Phase-Variation Microwave Sensor for Permittivity Measurements Based on a High-Impedance Half-Wavelength Transmission Line

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Abstract—A phase-variation microwave sensor operating in transmission and implemented by means of a high-impedance half-wavelength sensing line is reported in this paper. The sensor is useful for dielectric constant measurements and dielectric characterization of materials. By forcing the electrical length of the sensing line to be a half-wavelength when it is loaded with the so-called reference (REF) material, perfect matching is obtained regardless of the characteristic impedance of the line. This fact can be used to enhance the sensitivity for small perturbations, by merely increasing the characteristic impedance of the sensing line. An exhaustive analysis that supports such conclusion is reported in the paper. Then, two prototype sensors are designed and fabricated for validation purposes. As compared to the ordinary phase-variation permittivity sensor implemented by means of a matched (50-Ω) line with identical length, the sensitivity for small perturbations in the proposed sensor is 2.1 times larger. Further advantages of these sensors are low-cost, small size, implementation in planar technology, and very simple design and fabrication, derived from the fact that the sensing region is a half-wavelength transmission line.

Index Terms—Dielectric characterization, microstrip technology, microwave sensors, permittivity sensors, phase-variation sensors.

I. INTRODUCTION

The research activity devoted to performance improvement and size/cost reduction in planar sensors based on microwave technologies has experienced a significant growth in recent years. Microwave sensors are especially suited for material characterization since microwaves are very sensitive to the properties of the materials to which they interact. Coaxial probes [1]-[4] or resonant cavities [5]-[10] are well-known examples of microwave devices useful for material sensing and dielectric characterization. However, many applications demand sensor implementation in planar technology. The reasons are multiple, including low cost, low profile, possibility of sensor implementation in flexible substrates (e.g., for conformal sensors [11],[12]), compatibility with fully planar fabrication technologies (including also additive, i.e., printing, processes), and easy combination with other sensing technologies (e.g., microfluidics [13]-[16], wearables [17], lab-on-a-chip [18], organ-on-a-chip [19], organic and paper based sensors [20], etc.).

Many planar microwave sensors devoted to dielectric characterization and permittivity measurements are based on resonator-loaded transmission lines and exploit frequency variation [21]-[36]. In such devices, the sensing element is a planar resonator, sensitive to the complex permittivity of the material, or liquid, under test (MUT), which should be placed in contact with the sensing resonator, or in close proximity to it. The main limiting aspect of frequency-variation sensors is the need to generate a sweeping frequency signal able to cover the entire output dynamic range (the output variable in such sensors is the resonance frequency and, eventually, the peak or notch magnitude). For the generation of such frequency-varying signals, voltage controlled oscillators (VCOs) are typically used, representing a high cost solution for wideband output dynamic ranges. Wideband VCOs are also needed in the
so-called frequency-splitting sensors [37]-[44]. In these sensors (similar to differential-mode sensors [45]-[57]), a line is loaded with a pair of identical resonators, sensitive to the difference in the material characteristics between the MUT and the reference (REF) sample.

It is clear that a significant reduction in the associated sensor electronics is achievable if a single-tone harmonic (interrogation) signal suffices for sensing. Both the phase and the magnitude of either the transmission or the reflection coefficient of a resonator-loaded, or ordinary, line at a fixed (predefined) frequency are expected to vary with the complex permittivity of the MUT. Therefore, such variables can be used for sensing in the so-called single-frequency sensors. Coupling-modulation sensors [58]-[68] and phase-variation sensors [69]-[78] belong to the category of single-frequency sensors. The magnitude and the phase of the transmission or reflection coefficient are the output variables in coupling-modulation and phase-variation sensors, respectively.

Coupling-modulation sensors are robust against cross-sensitivities caused by variations in environmental factors (such as temperature and humidity) [79]. However, such sensors exhibit limited tolerance against the effects of electromagnetic interference and noise, as compared to phase-variation sensors. Thus, in applications where the ambient conditions are not expected to vary significantly (or are not expected to significantly cause appreciable changes in the properties of the MUT), phase-variation microwave sensors constitute a very good option.

Single-ended [69] and differential-mode [70] phase-variation microwave sensors operating in transmission and implemented by means of matched ordinary or meandered lines have been recently reported. In such sensors, the sensitivity is proportional to the length of the sensing lines. Thus, for size reduction, strategies based on controlling the dispersion (dispersion engineering) in artificial lines, including composite right/left-handed lines [45], electro-inductive-wave transmission lines [53], and slow-wave transmission lines [77],[78], have been reported. The resulting sensors exhibit competitive size and sensitivity. However, the design of these sensors is not exempt of certain complexity, as far as artificial lines [80] are involved. Recently, reflective-mode phase-variation microwave sensors based on either a half- or a quarter-wavelength open-ended sensing line cascaded to a set of 90° line sections with alternating high/low characteristic impedance, have been reported [73]. Excellent sensitivities with small sensing regions are achieved in these sensors, thanks to the multiplicative effect on the sensitivity caused by the impedance contrast of the high/low impedance 90° line sections.

In this paper, the objective is to optimize the sensitivity in transmission-mode sensors based on ordinary lines, by considering as design variables the characteristic impedance and the electrical length of the line section corresponding to the sensing region, not necessarily coincident with the whole sensor line. The work is organized as follows. Section II presents the proposed phase-variation microwave sensor and the working principle. The sensitivity analysis, devoted to find the optimum conditions for sensitivity optimization is carried out in Section III. From the conclusions inferred from such analysis, two microstrip prototype sensors are designed and validated by simulation and experiment, and the results are presented in Section IV. Finally, the main conclusions of the work are highlighted in Section V.

II. THE PROPOSED PHASE-VARIATION SENSOR AND WORKING PRINCIPLE

The schematic of the proposed sensor is depicted in Fig. 1. It is a two-port device consisting of a pair of cascaded line sections. The sensitive part of the sensor is the line section with characteristic impedance and electrical length designated as $Z_s$ and $\phi_s$, respectively. The MUT should be placed on top of the sensing region only, delimited by the dashed rectangle in Fig.1. The line section with impedance $Z$ and electrical length $\phi$ is cascaded to the sensing line in order to increase the degrees of freedom, with the aim of enhancing the sensitivity as much as possible without the need of elongating the length of the sensing line (the trivial approach [70]). It is obvious that the electrical length and characteristic impedance of the sensing line depend on the dielectric constant of the MUT placed on top of it. These parameters, in turn, contribute to the value of the phase of the transmission coefficient, the considered output variable of the proposed sensor, and an easily measurable quantity.

![Fig. 1. Schematic (a) and typical topology in microstrip technology (b) of the proposed transmission-mode phase-variation dielectric constant sensor.](image)

In uniform lines matched to the ports, the phase of the transmission coefficient coincides with the electrical length of the line. When designing a sensor based on a uniform line, the frequency of operation, $f_0$, must be set to a certain value, and a reference (REF) MUT should be decided. The electrical length of the line depends on that frequency, and also on the effective dielectric constant (determined by the transverse geometry of the line, by the dielectric constant of the substrate, and by the dielectric constant of the REF MUT). The characteristic impedance of the line, the relevant parameter for matching purposes, is also determined by the dielectric constant of the MUT. Therefore, in order to obtain line matching, thereby avoiding reflections, the line impedance should be forced to coincide with the reference impedance of the ports, $Z_0$, when the line is loaded with the REF MUT. These sensors are especially suited for the characterization of materials...
experiencing small changes (equivalent to small perturbations) with regard to the REF MUT, since, in this case, quasi-matching conditions are preserved.

However, by considering a phase-variation sensor based on a cascade of several line sections, where at least one of such sections is the sensitive part, it is expected that the phase of the transmission coefficient, the output variable, exhibits a more complex dependence with the phase (electrical length) and characteristic impedance of the sensing line(s). Thus, it is potentially possible to obtain a stronger dependence of the output variable (the phase of the transmission coefficient) with the dielectric constant of the MUT sample, the input variable. This is the main aspect to study in the next section, devoted to the sensitivity analysis of the proposed sensor, which specifically consists of two line sections, one of them acting as sensitive part, as indicated before.

III. SENSITIVITY ANALYSIS

Let us first consider a sensor based on a single uniform sensing line with impedance $Z_s$ and phase $\phi_s$ (i.e., let us exclude the presence of the cascaded line of Fig. 1). The phase of the transmission coefficient, $\phi_T$, depends on both $Z_s$ and $\phi_s$, which, in turn, are influenced by the dielectric constant of the MUT, $\varepsilon_{MUT}$. Thus, the sensitivity can be expressed as

$$S = \frac{d\phi_T}{d\varepsilon_{MUT}} = \frac{d\phi_T}{d\varepsilon_{MUT}} \frac{d\phi_s}{d\varepsilon_{MUT}} + \frac{d\phi_T}{dZ_s} \frac{dZ_s}{d\varepsilon_{MUT}}$$

(1)

In order to calculate (1), it is first necessary to obtain the transmission coefficient of the $S_{21}$ matrix according to (80), (81)

$$S_{21} = \frac{2}{A + B + CZ_0 + D}$$

(2)

with

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos \phi_s & jZ_s \sin \phi_s \\ jY_s \sin \phi_s & \cos \phi_s \end{pmatrix}$$

(3)

and $Y_s = 1/Z_o$. Introducing the elements of (3) in (2), the transmission coefficient is found to be

$$S_{21} = \frac{1}{\cos \phi_s + j \frac{1}{2} \left( \frac{Z_s}{Z_0} + \frac{Z_0}{Z_s} \right) \sin \phi_s}$$

(4)

and the phase of $S_{21}$ is

$$\phi_T \equiv \phi_{S_0} = -\arctan \left( \frac{1}{2} \left( \frac{Z_s}{Z_0} + \frac{Z_0}{Z_s} \right) \tan \phi_s \right)$$

(5)

Using (5), the derivatives of $\phi_T$ with $\phi_s$ and $Z_s$ that appear in the right-hand side member of (1) are found to be

$$\frac{d\phi_T}{d\phi_s} = -\frac{1}{M^{\frac{1}{2}} \cdot \cos^2 \phi_s + M \cdot \sin^2 \phi_s}$$

(6)

and

$$\frac{d\phi_T}{dZ_s} = -\frac{1}{1 + M^2 \cdot \tan^2 \phi_s} \cdot \tan \phi_s \cdot \left( \frac{1}{Z_0} - \frac{Z_s}{Z_s} \right)$$

(7)

respectively, where the dimensionless factor $M$, defined as follows

$$M = \frac{1}{2} \left( \frac{Z_s}{Z_0} + \frac{Z_0}{Z_s} \right)$$

(8)

is used for simplification purposes. The remaining two derivatives of the right-hand side member of (1) can be easily calculated. By considering sensor implementation in microstrip technology (the interest in this work), such terms are given by

$$\frac{d\phi_T}{d\varepsilon_{MUT}} = \frac{\phi_s}{4\varepsilon_{eff}} (1 - F)$$

(9)

and

$$\frac{dZ_s}{d\varepsilon_{MUT}} = -\frac{Z_s}{4\varepsilon_{eff}} (1 - F)$$

(10)

where the effective dielectric constant is [81]

$$\varepsilon_{eff} = \frac{\varepsilon_e + \varepsilon_{MUT}}{2} + \frac{\varepsilon_e - \varepsilon_{MUT}}{2} F$$

(11)

In (11), $\varepsilon_e$ is the dielectric constant of the substrate and $F$ is a geometry factor given by

$$F = (1 + 12 \frac{h}{W_s}) \frac{1}{2} + 0.04(1 - \frac{W_s}{h})^2$$

(12a)

for $W_s/h \geq 1$, or by

$$F = (1 + 12 \frac{h}{W_s}) \frac{1}{2} + 0.04$$

(12b)

for $W_s/h < 1$, where $h$ and $W_s$ are the substrate thickness and the width of the sensing line, respectively, and it is assumed that $t \ll h$, where $t$ is the thickness of the metallic layer. Moreover, the validity of (11) is subjected to the semi-infinite MUT approximation. That is, the MUT must be thick enough in the vertical direction, so that the electromagnetic field generated by the line does not reach the MUT-air interface. Note that for CPW technology, and considering both a semi-infinite substrate and MUT, the effective permittivity is given by (11) with $F = 0$. Thus, it follows that potentially higher sensitivity can be achieved by implementing the sensors in CPW (the fringing fields are higher in a CPW, and therefore the MUT has major effect on the phase and impedance of the sensing line).

Inspection of expression (8) indicates that if $Z_s = Z_0$ (a condition that guarantees perfect matching for the line loaded with the REF MUT sample), $M = 1$, and, consequently, $d\phi_T/d\phi_s = 1$. Moreover, $d\phi_T/dZ_s = 0$ for $Z_s = Z_0$, as it can be inferred from (7). Thus, for uniform matched sensing lines, the sensitivity is simply

$$S = \frac{d\phi_T}{d\varepsilon_{MUT}} = \frac{\phi_s}{4\varepsilon_{eff}} - (1 - F)$$

(13)
and it increases proportionally with $\phi$.

For $Z_1 \neq Z_0$, the line is mismatched and part of the injected power is reflected back to the source, unless $\phi = n \pi$, $n$ being an integer number. For $\phi = n \pi$, it follows that $d\phi/d\phi = M$ and $d\phi/dZ_s = 0$. Thus, the sensitivity is found to be

$$S = \frac{M \cdot n \pi}{4 \varepsilon_{\text{eff}} (1 - F)}$$ (14)

Thus, as compared to the sensitivity of the matched line (i.e., with $Z_1 = Z_0$) with identical electrical length, the sensitivity is multiplied by a factor that depends on $M$, given by (8) (note that the geometry factor, $F$, is not identical in a matched and in a mismatched line, and, for this reason, the sensitivity is not exactly multiplied by $M$). Indeed, $M$ increases either by increasing or decreasing the normalized impedance of the sensing line, $Z_s/Z_0$. However, a high line impedance is preferred since the width of the line, $W$, decreases by increasing $Z_s$, and this, in turn, reduces the geometry factor $F$, thereby boosting up the sensitivity.

Let us next consider the complete structure of Fig. 1. The $ABCD$ matrix of the whole two-port network can be inferred from the products of the $ABCD$ matrices of each line section. We do assume that the step-impedance discontinuities are ideal, and therefore not included in the schematic model of Fig 1(a), thereby avoiding an excessively complex formulation. Nevertheless, the main effects of such impedance variations are accounted for by the model, as inferred from the reasonable prediction of the sensitivities, as will be shown later. Introducing the resulting elements of the $ABCD$ matrix in (2), the transmission coefficient can be easily inferred, and, from it, the phase is found to be

$$\phi = -\arctan \left\{ \frac{(Z + Z_0)\cos \phi \sin \phi_s + (Z_s + Z_0)\sin \phi \cos \phi_s}{2 \cos \phi \cos \phi_s - (Z_s + Z_0)\sin \phi \sin \phi_s} \right\}$$ (15)

The sensitivity is also given by expression (1), where the terms (9) and (10) are valid, as well. Calculation of the derivatives $d\phi/d\phi_s$ and $d\phi/dZ_s$ from (15) gives

$$\frac{d\phi}{d\phi_s} = -\frac{1}{1 + \left( \frac{N}{D} \right)^2} \cdot \frac{D \cdot N_{\phi_s} - N \cdot D_{\phi_s}}{D^2}$$ (16)

$$\frac{d\phi}{dZ_s} = -\frac{1}{1 + \left( \frac{N}{D} \right)^2} \cdot \frac{D \cdot N_{\phi_s} - N \cdot D_{\phi_s}}{D^2}$$ (17)

where $N$ and $D$ are the numerator and the denominator, respectively, of the argument of the arctan in (15), and the following derivatives have been defined

$$N_{\phi_s} \equiv \frac{dN}{d\phi_s} = \frac{(Z_s + Z_0)\cos \phi \sin \phi_s + (Z + Z_0)\sin \phi \cos \phi_s}{2 \cos \phi \cos \phi_s - (Z_s + Z_0)\sin \phi \sin \phi_s}$$ (18)

$$D_{\phi_s} \equiv \frac{dD}{d\phi_s} = -2 \cos \phi \sin \phi_s - \frac{(Z_s + Z_0)\sin \phi \cos \phi_s}{2 \cos \phi \cos \phi_s - (Z_s + Z_0)\sin \phi \sin \phi_s}$$ (19)

$$N_{\phi_s} \equiv \frac{dN}{d\phi_s} = \frac{(Z_s + Z_0)\cos \phi \sin \phi_s + (Z + Z_0)\sin \phi \cos \phi_s}{2 \cos \phi \cos \phi_s - (Z_s + Z_0)\sin \phi \sin \phi_s}$$ (18)

$$D_{\phi_s} \equiv \frac{dD}{d\phi_s} = -2 \cos \phi \sin \phi_s - \frac{(Z_s + Z_0)\sin \phi \cos \phi_s}{2 \cos \phi \cos \phi_s - (Z_s + Z_0)\sin \phi \sin \phi_s}$$ (19)

In order to ensure the absence of mismatching reflections it is necessary that either $Z_1 = Z = Z_0$, or $Z_1 = Z \neq Z_0$ with $\phi + \phi_s = n \pi$. In the former case, the sensitivity is also given by (13), and this trivial case lacks interest. In the second case ($Z_1 = Z \neq Z_0$), (16) and (17) are found to be

$$\frac{d\phi}{d\phi_s} = \frac{1}{M \cdot \cos^2(\phi + \phi_s) + M \cdot \sin^2(\phi + \phi_s)}$$ (20)

and

$$\frac{d\phi}{dZ_s} = -\frac{1}{Z_s} \cdot \frac{\cos(\phi + \phi_s) \cdot \cos \phi \sin \phi_s \cdot (Z_s^2 - Z_0^2)}{Z_s^2 + Z_0^2}$$ (23)

respectively, and using $\phi + \phi_s = n \pi$ one obtains

$$\frac{d\phi}{d\phi_s} = -M$$ (24)

and

$$\frac{d\phi}{dZ_s} = -(1)^n \cdot \frac{M \cdot \cos(n \cdot \pi - \phi_s) \sin \phi_s \cdot (Z_s^2 - Z_0^2)}{Z_s^2 + Z_0^2}$$ (25)

where (25) is null if $\phi = (2m + 1) \pi/2$ [and consequently $\phi_s = (2n - 2m - 1) \pi/2$] or $\phi_s = m \pi$ (and consequently $\phi = (n - m) \pi$), where $m$ is an integer satisfying $m < n$. If (25) is null, the sensitivity is given by (14), and, obviously, the phase combination $\phi_s = m \pi$ and $\phi = (n - m) \pi$ provides a better sensitivity (note that this case is, indeed, the one where the line section with impedance $Z$ and phase $\phi$ is absent). Nevertheless, the optimum values of the phases of the lines (which should satisfy $\phi + \phi_s = n \pi$ for matching requirements) are those that maximize the whole sensitivity, which can be expressed as

$$S = \frac{d\phi}{d\phi_s} = \frac{M (1 - F)}{4 \varepsilon_{\text{eff}}} \left\{ \phi_s - (1)^n \cdot \cos(n \cdot \pi - \phi_s) \sin \phi_s \cdot \frac{Z_s^2 - Z_0^2}{Z_s^2 + Z_0^2} \right\}$$ (26)

In order to find the optimum value of $\phi_s$, the derivative of (26) with regard to $\phi_s$ is obtained, i.e.,

$$\frac{dS}{d\phi_s} = \frac{M (1 - F)}{4 \varepsilon_{\text{eff}}} \left\{ 1 - \frac{Z_s^2 - Z_0^2}{Z_s^2 + Z_0^2} \cdot \cos(2\phi_s) \right\}$$ (27)

and by forcing it to be zero, one obtains

$$\cos(2\phi_s) = \frac{Z_s^2 + Z_0^2}{Z_s^2 - Z_0^2}$$ (28)
Since there are not real solutions of (28) for \( \phi_s \), it follows that the sensitivity does not exhibit a maximum (or a minimum). Indeed, the magnitude of the sensitivity monotonically increases with \( \phi_s \), as it can be appreciated in Fig. 2. Thus, the optimum phase values are those satisfying \( \phi_s = m \pi \) (and thereby \( \phi = (n - m) \pi \)), and the sensitivity is given by expression (14). Naturally, for sensor size optimization, the line with impedance \( Z \) and phase \( \phi = (n - m) \pi \) can be excluded from the design (corresponding to \( n = m \)), and the sensor is indeed the one analyzed in the previous case. However, this result was not a priori evident, but derived from the detailed analysis carried out in this section. The main conclusion is that the sensing line must be chosen to exhibit a phase of \( \phi_s = m \pi \), i.e., a half-wavelength or a multiple of it, when it is loaded with the REF MUT, and the characteristic impedance must be as high as possible. The phase condition \( \phi_s = m \pi \) prevents from mismatching reflections, whereas the high value of the characteristic impedance of the sensing line section increases the sensitivity.

\[
S(\phi_s, Z_0) = \text{constant}
\]

Fig. 2. Representation of the sensitivity, as given by (26), as a function of \( \phi_s \) for \( n = 1 \), and for different values of \( Z_0 \). For the evaluation of \( F \) and \( \varepsilon_{\text{REF}} \), the parameters of the Rogers 4003C substrate, with thickness \( h = 1.524 \text{ mm} \) and dielectric constant \( \varepsilon_r = 3.55 \), have been used.

IV. SENSOR DESIGN, FABRICATION, AND VALIDATION

For validation purposes, three different sensors have been designed and fabricated. All the sensors verify that the electrical lengths satisfy the matching requirement, i.e., \( \phi + \phi_m = n \pi \). In sensor A, \( \phi = 0, \phi_m = \pi \), and \( Z = Z_0 = 124 \Omega \), with the same REF MUT. Finally, in sensor C, part of the half-wavelength line is not covered by the MUT, specifically, \( \phi = \pi/4, \phi_m = 3\pi/4 \), and the impedances are set to \( Z = Z_0 = 124 \Omega \). The operating frequency in all the sensors is \( f_0 = 2 \) GHz. It should be mentioned that for sensor C the width of the line sections is different, despite the fact that the impedances are identical. The reason is that the 3\pi/4 sensing line should exhibit the design impedance, \( Z_0 = 124 \Omega \), when it is loaded with the REF MUT (the one indicated above), whereas the \( \pi/4 \) line should exhibit an identical impedance (\( Z = 124 \Omega \)) when it is surrounded by air. The sensors have been implemented on the Rogers 4003C substrate with thickness \( h = 1.524 \text{ mm} \), dielectric constant \( \varepsilon_r = 3.55 \), and loss tangent \( \tan \delta = 0.0022 \).

With the indicated substrate, REF MUT, phases, and impedances, the line dimensions have been calculated, and then the prototype sensors have been fabricated by means of a LPKF H100 drilling machine. The photographs, as well as the relevant dimensions, of the fabricated sensors are depicted in Fig. 3. Note that identical access lines have been added to the half-wavelength sensor lines for connector soldering.

![Photographs of the fabricated sensors and relevant dimensions](image)

(a) Sensor A; (b) Sensor B; (c) Sensor C. The sensing regions are given by the (imaginary) rectangles with vertexes corresponding to the positions of the holes, which are used for screwing the samples against the sensing line section, thereby minimizing the effects of the air gap.

Prior to measurements, the dependence of the phase of the transmission coefficient with the dielectric constant of the MUT for the three sensors has been obtained by means of electromagnetic simulation, using the CST Microwave Studio commercial software. The considered input dynamic range is \( 1 < \varepsilon_{\text{MUT}} < 10.2 \), limited by the dielectric constant of air and the dielectric constant of a Rogers RO3010 substrate available in our laboratory. The results are depicted in Fig. 4 (a), where the phase of the transmission coefficient is actually represented in
reference to the phase of the transmission coefficient when the
REF MUT is on top of the sensing region. Actually, the
simulation results of Fig. 4(a) were repeated by considering
values of the loss tangent of the MUT of 0.0025, 0.0050 and
0.0100, and the results (not shown) are indistinguishable. The
measured phases of the transmission coefficient for specific
MUT samples, with the dielectric constants indicated in the
caption of Fig. 4, are also depicted in the figure (such phases
have been inferred by means of the vector network analyzer
Keysight N5221A, see Fig. 5). As it can be seen, there is very
good agreement between the simulated and the measured data
points.

![Graph](image1)

Fig. 4. Measured and simulated phase of the transmission coefficient at
the operating frequency for the sensors of Fig. 3 (a), and simulated
sensitivity (b). The considered MUTs for measurements are 4.57-mm
slabs of uncoated PLA ($\varepsilon_{\text{MUT}} = 3$), Rogers RO4003C ($\varepsilon_{\text{MUT}} = 3.55$), FR4
($\varepsilon_{\text{MUT}} = 4.4$) and Rogers RO3010 ($\varepsilon_{\text{MUT}} = 10.2$) substrates. In (a), the
measured magnitude of the transmission coefficient at the operating
frequency for the considered MUT samples is also included.

Due to the lack of a numerous set of experimental data points,
the sensitivity has been inferred from the derivative of the
simulated data. The results are depicted in Fig. 4(b). The figure
provides the value of the sensitivity for the REF MUT that
results from the simulated data points, as well as the one
inferred by means of expression (26), and designated as $S_0$. As
it can be appreciated, the agreement is reasonably good for all
three sensors, pointing out the validity of the sensitivity
analysis carried out in the previous section. Indeed, Fig. 4(b)
includes also the sensitivity as a function of $\varepsilon_{\text{MUT}}$, inferred
theoretically by means of expression (1), with the different
derivative terms calculated according to (9), (10), (16) and (17).
Note that (22) and (23) are only valid for $Z = Z_r$, which is
satisfied when $\varepsilon_{\text{MUT}} = \varepsilon_{\text{REF}}$, and expressions (24) and (25) need
that the phases satisfy $\phi + \phi = n \cdot \pi$, a condition fulfilled when
$\varepsilon_{\text{MUT}} = \varepsilon_{\text{REF}}$, as well. Obviously, for the calculation of the
theoretical sensitivity over the input dynamic range, it is
necessary to previously obtain the involved line phases and
impedances for each value of $\varepsilon_{\text{MUT}}$.

As it can be seen from Fig 4(b), the agreement between the
theoretical and simulated sensitivity is reasonably good within
the whole considered range. This further strengthens the validity
of the theoretical analysis of Section III. The slight
discrepancies between the results predicted by the theory and
those inferred from the electromagnetic simulations are attributed to non-idealities, such as the limited thickness of the
considered MUTs (not semi-infinite in practice), and to the fact
that the calculation of $Z_r$ and $\phi$, with a certain MUT on top of
the sensing line section is subjected to some inaccuracies.

It should be mentioned that the proposed sensors are
especially suited for applications where accurate measurements
of the dielectric constant of the MUT in the vicinity of that of
the REF MUT sample, or a related material magnitude, are
required. Thus, the present study reports a method to optimize
the sensitivity, for a given reference sample, in a kind of sensors
where design simplicity and size are key aspects. We would
like to clarify that the fact that the sensitivity is optimized for a
given value of the dielectric constant of the MUT (the REF
MUT) does not necessarily mean that the maximum magnitude
of the sensitivity over the considered input dynamic is the one
corresponding to such value of the dielectric constant of the
MUT ($\varepsilon_{\text{REF}}$). As it can be seen in Fig. 4, the maximum value of
the magnitude of the sensitivity actually takes place for $\varepsilon_{\text{MUT}} = 1$
(simulated values) in the three designed and fabricated sensors.
In the sensors reported in [73], the maximum magnitude of the
sensitivity for the considered input dynamic range was also

![Photograph](image2)

Fig. 5. Photograph of the experimental setup used for the phase
measurements, including the vector network analyzer Keysight N5221A
and the sensor with a MUT on top of the sensing region.
found to occur at \( \varepsilon_{\text{MUT}} = 1 \), but the sensitivity in such sensors was optimized for this value of the dielectric constant of the MUT (i.e., \( \varepsilon_{\text{REF}} = 1 \)). By contrast, in the sensors reported in [76], the magnitude of the sensitivity is a maximum for a dielectric constant value of roughly 3.55, the value of \( \varepsilon_{\text{REF}} \).

As expected, the results of Fig. 4 confirm that the optimum sensitivity in the limit when \( \varepsilon_{\text{MUT}} \rightarrow \varepsilon_{\text{REF}} \) is achieved for Sensor B, the one where the entire half-wavelength line, with significant impedance contrast \( (Z_{\text{L}}/Z_{\text{O}} = 2.48) \), acts as sensing line. The worst sensitivity corresponds to sensor A, based on a half-wavelength matched line (with \( Z_{\text{L}} = Z_{\text{O}} = 50 \text{ }\Omega \)). Increasing \( Z_{\text{L}} \) enhances the sensitivity. Concerning the limit of such impedance, it is dictated by the minimum width of the line that can be achieved with the technology in use, typically around 200 \( \mu \text{m} \) with most drilling machines (this is the case of the sensing line width in sensor B). Nevertheless, it is possible to increase the sensitivity by adding further 180° sensing line sections to the sensor (at the expense, of course, of increasing the size of the sensing region).

Concerning resolution, it is defined as the minimum input variable (in our case the dielectric constant) that can be resolved. Obviously, resolution is intimately related to the sensitivity. Thus, if we consider that the phase resolution (output variable) is of the order of 5°, a very conservative value, the resolution for discriminating variations in the dielectric constant of the MUT with regard to the one of the REF sample is roughly \( \Delta \varepsilon_{\text{MUT}} = 0.35 \). Obviously, such resolution can be improved (reduced) by enhancing the sensitivity by adding further 180° line sections to the sensing line.

A figure of merit (FoM) in phase-variation sensors is the ratio between the maximum sensitivity and the size of the sensing region expressed in terms of the squared guided wavelength, \( \lambda \). Table I contains the FoM, as well as other sensor parameters, for the three sensors designed and fabricated in this work, and for other phase-variation dielectric constant sensors available in the literature and operating in transmission. Actually, for the calculation of the FoM in the sensors of this work, the value of the sensitivity for \( \varepsilon_{\text{REF}} = 3.55 \), rather than the maximum sensitivity, has been considered. Also, for the calculation of the size of the sensing region, a width of 0.1 \( \lambda \) has been considered for the sensors of this work. This value is very conservative, which means that the FoM is indeed underestimated (note that with such width, it is by far guaranteed that the electric field lines generated by the line do not extend beyond the MUT boundaries, a necessary condition). As it can be seen from Table I, sensors B and C are competitive in terms of the FoM (though sensor B exhibits better sensitivity). Moreover, in sensor B, except by the access lines and connectors, the whole sensor is the sensing region, thereby corresponding to an optimized design. The sensitivity in sensor [70] is very high, but at the expense of a significantly large sensing area. By contrast, any of the designed and fabricated sensors of this paper is very small, as far as the whole sensor (obviously excluding the associated electronics) merely consists of a half-wavelength line section, the sensitive part being the whole line (Sensors A and B) or a fraction of it (Sensor C). Obviously, the sensitivity of Sensors A, B and C can be further enhanced by considering additional half-wavelength line sections.

In the sensor reported in [78], the sensitivity and the FoM are very competitive. In such sensor, a slow-wave transmission line based on a host line periodic loaded with patch capacitors is used for sensing (the sensitive elements are the patch capacitances). These slow-wave transmission line based sensors exhibit good performance, but their design is not exempt of certain complexity, at least as compared to the sensors reported in this paper. Indeed, the characteristic (or Bloch) impedance of the sensor in [78] is 50 \( \Omega \), whereas the phase is not subjected to a specific value. Namely, impedance matching is considered in [78], contrary to the sensor proposed in this paper, where phase matching is forced (i.e., the sensing line is a half-wavelength line when it is loaded with the REF sample). However, it is possible to apply phase matching to the sensors of [78], by considering a slow-wave transmission line with a phase of 180°, or multiple of it, and high value of the Bloch impedance, with potential sensitivity enhancement. Nevertheless, this aspect requires further investigation and is left for a future work.

It should be mentioned that in some of the sensors in Table I ([45],[53],[54]), the phase information was transformed to magnitude information. For this reason, the sensitivity in these sensors is given in dB, in coherence with the output variable, i.e., the magnitude of the transmission coefficient. The sensors reported in [45],[53], are based on artificial transmission lines (i.e., composite right left/handed lines in [45], and electro-inductive-wave transmission lines in [53]). Such lines exhibit significant (and controllable) dispersion, useful to enhance the sensitivity, but, similar to those sensors based on slow-wave transmission lines [77],[78], the design of sensors based on artificial lines is not straightforward.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Mode</th>
<th>Size (( \lambda ^2 ))</th>
<th>Max. Sensitivity (º / dB)</th>
<th>FoM (º / ( \mu \text{K} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>[45]</td>
<td>TRANSMISSION</td>
<td>---</td>
<td>600 dB</td>
<td>---</td>
</tr>
<tr>
<td>[69]</td>
<td>TRANSMISSION</td>
<td>---</td>
<td>54.8°</td>
<td>---</td>
</tr>
<tr>
<td>[70]</td>
<td>TRANSMISSION</td>
<td>12.90</td>
<td>415.6°</td>
<td>32.2</td>
</tr>
<tr>
<td>[53]</td>
<td>TRANSMISSION</td>
<td>0.075</td>
<td>25.3 °</td>
<td>---</td>
</tr>
<tr>
<td>[54]</td>
<td>TRANSMISSION</td>
<td>0.020</td>
<td>17.6 °</td>
<td>---</td>
</tr>
<tr>
<td>[77]</td>
<td>TRANSMISSION</td>
<td>0.030</td>
<td>7.9°</td>
<td>257</td>
</tr>
<tr>
<td>[78]</td>
<td>TRANSMISSION</td>
<td>0.040</td>
<td>20.0°</td>
<td>500</td>
</tr>
<tr>
<td>Sensor A</td>
<td>TRANSMISSION</td>
<td>0.050</td>
<td>6.64°</td>
<td>132.8</td>
</tr>
<tr>
<td>Sensor B</td>
<td>TRANSMISSION</td>
<td>0.050</td>
<td>14.24°</td>
<td>284.8</td>
</tr>
<tr>
<td>Sensor C</td>
<td>TRANSMISSION</td>
<td>0.038</td>
<td>12.25°</td>
<td>322.4</td>
</tr>
</tbody>
</table>

We would also like to mention that the reflective-mode phase-variation sensors reported in [73],[74] are very competitive in terms of sensitivity and FoM. However, measuring the phase of a transmission coefficient in a real scenario, that is, avoiding the use of a VNA, seems to be more feasible than measuring the phase of the reflection coefficient. This is the main advantage of the reported sensor, as compared to the sensors reported in [73],[74].

Finally, let us indicate that the loss tangent of solids samples with moderate- and low-loss factors cannot be accurately estimated with this approach. Nevertheless, the reported
sensors are useful to measure other variables related to the dielectric constant, e.g., material composition, or to detect defects in samples, typically manifested by a variation in the effective dielectric constant. In general, resonant methods [24]-[28],[82]-[84] offer a good option to estimate the loss tangent with reasonable accuracy. However, the proposed sensors, or more generally, phase-variation sensors, are interesting because the sensitivity can be enhanced by means of specific methods, like the one reported in this work, or by simply elongating the sensing line.

V. CONCLUSIONS

In conclusion, it has been demonstrated that the sensitivity in transmission-line-based phase-variation permittivity sensors operating in transmission can be enhanced through impedance contrast, and by setting the length of the sensing line to a half-wavelength (or a multiple of this length) at the operating frequency. In particular, in one of the designed and fabricated prototype sensors, implemented in microstrip technology, the sensitivity for small perturbations has been found to be 2.1 times larger than the one of the sensors based on an ordinary matched (50-Ω) half-wavelength sensing line. This sensitivity optimization has not been only demonstrated by simulation and experiment, but also through a detailed analysis of the structure, useful to predict the value of the sensitivity. The proposed sensors are very simple and small, since the sensing part merely consists of a half-wavelength transmission line. This facilitates sensor design and fabrication, and provides robustness against fabrication related tolerances (more critical in designs involving metallic or slotted resonant elements, or other complex structures, such as artificial lines, etc.). The fact that the sensors operate at a single frequency, and that the phase, the output variable, is robust against the pernicious effects of electromagnetic interference and noise are additional advantageous aspects of the reported sensors.

REFERENCES


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