, distributional effects

1. Introduction

In many urban areas of the developed world, public transport is greatly subsidised. The main objective of this policy is to reduce both the level of congestion and the environmental externalities caused by private transport by shifting demand from cars to public transport. When cars do not pay for the negative externalities they cause, the subsidy is justified on second-best grounds. However, the low sensitivity of car demand with respect to public transport price highly reduces the effectiveness of such a policy. In light of these results, transport authorities have implemented fare schemes with additional incentives to promote public transport use. Among them, the integrated fare system is a common option.

Essentially, an integrated fare allows free transfers between all public transport modes operating in a geographical area within a certain period of time. The system can be based either on a flat fare – the price does not depend on distance travelled – or on a zonal fare – the price increases according to the number of zones crossed. An integrated fare system reduces both the transaction and monetary costs of the trip. Likewise, as long as there is an increase in the proportion of off-bus sales, it also manages to reduce boarding times. Thus, this pricing strategy generally succeeds in raising the total number of passengers.¹ There are a number of empirical papers, reviewed in section 2, that confirm the positive, albeit moderate, effect of integrated fares on ridership.

An integrated fare system often implies a reduction in the average revenue per passenger, however, and, if a rise in the number of users does not compensate for this decrease, an integrated fare system will require an increase in financial support. In the presence of budget constraints, transport authorities must adopt restrictive measures consisting of either reducing quality or raising the general level of fares.

The objective of this paper is to assess how different fare levels and system designs affect the mode shares of public and private transport, as well as the corresponding impact on public transport company revenues. Because the policy we want to evaluate aims at favouring more environmentally friendly modes, we focus on modal switch between public transport and car and do not account for generated or suppressed traffic. Specifically, we provide evidence for how switching from flat to distance fares or from integrated to non-integrated tickets affects both ridership and the financial situation of public transport companies. It has to be acknowledged that we can only model changes in public transport use through changes in prices; other characteristics of the fare system, such as transaction costs, cannot be accounted for.

Additionally, because subsidies are also favoured as a way to address income distribution inequality, we evaluate the impact of alternative fare schemes on equity.

¹ Chowdhury and Ceder (2016) provide a literature review of the factors that influence commuter willingness to use public transport routes with transfers.

The ultimate goal of this paper is to help policymakers make decisions related to alternative urban pricing policies.²

As a case study, we focus on the Metropolitan Region of Barcelona. With the objective of promoting public transport, the Metropolitan Transport Authority in Barcelona introduced a multimodal integrated fare system in 2001, with a zone fare structure. The fare scheme was progressively extended and at present covers 296 municipalities and 4.5 million people. The area is divided into six rings and different tariff zones. The ticket price depends on the number of zones crossed, up to a maximum of six.

The severe economic crisis that hit the Spanish economy after 2007 caused a fall in the number of passengers and, consequently, the need for a subsidy increase from 50% to 60% of total operating costs. At the same time, the budget constraint was tightened. The transport authorities reacted by taking measures both to increase revenue and to reduce costs. From 2008 to 2014, fares in real terms went up by 31.6%. The quality of services was reduced in several dimensions. At an aggregate level, for example, the total number of seats-km offered was reduced by 5% between 2011 and 2014. Although these measures were to a large scale reversed after 2014, what happened illustrates how financial problems may force transport authorities to take measures that jeopardise public transport services.

Our empirical strategy consisted of estimating a probabilistic modal choice equation between public and car transport and using the estimated equation to simulate the consequences of alternative fare systems. For each individual in the sample, we calculated the fare under the current and the simulated schemes, as well as the corresponding subsidy in each scenario. The subsidy was measured as the difference between the fare paid and the operating costs per passenger of the transport modes used. It was thus possible to compute the effect of several alternative pricing strategies

² To avoid any potential misunderstanding of our goals, it must be highlighted that this paper does not address the question of the optimal subsidy level in urban areas. There is already a vast body of literature on that topic (e.g., Small and Verhoef, 2007; Parry and Small, 2009; Basso and Silva, 2014; Kilani, Proost and van der Loo, 2014). The ultimate solution for this depends on the urban form, the degree of unpriced externalities generated by cars, and individuals' responses to fare changes, among other factors. We seek to provide insight into the last issue by looking not only at the fare level but also at alternative fare schemes.

on ridership, revenue and the distributional profile of the subsidy. Unlike other studies, we computed the effects on equity taking into account the impact of a fare change on individuals' choice between public and private transport. We also distinguished between commuting and personal travel purposes.

However, data availability has forced us to introduce some simplifications related to the potential responses to fare changes. Firstly, we have not allowed for potential changes in residential location that might follow variations in public transport pricing policy. In our view, given the characteristics of the Spanish housing market and household's expenditure on public transport, this assumption would not essentially modify our conclusions. Firstly, 80% of the population lives in owner-occupied dwellings. Secondly, urban public transport expenditure accounts for less than 1 % of total household expenditure. We can therefore expect that variations in pricing policy will not significantly affect residential location. Thirdly, we have assumed that the origin–destination matrix is fixed. That is, we do not allow for redistribution across origins and destinations after a price change. This assumption is more realistic for work-related trips than for other travel purposes. Finally, given the difficulties in measuring the public transport alternatives for those actually walking or cycling, these two transport modes are excluded from the choice set for the individuals in the sample. We will come back to this point when describing the dataset. Although the previous assumptions would certainly affect the modal choice equation, in our view they would not essentially modify the results. This will be particularly true for work related trips.

The rest of the paper is organised as follows: in Section 2, we briefly describe the related literature, and in Section 3 we show the data. Section 4 presents the estimated probabilistic model with the modal choice elasticities. Section 5 explains the preliminary calculations needed to simulate different scenarios, while Section 6 presents the results of these simulations. Finally, Section 7 provides our conclusions.

2. Related literature

This literature review focuses on two different topics directly related to our work. First, we review studies that analyse the effects of alternative fare schemes on public

transport demand. Second, we look at the research assessing the impact of different pricing strategies on income distribution.

While there are many studies that estimate the relationship between public transport demand and fares, only a few of them analyse the consequences of changing the fare schemes. Most of the available literature concerns the implementation of either an integrated fare system or a travel pass scheme. FitzRoy and Smith (1998) estimated that the introduction of two heavily subsidised travel cards in the city of Freiburg raised the number of bus trips per capita by 9.4% and 13.9%, respectively. FitzRoy and Smith (1999) reported that the impact on public transport patronage of the introduction of a season ticket scheme in four Swiss cities ranged from 4.5% to 16%. Matas (2004) examined the impacts on demand resulting from the introduction of an integrated fare system in the metropolitan area of Madrid (Spain) based on a monthly travel card. Her results suggest that the non-pecuniary effects of the travel card created an increase in bus and underground patronage of 3.4% and 5.3%, in the short run, and 7% and 15% in the long run, respectively. Abrate et al. (2007), using panel data from 69 Italian public transport companies, found that introducing an integrated fare scheme could increase the number of passengers by 2.2% in the short run and 12.0% in the long run. However, this impact varied with the specific features of the integrated tariff system. Specifically, they showed that the impact was higher when, in addition to the season ticket, an integrated ticket for single trips was offered to accommodate occasional users; a zonal pricing system was introduced to better discriminate among users according to the distance travelled and, finally, when the integrated system was extended outside the urban area. Sharaby and Shiftan (2012) reported that the introduction of an integrated fare system in Haifa (Israel) increased the number of passenger trips by 7.7%. It has to be highlighted, however, that none of the previous studies directly addresses the effectiveness of integrated fares to achieve its main objective – that is, shifting passengers from cars to public transport.³

³ Sharaby and Shiftan (2012) provide indirect evidence of people shifting from car or taxi to bus transport after the implementation of an integrated fare system. Dargay and Pekkarinen (1997) have also reported, based on direct users' responses to a survey, that the introduction of a new integrated fare system achieved a modal shift between 10% and 20% from car to bus use in Finland. However, the authors pointed out that the information was limited and there was a large degree of uncertainty.

Only a small number of papers have addressed the distributional effects of different pricing alternatives, and their results were not conclusive. Among studies carried out in US, an early work by Cervero (1981) examined several alternative fare policies in terms of efficiency and equity for three California transit operators. He concluded that switching from a flat to a differentiated fare structure by distance and time-of-day could improve the efficiency and equity of fares. Nonetheless, he pointed out that the redistributive effects of the flat fare structure appeared to be only slightly regressive and cross-subsidies tended to be rather small. A more recent paper by Farber et al. (2014) developed a new method for assessing the social equity impacts of distance-based public transport fares that took into account trip generation and distance travelled. Applying this method to a case study in Wasatch Front, Utah, they found that overall distance-based fares benefited low-income, elderly and non-white populations. However, the authors noticed that the results were not transferable to other geographical areas with different spatial demographic patterns. Sanchez et al. (2007) found that distance-based fares would harm low-income groups more than high-income ones for several US metropolitan areas.

Bandegani and Akbarzadeh (2016) quantified the effect of distance-based fare structures on equity in the public transportation system in the city of Isfahan (Iran). Using the Gini index, they showed that switching from a flat to a distance-based had a progressive effect.

Finally, Borjesson et al. (2018), using data from Stockholm, analysed the distribution of effective subsidies across population groups for several alternative fare schemes. Their main finding was that the redistribution effects among income groups were small. This result held true independently of the actual fare structure (i.e. flat fare system, distance-based fares and fares with constant subsidy rates).

Overall, extant research has concluded that the distributional effects tend to be small and that distance-based schemes seem to be more progressive than flat fare schemes. However, these findings depend on the spatial distribution of the population across the metropolitan area and on the pattern of trips.

3. Data and descriptive statistics

The data was taken from the 2006 Daily Mobility Survey, which was a large cross-sectional travel survey, representative of trips in the Metropolitan Region of Barcelona. The survey provided information at an individual level on the number and characteristics of each journey, as well as socioeconomic variables. Although the data are from 2006, it is the latest available survey which fulfils the representative requirements. This survey also defined the origin and destination of each trip at a census tract level, making it possible to approximate travel times in a very precise way. We considered several travel purposes and grouped them into two categories: commuting and personal travel. The latter category includes shopping, medical consultations, visiting friends and family, escort trips, personal business, sport activities, entertainment, day trips, and other leisure activities. Commuting was treated as a separate category because sensitivity to prices and service levels is different when individuals face tighter time schedules for their work-related journeys. We excluded business and education trips. The first category was excluded because it corresponds to personal trips in the course of work that very often involve several stages and require a private vehicle. Students were excluded because this group includes individuals below the minimum age to drive cars. The final sample contains 8,006 observations for commuting and 11,105 for personal trips. Due to dataset information, it was not possible to account for the full choice set of transport modes. Specifically, we excluded walking and cycling from the sample due to the lack of data on their corresponding public transport alternatives from the geographical information system used (GIS). Although walking absorbs a high percentage of intra-municipal non-work related trips, for the rest of the observations this percentage is low. Besides, the available literature shows that changes in public transport fares for rail and metro result in a very low diversion factor for walking trips, although for bus fares the diversion rate might take higher values;⁴ however, in our sample urban buses only account for 5% of total trips⁵. As a result, in our view, exclusion

⁴ See, for instance, Fearnley et al. (2017) and Dunkerley et al. (2018).

⁵ This percentage has been calculated for the whole metropolitan area including both motorised and non-motorised trips

of the walking option will not essentially modify how a change in public transport fares affects the demand for private cars, which is the main focus of our work.

The dependent variable is a binary variable that takes the value 1 for public transport and 0 for private transport. For commuting purposes, 38% of individuals used public transport, and this fell to 33% for personal travel. However, if we look at public transport use according to income level (Table 1), it becomes clear that public transport use decreases as income increases. Besides, when using public transport, commuters make longer trips, cross a greater number of zones, and make more transfers between modes than other travellers.

Table 1. Market shares and main characteristics of public transport trips

	Commuting	Personal travel
Mode choice by income (monthly household income, €)		
Less than 1000	58.3%	58.3%
1000-2000	40.1%	34.3%
2000-3000	36.1%	24.2%
3000-4000	33.1%	22.5%
4000-5000	29.7%	24.3%
More than 5000	30.2%	13.7%
Total	38.4%	33.4%
Average trip length (kms)	11.8	8.3
Number of zones crossed		
1 zone	80.7%	89.4%
2 zones	12.1%	6.7%
3 or more zones	7.2%	3.9%
Number of transfers*		
No transfer	67.6%	78.8%
1 transfer	28.8%	19.1%
2 or more transfers	3.5%	2.1%

* Changes between metro lines are not counted as transfers

Source: Daily Mobility Survey, 2006

The explanatory variables were selected according to standard practice. First of all, each trip was characterised by the time costs both by public and private transport. Travel time matrices were constructed using a geographical information system according to the minimum travel time route observed between origin and destination, which were defined at census-tract level. For public transport, we considered access and egress time, in-vehicle time and waiting time. All variables included the costs of the subsequent

modes used to complete the trip. For private transport, we only considered in-vehicle time. Given the lack of information, we assumed that access and waiting time took the same value for all individuals in the sample. Regarding public transport waiting time and private in-vehicle time, we computed different time matrices for peak and off-peak hours and assigned the corresponding value according to the timing of the trip⁶. Table 2 provides the descriptive statistics. Note that there were some extreme values for waiting time because we measured waiting time as half the headway, even when frequency was lower than 30 minutes.⁷

Table 2. Travel time for public and private transport (in minutes)

Travel time (minutes)					
Variable	Commuting				
	Obs	Mean	Std. Dev.	Min	Max
In vehicle time (private)	8,006	18.62	12.36	0.41	82.67
peak	4,865	20.01	12.26	1.81	78.54
off-peak	3,141	16.47	12.19	0.41	82.67
In vehicle time (public)	8,006	23.28	17.06	1.00	124.00
Access and egress time (public)	8,006	24.34	15.16	2.73	99.89
Waiting time (public)	8,006	9.31	10.44	0.71	220.00
Variable	Personal travel				
	Obs	Mean	Std. Dev.	Min	Max
In vehicle time (private)	11,105	13.50	11.63	0.57	120.17
peak	2,137	16.61	12.04	2.02	81.54
off-peak	8,968	12.76	11.41	0.57	120.17
In vehicle time (public)	11,105	18.57	15.68	1.00	126.50
Access and egress time (public)	11,105	21.88	15.50	2.62	99.17
Waiting time (public)	11,105	7.90	8.27	0.71	128.00

Source: Time values computed using a Geographical Information System (GIS)

The second explanatory variable included in the choice model was the monetary cost. Unfortunately, with respect to public transport, no information was available about the type of fare used by each individual. To solve this limitation, we selected the ten-ride multimodal ticket which corresponds to the most common type of ticket (72% of all

⁶ Another important determinant of the choice between public and private transport is the number of inter-modal or intra-modal changes required. So, we included the number of transfers as an additional explanatory variable. However, this variable didn't show to be significant, pointing out that, in our case, the disutility of transfers was already accounted for by the access and waiting time variables.

⁷ Alternatively, we used the rule of half of the headway only for those services with headways lower than 30 minutes and included a dummy variable for those with lower frequencies. The estimation results were very similar.

tickets used, including social titles) and allows transfers between up to three public transport modes for a limited period of time. The price paid for the ticket depends on the number of zones travelled. As shown in Table 3, the fare increases less than proportionally with the number of zones traversed. It is interesting to note that the average fare paid per kilometre travelled clearly decreases with the number of zones crossed. For those individuals paying the one-zone fare, the average fare was 13 cents per kilometre for commuters and 17 cents for other travel purposes, whereas for those travelling across four zones, the average fare was only 5 cents for both groups.

For private transport, monetary costs depend essentially on the price of fuel and the distance travelled. Because we used a cross-section and no information was available about vehicle characteristics, it was necessary to assume the same fuel price for all individuals. In this case, the monetary cost was directly proportional to distance and, therefore, highly correlated with time cost. We decided not to include the monetary cost for private transport and, consequently, the time cost coefficient captures both time and monetary cost.

Regarding socioeconomic variables, the Daily Mobility Survey reported age, level of education, gender and income at individual level. Income referred to monthly household income and was divided into six categories from less than 1000€ to more than 5000€. The main descriptive statistics for these variables are in Table A.1 in Appendix 1.

Table 3. Fare levels

Zone	Fare (€)	Fare per km by transport zone (€)	
		Commuting	Personal travel
1	0.66	0.134	0.172
2	1.33	0.089	0.102
3	1.83	0.056	0.057
4	2.35	0.053	0.050
5	2.69	0.044	0.043
6	2.89	0.038	0.059

Source: Metropolitan Transport Authority

4. Probit equation and elasticities

4.1 Probit estimation

We estimated a probit equation where the dependent variable took the value 1 for public transport and 0 for private car. Those individuals for whom public transport modes were not available were excluded from the sample.

Table 4. Probit estimation results (Public transport=1)

	COMMUTING			PERSONAL TRAVEL		
Transport mode attributes	Coef.	Std. Err.	z	Coef.	Std. Err.	z
In-vehicle time (private)	0.0454	0.0031	14.59	0.0362	0.0033	11.04
In-vehicle time (public)	-0.0163	0.0024	-6.94	-0.0082	0.0022	-3.70
Access/egress time (public)	-0.0209	0.0031	-6.67	-0.0222	0.0025	-8.94
Waiting time (public)	-0.0930	0.0088	-10.62	-0.1187	0.0080	-14.78
Fare	-0.2604	0.0733	-3.55	-0.4172	0.0807	-5.17
Socioeconomic variables						
Age	-0.0859	0.0098	-8.81	-0.0544	0.0060	-9.01
Age square	0.0010	0.0001	8.88	0.0007	0.0001	10.99
Education (reference category: no degree)						
Primary education	-0.3209	0.1654	-1.94	-0.1637	0.0838	-1.95
Secondary education	-0.2073	0.1657	-1.25	-0.0843	0.0917	-0.92
University degree	-0.0024	0.1637	-0.01	-0.0165	0.0958	-0.17
Gender (female=1)	0.5415	0.0357	15.16	0.3697	0.0357	10.35
Household monthly income (reference category: less than 1000€)						
1000–2000	-0.3685	0.0774	-4.76	-0.3460	0.0448	-7.73
2000–3000	-0.5735	0.0811	-7.08	-0.6167	0.0525	-11.74
3000–4000	-0.7619	0.0880	-8.65	-0.7153	0.0720	-9.93
4000–5000	-0.9298	0.1075	-8.65	-0.6963	0.0936	-7.44
More than 5000	-0.9484	0.1294	-7.33	-0.9545	0.1176	-8.11
Trip purpose (reference category: daily shopping)						
Occasional shopping				0.3621	0.0661	5.48
Medical consultations				0.6287	0.0649	9.69
Visiting friends and family				0.3336	0.0602	5.54
Escort trips				-0.1929	0.0635	-3.04
Personal business				0.5404	0.0646	8.36
Sports activities				0.1445	0.0946	1.53
Entertainment				0.4639	0.0943	4.92
Other leisure activities				0.2056	0.0664	3.09
Day trips				0.4060	0.0794	5.11
Constant term	2.1655	0.2833	7.64	1.3203	0.2172	6.08
Nº observations	8,006				11,105	
Pseudo R ²	0.2485				0.2755	

Note: Standard errors are clustered at the origin zone to take into account potential problems derived from the autocorrelation and/or heteroscedasticity of the random disturbance terms

As shown in Table 4, all the coefficients take the expected sign and are statistically significant. As expected, the probability of using public transport increases with in-vehicle time for private transport and decreases with access, in-vehicle and waiting time

for public transport. Commuters are also more sensitive to variations in in-vehicle time, while individuals travelling for personal purposes are more sensitive to fare increases.

With respect to individual characteristics, age has a non-linear effect, so that the probability of using public transport peaks for individuals aged around 40 for both samples. Additionally, the probability of using public transport falls with the level of income and is higher for women. Once we take into account household income, the degree of education does not appear to affect significantly the modal choice between public and private transport.

For personal travel purposes, we included a dummy variable to account for the specific purpose of the trip. Taking daily shopping as the reference category, everything else being equal, the highest probability of using public transport corresponds to trips for medical consultation and the lowest to those made to accompany someone else.

Because household decisions regarding residential location are not exogenous, the estimated equation may face a problem of endogeneity. That is, the explanatory variables that capture journey costs may be correlated with the error term of the equation and, if this is the case, the estimated coefficients may be biased. Dealing with this econometric problem using Instrumental Variables is difficult due to the lack of sufficiently good instruments. In this study, we tried to soften this correlation by including in the equation a set of variables reflecting the degree of satisfaction of travellers with the available transport modes. The mobility survey included a questionnaire related to individual attitudes towards different transport modes, which included their degree of satisfaction.

For each individual we included their degree of satisfaction with public transport, private transport and walking. The results are presented in Table A.2 in Appendix 1. In both equations, the three variables take the expected sign and are highly statistically significant, whereas the coefficients for the time and fare remain approximately constant. This procedure provides some evidence that the problem of endogeneity is not severe in this equation.

4.2 Elasticities

From the estimation results of the probit model, we computed the modal choice elasticities for the variables of interest in the equation.⁸ The average elasticity was calculated by weighting the individual elasticity values with the expected probability of choice.

Table 5. Modal choice elasticities of public transport

	Commuting	Personal travel
Public transport fare	-0.172	-0.263
Public transport in-vehicle time	-0.255	-0.107
Public transport access and egress time	-0.337	-0.322
Public transport waiting time	-0.483	-0.531
Private transport in-vehicle time	0.622	0.361

Demand appears to be inelastic with respect to all transport mode attributes. The magnitude of the own-price elasticity is in line with available evidence from mode choice models. For instance, for the metropolitan area of Barcelona, Matas (1991) and Asensio (2002) also estimated, respectively, a price elasticity of public transport equal to -0.15 and -0.21 for commuting trips. More recently, from a revision of 41 studies, Fearnley et al. (2017) reported a modal choice elasticity equal to -0.22 for commuting and -0.31 for leisure. Using data from Greater Oslo, the same authors estimated own-price elasticities equal to -0.23 and -0.32 for the same groups of travellers.

Individuals are more sensitive to changes in travel time than to the cost of public transport fares. Among public transport travel times, they are more sensitive to waiting time, access/egress time and, finally, in-vehicle time. These results agree with those reported by Frank et al. (2008) and Paulley et al. (2006). Additionally, people are less sensitive to fares and more sensitive to in-vehicle time as commuters than as non-work travellers. The latter is explained by the tighter time constraints faced by those travelling to work. Increasing private in-vehicle time by 1% is associated with an increase in public transport use of 0.62% for work trips and 0.36% for non-work trips. Fearnley et al. (2017)

⁸ Modal choice elasticities correspond to the change in probability of choosing a particular alternative with respect to a given percentage change in an attribute of this alternative, holding fixed the total number of trips. In our case, we have calculated the elasticities for a 1% increase in current fares.

also showed that the cross-elasticity of public transport demand with respect to private car time is higher for commuter trips.

According to Wardman (2014), our estimated modal choice elasticities are a reasonable approximation to ordinary elasticities for commuting trips. For other travel purposes, the potential existence of a generation/suppression effect might cause an underestimation of ordinary elasticities.

Overall, the estimated elasticities support the conventional view that to obtain significant modal shift from car to public transport considerable reductions in public transport fares will be necessary. Improving public transport speed and access time would achieve better results. Otherwise, any reduction in travel time for cars can easily favour a modal change.

5. Simulation of changes in fares: Preliminary calculations

Once the probit model was estimated, the next step was simulating the impacts of alternative fare schemes and levels. First, we simulated the consequences on ridership and the corresponding impact on the revenues of transport companies. Second, we compared the distributional effects of different fare systems across population groups. Regarding the latter, as a preliminary step, we calculated the subsidy for each trip.

For each individual in the sample, we computed the subsidy as the producer costs minus the fare paid.

$$S_i = C_i - F_i$$

Production costs were calculated according to the operating costs for each transport mode, including depreciation and maintenance for vehicles. We used data from operating companies and different transport authorities. Due to difficulties in measurement, infrastructure costs for road and rail were not included. Although the latter could be considered a caveat, we applied homogeneous treatment to rail and road costs.

To compute unit operating costs, we assumed constant returns to scale and split annual costs over all passenger-kilometres on the service.

$$C_k = \frac{(\text{annual operating costs})_k}{(\text{passenger} - \text{km})_k}$$

where k is the transport mode. Therefore, the unit transport costs are dependent on the specific operating costs of the mode but also on the passenger-kilometre. The higher the number of passengers using a mode, the lower the unit cost and, in turn, the subsidy required. Table A.3 in Appendix 1 presents the unit operating costs for each public transport mode.

For each individual, the total trip cost was computed by multiplying the cost per passenger-kilometre by the journey length. When more than one mode was needed, we added the specific costs for each mode according to the kilometres travelled.

$$C_i = \sum_k C_k * t_{ik}$$

where t_{ik} is the distance travelled by mode k . Finally, we calculated the subsidy paid for each individual trip as total operating costs less the fare paid. The subsidy therefore depends on the transport modes used, the load factor of each mode, the trip length and the fare paid.

It has to be acknowledged that simplifying assumptions about transport costs, forced by data limitations, may affect the distributional impacts of the fare changes. Firstly, it may be argued that when breaking costs down at the passenger level, we should distinguish according to time of day to account for higher costs during peak hours and also higher load factors. Secondly, for each transport mode, the operating costs are spread evenly across passenger-kilometres. However, it may well be that costs and load factors differ between different segments along the service line. Operating costs may be lower in peripheral segments due to higher speeds but, at the same time, load factors would be lower. Finally, operating costs are based on the costs reported by the firms. Hence, we are assuming that the different companies operate with the same level of efficiency. More precise data on costs would allow us a better approximation of unit cost and,

accordingly, to account for potential cross-subsidisation between times of day or segments of a service line, as well as for different efficiency levels.⁹ However, given that we take into account both the transport modes used and the distance travelled, the unit costs used can be considered a reasonable measure of the true unit costs.

Figure 1 presents the histograms of subsidies in the commuting and personal travel samples. The average subsidy was 0.93€ for commuters and 0.64€ for personal travel purposes that represent, respectively, 52% and 46% of total operating costs. The lower subsidy for personal travel purposes is mainly explained by the shorter trip distance. As plotted in the histograms, there are significant differences between individuals. For instance, 23% of personal travel trips and 15% of commuting trips are not subsidised, but pay a fare higher than the cost. Therefore, it could be argued that short-distance travellers subsidise long distance travellers. The average subsidy for different groups of population is shown in Table A.4. For commuters, the data show a higher subsidy for men, for those aged less than 44, with primary or secondary education and living in the inner suburbs. The pattern is similar for personal travel trips, although people aged less than 29 with primary education benefit more from subsidies.

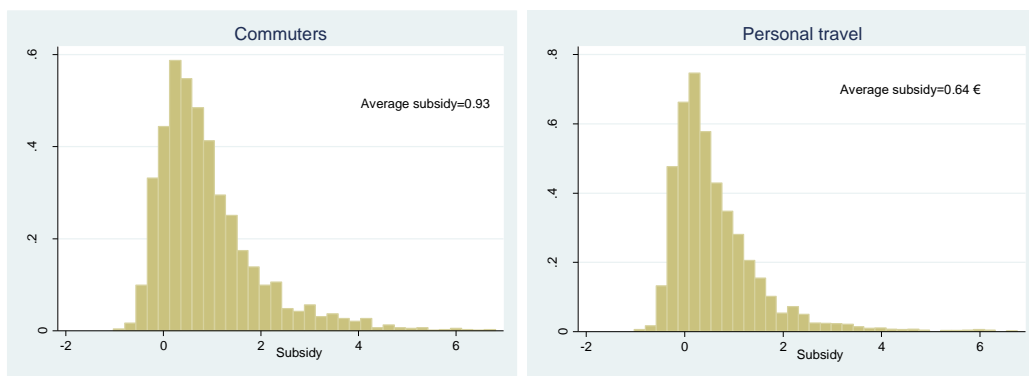


Figure 1. Distribution of public transport subsidies per trip

6. Simulation of changes in fare schemes: Results

We evaluated the effects of changing the fare schemes and levels by simulating different scenarios. When comparing scenarios, the fact that both the fare level and the fare

⁹ Note that the assignment of fixed costs between different time periods and segments of a service line is always a complex task involving a certain degree of arbitrariness.

structure affect the total number of passengers should be taken into account. In the first scenario, we were interested in the impact of a change in the price level, whereas in Simulations 2 and 3 we assessed the effects of alternative fare designs, keeping constant the average fare per passenger. Finally, to simulate a change to a non-integrated fare system, we recovered the structure and level of prices for all transport modes before the integrated fare scheme was implemented in 2006.

- Scenario 1: Extension of the multimodal flat fare to the entire metropolitan area with a price equal to the current fare in Zone 1 (0.66€). This scenario responds to the current policy of the transport authority to extend the Zone 1 fare outside of its initial limits.
- Scenario 2: Extension of a multimodal flat fare to the entire metropolitan area with a price equal to the average observed fare per passenger. In this scenario, while setting the same flat fare for the whole area, the decrease in price for those crossing more than one zone is compensated for its increase for those travelling within one zone. In this way, the average fare is kept constant so that we can evaluate the implications of setting a single fare for the whole area. Because the pattern of trips and the distance travelled is different for the work-related and non-work-related samples, so is the average price paid. For commuters the average fare in 2006 was 0.845€, whereas for non-commuters it was 0.766€.
- Scenario 3: Substitution of the current zonal system with a multimodal distance-based fare. The price per kilometre was computed so that the average fare paid remained equal to the observed one in each subsample. That implied 0.072€/km for commuters and 0.092€/km for non-commuters. Thus, the implications derived from the new scenario on ridership, company revenues, and income distribution depend on the pattern of trips.
- Scenario 4: Suppression of the integrated fare system. We simulated the level and the fare scheme existing in 2000, prior to the fare integration. A flat fare was available for bus, underground, and tram in Barcelona and the surrounding municipalities. Rail and interurban buses charged a distance-based or a zonal

fare, while each municipality charged a specific price for their urban bus services. Only transfers between metro lines were free. We recovered all prices set in 2000 and adjusted them according to the Consumer Price Index to obtain the 2006 values.

6.1. Effects on passengers and revenue

Overall, the results detailed in Table 6 suggest that changing fares has a relatively small effect on the number of passengers, as could be expected from the low elasticity values. However, some policies do have a significant impact on revenues. For commuters, the extension of the current flat fare to the whole area would increase the expected number of passengers by 4%, while revenue per passenger would drop by 17% with a final effect on company revenue of -14%. If the flat fare is set equal to the average price paid per passenger (Scenario 2), the effect on the number of public transport users is very low, while a slight increase in revenue can be expected. Moving to a distance-based system while keeping constant the average fare (Scenario 3) does not essentially affect the total number of passengers but greatly reduces the revenue per passenger and, consequently, revenue for the company. The explanation for such a drop is that distribution of trips is clearly asymmetric with a high percentage of short-distance trips. Although the average revenue per commuter is 0.845€, the mode is only 0.34€. Probably, a purely distance-based fare system would not be a realistic option. A fixed amount should be added in order to account for the fixed cost each traveller causes to the company. Our objective, however, is to highlight the differences between alternative fare systems. Finally, suppressing the integrated fare system and returning to the system in force before 2006 would reduce demand by 3.6% and increase revenue per passenger by almost 11%, with a net effect on company revenue of almost 7%.

Although the pattern for non-work trips is similar, some specific traits are shown. First, extending the current flat fare to the whole area has a lower impact on revenue per passenger given that, as shown in Table 1, only 10.6% of travellers cross more than one zone compared with 19.3% for commuters. Second, due to the shorter length of their trips, a purely distance-based scheme would cause a drop in passenger revenue which would double that of commuters. Thirdly, removing the integrated fare system would

have a moderate increase in revenue per passenger because the percentage of transfers for personal travel is lower than for commuters (Table 1).

Table 6. Impacts on public transport passengers (PT) and revenue

	Commuting			Personal travel		
	Before	After	Percentage change	Before	After	Percentage change
SIMULATION 1						
PT users (%)	0.401	0.418	4.2%	0.352	0.368	4.7%
PT revenues (€ per trip)	0.335	0.278	-17.1%	0.267	0.245	-8.3%
SIMULATION 2						
PT users (%)	0.401	0.404	0.7%	0.352	0.357	1.4%
PT revenues (€ per trip)	0.335	0.341	1.8%	0.267	0.273	2.3%
SIMULATION 3						
PT users (%)	0.401	0.404	0.7%	0.352	0.355	1.0%
PT revenues (€ per trip)	0.335	0.300	-10.5%	0.267	0.213	-20.2%
SIMULATION 4						
PT users (%)	0.401	0.387	-3.6%	0.352	0.340	-3.2%
PT revenues (€ per trip)	0.335	0.371	10.7%	0.267	0.270	1.2%

6.2. Distributional effects

To analyse the distributional effects of changing the fare structure, we calculated the subsidy per individual under the four alternative scenarios.¹⁰ That is, we took into account how individuals would respond to changes in fares. Our approximation consisted of calculating the subsidy in the disturbed solution (simulated scenario) as the sum of the observed subsidy plus the difference between the expected subsidy in the disturbed solution and the expected subsidy in the control solution (base case). The justification for this procedure is detailed in Appendix 2.

Table 7 presents the average subsidy per trip by income group in the base case and in each of the four scenarios, together with the difference with respect to the base case. The first column shows that the subsidy clearly decreases as the income level increases. This result is driven almost entirely by the frequency of use of public transport. In Table A.5 in the Appendix 1, we report the results of a regression equation that makes it possible to test if there are statistical differences in the average subsidy per income

¹⁰ To adequately compute the full distributional effect, we should also take into account the degree of progressivity of the taxes used to finance the subsidies. Because our main interest lies in the change of the distributional profile of subsidies, the distributional effect of taxes in our case can be considered neutral.

groups for those using public transport. As can be observed, no statistically significant differences are shown in either of the two samples. We may therefore say that the progressive effect of subsidies is due to the fact that public transport use is higher for low-income groups. No other potential variables such as the distance travelled or the public transport modes used seem to have any effect across different income groups.

Table 7. Average subsidy per income level (euros per trip)

Commuting									
Income	Base case	Sim. 1	Difference	Sim. 2	Difference	Sim. 3	Difference	Sim. 4	Difference
Less than 1000	0,535	0,639	0,104	0,518	-0,018	0,571	0,036	0,427	-0,108
1000-2000	0,389	0,497	0,108	0,407	0,018	0,381	-0,008	0,305	-0,084
2000-3000	0,331	0,448	0,117	0,364	0,033	0,311	-0,020	0,248	-0,083
3000-4000	0,304	0,431	0,127	0,351	0,048	0,274	-0,029	0,222	-0,082
4000-5000	0,221	0,326	0,105	0,255	0,034	0,216	-0,005	0,158	-0,063
More than 5000	0,236	0,360	0,124	0,285	0,049	0,187	-0,049	0,136	-0,100

Personal travel									
Income	Base case	Sim. 1	Difference	Sim. 2	Difference	Sim. 3	Difference	Sim. 4	Difference
Less than 1000	0,354	0,462	0,108	0,392	0,038	0,360	0,006	0,295	-0,059
1000-2000	0,229	0,309	0,080	0,263	0,034	0,220	-0,009	0,184	-0,045
2000-3000	0,143	0,208	0,065	0,173	0,030	0,135	-0,008	0,113	-0,030
3000-4000	0,155	0,224	0,069	0,191	0,037	0,148	-0,006	0,128	-0,027
4000-5000	0,197	0,274	0,077	0,238	0,041	0,200	0,003	0,168	-0,029
More than 5000	0,087	0,147	0,060	0,123	0,037	0,072	-0,015	0,060	-0,026

Looking at the distributional effects derived from changing the fare scheme, we observe that the distributional profiles of the different alternatives are very similar. The differences between the simulated scenario and the base case are rather small in all cases. For commuters, extending the current flat fare over the whole area yields a pretty homogenous increase in the average subsidy among income groups with a very mild regressive effect. When the flat fare is set equal to the average price per passenger, the magnitude of the effect is lower and the regressive effect persists. It is worth pointing out that for the poorest group the subsidy received decreases due to the fact that their trip length is shorter and the percentage of trips they make within one zone is higher. Thus, on average they pay a higher fare than in the base scenario. Likewise, the distance-based fare reduces the need for a subsidy for all income groups, except for the poorest one which causes a mild progressive effect. Finally, eliminating fare integration between transport modes reduces the subsidies for each income group, with a slight regressive impact on income distribution.

The pattern of changes is similar for personal travel purposes but some particular features arise. The implementation of a flat fare system suggests a progressive effect when the fare is set equal to the current level and neutral when the fare increases up to the average price paid. For the scenario with a distance-based fare, the impact on average subsidy is almost nil and so too are the distributional effects. Moreover, the removal of the integrated fare system leads to a milder regressive effect than in the case of work trips, but with a clearer pattern across income groups.

7. Conclusions

This paper confirms that travellers are more sensitive to changes in quality –mainly waiting and access time – than to changes in prices. Any reduction in quality will thus quickly imply a decrease in public transport use counterbalancing any effect of fare decreases. Moreover, the insensitivity to fares is higher when commuting than when travelling for personal purposes. The simulation exercises carried out show that different pricing structures – flat fares, distance-based and integrated tickets – have only a moderate effect on demand while the potential for revenue changes is higher.

Computing the effective subsidy for each individual in the sample confirms that subsidies have a progressive effect, albeit moderate. This effect is explained almost entirely by the fact that the use of public transport is higher for low-income groups. No other variables such as the distance travelled or the public transport modes used seem to play a significant role. Additionally, commuters benefit from higher subsidies compared to individuals travelling for other purposes. The simulated distributional profile of alternative pricing strategies appears to be quite homogenous among income groups. There appears, however, to be a mild regressive effect when removing the integrated fare scheme or extending the flat fare to the whole area.

The choice of either the pricing scheme or the level of subsidy corresponds to the political arena. However, our results provide guidance on the consequences of implementing alternative fare schemes. An extension of a flat fare system can, for example, negatively affect the financial situation of the operating companies without

succeeding in shifting travellers from private car to public transport. Nonetheless, decisions on pricing structure have to be based on a welfare analysis that takes into consideration a full range of factors beyond the scope of this paper. Our goal has been to provide useful information to help in the decision-making process.

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APPENDIX A. Complementary Tables

Table A.1. Descriptive statistics for socioeconomic variables

	Commuting		Personal Travel	
	Freq.	Percent	Freq.	Percent
Age				
16-29	1,564	19.54	2,074	18.68
30-44	3,940	49.21	4,107	36.98
45-64	2,461	30.74	3,328	29.97
65 -79	41	0.51	1,596	14.37
Total	8,006	100	11,105	100
Gender				
Male	4,399	54.95	4,959	44.66
Female	3,607	45.05	6,146	55.34
Total	8,006	100	11,105	100
Education				
No degree	108	1.35	448	4.03
Primary education	2,041	25.49	3,673	33.08
Secondary education	3,140	39.22	4,090	36.83
University degree	2,717	33.94	2,894	26.06
Total	8,006	100	11,105	100
Monthly household income (€)				
Less than 1000	525	6.56	1,829	16.47
1000-2000	3,330	41.59	4,435	39.94
2000-3000	2,539	31.71	3,028	27.27
3000-4000	1,027	12.83	1,089	9.81
4000-5000	337	4.21	374	3.37
More than 5000	248	3.1	350	3.15
Total	8,006	100	11,105	100
Travel purpose				
Daily shopping			1,220	10.99
Occasional shopping			791	7.12
Medical consultations			1,071	9.64
Visiting friends and family			2,156	19.41
Escort trips			1,868	16.82
Personal business			1,439	12.96
Sports activities			462	4.16
Entertainment			283	2.55
Other leisure activities			1,311	11.81
Pleasure trips			504	4.54
Total			11,105	100

Source: Daily Mobility Survey, 2006

Table A.2. Estimation results including the degree of satisfaction

	Commuting			Personal travel		
	Coef.	Std. Err.	z	Coef.	Std. Err.	z
Transport mode attributes						
In vehicle time (private)	0.0465	0.0033	14.3	0.0360	0.0033	11.1
In vehicle time (public)	-0.0174	0.0024	-7.2	-0.0086	0.0022	-3.9
Access time (public)	-0.0211	0.0033	-6.4	-0.0222	0.0026	-8.6
Waiting time (public)	-0.0900	0.0085	-10.6	-0.1144	0.0077	-14.8
Fare	-0.2389	0.0745	-3.2	-0.4094	0.0800	-5.1
Socioeconomic variables						
Age	-0.0941	0.0104	-9.0	-0.0614	0.0062	-10.0
Age square	0.0011	0.0001	9.0	0.0007	0.0001	11.3
Gender (female=1)	0.5083	0.0359	14.2	0.2721	0.0369	7.4
Education (reference category: no degree)						
Primary education	-0.2337	0.1744	-1.3	-0.1096	0.0846	-1.3
Secondary education	-0.1159	0.1750	-0.7	0.0101	0.0920	0.1
University degree	0.0428	0.1738	0.3	0.0503	0.0966	0.5
Household monthly income (reference category: less than 1000€)						
1000-2000	-0.3471	0.0796	-4.4	-0.3053	0.0478	-6.4
2000-3000	-0.5375	0.0864	-6.2	-0.5724	0.0552	-10.4
3000-4000	-0.7309	0.0909	-8.0	-0.6636	0.0777	-8.5
4000-5000	-0.8925	0.1109	-8.1	-0.6607	0.0963	-6.9
More than 5000	-0.9083	0.1358	-6.7	-0.8986	0.1267	-7.1
Trip purpose (reference category: daily shopping)						
Occasional shopping				0.3300	0.067684	4.88
Medical consultations				0.6122	0.067705	9.04
Visiting friends and family				0.3256	0.058415	5.57
Escort trips				-0.1997	0.063378	-3.15
personal business				0.4868	0.064536	7.54
Sport				0.1264	0.096838	1.31
Cultural activities (entertainment)				0.4284	0.092351	4.64
Other leisure				0.2030	0.066315	3.06
Pleasure trips				0.4506	0.079615	5.66
Degree of satisfaction with						
Private transport	-0.1805	0.0100	-18.1	-0.2103	0.010021	-21.0
Public transport	0.0812	0.0087	9.3	0.1134	0.013475	8.4
Walking and cycling	0.0895	0.0126	7.1	0.0980	0.008802	11.1
Constant term	2.2635	0.3137	7.2	1.4685	0.227912	6.4
Nº observations	8006			11105		
R ²	0.2485			0.2755		

Table A.3. Transport costs per passenger-km (€)

Interurban buses	
Origen or destination in Barcelona	0.0972
Outside Barcelona	0.1591
Urban buses	
Barcelona	0.4022
Barcelona conurbation	0.3117
Other urban buses	
<i>Population > 100,000 inhab</i>	0.2879
<i>20.000 - 100.000 inhab</i>	0.5047
<i>Less than 20.000 inhab</i>	0.8592
Rail	
Metro	0.1394
Tram	0.2632
Train (Renfe)	0.0757
Train (FGC)	0.1073

Table A.4. Average subsidy for different groups of population

	Commuting	Personal travel
Gender		
Men	1.009	0.706
Women	0.88	0.601
Age		
16-29 years	0.948	0.768
30-44 years	0.972	0.615
45-64 years	0.87	0.628
65-74 years	0.695	0.62
More than 75 years	-	0.525
Education		
No degree	0.771	0.598
Primary education	0.982	0.725
Secondary education	0.948	0.63
University degree	0.896	0.542
Residential area		
Barcelona	0.633	0.455
Inner suburbs	1.263	0.904
Outer suburbs	0.982	0.746

Table A.5. Average subsidy for income groups (public transport users)

	Commuting		Personal travel	
	Coefficient	t	Coefficient	t
Less than				
1000	0.919	14.06	0.607	20.56
1000-2000	0.052	0.72	0.060	1.55
2000-3000	-0.001	-0.02	-0.016	-0.35
3000-4000	-0.002	-0.02	0.080	1.17
4000-5000	-0.174	-1.32	0.202	1.91
More than				
5000	-0.138	-0.94	0.026	0.18
Observations	3072		3705	

APPENDIX B. Computing changes in subsidy for each simulation

As stated in the main text, we needed to compute the subsidy after each simulation. We calculated the subsidy in the disturbed solution as the sum of the observed subsidy plus the difference between the expected subsidy in the disturbed solution and the expected subsidy in the control situation. To do so, we used the following procedure:

- we defined the observed subsidy as the sum of the expected subsidy in the base case (control solution) plus a fixed effect term that makes it possible to recover the observed subsidy;
- we computed the expected subsidy in the simulated scenario (disturbed solution);
- we defined the subsidy in the disturbed solution as the sum of the expected subsidy plus the fixed term (which we assumed was the same in both situations); and
- finally, the subsidy in the disturbed solution was obtained as the observed subsidy plus the difference in the expected values.

$$\begin{aligned} S_i &= (c_i \cdot \pi_i - p_i \cdot \pi_i) + [(c_i - p_i) \cdot D_i - (c_i \cdot \pi_i - p_i \cdot \pi_i)] \\ &= (c_i \cdot \pi_i - p_i \cdot \pi_i) + u_i \\ \hat{S}_i &= (c_i \cdot \pi_i^* - p_i^* \cdot \pi_i^*) + u_i \end{aligned}$$

where,

$$(c_i \cdot \pi_i - p_i \cdot \pi_i) = \text{Expected subsidy control solution}$$

$$(c_i - p_i) \cdot D_i = \text{Observed subsidy}$$

$$(c_i \cdot \pi_i - p_i \cdot \pi_i) = \text{Expected subsidy}$$

$$u_i = \text{Individual fixed effect}$$

$$(c_i \cdot \pi_i^* - p_i^* \cdot \pi_i^*) = \text{Expected subsidy disturbed solution}$$

then,

$$\hat{S}_i - S_i = (c_i \cdot \pi_i^* - p_i^* \cdot \pi_i^*) - (c_i \cdot \pi_i - p_i \cdot \pi_i)$$

$$\hat{S}_i = S_i + c_i(\pi_i^* - \pi_i) - p_i^* \cdot \pi_i^* + p_i \cdot \pi_i$$

Here c_i is the operating cost, p_i is the fare paid under the control solution, π_i is probability of using PT in the control solution, d is a dummy variable that takes the value 1 if the individual chooses PT and 0 otherwise and p_i^* and π_i^* are the fare and probability in the disturbed solution.