

## Revisiting the Role of Irradiance in the Determination of Sunscreens' Sun Protection Factor



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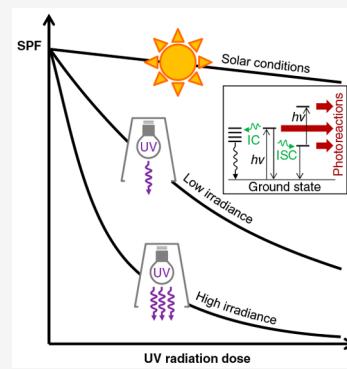
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**ABSTRACT:** The efficacy of a sunscreen tends to be associated with its sun protection factor (SPF) value, a figure determined in a test that relies on the independence of the SPF value to both UV radiation dose and irradiance. We probe these assumptions when photoinduced product degradation is present, and we estimate that the theoretical limit for their validity is when the sunfilter active molecule relaxation time is faster than  $\sim 10$  ns. While such threshold relaxation time should be compatible with the expected ultrafast relaxation mechanisms of sunfilter molecules (picoseconds), recent research on sunfilter photodynamics has identified the existence of much longer-lived molecular states. Such long lifetimes could compromise sunscreen performance and make the SPF value very different in natural sun irradiance conditions than in the solar simulated conditions typically used in SPF determination tests.



The sun protection factor (SPF) value has become a standard to benchmark UVB protection for sunscreens.<sup>1–4</sup> Routine SPF tests implicitly assume a linear response between the transmitted and irradiated doses (the ratio being the SPF) because the individual patients' SPFs are averaged and those falling outside a confidence interval are either dismissed or the averaged SPF value is deemed not valid. SPF tests also rely on the so-called reciprocity law, where the SPF value is assumed to be independent of the irradiance used for measuring it.<sup>2,5,6</sup>

The function of a sunscreen is to reduce incident solar radiation as it traverses the product in such a way that the transmitted radiation is lessened.<sup>1,3,7–11</sup> This is mostly achieved through scattering and absorption of UV radiation.<sup>1,3–10,12,13</sup> Absorption of radiation in a medium was first described in what is known as the Beer–Lambert law, which is commonly used and implied in the field of sunscreen testing.<sup>7,8</sup> Absorption by organic molecules involves a reversible process whereby molecules are excited from a ground state to an excited state before they return to the ground state and are able to absorb radiation again.<sup>3,4,13–15</sup>

There is plenty of research about the various factors impacting sunscreen performance,<sup>1,3,7–9</sup> but the time it takes for photoexcited sunfilter molecules to relax back to the ground state is rarely taken into account. This is possibly due to the ultrafast photodynamics expected for efficient sunfilter molecules.<sup>10–12,16–18</sup>

In light of recent research<sup>10,11,16,19–27</sup> we challenge the validity of the linearity and reciprocity assumptions during SPF tests. Purposefully ignoring all other factors which may impact sunscreen performance, in this work we aim to develop a simplified model for sunscreen performance during irradiation, to determine a more generalized SPF equation, and to estimate

the limits for molecular relaxation times for the validity of the linear and reciprocity assumptions under the typical irradiances and doses applied during SPF tests.

The Beer–Lambert law (for a given wavelength  $\lambda$ ) can be expressed in differential form as<sup>28</sup>

$$\frac{dE}{E} = -\sigma \cdot N \cdot dx \quad (1)$$

where  $E$  is the irradiance (i.e., number of photons per unit time and area),  $N$  the concentration of molecules with capacity to absorb photons,  $x$  the spatial dimension along the direction of incident radiation, and  $\sigma$  the photoabsorption cross section.

We start by noting that, when  $N$  is independent of both space and time, solving eq 1 requires a trivial integration in  $x$  with the solution being the well-known absorbance ( $A$ ) expression  $A = \sigma \cdot N \cdot \chi$  (with  $\chi$  being the sunscreen's film thickness, and assuming a perfectly distributed sunscreen on a perfectly uniform surface).<sup>7–9,28,29</sup> Such absorbance expression yields a linear SPF value that is independent of both radiation dose and irradiance.

However, for a sunfilter to be able to absorb, molecules which just had an absorption event will be disabled from further absorption for a little while (i.e., reversible photo-bleaching), specifically during the time between a molecule getting excited by a photon and relaxing back to the ground state.<sup>3,4,10,12,16,24</sup>

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Therefore,  $N$ , which is understood as the concentration of molecules with capacity to absorb, is not constant in time, and by the nature of the exponential decay of  $E$  in eq 1, the number of interacting photons with absorbing molecules will also be different at different depths of the sunscreen. Correspondingly, the exposure of  $N$  to  $E$  is different at different depths, and therefore,  $N$  is also not constant in space. With  $N$  not being constant in space, solving the integral in eq 1 requires an understanding of  $N(x, t)$ .

Considering a sunscreen with one sunfilter molecule type only, the simplest way to incorporate in eq 1 photoexcitation and nonradiative relaxation dynamics can be represented as

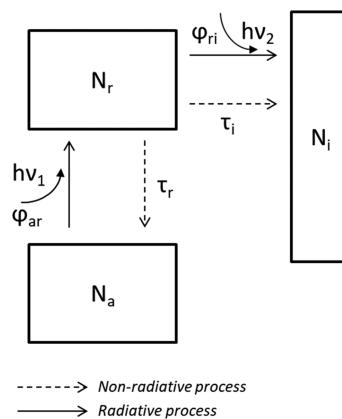
$$\frac{\partial N(x, t)}{\partial t} = -\varphi \cdot \sigma \cdot N(x, t) \cdot E(x, t) - (N(x, t) - N_0)/\tau \quad (2)$$

where  $N_0$  is the initial concentration of absorbing molecules before the sunscreen is irradiated ( $N$  at time 0);  $\varphi$  is the probability that one absorbed photon excites one molecule (often referred to as quantum yield), and  $\tau$  represents the relaxation time for excited molecules to return to the ground state.

The steady state ( $\partial N/\partial t = 0$ ) solution for  $N$  in eq 2 [ $N = N_0/(1 + \tau \cdot \varphi \cdot \sigma \cdot E)$ ] reveals that when  $\tau \cdot \varphi \cdot \sigma \cdot E \ll 1$ , then  $N \approx N_0$ ; that is, relaxation happens so fast that effectively all molecules  $N$  are available to absorb photons at any given point in time, and dose-response linearity should be expected. However, when  $\tau \cdot \varphi \cdot \sigma \cdot E \gg 1$ , then  $N \approx 0$  and dose-response linearity would break because of reversible photobleaching.<sup>14,28</sup> Although such nonlinearities are unlikely to be observed during SPF tests because they would appear when relaxation times are of the order of milliseconds or longer (because during SPF tests  $E(x = 0, t) = E_0$  can reach up to  $4.66 \times 10^{-3}$  mol·m<sup>-2</sup>·s<sup>-1</sup><sup>16</sup> and  $\sigma$  can be up to  $27\,516\text{ m}^2\cdot\text{mol}^{-1}$ <sup>13,21</sup>), which should be rare for the sunfilter molecules typically used in sunscreens. However, when irreversible photodegradation is taken into account, nonlinear dynamics can appear at much smaller relaxation times, given that while the effect of decreasing  $\tau$  is to linearize the dose-response curve, the effect of irreversible photodegradation is to bend it, as we shall see below.

Considering both dynamics simultaneously, i.e., that sunfilters will be temporarily disabled from absorbing because of the relaxation time needed to dissipate excess energy, and that sunfilters may be irreversibly degraded by radiation, we can generalize sunscreen photodynamics (excluding radiative decays to the ground state, which are normally regarded as undesirable<sup>3,4,9,14,15,17,24,27</sup>) as depicted in Figure 1.

In Figure 1,  $N_r$  ( $N_i$ ) is the concentration of molecules which have reversibly (irreversibly) interacted with photons, and  $N_a$  is the concentration of molecules which, not having been degraded (reversibly or irreversibly), actively contribute to absorption;  $\varphi_{ar}$  represents the quantum yield associated with the process of photon absorption (at frequency  $\nu_1$ ) from the ground to an excited state;<sup>4,9,13,16,17,22</sup>  $\tau_i$  represents the time taken for photoexcited molecules to irreversibly degrade (e.g., chemical reactions, photolysis, sensitization<sup>3,9,10,12,13</sup>);  $\varphi_{ri}$  represents the irreversible photodegradation quantum yield (at frequency  $\nu_2$ ) from an excited state,<sup>23,30,31</sup> and  $\tau_r$  encompasses nonradiative decays to the ground state (e.g., internal conversions, intersystem crossings returning to the ground state<sup>3,4,9,10,12</sup>).



**Figure 1.** Schematic representation of different molecular states during the process of photon absorption, relaxation and photo-degradation.

In the general model represented in Figure 1, and without entering into the specific photodynamics and photochemistry of specific molecules, we can establish

$$N_0 = N_a(x, t) + N_r(x, t) + N_i(x, t) \quad (3a)$$

$$N_r(x, t = 0) = N_i(x, t = 0) = 0 \quad (3b)$$

The equations governing the evolution for each state of the molecules become

$$\frac{\partial N_a}{\partial t} = -\Psi_{a \rightarrow r}(\nu_1, x, t) \cdot N_a(x, t) + \frac{N_r(x, t)}{\tau_r} \quad (4)$$

$$\begin{aligned} \frac{\partial N_r}{\partial t} = & \Psi_{a \rightarrow r}(\nu_1, x, t) \cdot N_a(x, t) - \Psi_{r \rightarrow i}(\nu_2, x, t) \cdot N_r(x, t) \\ & - \frac{N_r(x, t)}{\tau_r} - \frac{N_r(x, t)}{\tau_i} \end{aligned} \quad (5)$$

$$\frac{\partial N_i}{\partial t} = \frac{N_r(x, t)}{\tau_i} + \Psi_{r \rightarrow i}(\nu_2, x, t) \cdot N_r(x, t) \quad (6)$$

With  $\Psi$  conveniently defined as

$$\Psi_{a \rightarrow r}(\nu_1, x, t) \equiv \varphi_{ar}(\nu_1) \cdot \sigma_{ar}(\nu_1) \cdot E(\nu_1, x, t) \quad (7a)$$

$$\Psi_{r \rightarrow i}(\nu_2, x, t) \equiv \varphi_{ri}(\nu_2) \cdot \sigma_{ri}(\nu_2) \cdot E(\nu_2, x, t) \quad (7b)$$

Although eqs 4–6 are a simplification of the real dynamics taking place when a sunscreen is irradiated, they represent a more general model than the “linear” assumption made for  $N$  being constant in eq 1. Equations 4–6 and their corresponding integrated versions (eqs 8–10) are rarely linear.

$$\begin{aligned} N_a(x, t) = & \left\{ N_{a,i} + \tau_r^{-1} \int_0^t N_r(x, t') \cdot \right. \\ & \left. \exp \left[ \int_0^{t'} \Psi_{a \rightarrow r}(\nu_1, x, t'') \cdot dt'' \right] \cdot dt' \right\} \cdot \\ & \exp \left[ - \int_0^t \Psi_{a \rightarrow r}(\nu_1, x, t') \cdot dt' \right] \end{aligned} \quad (8)$$

$$N_r(x, t) = \left\{ N_{r,i} + \int_0^t \Psi_{a \rightarrow r}(\nu_1, x, t') \cdot N_a(x, t') \cdot \exp[\int_0^{t'} [\Psi_{r \rightarrow i}(\nu_2, x, t'') + \tau_r^{-1} + \tau_i^{-1}] \cdot dt''] \cdot dt' \right. \\ \left. \exp[-\int_0^t [\Psi_{r \rightarrow i}(\nu_2, x, t') + \tau_r^{-1} + \tau_i^{-1}] \cdot dt'] \right\}. \quad (9)$$

$$N_i(x, t) = N_{i,i} + \tau_i^{-1} \int_0^t N_r(x, t') \cdot dt' \\ + \int_0^t \Psi_{r \rightarrow i}(\nu_2, x, t') \cdot N_r(x, t') \cdot dt' \quad (10)$$

where  $N_{a,i}$ ,  $N_{r,i}$  and  $N_{i,i}$  are the initial number of molecules  $N$  in the active, reversible, and irreversible states, respectively.

Taking the customary<sup>8,9,29,32</sup> irregular film structure approximation modeled with the  $\Gamma$  probability density function  $\Gamma(h, \alpha, \beta)$ , where  $h$  is the random variable associated with the film thickness,  $\alpha$  the shape parameter, and  $\beta$  the inverse scale parameter,<sup>29,32</sup> the equation describing the SPF can be determined by inserting eqs 8 and 9 into eq 1 and integrating  $E$  over the time period until erythema is reached  $t = T$ ,<sup>9,29</sup> which yields

$$\text{SPF}(\chi, T) = \left( \int_0^T \int_{290}^{400} A_e(\lambda) \cdot E_0(\lambda) \cdot d\lambda \cdot dt \right) \\ \left( \int_0^T \int_{290}^{400} A_e(\lambda) \cdot E_0(\lambda) \cdot \int_0^\infty \exp[-\sigma(\lambda) \cdot \int_0^h N_a(x, t) \cdot dx] \cdot \Gamma(h, \alpha, \beta) \cdot dh \cdot d\lambda \cdot dt \right)^{-1} \quad (11)$$

where  $A_e(\lambda)$  is the erythema action spectrum<sup>5,6</sup> and where the irradiated UV dose is the numerator and the transmitted dose is the denominator of eq 11.

Because our purpose is to identify the order of magnitude of molecular relaxation times ( $\tau_r$ ) for the appearance of SPF nonlinearities, as opposed to modeling the dynamics of specific sunfilter molecules, we assume flat absorption curves (i.e.,  $\sigma$  and  $\varphi$  not changing with wavelength) and the same  $\sigma$  for all processes of Figure 1. While these assumptions are unlikely for a single sunfilter molecule, they are quite representative of broad band sunscreen products, which contain several sunfilters together achieving an overall flat absorbance spectrum in the 290–400 nm wavelength range.<sup>33</sup> On this basis the expected transmitted irradiance  $\bar{E}$  under the linear approximation ( $N_r, N_i \ll N_0; N_a \approx N_0$ ) is given by

$$\frac{\bar{E}}{E_0} = \int_0^\infty e^{-\sigma \cdot h \cdot N_0} \cdot \Gamma(h, \alpha, \beta) \cdot dh = (1 + \beta \cdot \sigma \cdot N_0)^\alpha \quad (12)$$

and given that the mean  $h$  of the  $\Gamma$  distribution is  $\alpha \beta$ , the expected SPF under the linear approximation is  $\text{SPF} = (1 + \chi \cdot \sigma \cdot N_0 / \alpha)^\alpha$  (with  $\chi$  being the average film thickness).

Considering that the reversible processes from  $N_a$  to  $N_r$  (and vice versa) should be much faster than the irreversible degradation processes (from  $N_r$  to  $N_i$ ), a quasi-equilibrium state between  $N_a$  and  $N_r$  is rapidly reached, whereby we can approximate eq 4 to  $\partial N_a / \partial t = 0$  and, correspondingly,  $N_a = N_r / \tau_r \cdot \varphi_{ar} \cdot \sigma \cdot E$ . In this regime we can transform eq 6 into

$$\frac{\partial N_i}{\partial t} \approx -\frac{(\tau_i^{-1} + \varphi_{ri} \cdot \sigma \cdot E)}{\tau_r^{-1} + \varphi_{ar} \cdot \sigma \cdot E} (\varphi_{ar} \cdot \sigma \cdot E) \cdot (N_i - N_0) \quad (13)$$

In this form, eq 13 clearly shows that, when  $\tau_i^{-1} \ll \varphi_{ri} \cdot \sigma \cdot E$ , reciprocity will be lost because the number of irreversibly transformed molecules will depend on  $E^2 \cdot \Delta t$ , as opposed to  $E \cdot \Delta t$  (typically  $\tau_r^{-1} \gg \varphi_{ar} \cdot \sigma \cdot E$ , so the term in  $E$  in the denominator does not play a role). Moreover, for nonreciprocity to be observable, the transmitted dose curve must be nonlinear.

Breakdown of linearity is associated with a significant increase of the number of irreversibly degraded sunfilter molecules during exposure to radiation. Thus, in order to identify the onset of nonlinearities, our strategy will be to assume linearity and determine under what conditions  $N_i$  has become large enough to be inconsistent with the assumption of linearity. Because under the linear approximation irradiance will not change significantly over time, eq 13 can be integrated, yielding

$$N_i \approx \frac{(\tau_i^{-1} + \varphi_{ri} \cdot \sigma \cdot E)}{\tau_r^{-1}} (\varphi_{ar} \cdot \sigma \cdot E) \cdot N_0 \cdot \Delta t \quad (14)$$

where  $\Delta t$  is the irradiation time interval and, because of the linearity assumption,  $N_0 - N_i \approx N_0$ .

In order to establish the limits for the validity of the linear approximation, it is necessary to define a criterion for SPF linearity. We establish it through an auxiliary magnitude  $K$ , so that the obtained SPF is considered linear if its value is greater than  $1/K$  times the nominal SPF in the linear approximation (eq 12). Thus, this criterion is equivalent to

$$N_i \leq N_0 \cdot \left( \frac{1 - K^{-1/\alpha}}{1 - \text{SPF}^{-1/\alpha}} \right) \quad (15)$$

When eqs 14 and 15 are combined, linearity will be maintained when

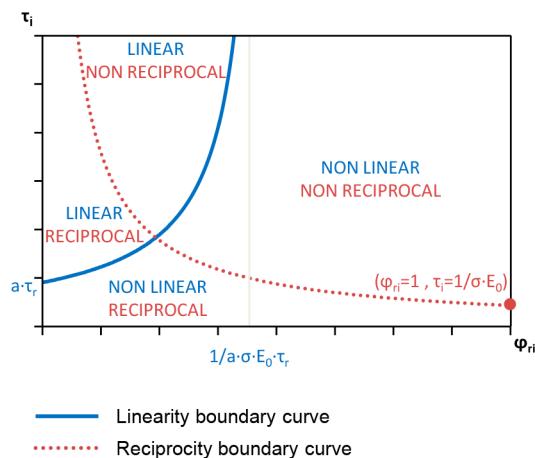
$$\tau_i \gg \frac{a}{\frac{1}{\tau_r} - a \cdot \sigma \cdot E_0 \cdot \varphi_{ri}} \equiv \tau_{i,\text{th}} \quad (16)$$

holds, where  $\tau_{i,\text{th}}$  is the threshold value of  $\tau_i$  separating the linear and nonlinear behavior of the system, and we have assumed the maximum possible value  $E = E_0$ .  $a$  is given by

$$a \equiv \frac{\Delta t \cdot \varphi_{ar} \cdot \sigma \cdot E_0 \cdot (1 - \text{SPF}^{-1/\alpha})}{(1 - K^{-1/\alpha})} \quad (17)$$

A graphical representation (Figure 2) of  $\tau_{i,\text{th}}$  versus  $\varphi_{ri}$  for eq 16 shows that  $\tau_{i,\text{th}} = a \cdot \tau_r$  when  $\varphi_{ri} = 0$ , and then  $\tau_{i,\text{th}}$  grows asymptotically with  $\varphi_{ri}$  until  $\varphi_{ri} = 1/a \cdot \sigma \cdot E_0 \cdot \tau_r$  (solid blue line in Figure 2). The effect of lowering  $\tau_r$  is to move the  $\tau_{i,\text{th}}$  linearity boundary curve to the right. For a given  $\tau_r$ , linearity during SPF test irradiation will be maintained when the values of  $\tau_i$  and  $\varphi_{ri}$  are above the solid blue curve, whereas linearity will break if the values are below the solid blue curve.

Overlaying the condition for nonreciprocity  $\tau_i^{-1} \ll \varphi_{ri} \cdot \sigma \cdot E_0$  in this map we can establish the area where both linearity and reciprocity assumptions break: for a given  $\tau_r$ , SPF will not be constant and reciprocity will not be maintained when the values of  $\tau_i$  and  $\varphi_{ri}$  are both below the linearity boundary curve (solid blue line in Figure 2) and above the reciprocity boundary curve (dotted red line in Figure 2). The threshold (lowest)  $\tau_r$  for which failure of both linearity and reciprocity could be first noticed will be given by



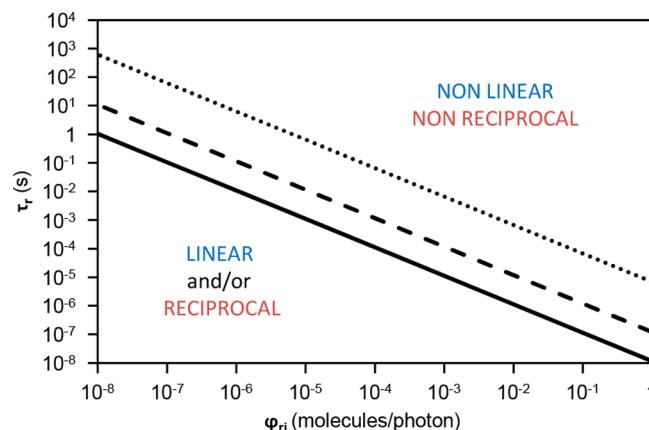
**Figure 2.** Map showing the different linearity–reciprocity regimes as a function of the time taken for photoexcited molecules to irreversibly degrade ( $\tau_i$ ) and irreversible photodegradation quantum yield from the excited state ( $\varphi_{ri}$ ).

$$\tau_r \approx \frac{1}{2 \cdot a \cdot \sigma \cdot E_0 \cdot \varphi_{ri}} \quad (18)$$

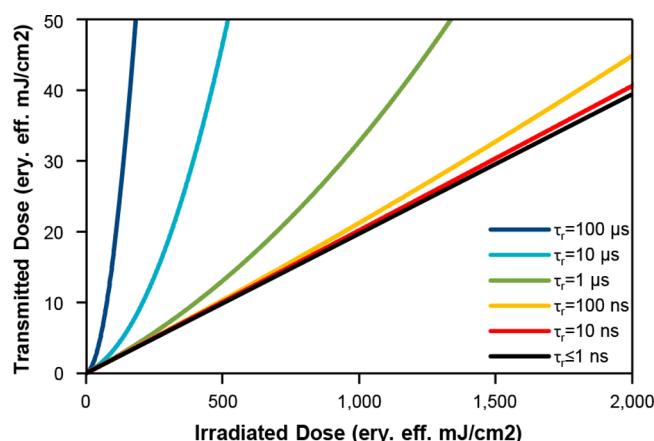
In order to estimate these limits numerically we consider a typical irradiation time of  $T = \Delta t = 1754$  s, after which a solar simulator (SS) at  $E_0 = 4.66 \times 10^{-3}$  mol·m<sup>-2</sup>·s<sup>-1</sup> (equivalent to 1600 W/m<sup>2</sup>, delivered in the 290–400 nm range)<sup>6</sup> delivers 2000 erythematic effective mJ·cm<sup>-2</sup> for 40 erythematic effective mJ·cm<sup>-2</sup> transmitted through an SPF50 sunscreen. We assume  $N_0 = 125$  mol·m<sup>-3</sup>, which given typical molar weights of sunfilter molecules (300–900 gr·mol<sup>-1</sup><sup>14,18</sup>) represents a concentration of 4–11%. With a homogeneously distributed sunscreen at a rate of 2 mg·cm<sup>-2</sup> average film thickness is  $\chi = 2 \times 10^{-5}$  m,<sup>3,7,9,29</sup> and taking  $\alpha = 1.105$ <sup>32</sup> an expected SPF of 50 under the linear approximation is achieved with  $\sigma = 15\,000$  m<sup>2</sup>·mol<sup>-1</sup> (which is well in range with typical sunfilter cross sections<sup>13</sup>). Because  $\varphi_{ar}$  also varies per molecule, we assume an efficient product and set  $\varphi_{ar} = 0.8$ <sup>17,34</sup> because we are interested in estimating the minimum  $\tau_r$ . We establish the tolerance level for the maintenance of the linearity assumption to  $(1 - 1/K) = 17\%$  (i.e., a sunscreen is generally considered photostable if its SPF does not change more than 17% during SS irradiation<sup>6</sup>). With these parameters in eq 18, the lowest possible  $\tau_r$  is about 10 ns (solid line in Figure 3). Using a smaller  $\sigma$  (dashed line in Figure 3) or a smaller  $E_0$  (dotted line in Figure 3) increases the  $\tau_r$  limit.

Through the numerical integration of eqs 4–6 we can validate the appropriateness of the various approximations made in eqs 12–18. Figure 4 shows the effects on the transmitted dose at irradiated doses up to 2000 erythematic mJ·cm<sup>-2</sup> for different values of  $\tau_r$ , with the other parameters set as per the limits of the validity of the linearity and reciprocity assumptions considered before. It can be observed that the larger the relaxation time, the stronger the deviation from the linear approximation, and that relaxation times longer than about 10–100 ns exhibit a nonlinear behavior whereas faster relaxation times approximate to the linear assumption.

Figure 5 shows a simulation with the same parameters for a SPF50 sunscreen except for sunfilter relaxation time  $\tau_r = 10 \mu\text{s}$ . After irradiating 400 erythematic effective mJ·cm<sup>-2</sup>, SPF would be 34 under natural sun spectrum (in the 290–400 nm range) irradiance of 61 W·m<sup>-2</sup><sup>35</sup> but SPF would be 13 if evaluated at



**Figure 3.** Map showing the different linearity–reciprocity regimes as a function of the relaxation time from the excited to the ground state ( $\tau_r$ ) and irreversible photodegradation quantum yield from the excited state ( $\varphi_{ri}$ ). The different lines separating the two regimes correspond to different model parameters (see main text and eq 18). Parameters for the solid line set as in the main text. For the dashed line,  $E_0 = 4.66 \times 10^{-3}$  mol·m<sup>-2</sup>·s<sup>-1</sup>,  $N_0 = 405$  mol·m<sup>-3</sup>, and  $\sigma = 4605$  m<sup>2</sup>·mol<sup>-1</sup>. For the dotted line,  $E_0 = 1.88 \times 10^{-4}$  mol·m<sup>-2</sup>·s<sup>-1</sup>,  $N_0 = 125$  mol·m<sup>-3</sup>, and  $\sigma = 15000$  m<sup>2</sup>·mol<sup>-1</sup>.



**Figure 4.** Transmitted dose versus irradiated dose for different simulations varying relaxation time  $\tau_r$ . Simulation for eqs 11 with parameters  $E_0 = 4.66 \times 10^{-3}$  mol·m<sup>-2</sup>·s<sup>-1</sup> (equivalent to 1600 W·m<sup>-2</sup>),  $\sigma = 15\,000$  m<sup>2</sup>·mol<sup>-1</sup>,  $N_0 = 125$  mol·m<sup>-3</sup>,  $\chi = 2 \times 10^{-5}$  m,  $\alpha = 1.105$ ,  $\varphi_{ar} = 0.8$ ,  $\varphi_{ri} = 0.8$ , and  $\tau_i = 1754$  s.

SS irradiance of 1600 W·m<sup>-2</sup>. A product that would appear to be quite photostable up to 5 h under natural sun conditions (in the 290–400 nm range) would not behave as such having received the same irradiation dose under accelerated SS conditions.

The study of the photodynamics of sunfilter molecules is a growing field of research.<sup>10,16,15,18</sup> Efforts have been aimed at understanding single sunfilter molecules in isolation, and ultrafast (picoseconds) relaxation times have been determined for many of the commonly used sunfilters.<sup>10,11,15–18</sup> However, research has also shown the dependence of these photodynamics on the characteristics of the neighboring environment (e.g., polarity and pH) and molecules.<sup>3,7,9–11,14–16,22–24,36</sup> It is not well-known whether the ideally ultrafast relaxation photodynamics of sunfilter molecules is maintained when they are combined in the complex formulation of a sunscreen product.<sup>10,11,23,30,34,36</sup> Many of

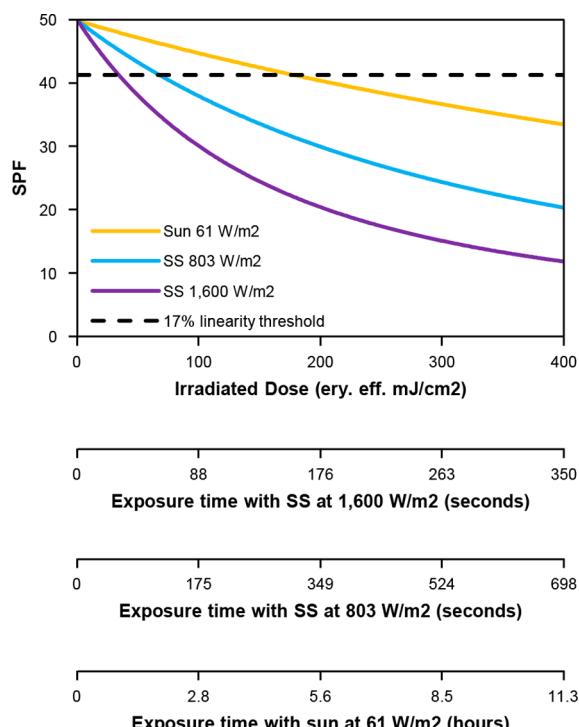


Figure 5. SPF versus irradiated dose for different irradiances  $E_0$ .

these confounding factors have been shown to alter sunfilter photodynamics and to create long-lived states that can be of the order of nanoseconds for cinnamate derivatives<sup>10,11,17,27</sup> and for ethyl ferulate,<sup>10,19</sup> microseconds for methyl anthranilate<sup>17,25,26</sup> and for avobenzone<sup>24</sup> (although avobenzone in nonprotic solvents has been shown to have long-lived states with lifetimes of milliseconds<sup>22,23</sup>), or as long as seconds for methyl anthranilate<sup>16,20</sup> and for triazones.<sup>21</sup> Additionally, other processes such as the darkening of photographic emulsions, inactivation/mutation of bacteria or viruses or the fading of dyes have shown the failure of reciprocity in some instances.<sup>37</sup>

In conclusion, applying basic photodynamic concepts in a simple form we established the boundaries for the validity of the SPF linearity and reciprocity assumptions under irreversible photodegradation in the dose ranges and irradiances typically used during SPF tests. We find that, excluding other factors, both assumptions could theoretically break when relaxation times are 10 ns or longer, which, according to recent research, appears to be well within the long-lived excited-state lifetimes of many sunfilter molecules. The breakdown of reciprocity is associated with a prevalence of pathways to irreversible sunfilter degradation involving photoactivation from an excited state. Until further research is conducted we should be mindful of the way sunscreens' SPFs are evaluated and how the SPF value is interpreted, because different test UV dose and irradiance conditions could yield different SPF values.

## COMPUTATIONAL METHODS

Equations 1, 4, and 5 have been integrated using Euler's method. Three two-dimensional meshes have been numerically integrated to model dependent variables  $E$ ,  $N_a$ , and  $N_r$  to independent variables space ( $x$ ) and time ( $t$ ). For the space ( $x$ ) dimension we have assumed  $2 \text{ mg} \cdot \text{cm}^{-2}$  uniformly distributed sunscreen product (average sunscreen film thickness  $\chi = 2 \times$

$10^{-5} \text{ m}$ <sup>3,7,9,29</sup>), using a simulation mesh with  $\text{dx} = 10 \text{ nm}$  and integration limit of  $10 \mu\text{m}$ . For the time dimension we have adjusted simulation parameters to achieve accumulated doses of erythematic effective  $2000 \text{ mJ} \cdot \text{cm}^{-2}$ ; accordingly,  $\text{dt}$  has changed in each simulation. Boundary conditions are set at  $E(x = 0, t) = E_0$ ,  $N_a(x, t = 0) = N_0$ , and  $N_r(x, t = 0) = 0$ .

Using ISO Standard 24444:2010s example for solar simulator spectral irradiance in the 290–400 nm wavelength range<sup>6</sup> and converting the spectral energy to number of photons, we establish that when irradiance is  $803 \text{ W} \cdot \text{m}^{-2}$  the number of corresponding photons is  $2.34 \times 10^{-3} \text{ mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$  and when irradiance is  $1600 \text{ W} \cdot \text{m}^{-2}$  the number of SS photons is  $4.66 \times 10^{-3} \text{ mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$ . For natural sun irradiance we use the irradiance and spectrum given by ASTM Standard G 173-03,<sup>35</sup> which in the same wavelength range as in the SPF SS tests (290–400 nm) gives an irradiance of  $61 \text{ W} \cdot \text{m}^{-2}$  or  $1.88 \times 10^{-4} \text{ mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$ .

To convert UV doses to erythematic effective radiation doses we have used the erythema action spectrum  $A_e(\lambda)$ .<sup>6</sup> Given that we have assumed flat absorption curves, the spectral output of the transmitted radiation through the sunscreen will be identical to the spectrum of the incident radiation attenuated by the sunscreen's absorption. For SS irradiance of  $803 \text{ W} \cdot \text{m}^{-2}$ , the equivalent erythematic effective irradiance is  $5.7 \text{ W} \cdot \text{m}^{-2}$ ; for  $1600 \text{ W} \cdot \text{m}^{-2}$ , it is  $11.4 \text{ W} \cdot \text{m}^{-2}$ . For natural sun irradiance in the 290–400 nm range,  $61 \text{ W} \cdot \text{m}^{-2}$  is equivalent to  $0.1$  erythematic effective  $\text{W} \cdot \text{m}^{-2}$ .

The  $\Gamma$  function  $\Gamma(h, \alpha, \beta)$  is calculated as per eq 19 with selected parameters  $\alpha = 1.105$ <sup>32</sup> and  $\beta = \chi/\alpha = 1.81 \times 10^{-5} \text{ m}$  (so that the mean sunscreen film thickness is  $\chi = 2 \times 10^{-5} \text{ m}$ ):

$$\Gamma(h, \alpha, \beta) = \left( \frac{h}{\beta} \right)^{\alpha-1} \cdot e^{-h/\beta} \cdot \frac{1}{\beta \cdot \Gamma(\alpha)} \quad (19)$$

Molar decadic extinction coefficients ( $\epsilon$ ) for UV filters typically range from 3 000 to 119 500 decadic liters·mol<sup>-1</sup>·cm<sup>-1</sup>.<sup>13</sup>

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## Notes

The authors declare the following competing financial interest(s): Aleix Bacardit is an employee of Isdin. The authors report no other conflicts of interest in this work.

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