Socio-Economic Planning Sciences xxx (xxxx) xxx

Contents lists available at ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

Efficiency and quality in Colombian education: An application of the metafrontier Malmquist-Luenberger productivity index

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ARTICLE INFO

JEL classification:
JEL
C61
H52
I21

Keywords:
Education
Efficiency
Metafrontier malmquist-luenberger index
Inequality

ABSTRACT

This study uses the metafrontier Malmquist-Luenberger index to measure changes in the productivity of 4587 schools in the Colombian education system. Public and private schools are differentiated and all the schools participated in the Saber 11 standardized test between 2014 and 2017 in the subjects of mathematics, reading, sciences, social and citizen sciences and English. This methodology is appropriate to measure productivity while using good and bad outputs in the educational context. The general results indicate deterioration in both sectors; this behavior is due to the change in best practices and the change in efficiency. Large gaps are also found between departments.

1. Introduction

Educational efficiency is a topic of intense political, social and academic debate [25] for various reasons. First, education is considered as the main source of human capital accumulation of a country [39]. Second, education plays a fundamental role in the redistribution of living conditions in society [37]. Finally, because educational budgets are high and growing [28], it is important to ensure that educational spending is carried out with a high level of efficiency.

Different development plans around the world recognize education as a priority due to the social externalities it presents [52]. Colombia is no exception, as shown by its efforts to close inequality gaps through education. At the local level, the Ministry of National Education (MEN) of Colombia is the institution responsible for managing resources in education, one of the focuses of which is to reduce gaps in access and quality, thus improving the level of human capital and, in turn, promoting economic growth and development [5].

The main objective of recent educational policies has been to close social gaps [40]; again, Colombia is no exception. To this end, the *Estimulos a la Calidad Educativa* (Incentives to Educational Quality) incentive system was designed under MEN Decree 501 of 2016, which uses the *Índice de Calidad* (Quality Index) as a single measurement tool for the granting of stimuli. In turn, this index comprises the Synthetic Index of Educational Quality (ISCE) and the Management Index for Educational Quality (IGCE). The first focuses on the results of the educational process (progress, performance, efficiency and school environment), and the second, on the schools' resources (efficiency in the infrastructure). The application of this index reflects the importance that the government attributes to the efficient management of resources in education, showing that educational quality is being measured from different perspectives [68]. However, although the quality index aims to measure the efficiency in both components, it should be noted that there is no input-output logic in its construction.

The educational reform carried out with the implementation of the Quality Index is relevant for the context in which it is developed and for the objective it is intended to achieve; however, the methodology of the index is not robust. The main characteristics that can be improved and that motivate the present study are: first, the components should not have a priori weights within the indicators; second, the conceptualization of efficiency must be underpinned on production theory [8]; and finally, there must be a global vision that can have multiple benchmarks for comparison, and not a partial vision through two weighted indexes. It is worth pointing out that estimation of models based on data envelopment analysis methods for the calculation of composite indicators has gained prominence [33], among other reasons, to avoid the subjectivity of choosing the weights of the components a priori [61] and to have the

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https://doi.org/10.1016/j.seps.2021.101122
Received 3 November 2020; Received in revised form 30 May 2021; Accepted 6 July 2021
Available online 7 July 2021

possibility of estimating different groups under the same approach [6].

The main objective of the study is to evaluate the change in the productivity of 4587 schools over time, differentiating between public[1] and private sectors. In addition, there are two specific objectives: first, to propose a robust and integrative methodology of the concepts measured partially by the incentive system *Estimulos a la Calidad*; and second, to analyze the change in the efficiency of the education system with the introduction of the ISCE. The analysis takes into account the political–administrative division of Colombia, which is divided into 32 departments and its capital district, to determine if there are differences. This methodology also allows the analysis to consider three different orientations in the results, towards performance, towards inequality, or both at the same time.

The education sector in Colombia is a representative case of a developing economy with high social inequity, where there are large gaps between public and private education [15]. Therefore, rigorous analysis of the differences between the sectors is important for developing educational policies. The gaps between the public and private sectors are worrisome for two main reasons: first, there is high private spending by households that want to access a better education for their children, which generates strong pressure on their well-being [53]. Second, there is evidence that the difference in resources between schools in the public and private sectors is one of the most relevant drivers of gaps [13], which depends mainly on the differences in their funding.

One of the main motivations of this study is to use efficiency measures with an input-output logic, and in turn, incorporate inequality in the education system by using undesirable or bad outputs, thus improving the approach used by the government's quality index. Inequality and inequity in education are two problems of constant concern around the world and especially in developing countries. However, this study focuses on inequality as a bad output for various reasons. First, it is a relevant topic of constant debate that is attracting increasing interest in the literature [46]. Second, it is a problem in which schools play a fundamental role, for example, by running additional classes or by grouping students according to levels [2]. And finally, the effects of inequality are not only localized, but are also externalized and affect the whole economic system [16].

Educational efficiency has gained relevance in the measurement of different problems [25], among which the temporal evolution of educational productivity is highlighted [3,27]. Only three studies have focused on measuring changes in educational productivity by incorporating desirable and undesirable outputs [7,31,32]. Tsai et al. [68] state that the correct evaluation of the productivity of an education system must consider both outputs (good and bad) while controlling the inputs and environmental variables. As far as we are aware, no evaluation has these characteristics, nor considers the differences between groups (public and private). As this paper offers an initial approach, it is therefore relevant to all contexts, but even more so for Colombia due to the large inequality gaps and the country's demonstrated interest in education policy.

To carry out this approach, we use the metafrontier Malmquist Luenberger (MML) index developed by Oh [54] because it helps to incorporate a temporal dimension in the analysis while considering good and bad outputs in the process. Additionally, directional distance functions (DDF) are applied since they allow efficiency to be measured by improving the academic average while reducing the variance of the results. This paper analyzes the changes in the productivity of 4587 schools through the result in the standardized exams of senior high-school students who participated in 2014 and 2017. This period is relevant to analyze the evolution of productivity due to the change in the regulations related to the *Índice de Calidad*, which proposes incentives to schools in matters of management and budget, and acts as a market signal.

The contribution of this work is threefold. First, it opens the way for multiple applications in educational policy, since it is proposed as a benchmark for delivering incentives in the public sector, and it acts as a market signal in the private sector. Second, it responds to calls for research related to educational efficiency which applies variables that take into account all the dimensions of standardized tests [41], and incorporates both performance and inequality in the educational process [68]. Third, comparing the application employed in this study with other similar studies in the literature [7,31,32], the Saber11 database is used instead of TIMSS (Trends in International Mathematics and Science Study) to measure the outputs of the process, and the C600 (census of schools in Colombia) provided by the DANE is used for the inputs. It is the first application of the metafrontier Malmquist Luenberger index for a specific education system that applies partial frontiers for different sectors.

The results show, on average, better performance change in the public than the private sector, although there is a general deterioration in the education system regardless of the orientation used. The public sector has a better performance change when there is an orientation towards equality (bad outputs), driven mainly by the change in efficiency. Additionally, departments show different approaches to working on educational performance, and present results that vary significantly.

The study is organized into five sections. This introduction is followed by the literature review (Section 2). The methodological aspects of the MML index and its decomposition are then described (Section 3), after which the databases used in the education system evaluation process are presented, the variables are explained, and the main results are reported (Section 4). Finally, the main conclusions are detailed (Section 5).

2. Literature review

The study of efficiency of schools was strongly motivated by the Coleman report, which highlighted the lack of participation of educational institutions in the struggle for equal opportunities in the United States [19]. This research line has been approached from different perspectives [65]. In this study, we take the public economics perspective, where emphasis is placed on technical efficiency, generally through a non-parametric approach, and the units evaluated are compared with their peers according to the levels of inputs and outputs, mainly using non-parametric frontier models.

The non-parametric approach is highlighted for the major advantages it has over the other methods used in the literature. First, it is less vulnerable to specification problems that affect econometric models [62]. Second, it is not necessary to define assumptions about the distribution of errors and the production function [73]. And third, multiple inputs and outputs can be used [66], which for the objective of this study is fundamental, since it takes into account performance and inequality at the same time. In addition, in the field of educational efficiency, non-parametric methods have been the most frequently applied in the literature [25].

Within the line of public economics, the Malmquist index has been used to analyze temporary changes in productivity [31,60,64], which can be split in two components: technological change and change in efficiency. Alternatively, the Hicks-Moorsteen index [4] has also been used with an approach based on performance and total factor productivity ratios. Studies addressing the evolution of productivity using the Malmquist index [14] have applied methodological complements that enhance the scope; for example, by using directional distance functions (DDF) [18] efficiency can be measured with multiple approaches and flexible orientations. Moreover, the use of metafrontiers [22,66], based

[1] The National Administrative Department of Statistics (DANE) of Colombia uses the terms 'official' and 'unofficial' to differentiate schools managed by the public administration from those that are not; however, in this article we refer to these schools as 'public' and 'private', respectively, as these terms will be more familiar to international readers.

on the approach of Battese et al. [6]; allows decompositions of different categories (Thieme et., 2013) and analyzes different groups or technologies [24] in the same context.

In this study, the metafrontier Malmquist Luenberger (MML) index developed by Oh [54] is adopted, since it incorporates a temporal dimension in the analysis while considering the good and bad outputs of the process, and in turn, it focuses on metafrontiers. To date, only three studies have measured changes in educational productivity considering good and bad outputs [7,31,32], all of which prioritize the educational quality orientation. However, the authors are unaware of any other applications of the MML approach to the education sector, making this study an innovation in this field.

Although the MML has not been applied to the education sector, similar studies have been undertaken. In the first, Giménez et al. [31] used a global (non-radial) Malmquist index to measure the change in the productivity of 29 education systems from 2003 to 2012, considering as variables the results of mathematics and reading and their standard deviation. Subsequently, Ben Yahia et al. [7] conducted research using DDF to work with bad outputs and non-discretionary inputs, based on a sample of Tunisian schools in 2012. Finally, Giménez et al. [32] used TIMSS data for 28 countries in an efficiency analysis from 2007 to 2011 with a global Malmquist-Luenberger model due to the presence of bad outputs. These authors conclude that although there are large differences between countries, on average educational performance declined during those years.

The quality of an education system does not depend exclusively on academic performance. For this reason, the present study highlights not only educational achievement but also equality in the education process. Tsai et al. [68] frame two objectives as a golden rule in educational policy: excellence (high performance) and equality (low variability in performance) in the results. The first objective has been thoroughly studied in the literature [45], for example, to find its determinants [41], the role of environmental variables [49] and the differences between education systems [56]. Additionally, most transnational studies that examine performance consider each area of interest separately [39]. However, it is desirable to have composite measures of academic performance to produce more reliable studies [41].

The second objective, equality, has been analyzed in terms of the role of education systems in the standardization of opportunities [44]. Equality in educational processes has often been measured through the total variance of academic performance [43]. However, how to measure and treat inequality among students within an educational system is a topic discussed in the literature [9], which has been debated in many countries, with positions between a selective system (for example, Germany, Hungary, Austria) or a comprehensive system (for example, Japan, Canada, Norway) [38]. The main policies have focused on early follow-up of students [26,39], grouping of skills and/or performance in the classroom [42] and individualized support [30]. In general, those in favor of homogenizing classes affirm an increase in efficiency, and those who are not in favor affirm that the level of low ability students is affected because of lowered expectations and self-esteem due, among other reasons, to the peer group effect [9].

Based on the above, although there are different positions in the literature regarding student inequality, the present study takes it into account in terms of the standard deviation of students, mainly because it is a measure that can be used for different types of standardized tests [57], and because it is a proxy of other measures used in previous studies [43]. Also, the model jointly evaluates excellence and inequality, in pursuit of aspects of improvement based on the available inputs, following the quality education goal in the 2030 Agenda for Sustainable Development, which commits to "providing inclusive and equitable quality education and promoting lifelong learning opportunities for all" [70]".

This paper proposes an approximation through the MML index, which responds to calls in the literature by prioritizing both the performance and the equality of the process, taking into account the differences between the types of schools, and considering composite measures that integrate all areas evaluated and their standard deviation. The present study is therefore the first approach of this type in this research field.

3. Methodology

The first part of this section explains the DDF, which are necessary to estimate the MML index. The second part presents the evolution of productivity measurement indices in general and explains the MML index model.

3.1. Directional distance functions

The technology that models the set of production possibilities assumes K groups $(k = 1,2 \ldots, K)$ for T periods of time $(t = 1, 2 \ldots, T)$. A set of inputs and outputs is used in the process: the vector of inputs is $x = (x_1, \ldots, x_N) \in R_+^N$ and the outputs are distinguished between desirable $y = (y_1, \ldots, y_M) \in R_+^M$ and undesirable $b = (b_1, \ldots, b_J) \in R_+^J$. The set of production possibilities meets the following axioms [29]:

$$P(X) = \{y, b \mid x \; can \; produce \; (y, b), \; x \in R_+^N\} \tag{1}$$

$$if \; (y, b) \in P(x) \mid y' < y, \; then \; (y', b) \in P(x) \tag{2}$$

$$if \; (y, b) \in P(x) \mid b = 0, \; then \; y = 0 \tag{3}$$

$$if \; (y, b) \in P(x) \mid 0 \leq \varnothing \leq 1, \; then \; \left(\frac{1}{\varnothing} y, \varnothing b\right) \in P(x) \tag{4}$$

$$if \; (y, b) \in P(x) \mid (y', b') \leq (y, b), \; then \; (y', b') \in P(x) \tag{4.1}$$

The first axiom suggests that the set of outputs is strongly disposable, while the set of bad outputs is only weakly disposable; this means that there are no additional costs to reduce the production of desirable outputs, but the reduction of undesirable outputs can require more input consumption or, alternatively, the reduction of good outputs. The second, known as 'null-jointness', indicates that decision making units (DMUs) cannot produce the desirable output without producing the undesirable output. The third axiom shows that bad outputs are weakly disposable, indicating that minimizing this type of output is expensive. The fourth states that it is feasible to reduce the set of good outputs while increasing the bad outputs proportionally by $\varnothing$; note that this axiom must be contrasted with equation (4.1) since it allows a non-proportional reduction of good and bad outputs.

To estimate the DDF used by the MML index, parametric and non-parametric frontier models can be applied. In this study we chose the non-parametric approach because it does not require the assumption of a functional form, nor a specific distribution of the error term [66]. It also allows us to work with multiple sets of inputs and outputs without having to assume factor prices, which is appropriate and applicable to the education sector, since they are unknown or difficult to estimate [25].

The main idea of the DDF, as can be seen in Fig. 1, is to maximize the desirable outputs while minimizing the undesirable ones by maintaining or reducing the level of inputs used. The mathematical expression of the DDF is as follows:

$$D(x, y, b; g) = \max\{\beta \mid (y + \beta g_y, \; b - \beta g_b) \in P(x) \; \} \tag{5}$$

where $g = (g_y, g_b)$ is a directional vector, which indicates the direction of approach to the frontier. Following Chung, Färe and Grosskopf [18]; and if $g = (y, b)$, the DDF can be rewritten as follows:

$$D(x, y, b; g) = \max\{ \beta : ((1 + \beta)y, (1 - \beta)b) \;) \in P(x) \} \tag{6}$$

In the previous expression, the utility of the DDF can be analyzed, where the objective is to increase the desirable outputs (y) in a β

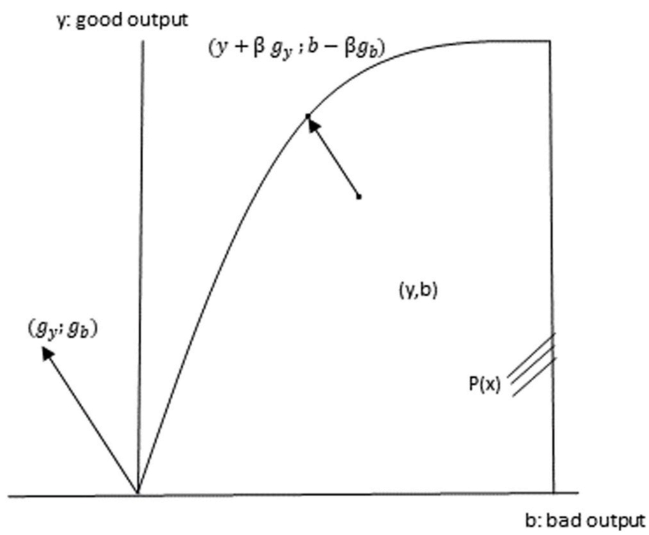

Fig. 1. Directional Distance Function. Source: [17,58].

proportion while reducing the undesirable outputs *(b)* in the same proportion.

3.2. Temporary productivity analysis

Changes in productivity have been quantified in the literature using the Malmquist Index, introduced by Caves et al. [14]; in a parametric framework. This index attributes the changes in productivity to two components: the change in efficiency (or catching up) and the technological change. The first refers to how the units have approached or moved away from the contemporary production frontier during the analyzed period, while the second quantifies how it has moved. Thus, the first of the components is the effect attributable to the capacity for management, organization, and coordination, and the second is related to the capacity for innovation [50].

The MML index arose from the Malmquist Luenberger (ML) index [18] and Oh's developments [54,55]. The ML evaluates changes in productivity when bad outputs are incorporated into the production function [29]. Later, based on the ML, Oh [55] proposed the global Malmquist Luenberger (GML), which focuses on comparing observations against a single global frontier, and finally, Oh [54] adapted the GML to the use of metafrontiers [6].

Fig. 2 shows the MML index and its components with two years and two groups. The intertemporal reference technology is the envelope (metafrontiers) of contemporary technology and the global reference technology is the envelope of the intertemporal reference technology. For the present study, the contemporary reference (*F (oft)*) is the closest reference against which each school in a specific group and year is compared (public schools in period *t*). The intertemporal reference technology (*F (Of)*) is the metafrontier that considers all the contemporary boundaries of a specific group (for example, public schools). Finally, the global reference technology (*F(G)*) is the metafrontier that includes all the groups and years under study (public and private schools in periods t and $t+4$).

In other words, to define the index and its decomposition, it must be considered that there are three types of possible references: technological, intertemporal and global [69]. Oh [54] defines the MML productivity index as follows:

$$MML^{t,t+1}\left(x^t, y^t, b^t, x^{t+1}, y^{t+1}, b^{t+1}\right) = \frac{1+D^G(x^t, y^t, b^t)}{1+D^G\left(x^{t+1}, y^{t+1}, b^{t+1}\right)} \tag{7}$$

$$= \frac{1+D^t(x^t, y^t, b^t)}{1+D^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}\right)} \times \frac{\left(1+D^I(x^t, y^t, b^t)\right)/\left(1+D^t(x^t, y^t, b^t)\right)}{\left(1+D^I\left(x^{t+1}, y^{t+1}, b^{t+1}\right)\right)/\left(1+D^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}\right)\right)} \times \frac{\left(1+D^G(x^t, y^t, b^t)\right)/\left(1+D^I(x^t, y^t, b^t)\right)}{\left(1+D^G\left(x^{t+1}, y^{t+1}, b^{t+1}\right)\right)/\left(1+D^I\left(x^{t+1}, y^{t+1}, b^{t+1}\right)\right)} \tag{8}$$

$$MML^{t,t+1}\left(x^t, y^t, b^t, x^{t+1}, y^{t+1}, b^{t+1}\right) = \frac{TE^{t+1}}{TE^t} \times \frac{BPR^{t+1}}{BPR^t} \times \frac{TGR^{t+1}}{TGR^t} \tag{9}$$

$$MML^{t,t+1}\left(x^t, y^t, b^t, x^{t+1}, y^{t+1}, b^{t+1}\right) = EC \times BPC \times TGC \tag{10}$$

Following Oh's [54] proposal, it should be taken into account that the distances (D) shown in the previous equations (7) and (8) are the DDF explained in the previous section, where a comparison is made with a global (D^G), an intertemporal (D^I) and a contemporary (D^t) frontier. Additionally, Oh [55] proposes decomposing the MML index into three components: the first is the change in efficiency (EC), the second is the change in the best practices gap (BPC) and the third, the change in technology gap (TGC). If the MML index is greater than unity, it indicates a positive change in productivity between periods t and $t+1$; in

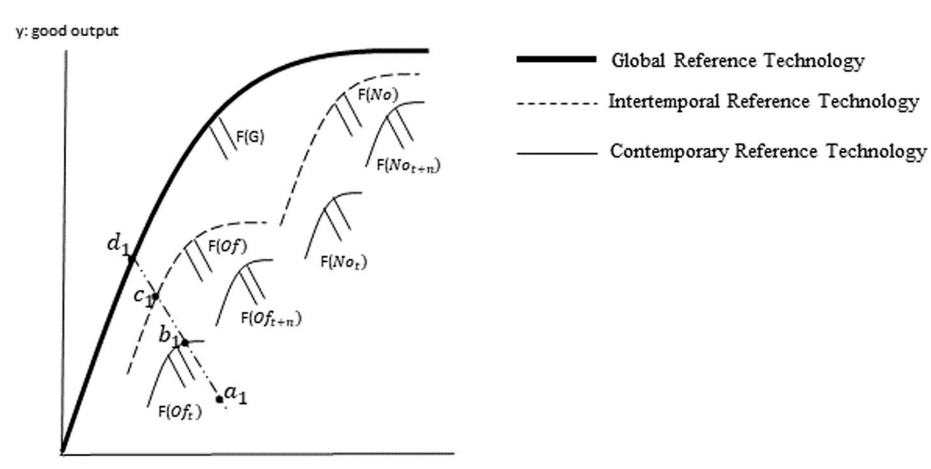

Fig. 2. MML productivity index diagram. Source: [36].

other words, the distance to the global frontier is less in the period $t+1$ than in the period t. An MML index value less than unity is a deterioration in the productivity of the units evaluated.

If EC is greater than unity, it shows an improvement in efficiency, which is the measure of recovery of technical efficiency within a group during the added period, indicating how fast a DMU moves towards the reference technology. Referring to the second component, BPC measures the change between the best contemporary reference technology and intertemporal technology: when BPC is greater than unity, it indicates that the contemporary approaches the intertemporal frontier. Finally, TGC is the technical gap between the frontier of intertemporal reference technology in moment t and the global frontier reference in moment $t+1$. When TGC is greater than unity, it indicates a narrowing of the global technology gap from the frontier of a specific group.

The DDF can be calculated from different types of frontier models, following different programming systems $\overrightarrow{D}^{G}(x^{w}, y^{w}, b^{w})$, $\overrightarrow{D}^{I}(x^{w}, y^{w}, b^{w})$, $\overrightarrow{D}^{w}(x^{w}, y^{w}, b^{w})$ where $w = t$ and $t+1$. We chose to calculate the DDF with DEA type models because it avoids the need to assume a functional form for the production function a priori and it is possible to work with a combination of multiple inputs and outputs. To calculate these distance functions using DEA type models, the following optimization program must be solved [17,55]:

$$D^{d}\left(x^{k\acute{w}}, y^{k\acute{w}}, b^{k\acute{w}}\right) = \max\beta \tag{11}$$

s.t.

$$\sum \lambda^{k,w} y_{m}^{k,w} \geq (1+\beta) y_{m}^{k\acute{,}w}, \quad m = 1, \ldots M. \tag{12}$$

$$\sum \lambda^{k,w} b_{j}^{k,w} \leq (1-\beta) b_{j}^{k\acute{,}w}, \quad j = 1, \ldots J. \tag{13}$$

$$\sum \lambda^{k,w} x_{n}^{k,w} \leq x_{n}^{k\acute{,}w}, \quad n = 1, \ldots N. \tag{14}$$

$$\lambda^{k,w} \geq 0 \tag{15}$$

The superscript d in the objective function is intended to identify the type of directional function since it can be contemporary, intertemporal or global. A vector $\lambda^{k,w}$ appears in the restrictions; this vector shows the intensity to construct the restriction, where k indicates the education sector. The estimation of the DDF through a DEA is an optimal solution for the calculation and decomposition of the index, where a β coefficient is obtained, which is interpreted as the simultaneous increase (decrease) that can be achieved in the good (bad) outputs, given the input consumption.

The next section presents the empirical study, and describes the variables and sources used in the analysis. The main results and decomposition are also shown according to the multiple directions offered by the index.

4. Empirical study: data and results

This section describes the data used in the model. It is focus on the variables and sources used for the analysis.

4.1. Sample and variables

The study considers 4587 schools participating in Saber 11 (standardized test) in the years 2014 and 2017. These are the schools for which complete information is available in all the variables during the years of study. The sample is 75.5% from the public sector and 24.5% from the private sector.

Based on the literature review, and in accordance with the proposal in the methodological section, this study uses seven variables: one good output, one bad output, and five inputs. The good output is the sum of the generalized global score for the school [7,22,31,32,63,66]. The global score is the weighted average[2] of the individual scores of each of the tests that students take in the exam, divided into the total weighting (13) and multiplied by the number of tests (5). To generalize the variable at the school level, since the inputs are at this level, the sum of the global score is divided into the number of students who took the exam and multiplied by the total enrollment of the school.

The bad output, following the scale with which the global score is built, is the standard deviation of the generalized global score for schools. To date, equality in educational processes has been measured through the total variance of academic performance [43], whereas other approaches have used the percentage of students who do not reach minimum scores on standardized tests [31,32]. However, standard deviation is used because it is a measure that can be applied for different types of standardized tests [57], and according to the literature, having homogeneous groups improves efficiency, which is the objective of the analysis of this study [9].

The five inputs used are frequent in the literature [25]: the amount of electronic equipment, number of teachers working as managers, number of teachers in classrooms, number of students enrolled, and the average socioeconomic and cultural level of the school as an environmental variable. Electronic equipment is quantified as all desktops or laptops and tablets available and in use [1,51].

The next two inputs are the number of teachers in managerial roles and teachers in classrooms. The first one is used as a proxy for variables commonly used in the literature such as managers or administrative staff [11,34,35], and the second is teachers in classrooms [3,21,48,67], whose main function is teaching. These two variables are included to represent the human capital of the school, mainly due to the different functions of the two roles. The fourth input refers to the number of students enrolled in the school [23,59].

Finally, there is a debate in the literature about how to include the socioeconomic index of the students in the estimations, where some authors use it as an input and others as an environmental variable. However, despite this debate, most scholars use it as input [20,21,31,66]. On the other hand, the methodological relevance of the conditional models is recognized as having opened up the possibility of adopting the socioeconomic index as an environmental variable; however, even so, some authors treat this variable as an input when using conditional models [20]. In this article, as in other research, the socioeconomic index is included as an input, since the evaluation is focused on the capacity of schools to make the most of the inputs [10,12,21]. In addition, because the databases do not offer the index, following Thieme et al. [66]; this is defined as an estimated latent variable through a multiple correspondence analysis, considering the educational level of parents and the socioeconomic level of the household. All the goodness of fit statistics yield results according to the calculation of the indicator.

Table 1 reports a summary of descriptive statistics for the variables used. The overall score shows an average decrease of 1.16%, and its standard deviation reflects a growth of 5.41%. It should be noted that statistics show an inverse behavior to the ideal in the educational process. The inputs have the following behavior: electronic equipment and teachers in classrooms increase on an average by 97.27% and 0.51%, respectively. The total enrollment of students, the number of teachers in

[2] A weighted average is used (three points each for mathematics, reading, social studies and natural sciences and one for English language) for two reasons: first, the literature recommends using multiple areas of study [41] and not just math and/or language, as is common; and second, because it is the measure used for admission to higher education and for the design of educational policies in Colombia. In addition, the weights of the areas are defined by the ICFES and are therefore maintained, mainly because one of the largest gaps between public and private education in Colombia is in the area of English and the global score smooths it out.

Table 1
Descriptive statistics of inputs and outputs.

Variables	Year	Average	Standard deviation	Private	Public
Output					
y_1: global score	2014	202,410	174,972	157,323	217,182
	2017	200,059	172,283	159,337	213,277
y_2: standard deviation of the global score	2014	474.28	304.14	557	447
	2017	499.95	275.68	552	483
Input					
x_1: electronic equipment	2014	79.88	78.65	53.62	88.49
	2017	157.58	185.06	58.32	189.8
x_2: enrollment of school	2014	798.19	670.16	553.14	878.48
	2017	768.46	639.71	540.83	842.34
x_3: teachers in management roles	2014	2.93	1.95	3.33	2.8
	2017	2.88	1.72	3.11	2.81
x_4: teachers	2014	31.64	22.90	27.57	32.97
	2017	31.8	22.63	27.84	33.08
x_5: socio-economic and cultural index	2014	5.04	0.85	3.97	5.39
	2017	4.96	0.73	5.9	4.66

Source: self-devised.

management roles and the socioeconomic and cultural index show respective average decreases of 3.72%, 1.71%, and 1.59%.

Due to the considerable heterogeneities in access to and quality of education among Columbia's 33 departments[3] and educational sectors (public/private), the results are presented with this disaggregation. The average global score for the private sector shows a growth rate of 1.28%. Bad output shows positive behavior in the private sector and negative behavior in the private one.

The inputs show different behaviors according to education sector. For example, electronic equipment had a growth rate of 114% in the public sector and 8.77% in the private. The teaching staff presented a relatively stable trend in both sectors, except for teachers in management roles in the private sector (−6.61%). Overall, the socioeconomic and cultural conditions of the students improved significantly: average growth for private schools was 48.61%, whereas these conditions deteriorated by 13.54% in the public schools. In next section, the inputs and outputs are used to estimate the metafrontier Malmquist Luenberger index. The results are presented by component and educational sector.

5. Results

The results section presents the analysis of educational efficiency[4] for Colombia at the departmental level and by education sector from two points of view: first, a static analysis through a DEA to provide an initial reference point. And second, a temporary analysis of productivity, which focuses on the evolution over time.

5.1. Static efficiency analysis

The static analysis[5] was carried out through a DDF and DEA described in the methodological section for the years 2014 and 2017, disaggregated by education sector and type of orientation. The analysis was performed for all departments of Colombia according to education sector and considering alternative orientations (good and bad output, bad output and, finally, good output). The DEA estimation in the static analysis shows high variability in the results among the education sectors, types of orientation, and departments at different times. For example, in 2014 Casanare is the most efficient department when it has an orientation towards good and bad outputs simultaneously, with a coefficient of 0.1026, which means that good outputs can be increased by 10.26% while bad outputs are decreased in the same proportion while maintaining the level of inputs.

As we see in Table 2, the static analysis on average shows higher levels of inefficiency in the public than in the private sector. Following the orientation towards good and bad output simultaneously, the public sector presents potential improvements of 0.2101 and 0.2079 in 2014 and 2017, respectively, and the private sector presents values of 0.1969 and 0.1562, showing better performance at 1.32% and 5.16%. The estimate towards bad output shows inefficiencies of 0.6549 and 0.5715 in the private sector and 0.6754 and 0.6613 in the public sector for the years 2014 and 2017, respectively; these results reveal gaps of 2.06% and 8.98%, respectively. Finally, the estimate with a good output orientation shows inefficiencies of 0.2121 and 0.1618 for the private sector and 0.2227 and 0.2169 for the public sector, highlighting gaps of 1.05% and 5.51%. In summary, three results are evident in the education system. First, the private sector is more efficient in both 2014 and 2017. Second, the gaps between sectors widen in this period. And third when the focus is on performance (good output) and equality (bad output) simultaneously, inefficiency values are lower.

At the departmental level, there are considerable differences in the results. In the private sector with simultaneous orientation towards good and bad outputs, the most efficient department (Casanare) has a value of 0.1026 and the least efficient (Putumayo), a value of 0.4514, showing a 35% gap in 2014 compared to the 23% gap for 2017 in the public sector. Following the orientation towards bad outputs and good outputs, the gaps in 2014 and 2017 are 39% and 41%, and 38% and 24%, respectively. Finally, the Li test [47] is used to check whether there is a significant difference between the general distributions of the results (it is used to compare all the results calculated in this article). This nonparametric statistical test compares two unknown distributions using kernel densities. Its main advantage is that unlike most statistical tests, the Li test is not based on comparisons of means or medians, but compares two complete distributions against each other.

The results of this static analysis provide an initial overview of the efficiency of the Colombian education system in two moments, and show the heterogeneity and the diverse patterns between the departments and types of orientation.

5.2. Temporary analysis of educational efficiency

Temporary analysis is carried out through a MML index described in the methodology section for the years 2014 and 2017, disaggregated by education sector and type of orientation. Table 3 shows the results of the scores for the metafrontier Malmquist Luenberger index and its decomposition for all departments of Colombia according to education sector. Table 4 shows the summary of applying the MML with three orientations. First, following the orientation g = (y, b); that is, it aims to increase the desirable outputs (y) by percentage β and reduce the undesirable outputs (b) in the same proportion. Second, with orientation g = (0, b); in this case, the objective is to minimize the bad outputs

Table 2
Static analysis of educational efficiency with different orientations between 2014 and 2017.

Orientation	Year	Private	Public
Good and bad outputs	**2014**	0,1969[a]	0,2101[a]
	2017	0,1562[a]	0,2079[a]
Bad outputs	**2014**	0,6549[a]	0,6754[a]
	2017	0,5715[a]	0,6613[a]
Good outputs	**2014**	0,2121[a]	0,2227[a]
	2017	0,1618[a]	0,2169[a]

Source: self-devised.

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[a] 1%.

[3] Descriptive statistics at the departmental level are available upon request.

[4] Before the estimation, a procedure was carried out to detect the extreme values and outliers using, among other methods, super efficiency [72] and a multivariate method, bacon [71], after which the estimation was performed without these extreme values.

[5] Departmental results are available upon request.

Table 3
Educational improvement (orientation towards good and bad output) between 2014 and 2017.

Department	Private				Public			
	EC	BPC	TGC	MML	EC	BPC	TGC	MML
Amazonas	1.1378	0.9129	0.9766	1.0145	0.9859	1.0111	1.0072	1.0040
Antioquia	1.0153	0.9566	0.9580	0.9304	0.9984	0.9881	0.9924	0.9790
Arauca	1.0507	0.9395	0.9897	0.9769	1.0366	0.9682	0.9928	0.9964
Archipiélago de San Andrés	0.9776	1.0595	0.9666	1.0011	0.9469	1.0302	0.9983	0.9739
Atlántico	1.0403	0.9508	0.9809	0.9703	1.0129	0.9720	0.9976	0.9823
Bogotá, D.C	1.0281	0.9649	0.9611	0.9534	1.0051	0.9682	0.9815	0.9551
Bolívar	1.0222	0.9774	0.9686	0.9677	0.9953	0.9964	0.9964	0.9882
Boyacá	1.0499	0.9582	0.9517	0.9575	1.0143	1.0287	0.9795	1.0220
Caldas	1.0352	0.9753	0.9537	0.9629	0.9811	1.0104	0.9934	0.9847
Caquetá	1.0265	1.0185	0.9771	1.0216	0.9882	1.0273	0.9933	1.0084
Casanare	1.0436	0.9912	0.9804	1.0141	1.0044	1.0106	0.9967	1.0117
Cauca	1.0849	0.8755	1.0200	0.9688	0.9385	1.0402	1.0091	0.9851
Cesar	1.0521	0.9184	0.9861	0.9529	1.0194	0.9743	0.9964	0.9896
Chocó					0.9615	1.0308	0.9973	0.9885
Córdoba	1.0808	0.9698	0.9607	1.0070	0.9895	1.0089	0.9912	0.9894
Cundinamarca	1.0352	0.9589	0.9720	0.9649	1.0080	0.9931	0.9879	0.9889
Guainía					1.0332	0.9590	0.9990	0.9899
Guaviare	1.0056	0.9473	1.0128	0.9648	0.9286	0.9630	0.9987	0.8931
Huila	1.0668	0.9552	0.9449	0.9628	1.0175	1.0258	0.9849	1.0279
La Guajira	0.9838	1.0268	0.9786	0.9886	0.9541	1.0376	0.9986	0.9886
Magdalena	1.0156	0.9544	0.9703	0.9405	0.9861	1.0033	0.9898	0.9793
Meta	1.0453	0.9401	0.9522	0.9356	0.9886	0.9911	1.0046	0.9843
Nariño	1.0299	0.9603	0.9653	0.9547	1.0050	1.0167	0.9745	0.9957
Norte de Santander	1.0695	0.9066	0.9798	0.9500	1.0328	0.9710	0.9909	0.9938
Putumayo	1.3821	0.7326	1.0089	1.0216	1.0181	1.0175	0.9859	1.0213
Quindío	1.0805	0.9194	0.9629	0.9566	0.9833	1.0034	0.9922	0.9790
Risaralda	1.0457	0.9386	0.9535	0.9360	0.9875	0.9950	0.9950	0.9777
Santander	1.0491	0.9482	0.9506	0.9457	1.0070	0.9968	0.9856	0.9893
Sucre	1.0348	0.9676	0.9702	0.9714	0.9826	0.9952	0.9936	0.9715
Tolima	1.0464	0.9691	0.9587	0.9722	0.9781	1.0184	0.9919	0.9880
Valle del Cauca	1.0462	0.9436	0.9954	0.9827	0.9932	0.9777	0.9984	0.9695
Vaupés					1.0065	0.9984	0.9987	1.0035
Vichada					0.9921	1.0205	0.9963	1.0087
Total	1.0374[b]	0.9546[a]	0.9680[b]	0.9586[a]	0.9977[b]	0.9958[a]	0.9909[b]	0.9845[a]

MML: Metafrontier Malmquist Luenberger.
EC: Efficiency Change.
BPC: Best Practices Change.
TGC: Technical Change Gap.
Source: self-devised.
[a] 5%.
[b] 1%.

Table 4
Summary of educational improvement (three orientations) between 2014 and 2017.

Orientation	Component	Private	Public
Good and bad outputs	**EC**	1.0374[b]	0.9977[b]
	BPC	0.9546[a]	0.9958[a]
	TGC	0.9680[b]	0.9909[b]
	MML	0.9586[a]	0.9845[a]
Bad outputs	**EC**	1.0302[b]	1.0005[b]
	BPC	0.9583	0.9973
	TGC	0.9655[b]	0.9908[b]
	MML	0.9680[b]	0.9909[b]
Good outputs	**EC**	1.0396[b]	0.9985[b]
	BPC	0.8911[b]	1.0080[b]
	TGC	0.9024[b]	0.9258[b]
	MML	0.8360[b]	0.9318[b]

MML: Metafrontier Malmquist Luenberger.
EC: Efficiency Change.
BPC: Best Practices Change.
TGC: Technical Change Gap.
Source: self-devised.
[a] 5%.
[b] 1%.

without considering the desirable outputs. Finally, the results with the orientation $g = (y, 0)$, where the objective is to maximize good outputs, and bad outputs are not considered.

Table 3 shows the results of the estimation of the MML index and its components with a simultaneous orientation towards good output and bad output. Changes in productivity are positive when the coefficients are greater than unity and negative when lower values are obtained. The aggregate results offer an overview of the whole system, although the analysis is broken down at the departmental level for a more detailed analysis. The education system, in general, shows a decrease in productivity of 4.14% (column 4) in the private sector and 1.55% (column 8) in the public sector, the latter showing a better performance by 2.59%. The behavior in the private sector is mainly driven by a decrease in the BPC and TGC of 4.54% (column 2) and 3.20% (column 3) respectively, offset by an EC improvement of 3.74% (column 1). In the public sector, the behavior of the TGC (0.91%), the BPC (0.42%) and the EC (0.23 deteriorates in all cases.

The EC is the component with the best performance in the private sector as behavior improves in 84% of the departments. In the BPC (Archipelago of San Andrés, Caquetá, La Guajira) and TGC (Cauca, Guaviare, Putumayo), only 9% of the departments show positive behavior. In the public sector, the component with the highest number of departments with positive behavior is the BPC (51%), followed by the EC (41%) and the TGC (9%).

The gaps between departments are different for each of the components, although the difference between the best and worst department evaluated in each component is always greater in the private than in the public sector. The gaps are 40.5%, 32.7% and 7.5% in the private sector and 10.8%, 8.1% and 3.5% in the public sector for the EC, BPC, and TGC, respectively, and the MML index presents a 9.1% gap for the private and 13.5% for the public sector. The results of the estimation of the index oriented to good output and bad output simultaneously show a better change in public sector performance except for the component directly related to the change in efficiency.

Table 4 [6] presents the summary of the results of the estimation of the MML index and its components with all orientations. On average, for the orientation towards bad output, the public sector has a better change in performance in both the MML index and the BPC and TGC components than the private sector. The differences in favor of the public sector considering the orientation to bad output are 3.90%, 2.53% and 2.30% for the components BPC, TGC, and MML, respectively, and the difference in EC in favor of the private sector is 2.97%. When the orientation to good output is followed, the differences in average increase to 11.69%, 2.34% and 9.58% for the BPC, TGC and MML components, and the EC has a difference in favor of the private sector of 4.11%. The orientation towards bad outputs focuses on equality in the education system, showing differences between sectors and highlighting the gaps. The results reveal that for both sectors the greatest participation in increasing productivity is provided by the EC.

The results of estimates with a static and dynamic approach considering multiple orientations reveal a difference between the approaches and orientations; four common patterns emerge. First, overall performance deteriorates in the private sector, and to a greater extent when it is oriented towards performance. Second, the EC is the component that drives positive changes in productivity. Third, the least efficient departments (Amazonas, Putumayo) at the initial moment, considering the static approach (DEA), are those that are marginally more easily able to improve their productivity. Finally, large gaps are evident between the departments.

6. Conclusions

The present study evaluates the evolution in the productivity of the Colombian education system between 2014 and 2017, taking into account students' results in the Saber 11 test and the endowments of educational institutions, and differentiating between orientations towards the types of results (performance and equality) and the education sectors.

The methodological approach is based on the metafrontier Malmquist Luenberger index (MML), and to our knowledge, this study is the first application in the education sector. This is a relevant tool to analyze the educational context since it allows bad outputs to be incorporated into the process while considering both performance and different groups in the evaluation. From an empirical point of view, the study aimed to analyze the change in the productivity of the Colombian education system and the similarities and differences between the sectors, and to discover the performance of the different units in a context of quality incentives. It also seeks to offer a robust tool with greater scope for measuring efficiency than the Quality Index explained in the introductory section.

The results of the MML index according to the different orientations show, on average, deterioration in the private sector of 16.4% (good output), 3.20% (bad output) and 4.14% (good and bad output), depending on their orientation. For the public sector, there is a deterioration of 6.82% (good output), 0.91% (bad output) and 1.55% (good output and bad output). The results confirm that the public sector performs better than the private sector, regardless of the orientation. The superior productivity of the public sector is in line with the study by Mancebón et al. [51]; who find better levels of performance in public schools than in their private counterparts in the Spanish education system.

The components of the index show great differences in behavior depending on the sector analyzed. The private sector is mainly driven by the change in efficiency (EC), and to a greater extent when the performance orientation is followed (3.96%). At the departmental level and considering each of the components, the results are as follows: (i) high variability among departments and, in general, their level of productivity is correlated with the efficiency in the initial situation; (ii) less dispersion of the results in the public than in the private sector; (iii) greater variance in the index when focusing on performance.

The results of the educational tests, in general, are analyzed directly, without considering either bad outputs or the available resources. This is one of the reasons why the findings of the present study do not coincide with current discussions surrounding academic results of public and private schools. From a public policy point of view, 75% of the schools evaluated are public; therefore, there should be a concern for access to a more egalitarian education without sacrificing good performance. In conclusion, a simple analysis must be distinguished in absolute terms as in the present study, in which the focus is on the productivity and equality of the education system.

The MEN *Estímulos a la Calidad* program in Colombia, introduced in 2016 to evaluate and encourage academic quality, is one of the motivators of this study. The results confirm that, on average, the program had no positive effects on the evolution of productivity of schools in the education system. That is, school productivity decreased from 2014 to 2017. This decrease may be due to various reasons, of which we identified two. First, the program applies a blanket educational policy to the whole country, regardless of the context. The second reason is related to the *Estímulos a la Calidad* program and the new measurement of schools' quality. One component of the measurement is the students that pass the academic year (promotion in the index), which can incentivize teachers to raise students' grades, which may not guarantee the minimum learning standards.

There are several implications for education policy. First, the *Estímulos a la Calidad* program and its adaptability should be restructured according to the context. Additionally, variables that cannot be biased by the MEN's design of incentives must be taken into account. Finally, public policy must focus on all students in the education system. For this reason, maximum attention should be paid to guaranteeing minimum learning standards for all students and not simply to the results of standardized tests.

Although the study met its objectives and the results are relevant to the context in which it was developed, three limitations should be addressed in future research. First, the availability of a restricted period in the databases; expanding this period would provide a better overview of the approach. Second, this study, like many others, carries out analyses with the schools as its unit of analysis; however, there is a tendency towards considering the student as the unit of analysis. Additionally, taking into account different academic courses, instead of standardized tests, could yield results with greater scope. Third, the effects of departmental characteristics should be analyzed, since, in an environment such as Colombia, institutions and location can play an important role in educational productivity.

Funding

Víctor Giménez, Sebastian López-Estrada and Diego Prior acknowledge the financial support of the Spanish Ministerio de Economía y Competitividad (ECO2017-88241-R).

[6] As the focus is on components and sectors, results at the departmental level for the second and third orientation are not presented here, but are available upon request. This decision was made at the suggestion of one of the reviewers and to reduce space.

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