Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 203 (2022) 105477
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Unemployment risks and intra-household insurance ☆

Javier Fernández-Blanco

Universitat Autònoma de Barcelona, Barcelona School of Economics and MOVE, Spain

Received 29 September 2020; final version received 5 May 2022; accepted 8 May 2022
Available online 13 May 2022

Abstract

To study the constrained efficient public insurance provision against unemployment risks, we build a directed search model with households where a spouse's ability to provide consumption insurance determines the risks job-seekers take on. The planner's transfers to the unemployed fall with the spouse's income because of concave preferences with limited complementarity between consumption and the spouse's labor. Due to the absence of such a transfers scheme in the laissez-faire equilibrium, too many too-low-wage jobs are created as jobless workers seek insurance in the labor market. Social welfare-maximizing policies with simple tax schemes exhibit unemployment benefits falling with spouse's income, and nearly fill up the welfare gap.

JEL classification: J08; J22; J64; J65

Keywords: Unemployment risks; Intra-household risk-sharing; Directed search; Constrained efficient insurance

☆ I thank two anonymous referees and the editor for their very helpful comments. I also want to thank Jeff Campbell, Yongsung Chang, Martin Kaae Jensen, Helmut Maurer, Jean-Baptiste Michau, Claudio Michelacci, Nicola Pavoni, Edgar Preugschat, Aysegul Sahin, Itay Saporta and Edward Schlee for their feedback. I am also grateful to numerous participants at conferences and seminars, and to Tommaso Santini for excellent research assistance. I acknowledge financial support from the Spanish Government under Grant No. PGC2018-094364-B-I00, from the Generalitat de Catalunya through Grant 2017-SGR1765, and from the Spanish Agencia Estatal de Investigación (AEI) through the Severo Ochoa Program for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S).
E-mail address: javier.fernandez@uab.cat.

https://doi.org/10.1016/j.jet.2022.105477

1. Introduction

How much public unemployment insurance should be provided in a world where consumers have various ways to buffer income shocks? The normative analysis of this question is typically restricted to simple policies such as a constant replacement rate scheme. By contrast, heterogeneity is the essence of modern macroeconomics, and insurance needs are likely to differ across households. In particular, there is a large body of empirical research documenting that the income of a worker's spouse plays a major role in consumption insurance. See e.g. Mincer (1962), Cullen and Gruber (1996), Browning and Crossley (2001) and Blundell et al. (2016b). In this paper, we address this question allowing for the public insurance provision to vary with the private insurance arrangements through the spouse's labor supply.

We consider a static frictional economy where households are risk-sharing institutions formed by a jobless worker and his spouse. Households are ex-ante heterogeneous in the spouse's market productivity. Jobless workers decide whether to participate in the labor force or not. We model job search as directed so that job-seekers trade off higher wages and lower employment chances. A worker's job-search strategies (and, hence, the income risks taken on in the labor market) are shaped by his spouse's ability to provide insurance, which is arranged by pooling income within the household and adjusting her labor supply. We assume that all decisions are made jointly by the worker and his spouse, and, hence, no moral hazard is generated within the household. In line with the search literature (e.g. Golosov et al. (2013)) as well as the optimal labor income taxation literature since Mirrlees (1971), a worker's labor force participation decision and job search strategies are private information and so are his spouse's productivity and labor supply. Employment and income are observable instead.

Our main contribution is the characterization of the *constrained efficient* provision of insurance. A social planner maximizes a household's expected utility subject to the technological constraints and the incentive-compatibility constraints related to the information frictions. Furthermore, to focus on the optimal allocation of consumption risks, the social planner is allowed to redistribute resources among ex-ante identical households, but not among ex-ante different ones. The planner sets a type-specific transfers scheme to insure away partially the consumption risks, and households with the two members unemployed receive the largest transfers. We show that the transfers to the unemployed would steadily fall with the spouse's income if a spouse's productivity were observable (or such information frictions were rather small). These patterns over the distribution of households result from the limited complementarity between consumption and labor and concavity of a household's preferences. These two assumptions on preferences ensure that the marginal utility gains from the transfers fall with the spouse's productivity, and that both consumption and leisure are normal goods. Thus, our theory not only questions the standard single-replacement-rate unemployment insurance scheme, but also provides a rationale for the dependency allowance jobless workers with an unemployed spouse are entitled to in eight states of the U.S. and Belgium.[1] This *tagging* feature in the sense of Akerlof (1978) is also supported by the empirical evidence. For example, Browning and Crossley (2001) estimate a small effect of the unemployment insurance replacement rate on household's expenditure, but significant for those households with an unemployed spouse.

[1] The states are Connecticut, Illinois, Iowa, Maine, Michigan, New Jersey, Ohio and Pennsylvania. See the Department of Labor documentation: https://oui.doleta.gov/unemploy/pdf/uilawcompar/2018/monetary.pdf. For Belgium, see https://www.onem.be/fr/documentation/feuille-info/t67.

The laissez-faire equilibrium allocation is incentive compatible. Likewise, the labor force participation decision is efficiently set in equilibrium as the planner's transfers are contingent to a household's (self-reported) type and, hence, can be adjusted as much as needed. However, constrained efficiency is not attained in the decentralized economy because of the market's inability to pool consumption risks (incomplete markets). Similar to Acemoglu and Shimer (1999), if the absolute risk aversion of a household's indirect utility function falls with the spouse's productivity, the smaller intra-household insurance provision, the more insurance the unemployed seek in the labor markets: workers married to less productive spouses apply to lower-wage jobs anticipating shorter queues as firms create more of those higher-profits jobs. Therefore, equilibrium exit rates from unemployment decline with the spouse's income, consistent with the falling pattern observed in the U.S. economy. Because of the partial insurance the social planner achieves through a transfers scheme, the constrained efficient exit rates are flatter and steadily increase in the spouse's income under some assumptions.

Our second contribution is a quantitative exploration of the planner's allocation. We find that the constrained efficient replacement rate starts at just below 30% for workers with an unemployed spouse and then steadily falls with the spouse's income, with a weighted average just below 22%. When the planner is further restricted to a constant replacement rate over the distribution of households instead, it is set at nearly 20%. Average welfare gains relative to the laissez-faire equilibrium are significantly lower in the latter case, 4.77% vs. 5.85%. Two subgroups of households are particularly harmed by a constant replacement rate policy. First, if transfers are not tailored to households with both members unemployed, welfare losses for this subgroup are sizable and their consumption falls by 5%. Second, the participation rate falls some 20 percentage points because incentive compatibility becomes more stringent under the constant rate regime. Being expelled from the labor force makes these households worse off than in the laissez-faire equilibrium with welfare losses reaching up to 4%.

Furthermore, our quantitative work shows that a social welfare-maximizing policy with a simple tax scheme exhibits falling unemployment benefits over the spouse's productivity distribution, and nearly closes up the welfare gap. Importantly, we find that the welfare implications of whether or not a spouse's productivity is observable are very minor.

This paper contributes to several branches of the labor literature. First, following the lead of Burdett and Mortensen (1978) in the search literature, Guler et al. (2012) and Pilossoph and Wee (2021) address the joint search of couples in a McCall setting with an exogenous distribution of wages, and Bacher et al. (2022) explore its lifecycle dimension in a random search model. Mankart and Oikonomou (2017) model the participation decision of the secondary earner as an insurance mechanism to account for the observed acyclicality of the labor force participation rate. Similarly, in our model, a worker's participation and search decisions are shaped by the spouse's (potential) income and vice versa.

Second, while we abstract from other sources of insurance, the optimal single replacement rate has been quantitatively studied in frictional settings where workers are allowed to save (and borrow). See e.g. Hansen and İmrohoroğlu (1992), Lentz (2009), Krusell et al. (2010), Lifschitz et al. (2018) and Braxton et al. (2019).[2] With the exception of the latter, either wages are taken exogenously or search is assumed to be random and savings observable. The optimal single replacement rate is typically found to be fairly low because of the large distortions on job creation

[2] Wealth holdings are typically fairly modest for newly unemployed workers (see e.g. Engen and Gruber (2001) and Chetty (2008) for the U.S. and Kolsrud et al. (2018) for Sweden). However, Braxton et al. (2019) document that the U.S. constrained job losers default on their credit obligations and the unconstrained ones borrow.

and the crowding out effects.[3] Haan and Prowse (2017) add the spouse's labor supply to a life cycle model with exogenous wages, and find that the optimal single replacement rate is 20% for Germany. Birinci (2019), using a random search model, and Wang (2019), in a setting with time-invariant exogenous wages, model both savings and the spouse's behavioral responses to quantitatively examine the cyclicality of optimal unemployment benefits. Ortigueira and Siassi (2013) and Choi and Valladares-Esteban (2020) have also addressed the role of a spouse's labor supply as an insurance mechanism in a Bewley framework with exogenous income risks. Our work complements this quantitative research by focusing on the constrained efficient allocation of risks, which differ over the distribution of households, thereby generating differences in replacement rates.[4] As claimed above, our quantitative analysis reaches similarly low (weighted average) replacement rates, but highlights the welfare losses through lower participation and lower consumption insurance under a single-replacement-rate policy. We also highlight the role of endogenous wages in pinning down unemployment risks through vacancy creation, and allow for complementarity between consumption and the spouse's labor.

The paper proceeds as follows. Section 2 describes the economy. In Section 3, we study the market equilibrium. Section 4 analyzes the planner's solution. In Section 5, we undertake a quantitative exercise. The last section concludes.

2. Economy

In this section, we describe a frictional model of the labor market that will be used to examine both the laissez-faire equilibrium and the constrained efficient insurance provision against unemployment risks. We consider a static economy populated by a measure one of two-member households and a large continuum of risk-neutral firms. Each household is formed by a male and a female. Different roles are assigned to each household member, but the gender labels are just used for the sake of clarity. The static setting is fairly asymmetric across genders because of the focus on the unemployment risks males face, while the female side is quite stylized to model endogenous differences in the spouse's ability to provide consumption insurance.[5]

Males start jobless and can search for a job in a frictional labor market, supplying indivisible labor (one unit of time) at market productivity $\overline{x}_m$. A female's employment status is randomly determined at the beginning of the period. Employed females draw a market productivity x per unit of time. Let G denote the cumulative distribution function of employed female's types with support $[\underline{x}, \overline{x}]$, and $\underline{x} > 0$. An employed female chooses labor supply, ℓ_x, and obtains market income $y_x = x\ell_x$. For notational simplicity, we denote the market productivity and labor supply of an unemployed female by 0, $y_x = \ell_x = 0$ for $x = 0$. Differences in productivity translate into differences in intra-household consumption insurance through the females' labor supply

[3] For example, Krusell et al. (2010) find that the optimal replacement rate is 30%. In Lifschitz et al. (2018), the welfare gains of unemployment insurance stem largely from redistribution across exogenously different workers, and the optimal replacement rate is around 10% if such exogenous heterogeneity is absent.

[4] More generally, in a static environment fairly similar to ours, Kleven et al. (2009) also find that optimal tax rates on an individual's labor income differ by the earnings of his or her spouse.

[5] As opposed to a dynamic environment where symmetry would be a natural assumption, a static setting relates naturally with various asymmetries between the two household members. Indeed, the asymmetry in this static setup quite resembles the one modeled by Kleven et al. (2009) to study optimal joint taxation. In their case, primary earners are endowed with some unobservable productivity and choose labor supply, whereas secondary earners decide whether or not to participate at a fixed number of hours. The case with a non-degenerated distribution of male productivities is examined in the Online Appendix.

decision. As households differ ex-ante by the female's productivity while all males are equally productive in the economy, we will refer to the household's type as its female's productivity.

Following e.g. Golosov et al. (2013) and Fernández-Blanco (2013) as well as the optimal unemployment insurance literature since Shavell and Weiss (1979), search strategies are assumed to be workers' private information, while an individual's labor market status and output are publicly observable. Likewise, in line with the optimal labor income taxation literature since Mirrlees (1971), a female's productivity per unit of time and labor supply are also private information, while income is not.

Preferences. We assume a unitary model of the household as in e.g. Guler et al. (2012) and Krueger and Wu (2018). That is, the two members of a household make all the decisions jointly, income is pooled, and the household derives utility from consumption, c, and leisure of the female.[6] We impose the following assumptions on the utility function $\upsilon(c,\ell)$ that describes a household's preferences:

A1. Thrice continuously differentiability.
A2. Strict monotonicity: $0 < \frac{\partial \upsilon}{\partial c}$ and $\frac{\partial \upsilon}{\partial \ell} < 0$.
A3. True concavity: $\frac{\partial^2 \upsilon}{\partial c^2}, \frac{\partial^2 \upsilon}{\partial \ell^2} < 0$ and $0 < \frac{\partial^2 \upsilon}{\partial c^2}\frac{\partial^2 \upsilon}{\partial \ell^2} - \left(\frac{\partial^2 \upsilon}{\partial c \partial \ell}\right)^2$.
A4. Limit conditions: $0 < \lim_{\ell \to 0} \frac{\partial \upsilon}{\partial c}x + \frac{\partial \upsilon}{\partial \ell}$ and $\lim_{\ell \to \bar{\ell}} \frac{\partial \upsilon}{\partial c}x + \frac{\partial \upsilon}{\partial \ell} < 0, \forall x \in [\underline{x}, \overline{x}]$.
A5. Limited complementarity between consumption and labor: $\frac{\partial^2 \upsilon}{\partial c \partial \ell} < \frac{\partial^2 \upsilon}{\partial c^2}\frac{\frac{\partial \upsilon}{\partial \ell}}{\frac{\partial \upsilon}{\partial c}}, \ \frac{\partial^2 \upsilon}{\partial \ell^2}\frac{\frac{\partial \upsilon}{\partial c}}{\frac{\partial \upsilon}{\partial \ell}}$.

The first four conditions are fairly standard. The limited Edgeworth-Pareto complementarity assumption is a necessary and sufficient condition for both consumption and leisure to be normal goods, and it shall prove to be key in deriving comparative results over the distribution of households. We will pay particular attention to the following families of preferences:

$$\mathcal{F}_1 \equiv \left\{ \upsilon(c,\ell) = \frac{c^{1-\theta}}{1-\theta} - \gamma \frac{\ell^{1+\xi}}{1+\xi} \quad \middle| \quad \theta, \xi, \gamma > 0 \right\}$$
$$\mathcal{F}_2 \equiv \left\{ \upsilon(c,\ell) = \frac{\left(c \cdot exp(-\psi \ell)\right)^{1-\theta}}{1-\theta} \quad \middle| \quad \theta > 1, \psi > 0 \right\}$$

As such additively separable preferences are extensively used in the macroeconomics literature, we will refer to them as the *standard macro* preferences. The second family of preferences exhibits non-separability and complementarity between consumption and labor, in line with some empirical evidence. See e.g. Hall and Milgrom (2008) and Blundell et al. (2016b). Such preferences are of particular interest as the female's labor supply adjustments provide full consumption insurance. We will refer to them as *LMP* preferences as they extend those assumed in Low et al. (2010) to the intensive margin of labor supply.[7]

[6] It can be shown that a cooperative model of the household -with an explicit representation of its members' preferences- and Pareto weights independent of income and policy delivers the same households' decisions as a unitary model does, while also ensuring Pareto efficient intra-household outcomes. See e.g. Chiappori and Mazzocco (2017).

[7] Variations of the LMP preferences are assumed e.g. by Blundell et al. (2016a) and Shephard (2019). Standard macro preferences in turn are used e.g. by Heathcote et al. (2014) and Gayle and Shephard (2019).

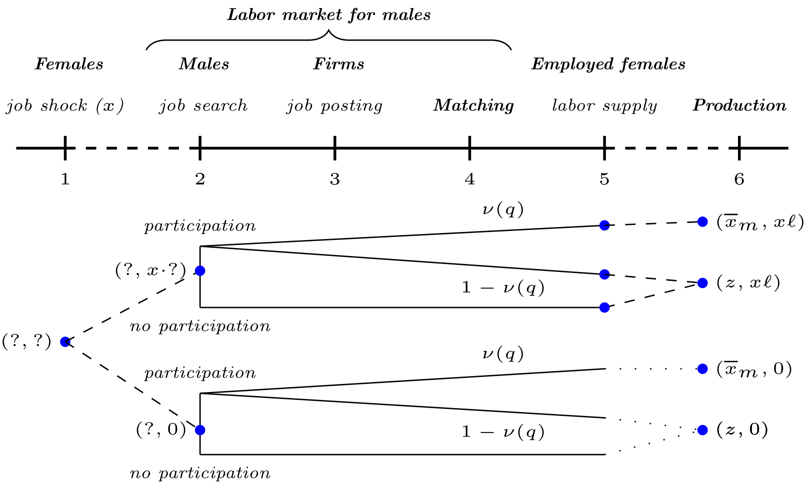


Fig. 1. Timing of events and a household's output vector. Note: A household's output vector consists of the male's and the female's production, which is her market productivity x times her labor supply if employed and 0 otherwise. These elements are unknown at the beginning of the period, and determined throughout the period. $\nu(q)$ denotes a male's job-finding probability.

Timing and labor markets. Fig. 1 presents the timing of events. There are six stages. In the first stage, the potential of the intra-household insurance is determined: females are assigned an employment status and market productivity, x, if employed. The next three stages refer to the labor market for males. In stage two, males decide whether to participate in the labor force. At this point, there are roughly speaking four groups of households: an employed female is married to either a participating or a non-participating male, and similarly for their unemployed ($x = 0$) counterparts. Participating males direct their search: choose a submarket or location where to submit one job application at utility cost k_w. Similarly, in stage three, recruiting firms decide on the submarket where to place their vacancies, and incur cost κ. Meetings take place in stage four. The matching probabilities are described below. In stage five, employed females decide their labor supply. Finally, production and consumption take place in stage six. As shown on the right of Fig. 1, the output vector of a couple formed by an employed male married to an employed (unemployed) female is $(\overline{x}_m, x\ell)$ ($(\overline{x}_m, 0)$), while unemployed males produce z units of output at home.[8]

As usual in the search literature, each firm holds a single vacancy. To ensure that vacancy creation is a profitable activity, we assume that $\overline{x}_m - z > \kappa$. We denote by q the expected queue length or ratio of job-seekers to vacancies at a particular location.[9] Males find a job with probability $\nu(q)$, whereas firms fill their vacancies with probability $\eta(q)$. It must be the case that $\nu(q) = \frac{\eta(q)}{q}$ in any given location since the mass of newly employed workers equals the mass of newly filled jobs. We assume that ν is a decreasing function to capture the intuition that it is easier to find a job in tighter labor markets. Similarly, η is assumed to be increasing. Likewise,

8 Unemployed females' home production is normalized to 0 for simplicity. Otherwise, it would be reasonable to impose that the employed females' market income were $y = z_f + \ell x$, without changing the results.

9 Although the ratio q depends on the characteristics of the jobs posted in a location, we eliminate this dependence notation unless necessary for the sake of readability.

the following limit conditions are necessary to ensure existence of the equilibrium and the planner's allocations: $\lim_{q\to 0} \nu(q) = \lim_{q\to\infty} \eta(q) = 1$ and $\lim_{q\to\infty} \nu(q) = \lim_{q\to 0} \eta(q) = 0$. Let $\phi(q) \equiv \frac{q\eta'(q)}{\eta(q)}$ denote the elasticity of the job-filling probability, which is assumed to be a decreasing function.

3. Market economy

In this section, we analyze an economy in which agents make decisions in a decentralized way. There is free entry of firms in the labor market for males, and potentially infinitely many submarkets. Each submarket is defined by a wage offer, w, and its associated queue length, q. Participating males choose a submarket to submit a job application, and recruiting firms decide the wage they commit to when creating a vacancy. We start describing how employed females make their labor supply decisions in stage five, and proceed backwards.

Stage 5. An employed female's labor supply decision. Consider a household formed by an employed female of productivity x and a male with wage w at the end of the period ($w = z$ if unemployed). We denote the indirect utility function of the household by

$$V_x(w) \equiv \max_y \ \upsilon\big(w + y, \frac{y}{x}\big) \tag{1}$$

$V_x(w)$ is well-defined because of the Weierstrass theorem together with Assumption A1. Likewise, the first order condition (2) is also sufficient because of Assumptions A3 and A4, and uniquely determines the female's income, $y_x(w)$.[10]

$$\frac{\partial \upsilon\big(w + y, \frac{y}{x}\big)}{\partial c} x + \frac{\partial \upsilon\big(w + y, \frac{y}{x}\big)}{\partial \ell} = 0 \tag{2}$$

Let $V_0(w) \equiv \upsilon(w, 0)$ and $y_0(w) \equiv 0$ denote the utility of a household with an unemployed female and her labor supply, respectively.

Stage 3. Job creation in the labor market for males. There is free entry of firms in all submarkets, both in and out of equilibrium. That is, the following condition must hold for all $w \in [z, \overline{x}_m]$:

$$\eta(q(w))(\overline{x}_m - w) \leq \kappa, \text{ and } q(w) \leq \infty, \text{ with complementary slackness} \tag{3}$$

Intuitively, the larger the wage, the higher the ratio of job-seekers to vacancies. In the limit, no positive mass of firms commit to a wage equal to the male's market productivity.

Stage 2. A male's participation and search decisions. Males decide whether to participate in the labor force and, if so, the type of job they apply to. The expected utility of a household of type x amounts to $V_x(z) + \max\{0, S_x - k_w\}$, where the second argument in the max operator denotes the search value (or the expected gains to searching in the market economy) net of the participation costs. The expected gains from a job application in a specific submarket w amount to $\nu(q(w))\big(V_x(w) - V_x(z)\big)$, and the search value is defined as the supremum of these expected gains across submarkets.

[10] In the case of LMP preferences, Assumption A4 does not hold for female's productivity values below $w\psi$. In that region, a female's optimal labor supply is 0.

3.1. Equilibrium

We now turn to the equilibrium definition. Let $X \equiv \{0\} \cup [\underline{x}, \overline{x}]$.

Definition 1. A directed search equilibrium consists of search values $\{S_x^*\}_{x \in X}$, the income of the employed females $\{y_x^*\}_{x \in [\underline{x}, \overline{x}]} : [z, \overline{x}_m] \to \mathcal{R}_+$, a set of labor force participants $X_p^* \subset X$ and their respective wages $\{w_x^*\}_{x \in X_p^*}$, and a queue length function $Q^* : [z, \overline{x}_m] \to \mathcal{R}_+$ such that:

i) Households' optimal decisions:
 (a) males' labor force participation:

$$x \in X_p^* \iff \nu(Q^*(w_x^*))\big(V_x(w_x^*) - V_x(z)\big) \geq k_w$$

 (b) males' job search: $\forall x \in X_p^*$,

$$\nu(Q^*(w))\big(V_x(w) - V_x(z)\big) \leq S_x^*, \quad \forall w \in [z, \overline{x}_m], \text{ and}$$
$$\nu(Q^*(w_x^*))\big(V_x(w_x^*) - V_x(z)\big) = S_x^*$$

 (c) employed females' income: $\forall x \in [\underline{x}, \overline{x}]$, $\forall w \in [z, \overline{x}_m]$, $y_x^*(w)$ solves the household's problem (1).

ii) Free entry of firms:
$\eta(Q^*(w))(\overline{x}_m - w) \leq \kappa$, $\forall w \in [z, \overline{x}_m]$, and $Q^*(w) \leq \infty$, with complementary slackness. In particular, the first inequality is an equality for all wages in set $\{w_x^*\}_{x \in X_p^*}$.

A male decides to participate in the labor force and seek job opportunities if the search value outweighs the participation costs. No submarket with a promised expected value below S_x^* attracts applications from type-x households. The second condition states that, before making their search decisions, males form rational expectations about firms' decisions in stage three. Specifically, they expect the ratio of job-seekers to firms in any submarket to be determined by the free entry condition. Thus, they trade off a higher wage and a lower job-finding probability.

A household's indirect utility function. Function V_x is a central object in this setting. Proposition 3.1 lists its properties as well as those of the optimal income of an employed female, which are inherited from the assumptions on preferences. Importantly for the comparative analysis across ex-ante different households, function $V_x(w)$ is concave in w and its cross partial derivative is negative (i.e. the marginal utility gains from a male's wage fall with his spouse's productivity) because of the assumption of limited complementarity between consumption and labor. Furthermore, a lower female's productivity can be interpreted as a more risk averse household if the absolute risk aversion of function V_x is decreasing in x. In particular, this is the case for both the standard macro and the LMP preferences. Likewise, the limited complementarity assumption also ensures that a female's leisure is a normal good as her income decreases with the male's wage.[11] While the reduction in her income is smaller than the increase in his wage for the standard macro preferences, it is one-to-one for the LMP preferences. In this latter case,

[11] The negative cross-wage elasticity is in line with the empirical evidence. Hyslop (2001) estimates that a \$1 increase in a husband's hourly wages reduces the wife's annual earnings by \$300 and her labor supply by 35 annual hours. Devereux (2004) estimates the cross-wage elasticity of wife's hours worked at -0.4, while Blau and Kahn (2007) at -0.2. Likewise, Blundell et al. (2016b) find the Marshallian cross-wage elasticity to be -0.75 for women and -0.22 for men.

there is full consumption insurance through the female's labor supply as equation (2) becomes $w + y_x(w) = \frac{x}{\psi}$.

Proposition 3.1. ***Household's indirect utility function and female's income.***

1. *For any productivity $x \in X$, a household's indirect utility function $V_x(w)$ is strictly increasing and concave in the wage w. Furthermore, a female's income $y_x(w)$ is strictly decreasing in w, while consumption $w + y_x(w)$ is non-decreasing.*
2. *For any wage $w \geq z$, function $V_x(w)$ is strictly increasing and its derivative with respect to wage w, $V_x'(w)$, strictly decreasing in the female's productivity x. Further, income $y_x(w)$ is increasing in x.*
3. *Let $x, x' \in [\underline{x}, \overline{x}]$ such that $x' < x$. There exists a function $\mathcal{V}$ such that $V_{x'}(w) = \mathcal{V}\big(V_x(w)\big)$, and $\mathcal{V}' > 1$. Furthermore, $\mathcal{V}$ is concave (convex) if and only if the absolute risk aversion of V_x, $\frac{-V_x''(w)}{V_x'(w)}$, is decreasing (increasing) in x. $\mathcal{V}$ is concave if a household's preferences are either standard macroeconomic or LMP preferences.*

Equilibrium characterization. Assume that a male's productivity, $\overline{x}_m$, is sufficiently high for all households with an unemployed female to engage in job search.

A type-x household participates in the labor force if and only if $S_x^* \geq k_w$, where the equilibrium search value is defined as

$$S_x^* \equiv \max_{q \geq 0,\, w \in [z, \overline{x}_m]} \nu(q)\big(V_x(w) - V_x(z)\big) \qquad \text{s. to} \quad \text{condition (3)} \tag{4}$$

Job-seekers choose the (q, w) pair that maximizes the expected gains from search within the set of submarkets characterized by the free entry condition (3). This constraint determines a negative relationship between wages and job-finding probabilities. In equilibrium, the search value decreases with female's productivity. This results from the difference $V_x(w) - V_x(z)$ falling with x because of the negative cross partial derivative of the indirect utility function. Thus, the labor force participation decision boils down to a reservation rule.[12] Then, conditional on participating, the equilibrium pair (q_x^*, w_x^*), with $q_x^* \equiv Q^*(w_x^*)$, is determined by the equilibrium condition (5) and the zero-profit condition (6). The former equates the costs of creating a vacancy to the expected profits, which amount to the probability of filling a vacancy times the share $1 - \phi(q)$ of the joint value of the firm-worker pair, i.e. the sum of the firm's profits, $\overline{x}_m - w$, and the household's surplus, $\frac{V_x(w) - V_x(z)}{V_x'(w)}$. Proposition 3.2 summarizes these results.

Proposition 3.2. ***Equilibrium characterization.***

Assume that a male's productivity is such that males with an unemployed spouse participate in the labor force. Then, there exists a unique equilibrium. Furthermore,

[12] As a referee pointed out, if search costs were modeled in units of the consumption good instead, males married to low productive females would decide not to participate in the labor force if their consumption if jobless were significantly reduced. If the model is modified along these lines and recalibrated using the same targets as those in Section 5, the equilibrium net value of participating in the labor force is always positive, but not monotone: an increase at the bottom of the female's productivity distribution is rapidly followed by a steady and larger decline.

1. *the equilibrium search value,* S_x^**, decreases with female's productivity. There exists a reservation productivity* x^* *such that a male married to a type-*x *employed female participates in the labor force if and only if* $x \leq x^*$.
2. *for any female's productivity* $x \in [\underline{x}, \overline{x}]$*, the equilibrium income* $y_x^*(z)$ *is determined by equation (2). Likewise, for any* $x \in X_p^*$*, the equilibrium tuple* $(q_x^*, w_x^*, y_x^*(w_x^*))$ *is characterized by equation (2) and*

$$\kappa = \eta(q)(1-\phi(q))\left(\frac{V_x(w) - V_x(z)}{V_x'(w)} + \overline{x}_m - w\right) \tag{5}$$

$$\kappa = \eta(q)\big(\overline{x}_m - w\big) \tag{6}$$

The subset of men with an unemployed wife becomes an interesting benchmark as such households are equivalent to the single job-seeker typically modeled in the search literature. While their labor market outcomes depend exclusively on their individual characteristics (i.e. productivity), condition (5) implies that wages and employment chances differ among males with an employed spouse insofar as the household's surplus depends on the female's productivity. Put differently, the equilibrium is generically separating. This highlights a source of wage dispersion that has been overlooked when using only individual data (e.g. in a Mincerian regression). Why does wage dispersion arise in equilibrium in the presence of equally productive workers and homogeneous firms? Recall that job-seekers trade off wages for employment chances. Thus, wage dispersion and differences in risk attitudes are the two sides of the same coin. To obtain some insights, consider first preferences that are quasi-linear in either consumption or leisure, which violate the concavity and limited complementarity assumptions. With such preferences, all households direct their search to the same submarket in equilibrium (to maximize expected income net of home production) because the household's surplus amounts to $w - z$. In the former case households are risk neutral in consumption, whereas full consumption insurance through the female's labor supply arises if preferences are quasi-linear in leisure. It is then tempting to conclude that wage dispersion results from consumption risk aversion and differences in private insurance arrangements within the household. However, recall that full insurance against consumption risks also takes place with LMP preferences, whereas a household's surplus falls with x.[13]

Then, why does job search differ across households? Fig. 2 displays the graphical representation of the household's problem (4). In addition to the zero-profit (concave) curve, it shows a solid indifference curve of households with female's productivity x, and a dashed one of households of type $x' > x$. Although it is shown that a higher female's productivity flattens the indifference curve, this needs not be the case. The slope of a household's indifference curve amounts to $\frac{-\nu'(q)}{\nu(q)}$ times the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$. Thus, higher wages and longer queue lengths are positively associated with higher female's productivities if and only if the household's surplus declines with female's productivity. Proposition 3.3 states that this holds if and only if the absolute risk aversion of the household's indirect utility function also declines with x, which is the case for both the standard macroeconomic and the LMP preferences. Still, why is the absolute risk aversion related to the equilibrium sorting outcome? It is widely accepted that the risk

[13] Notice that the quasi-linear utility function $\upsilon(c, \ell) = \log(c) - \psi\ell$ is the limit case of the LMP preferences (properly adjusted by a constant) as elasticity θ goes to 1. Also recall that a female's income is positive provided that $z\psi < x$ with LMP preferences. Therefore, all households with female's productivity above $z\psi$ apply to different jobs.

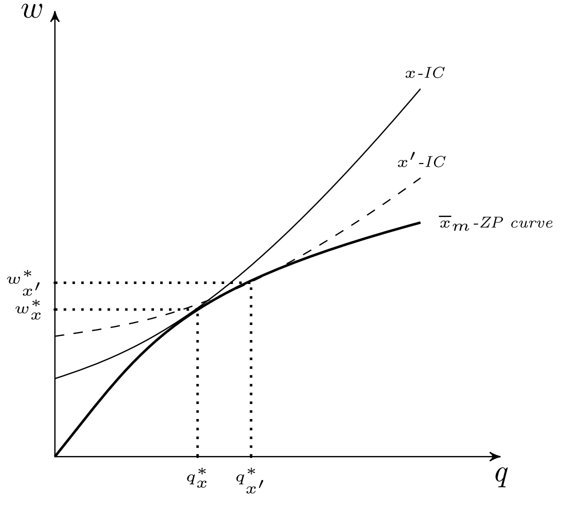


Fig. 2. Equilibrium sorting. Note: In equilibrium, a household's utility is maximized subject to firms obtaining expected zero profits. A household's indifference curves are shown as convex lines, and the concave line displays a firm's zero-profit condition. Households of different types locate in different markets.

premium we are willing to pay to get rid of a risk decreases with our wealth, which is the case when preferences exhibit decreasing absolute risk aversion. See Pratt (1964) and Gollier (2004). In a similar fashion, in our setting, males married to more productive females apply to higher-wage jobs that are harder to obtain if the absolute risk aversion of a household's indirect utility function decreases with female's productivity. This is also in line with the findings in Acemoglu and Shimer (1999), where workers have some wealth endowment and direct their search, and Guler et al. (2012), where married workers sequentially draw offers from a wage distribution and set a reservation wage as a function of the spouse's wage. In both economies, the transition rate from unemployment to employment decreases with the respective private insurance provision under DARA preferences. Interestingly, this analysis highlights an endogenous force for a positive correlation of earnings within the household.

Proposition 3.3. ***Sorting in labor markets.*** *Both equilibrium wages, w_x^*, and queue lengths, q_x^*, are strictly increasing (decreasing) in female's productivity if and only if the absolute risk aversion $\frac{-V_x''}{V_x'}$ decreases (increases) in x.*

Intra-household insurance. We have examined how much insurance risk-averse agents seek in labor markets, which is linked to the equilibrium wage dispersion characterized above. We now turn to the private provision of insurance within the household, abstracting from the labor market insurance, and how it evolves over the employed female's productivity distribution. First, consider the variation of the difference $V_x(w) - V_x(z)$ with respect to productivity x while keeping the market wage w fixed. Lemma 3.4 states that this difference declines with the female's productivity because of the negative cross partial derivative of the indirect utility function. This pattern results from the insurance provided through income-pooling, but not from changes in the spouse's labor supply because of the Envelope theorem.

To focus on the additional insurance that stems from the female's behavioral responses, which can be empirically tested, we study the female's income difference between a single-earner and a two-earner households, $\frac{y_x^*(z)-y_x^*(w_x^*)}{y_x^*(w_x^*)}$. This percentage difference measures how much extra female's income is optimal upon the realization of the unemployment risks.[14] Notice that this is equivalent to the percentage increase in a female's labor supply if her partner fails to find a job. To abstract from the variation in the income risk that the insurance itself induces, we examine how this measure changes with a female's productivity for a constant wage.[15] Nonetheless, it is worth underscoring that the female's income does not generate moral hazard insofar as the two members of the household make the decisions jointly.

Consider first the following monotonic transformation of the percentage income difference, $\ln(y_x^*(z)) - \ln(y_x^*(w_x^*)) = -\int_z^{w_x^*} \frac{\frac{dy_x^*(w)}{dw}}{y_x^*(w)} dw$. By differentiating it with respect to x, we obtain

$$\frac{\partial\big(\ln(y_x^*(z)) - \ln(y_x^*(w_x^*))\big)}{\partial x} = \underbrace{-\frac{\frac{dy_x^*(w_x^*)}{dw}}{y_x^*(w_x^*)}\frac{\partial w_x^*}{\partial x}}_{d_1(x)} \underbrace{-\int_z^{w_x^*} \frac{\frac{\partial \frac{dy_x^*(w)}{dw}}{\partial x} y_x^*(w) - \frac{\partial y_x^*(w)}{\partial x}\frac{dy_x^*(w)}{dw}}{y_x^*(w)^2} dw}_{d_2(x)} \tag{7}$$

Changes in productivity x affect both the upper limit w_x^* and the integrand of the log income difference. Term $d_1(x)$ captures the labor market responses to a higher female's productivity as reasoned above. When abstracting from such labor market responses, the term $d_2(x)$ in that expression is the primary object of our analysis. Consider the numerator of the integrand of $d_2(x)$. While the sign of the product of the two partial derivatives is negative as stated in Proposition 3.1, additional assumptions on higher order derivatives of the utility function are required to determine the sign of the cross derivative and of the sum itself. Nonetheless, Lemma 3.4 claims that the integrand is always positive if preferences are additively separable or of the LMP sort. As a result, the percentage income difference (abstracting from the labor market responses) falls with the female's productivity. The proof is in the Online Appendix.

Lemma 3.4. ***Differences in intra-household insurance: income pooling and behavioral responses.*** *Given a wage w,*

1. *the difference $V_x(w) - V_x(z)$ declines with female's productivity, x,*
2. *the percentage difference in female's income declines with x if preferences either are additively separable or belong to set $\mathcal{F}_2$.*

[14] Notice that this measure does not depend on the potential income loss $w_x^* - z$. This is because, as discussed in the previous section, males apply to different wages only because of the insurance provided within the household. Thus, when abstracting from the market insurance, males should be thought of as though applying to the same job.

[15] Consider the LMP preferences for the sake of the argument. In this case $\frac{y_x^*(z)-y_x^*(w_x^*)}{y_x^*(w_x^*)} = \frac{w_x^*-z}{\frac{x}{\psi}-w_x^*}$. This measure declines with x, given the wage w_x^*. However, as stated in Propositions 3.1 and 3.3, households with more productive females are less risk averse and apply to higher-wage, lower-meeting-rate jobs, thereby inducing a lower female's income and a higher percentage difference in female's income.

The bottom line is that job-seekers with a lower female's productivity use both insurance channels more intensely under some assumptions on preferences.[16] One interpretation of the equilibrium results is that a public provision based only on an individual's past income, discarding the spouse's income may be inefficiently biased towards those workers who value additional insurance less and benefit from it for a longer unemployment period.

3.2. Testable implications

We now test whether the theoretical predictions regarding these two channels of private insurance provision are supported by the empirical evidence. We use data from the Survey of Income and Program Participation (SIPP) for the U.S. Our dataset covers the period from 1996:6 to 2013:6, and comprises individuals aged 25 to 55 who have gone through non-employment and have been employed before and after such a non-employment spell within a 3-4 year period (a SIPP panel). In addition to demographics, we have precise information about e.g. their and their spouse's, if married, labor market status, earnings, and occupation. An individual's information is used as controls to homogenize the sample. In particular, we control for a household's net liquid wealth as the availability of other insurance sources and particularly savings may shape the insurance provided through the spouse's earnings. Labor market status is restricted to employment (E) and non-employment ($\not{E}$). An observation for any given variable of interest is linked to a $E\not{E}E$ spell. See the Online Data Appendix for further details.

The first theoretical implication (Proposition 3.3) refers to the relationship between the job-finding rate and a job value with a worker's spouse's income. Specifically, under some preferences-related conditions, they vary monotonically and in opposite directions with the spouse's income. We first use Cox proportional hazard models to estimate the relationship between a worker's likelihood of becoming employed and his or her spouse's earnings prior to the non-employment spell. The results are shown in Table 1. In line with Guler et al. (2012), this relationship is negative. In the second specification, we find that the job-finding hazard rate of a worker whose spouse's earnings are in the bottom quintile of the distribution is about 9 percent higher than its counterpart for the top quintile. The gap is even larger for those households with a spouse with no earnings, a group that amounts to just below 20% of our sample of married individuals. Furthermore, when interacting the spouse's log earnings with the household's wealth in the last specification, we find that the cross elasticity is significant in the bottom 80% of the wealth distribution, suggesting that the insurance channel may not be operative at high levels of wealth.

These findings can be rationalized through the lens of the model as preferences being such that the absolute risk aversion of the indirect utility function of a household declines with a spouse's productivity. Then, theory predicts that a spouse's earnings are positively related to the value of the job applied to. However, the longitudinal feature of SIPP does not allow us to track wages long enough to compute a reasonable present discounted value of the stream of wages at the job (and subsequent jobs). An indirect way to test this theoretical prediction is by examining the relationship between occupation-switching rates and a spouse's earnings. The intuition is that, to the extent that the returns to occupation-specific human capital are sizable, larger spouse's

[16] In the Online Appendix we show that public insurance provision crowds out private insurance both sought in labor markets and provided within the households, and such crowding-out effects appear to be stronger at the bottom of the female's productivity distribution.

Table 1
Cox hazard model estimates.

	Married workers		All workers		Married workers	
Married			0.065***	(.025)		
Zero Sp earnings	−0.124	(.092)			−0.165*	(.095)
Sp earnings	−0.021*	(.012)				
Sp earnings Q1			−0.024	(.030)		
Sp earnings Q2			−0.056*	(.031)		
Sp earnings Q3			−0.100***	(.032)		
Sp earnings Q4			−0.051*	(.031)		
Sp earnings Q5			−0.118***	(.034)		
Wealth Q1 × Sp earnings					−0.025**	(.012)
Wealth Q2 × Sp earnings					−0.033**	(.013)
Wealth Q3 × Sp earnings					−0.034**	(.013)
Wealth Q4 × Sp earnings					−0.021*	(.012)
Wealth Q5 × Sp earnings					−0.017	(.012)
Wealth Q2	−0.048*	(.027)	−0.059***	(.020)		
Wealth Q3	−0.040	(.036)	−0.029	(.025)		
Wealth Q4	0.035	(.027)	0.010	(.021)		
Wealth Q5	0.060**	(.029)	0.020	(.022)		
Female	−0.265***	(.023)	−0.139***	(.016)	−0.265***	(.024)
Previous log earnings	0.089***	(.010)	0.089***	(.008)	0.094***	(.010)
Number of observations	14,396		28,398		13,807	

Note.- Dataset comprises the first $E\not{E}E$ spell for all individuals in the sample. *Sp earnings* stands for the spouse's log (1+)earnings. The first specification is restricted to married individuals with zero or positive spouse's earnings. The coefficient reports the elasticity of the hazard rate with respect to the spouse's average earnings of the previous three months prior to the non-employment spell. The sample of the second specification includes all individuals. The first set of coefficients in this second column can be interpreted as the percentage change in hazard rate associated with the quintile of the spouse's average earnings of the previous three months prior to the $\not{E}$ spell relative to the reference group, which comprises workers with a spouse with zero earnings prior to the $\not{E}$ spell. In the last specification, the spouse's log earnings are interacted with the household's net liquid wealth, and the correspondent cross elasticities are reported. In addition to the unemployment rate, a time line and monthly dummies, all models also include additional worker's information: a quadratic polynomial of age, and dummies for the number of children under age 18, seam effects, white and black, high-school, college degree and post-college degree, homeownership, state, occupation and industry. The wealth dummies are indicator variables for whether the household's net liquid wealth falls into the corresponding quintile of the distribution, and the reference group in specifications 1 and 2 is the first quintile of the distribution. Standard errors are in parentheses.

earnings are related to a lower probability of forgoing such occupation-specific returns.[17] We report the Probit estimates in the Online Data Appendix. Jobless workers whose spouse's earnings are above the median are significantly more likely not to switch occupations upon reemployment despite their longer non-employment spells.[18] When interacting with a household's wealth, we find that the negative estimates are significant except for the bottom quintile of the wealth distribution.

Finally, and given the previous findings, theory (Lemma 3.4) predicts a negative relationship between a spouse's earnings and the difference between the income of the spouses of the un-

17 See e.g. Kambourov and Manovskii (2009). Huckfeldt (2021) also documents that the initial earnings losses of job-losers who switch occupations more than double those who are reemployed in the same occupation.

18 Longer spells are associated with higher occupation-switching rates. See e.g. Carrillo-Tudela and Visschers (2013).

employed and of those married to the employed. To test this prediction, we regress the change between the spouse's log earnings averaged over the three-month period after and before a $\not{E}$ spell of a worker on the distribution of the spouse's log earnings prior to the $\not{E}$ spell, and control for the duration of the non-employment spell to capture the insurance-inducing extra risks. As shown in the Online Data Appendix, spouses at the top of the distribution increase significantly less their market earnings than those at the bottom. It could be argued that spouses with higher-paid jobs have less margin to adjust their labor supply, while those in the lower tail of the distribution may be working part time. However, the aforementioned patterns still hold when further restricting the sample to those workers with full time spouses. When interacting the spouse's earnings with household's wealth, the estimated elasticities are negative, highly significant and rather similar for all the wealth quintiles. These findings are quite in line with Cullen and Gruber (1996), who use SIPP data from 1984 to 1992, a period with lower female labor force participation rates. They estimate a 5.5% rise in wives' hours worked conditional on working, and highlight that the small responses are due to the crowding-out effects of unemployment benefits. Unlike them, we do find significant effects on the extensive margin.

4. Centralized economy

In this section, we examine the constrained efficient insurance scheme. Because of our focus on the efficient allocation of risks, the social planner is only allowed to redistribute resources among ex-ante identical households. We show that constrained efficiency exhibits partial consumption insurance through type-specific transfers. Households with the two members unemployed receive the largest transfers. However, constrained efficiency cannot be attained in the market economy because of incomplete markets and information frictions. If females' productivity were observable instead, the planner's transfers to the unemployed males would decline monotonically with female's productivity, while job-finding rates rise unlike in equilibrium. The reason for transfers falling with the female's productivity is the limited complementarity between consumption and labor together with concavity in a household's preferences.

4.1. Formulation of the social planner's problem

As usually assumed in the search literature, the social planner maximizes a utilitarian welfare function subject to the same search and information frictions as agents encounter in the market economy.[19] While taking as given the employment status of females and the aggregate distribution G, the planner sets a mass of vacancies in the labor market for males, dictates a male's participation decision and job search strategies as well as his spouse's income, and assigns consumption bundles to households. Formally, the planner designs a symmetric incentive compatible revelation mechanism.

To be specific, a symmetric mechanism is a menu of contracts $\mathcal{C} \equiv \{(q_x, c_x^e, c_x^u, y_x^e, y_x^u)\}_{x \in X_p} \cup \{(c_x^n, y_x^n)\}_{x \in X_{np}}$ together with a subset X_p that comprises the types of those households engaged in job search.[20] The superscript $i \in \{e, u, n\}$ indicates the labor force status of the male at the end

[19] Specifically, the planner does not observe whether and where workers apply for a job, but does know the job type if employed. Likewise, a female's productivity per unit of time and labor supply are also private information, while income is not. Furthermore, to compare with the laissez-faire equilibrium, assume that the parameter values are such that all males with an unemployed spouse search in the labor market.

[20] Let $X_{np} \equiv X \setminus X_p$ be the complementary subset.

of the period. Contracts are indexed by a household's announcement of its type.[21] That is, for any reported x, the mechanism specifies 1) whether the male searches for a job, 2) conditional on participating, a location where to submit an application and the associated job-finding probability (q_x) as well as a consumption and female's income tuple contingent on the job search outcome ($c_x^e, c_x^u, y_x^e, y_x^u$), and 3) a consumption and female's income pair if not participating (c_x^n, y_x^n). The mechanism is symmetric in the sense that all households reporting the same type are treated identically.

The mechanism must be feasible and incentive compatible. First, the planner is allowed to pool and redistribute resources among ex-ante identical households, but not among ex-ante different ones. Thus, we say that a mechanism $\mathcal{C}$ is feasible if total consumption promises do not exceed total output net of vacancy creation costs for each household's type. That is, the following type-specific resource constraints must hold

$$\begin{aligned} &\frac{\kappa}{q_x} \leq \nu(q_x)\big(\overline{x}_m + y_x^e - c_x^e\big) + (1-\nu(q_x))\big(z + y_x^u - c_x^u\big), \quad \forall x \in X_p, && \text{(RC)} \\ &c_x^n \leq z + y_x^n, \quad \forall x \in X_{np} \end{aligned}$$

Second, the mechanism must also be compatible with households' incentives regarding both their type report and labor force participation. We start with the former. Households with an employed female must truthfully reveal their type.[22] That is, the following conditions hold to ensure that no household has incentives to misreport its type.

$$\begin{aligned} \mathcal{U}_x &\geq \mathcal{U}_x(x'), \upsilon(c_{x''}^n, \frac{y_{x''}^n}{x}), && \forall x, x' \in X_p \setminus \{0\}, x'' \in X_{np} && \text{(ICC}^1\text{)} \\ \upsilon(c_x^n, \frac{y_x^n}{x}) &\geq \mathcal{U}_x(x'), \upsilon(c_{x''}^n, \frac{y_{x''}^n}{x}), && \forall x, x_f'' \in X_{np}, x_f' \in X_p \setminus \{0\} \end{aligned}$$

where $\upsilon(c_{x''}^n, \frac{y_{x''}^n}{x})$ and $\mathcal{U}_x(x')$ denote the utility of a type-x household that reports $x'' \in X_{np}$ and $x' \in X_p \setminus \{0\}$, respectively, and

$$\begin{aligned} \mathcal{U}_x(x') &\equiv \upsilon\big(c_{x'}^u, \frac{y_{x'}^u}{x}\big) + \max\big\{0, \nu(q_{x'})\big(\upsilon\big(c_{x'}^e, \frac{y_{x'}^e}{x}\big) - \upsilon\big(c_{x'}^u, \frac{y_{x'}^u}{x}\big)\big) - k_w\big\}, \\ \mathcal{U}_x &\equiv \mathcal{U}_x(x), \quad \forall x \in X_p \end{aligned}$$

Because job search is not observable to the planner, the max operator on the right hand side of the $\mathcal{U}_x(x')$ expression reflects the possibilities of searching and not searching for a type-x' job.[23]

Finally, the net value of job search must exceed the search cost to ensure that participating in the labor force is desirable for those households who are asked to engage in job search. That is, for the mechanism to be compatible with households' incentives, the following set of labor-force-participation conditions must also hold:

[21] For notational simplicity, an unemployed female is asked to deliver no income, $y_0^e = y_0^u = 0$.

[22] Recall that a female's employment status is observable. Thus, households with an unemployed female cannot pretend to be a household with an employed wife, nor vice versa.

[23] For simplicity, the $\mathcal{U}_x(x')$ expression does not include the option of searching for a type-x'' job while reporting type x'. This option is excluded on the grounds that this behavior can be easily discouraged by considering mechanisms that include dissuasive penalties if the job type of an employed worker differs from the announced type as the former is observed by the planner.

$$\nu(q_0)\big(\upsilon\big(c_0^e,0\big)-\upsilon\big(c_0^u,0\big)\big),\ \nu(q_x)\bigg(\upsilon\Big(c_x^e,\frac{y_x^e}{x}\Big)-\upsilon\Big(c_x^u,\frac{y_x^u}{x}\Big)\bigg)\geq k_w,\quad \forall x\in X_p\setminus\{0\} \tag{ICC^2}$$

The social planner chooses a feasible and incentive compatible mechanism to maximize a household's expected utility taking as given the distribution of female's productivities, G.[24] Thus, the planner's problem is

$$\max_{X_p,\mathcal{C}} \int_{X_p} \mathcal{U}_x dG(x) + \int_{X\setminus X_p} \upsilon(c_x^n, \frac{y_x^n}{x}) dG(x)$$

s. to (RC), (ICC[1]) and (ICC[2])

The labor-force-participation constraints (together with the feasibility constraints) put a limit on the transfers that can be made across households. If they were eliminated, the planner would promise the same utility regardless of the search outcome, and then males would pretend to participate in the labor force without doing so to avoid the search costs. As shown in the Online Appendix, the first incentive compatibility conditions ensure that households with more productive females are promised higher expected values.

4.2. *Characterization of the social planner's allocation*

Proposition 4.1 characterizes the planner's allocation, $(\hat{X}_p, \hat{\mathcal{C}})$.[25] As in equilibrium, the planner sets a reservation productivity, $\hat{x}$, that determines the set of participating males. The threshold is pinned down by the net returns to participating being just equal to the participation costs. Males married to higher-productivity females are left out of the labor force. Because such households face no risks, no type-specific transfers are tailored to them, and they obtain value $V_x(z)$. Since this (autarky) value is a natural lower bound for all households, no household has incentives to pretend to be a non-participant of a different type.

The planner sets a transfers scheme among ex-ante identical households with a participating male to partially eliminate the consumption risks. Importantly, the transfers to households with the two members unemployed exceed the transfers promised to those formed by an unemployed male and an employed female. Furthermore, in contrast to the empirical evidence and the equilibrium allocation, there exists a positive mass of job-seekers married to employed females facing higher employment chances than their counterparts with an unemployed spouse. That is, fewer jobs are created for the husbands of the unemployed females as this result is independent of the distribution of female types. These observations are closely intertwined. Our intuition -driven by the quantitative work and the theoretical analysis with observable types below- is that not only are risks larger for households with an unemployed female, but so are the marginal utility gains from the transfers because of the properties of function V_x, particularly its negative cross partial derivative.

24 For simplicity, we will assume throughout this section that the cdf G is differentiable and has a continuous support. The support of G will be assumed to be discrete in the numerical analysis undertaken in Section 5.

25 Assuming quasi-linear preferences, like in Davoodalhosseini (2019), would greatly simplify the planner's problem. In the Online Appendix we show that if preferences are quasi-linear in consumption, then the constrained efficiency result in Moen (1997) carries over to an economy with households. With quasi-linear preferences in leisure, vacancy creation is also determined to maximize total output, yet the equilibrium allocation is not constrained efficient.

The laissez-faire equilibrium allocation is feasible and incentive compatible. Nonetheless, it does not attain constrained efficiency. This is because of the superior tools the planner has to confront the unemployment risks. While risk-averse households only rely on their female's labor supply and the insurance provided in the labor market to buffer the income shocks in the decentralized economy, the planner jointly sets the mass of vacancies, female's labor supply and a system of type-specific transfers. Put simply, notice that the equilibrium equation (5) only differs from its planner's counterpart (8) in that the social value of a filled vacancy is replaced by a household's surplus plus a firm's profits, which factor in wages but not transfers. Therefore, the equilibrium provision of insurance against consumption risks falls short of the constrained efficient level. Importantly, there is no other source of inefficiency since the equilibrium allocation would be constrained efficient if the planner were not allowed to make transfers to households with unemployed males.

Proposition 4.1. ***Planner's allocation.*** *Assume that a male's productivity is such that those with an unemployed spouse participate in the labor force. Then, there exists a solution of the planner's problem, $(\hat{X}_p, \hat{\mathcal{C}})$. Furthermore,*

1. *The value of non-participating households is $V_x(z)$. Consider differentiable mechanisms. Then, there exists a threshold $\hat{x} \geq x^*$ such that the participation set $\hat{X}_p = \{0\} \cup [\underline{x}, \hat{x}]$.*
2. ***Households with an unemployed female.*** *The tuple $(\hat{q}_0, \hat{c}_0^e, \hat{c}_0^u)$ is determined by the following equation, together with binding feasibility and labor-force-participation conditions:*

$$\kappa = \eta(q_0)\big(1 - \phi(q_0)\big)\left(\frac{v(c^e, 0) - v(c^u, 0)}{\frac{\partial v}{\partial c}(c^e, 0)} + \overline{x}_m - c^e - z + c^u\right) \tag{8}$$

   Furthermore, type-0 households with an unemployed male receive the largest transfers:

$$\hat{c}_x^u - \hat{y}_x^u \leq \hat{c}_0^u, \ \forall x \in \hat{X}_p$$

3. *The equilibrium allocation is not a solution of the planner's problem. If the planner is further constrained not to redistribute resources at all, then the laissez-faire equilibrium is constrained efficient.*

Interestingly, as stated below in Proposition 4.2, the participation margin is efficiently set in the market economy: the planner's threshold coincides with the equilibrium one. We provide some intuition for this result below. Now, to provide further insights on the inefficiency result, consider separately the different insurance margins for a household with an employed female.[26]

Regarding the insurance provided within the household, let $D_{x'}^e(x) \equiv \frac{\partial v}{\partial c}(c_x^e, \frac{y_x^e}{x'}) + \frac{\frac{\partial v}{\partial \ell}(c_x^e, \frac{y_x^e}{x'_f})}{x'}$ denote the marginal utility that a two-earner household of type x' derives from increasing the female's income while pretending to be of type x. Then, the planner's counterpart of equilibrium condition (2) is

$$D_x^e(x) = \int_{X_p} \hat{\lambda}_{x',x}^3 D_{x'}^e(x) dG(x') + \int_{X_{np}} \hat{\lambda}_{x',x}^5 D_{x'}^e(x) dG(x') \tag{9}$$

[26] See the Online Appendix for a detailed derivation of the expressions below.

where $\hat{\lambda}^3_{x',x}$ and $\hat{\lambda}^5_{x',x}$ are a composite of the Lagrange multipliers of the incentive-compatibility conditions (ICC^1) and (ICC^2). Since $D^e_x(x) = 0$ in equilibrium, information frictions introduce a wedge between the equilibrium female's labor supply decisions and their planner's counterparts.

The insurance provided through the vacancy creation margin is also affected by information frictions, but also by the planner's ability to redistribute resources as reasoned above. A similar expression to equation (9) can be written as the planner's counterpart of equilibrium equation (5). Indeed, if a household's type were observable to the planner (i.e. $\hat{\lambda}^3_{x',x} = \hat{\lambda}^5_{x',x} = 0$ for all $x \in \hat{X}_p$), the equilibrium allocation would satisfy these two planner's equations. Although the equilibrium insurance provision would still fall short of the optimal level due to the planner's differential ability to redistribute resources, this case raises the question of how large these information frictions are. Because if such frictions were rather small, as our quantitative exercise in Section 5 suggests, there may be lessons to learn from the study of a centralized economy with observable types. This is the goal of the next subsection before quantitatively characterizing the planner's allocation.

4.2.1. Planner's allocation with observable types

Consider the case in which the planner does observe a female's productivity, and refer to it as the scenario with observable types (OT). The planner's problem can then be rewritten as a sequence of type-specific maximization problems because of the elimination of the incentive compatibility condition (ICC^1). In particular, the problem associated to the type-x households with a participating male is

$$(P^{OT}_x) \qquad \max_{X_p, \mathcal{C}} \mathcal{U}_x$$

$$\text{s. to (RC) and (ICC}^2\text{)}$$

We will refer to the planner's solution in this alternative scenario, $(\hat{X}^{OT}_p, \hat{\mathcal{C}}^{OT})$, as the planner's OT allocation. Proposition 4.2 characterizes it. Our primary interest is on the differences across ex-ante different households. First, the planner's transfers to the unemployed (or one-earner households) decline with the female's productivity. Thus, this finding extends the constrained efficient result stated in Proposition 4.1 so that unemployed males with a jobless wife receive the largest transfers. Indeed, if the absolute risk aversion of the household's indirect utility function falls with female's productivity, so does the type-specific transfers to net income ratio $\frac{\hat{c}^{uOT}_x - \hat{y}^{uOT}_x - z}{\hat{c}^{eOT}_x - \hat{y}^{eOT}_x}$, which is a sort of replacement rate. Second, job-finding rates rise over the distribution of households in sharp contrast with the declining pattern observed in the data and in the equilibrium allocation. These patterns over the household distribution result from the concavity of preferences and the utility gains from marginally increasing the transfers declining with a female's productivity (i.e. a negative cross partial derivative of the indirect utility function).

Proposition 4.2. ***Planner's OT allocation.*** *Assume a female's productivity is observable to the planner. Then, there exists a solution of the planner's OT problem, $(\hat{X}^{OT}_p, \hat{\mathcal{C}}^{OT})$. Furthermore,*

1. *Households with an unemployed female and those with a non-participating male are offered the same contract as in the constrained efficient allocation. If differentiable mechanisms are considered, there exists a threshold $\hat{x}^{OT}$ such that the participation set $\hat{X}^{OT}_p = \{0\} \cup [\underline{x}, \hat{x}^{OT}]$.*

2. *The planner's OT solution for participating households with an employed female is determined by the resource constraint (RC) and the following conditions:*
   (a) *Female's income condition:*

$$\frac{\partial v}{\partial c}\left(c_x^i, \frac{y_x^i}{x}\right)x + \frac{\partial v}{\partial \ell}\left(c_x^i, \frac{y_x^i}{x}\right) = 0, \textit{ for } i \in \{e, u\} \tag{10}$$

   (b) *Vacancy creation condition:*

$$\kappa = \eta(q_x)\big(1 - \phi(q_x)\big) \times \left(\frac{V_x(c_x^e - y_x^e) - V_x(c_x^u - y_x^u)}{V_x'(c_x^e - y_x^e)} + \overline{x}_m + y_x^e - c_x^e - z - y_x^u + c_x^u\right) \tag{11}$$

   (c) *The labor-force-participation constraint (ICC*[2]*) is binding, and* $\hat{c}_x^{eOT} \geq \hat{c}_x^{uOT}$ *and* $\hat{y}_x^{eOT} < \hat{y}_x^{uOT}$.
3. ***Pattern over the household distribution:*** $\forall x, x' \in \hat{X}_p^{OT} \mid x' < x$,
   (a) *Declining transfers to one-earner households:* $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} \leq \hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT}$
   If the absolute risk aversion of the household's indirect utility function, $\frac{-V_x''}{V_x'}$*, falls in x, then so does the ratio* $\frac{\hat{c}_x^{uOT} - \hat{y}_x^{uOT} - z}{\hat{c}_x^{eOT} - \hat{y}_x^{eOT}}$.
   (b) *Increasing job-finding rates and female's income:* $\nu(\hat{q}_{x'}^{OT}) < \nu(\hat{q}_x^{OT}),\ y_{x'}^{uOT} < \hat{y}_x^{uOT}$

Lemma 4.3 compares the planner's OT allocation with the equilibrium. First, the set of participating households in the two planner's allocations coincides with the equilibrium one. This may look a somewhat surprising result. Certainly, all males participating in equilibrium are also better off participating in the planner's OT allocation because a transfers system raises a household's value due to risk aversion. Although participation is decided if the search gains outweigh the search cost and the former shrinks with the transfers, the planner can accommodate them as much as needed as transfers are type specific. In the limit, those transfers can be set at 0 as in the equilibrium allocation. Second, the laissez-faire equilibrium does not coincide with the planner's OT allocation. Although the planner's OT conditions (10) and (11) regarding the two insurance margins hold in equilibrium, the spouse's labor supply in one-earner households is inefficiently large and too many too-low-wage jobs are created in the market economy in the absence of a transfers system. The proof is in the Online Appendix.

Lemma 4.3. ***Comparison with the equilibrium allocation:***

$$\hat{x}^{OT} = \hat{x} = x^*, \quad \textit{and} \quad \nu(q_x^*) > \nu(\hat{q}_x^{OT}),\ c_x^{u*} < \hat{c}_x^{uOT} \textit{ and } \hat{y}_x^{uOT} < y_x^{u*}, \quad \forall x \in \hat{X}_p^{OT}$$

4.3. Implementation of the planner's allocation

A natural question is whether the planner's allocation can be attained in the market economy, and, if so, what fiscal instruments are necessary.

Unfortunately, we cannot derive theoretical results for the decentralization of the constrained efficient allocation. In the quantitative analysis undertaken in Section 5 we do show that some non-type-specific tax-and-transfers scheme may go a long way to fill up the welfare gap between the market equilibrium and the planner's allocation. Such a scheme exhibits a sizable decline in the replacement rate over the distribution of the female's productivity.

If a female's type were observable to the planner instead, a sufficiently rich public policy would suffice to implement the planner's OT allocation. In the Online Appendix we show that, while the planner's OT allocation cannot be implemented through non-type-specific schemes as they are not powerful enough once types are observable, a type-specific public policy featuring unemployment benefits declining over the distribution of the female's type and an affine tax scheme on male's income suffice to attain efficiency in the market economy. A female's income should not be taxed in order not to distort the efficiently set private insurance provision within the household (Proposition 4.2). The planner's OT allocation can also be obtained in the market economy without the government's intervention, however. This is the case e.g. in an alternative setting with a wider contracting space such that firms are allowed not only to commit to a wage to successful applicants, but also to reward unsuccessful applications with a payment s.[27] Such firms indeed offer labor and insurance contracts at once. Risk-averse households much value the application reward s when seeking jobs. Such payments per application reduce firms' profits and, hence, job-finding rates. Since marginal utility gains associated to the application rewards fall with female's productivity $\left(\frac{\partial V_x'}{\partial x} < 0\right)$, households of higher types apply to jobs with a lower s and higher employment rates. This is the case even if wages increase with x as in the baseline.

5. Quantitative exploration

The goal of this section is primarily to characterize quantitatively the constrained efficient insurance provision, and, in particular, the level and slope of the planner's transfers schedule, and study its possible implementation in the market economy. Furthermore, we aim to explore the welfare implications of private information as well as of a single replacement rate policy. The main result is fourfold: First, a sort of planner's replacement rate steadily falls from just below 30% to 0 over the household's distribution. Second, a simple tax-and-transfers scheme exhibiting falling transfers can deliver substantial welfare gains. Third, the costs of the private insurance arrangements being unobservable are fairly small. Finally, a constant replacement rate has strong effects both on participation and the level of insurance against consumption risks over the distribution. In the description of the results, we will refer to households with an unemployed (employed) male and an employed female as one- (two-)earner households.

5.1. Calibration

The calibration strategy is fairly standard and targets some labor market statistics of the U.S. economy. We set the model period to last 1 month. The following unemployment insurance policy is assumed: benefits b are collected by all unemployed males, and the insurance scheme is funded through a proportional tax rate, τ, on males' wages and females' income to balance the government's budget. We also assume the following functional forms. First, we consider standard macro preferences, $\upsilon \in \mathcal{F}_1$, that are consistent with the empirical evidence discussed in Section 3.2. Second, the matching function is assumed to be CES; hence, $\nu(q) = \frac{1}{(s_1+q^{s_2})^{1/s_2}}$. Third, as there is no direct empirical counterpart of the distribution of the female's productivity, G, we use the distribution of the earnings of the employed spouses to inform G in the same

[27] See the Online Appendix for further details. Golosov et al. (2013) argue that such contracts that trade job applications instead of labor services are not enforceable because of information frictions, and offer an alternative decentralization that consists of having an insurance market alongside the labor market.

Table 2
Parameter values.

Parameter	Value	Interpretation	Target
θ	1.730	inverse of Frisch elasticity of consumption wrt price	Blundell et al. (2016b)
ξ	1.451	inverse of Frisch elasticity of labor supply wrt own wages	Blundell et al. (2016b)
m_0	0.194	mass of unemployed females	mass of spouses with no earnings (SIPP)
γ	$5.9 \cdot 10^{-4}$	scale labor disutility parameter	ratio of avg hours worked by household members (SIPP)
s_1	10.371	scale factor of the matching function	monthly job-finding rate (SIPP)
s_2	1.908	elasticity of the matching function	elasticity of job-filling rate
(m_x, d_x)	(8.092, 1.022)	mean and standard deviation of G	mean and st. dev. of spouse earnings dist. (SIPP)
$\overline{x}_m$	7063.454	males' productivity	ratio of avg worker's and avg spouse's earnings (SIPP)
κ	1445.142	vacancy creation cost	14% average quarterly wage per hire
k_w	$2.6 \cdot 10^{-5}$	participation cost	percentage of non-emp. spells over 1 year (SIPP)
z	1642.020	home production	consumption ratio c_u/c_e
b	1242.841	unemployment benefits	replacement rate for average wage

spirit as Kleven et al. (2009). Typically the former is assumed log normal, with the right tail approximated as Pareto distributed. Consistently, we assume that (conditional on employment) the female's productivity is log-normally distributed, eliminate the top 2.5% of the actual spouse's earnings, and target the first and second moments of the actual spouse's earnings, m_x and d_x. We use 100 grid points with equal distance over $(0, \overline{x}]$, where $\overline{x}$ is set to rule out the top 2.5% of the earnings distribution.[28] Let m_0 denote the mass of households with no female's income.

Table 2 summarizes the calibration exercise. Regarding the preference parameters, and in line with Krueger and Wu (2018), we take the inverse of the Frisch elasticities θ and ξ directly from Blundell et al. (2016b),[29] and the scale factor γ is calibrated to match the ratio of average hours worked by the newly employed in the first month to their employed spouse's counterpart, which in our SIPP sample of married jobless individuals is 1.065. Furthermore, 19.38% of the spouses in our subsample have no earnings after the worker transits back to employment; hence, we set $m_0 = 0.1938$.

The remaining parameters are jointly calibrated together with the scale factor γ. We first refer to the targets determined using our SIPP dataset, and then to those values taken from the literature. To calibrate the parameters m_x and d_x of the distribution G, we target the mean and standard deviation (-0.109 and 0.960) of the empirical distribution of the spouse's log earnings at a worker's re-employment, conditional on earnings being positive and normalized to the median spouse's earnings. A male's market productivity $\overline{x}_m$ is set to match the ratio of the work-

[28] Notice that a grid of 100 points implies approximately 10^4 incentive compatibility constraints in the planner's problem. Increasing the number of grid points adds no much to the results while increasing dramatically the computation time. Likewise, bear in mind that in the model we use gender labels merely to distinguish the roles, whereas in our dataset we deal with workers (or job-seekers) and their spouses, who may or may not obtain earnings.

[29] As we do not distinguish by gender, we take averages of such estimates.

ers' average log earnings after re-employment within the first month to the spouses' average log earnings, 0.966. The proportion of jobless individuals after 1 year of non-employment (more than twice the average non-employment spell in our sample), 10.47%, informs the participation cost, k_w. Regarding the two parameters of the matching technology, we target the job-finding rate at the first month, 0.206,[30] and the elasticity of the job-filling rate with respect to the unemployment-to-vacancy ratio at 0.5 as widely used in the search literature. The average replacement rate informs the unemployment benefits, b, and is set at 25%.[31] Finally, we follow Hall and Milgrom (2008) to pin down the vacancy-creation cost, κ, and home production, z. We set the costs of vacancy-posting to match 14% of the average quarterly wages per hire, and home production to match the ratio of the average consumption in one-earner households to the average consumption in two-earner households at 0.85 as estimated by Browning and Crossley (2001).

The model matches the set of targets fairly well. In particular, the first and the second moments of the distribution of the spouse's log normalized earnings are very precisely matched. Nonetheless, the Kernel density estimates differ as the model-generated female's log income is close to normally distributed while the actual earnings appear to be highly negatively skewed and left long-tailed (with skewness and kurtosis equal to -2.294 and 15.226 respectively).[32]

5.2. *Comparison with the planner's allocations*

We now turn to the comparison of the market and centralized economies.[33] To this end, we take the calibrated parameter values except the policy ones, i.e. unemployment benefits and taxes. Fig. 3 displays the laissez-faire equilibrium as well as the two planner's allocations analyzed in the previous section.

The consumption gap between one- and two-earner households is larger in equilibrium than in the constrained efficient allocation. This is the case despite the fact that the two private insurance margins are excessively used in equilibrium as shown in Figs. 3b-3d: the female's labor supply in one- (two-) earner households is inefficiently large (small), and job creation is also excessive over the whole household distribution. Indeed, these two insurance margins are less used in equilibrium the higher the female's productivity in sharp contrast with the planner's allocation, with the equilibrium exit rates falling about 10 percent and the percentage difference in female's income reduced by a factor of 9.

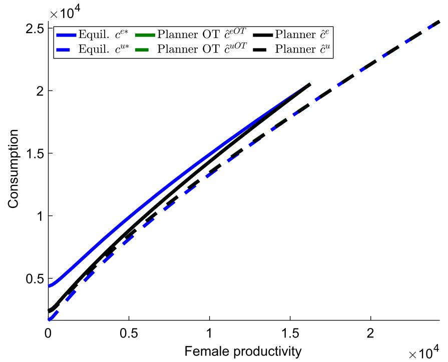
We are primarily interested in the variation of transfers over the distribution of female's productivity. While transfers are 0 in equilibrium, $\hat{c}_x^u - z - \hat{y}_x^u$ is positive and steadily declines in the planner's allocation. Fig. 3e plots the ratio $\frac{\hat{c}_x^u - z - \hat{y}_x^u}{\hat{c}_x^e - \hat{y}_x^e}$, which defines the type-specific replacement rate in the planner's allocation as the ratio of the net transfers received by an unemployed male to the net income earned by his employed counterpart. It steadily declines with the female's

[30] This rate is rather low (e.g. below the 0.248 reported by Krusell et al. (2011)) mostly because of the elimination of non-employment spells shorter than 3 weeks. Because of this, we compute the job-finding rate for the first 5 weeks of non-employment.

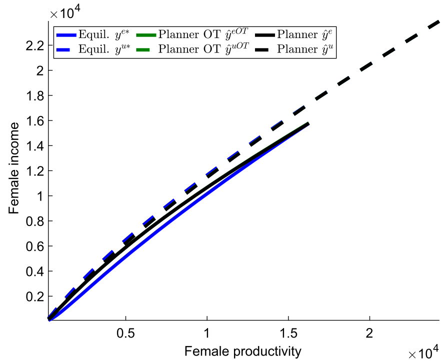
[31] We take a rather low estimate of the replacement rate because of our use of non-employment instead of unemployment spells. For example, Anderson and Meyer (1997) estimate a pre-tax rate to be around 40%, although they also document quite low take-up rates. By contrast, Hornstein et al. (2005) argue that 20% would be an upper bound since the unemployed workers' salaries are below average wages.

[32] Differences in the optimal Kernel widths also explain a substantial part of the differences in the Kernel densities.

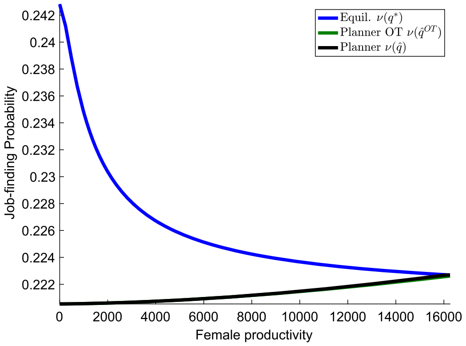
[33] To compute the planner's solutions, we use AMPL, which is a modeling language to solve large-scale non-linear optimization problems.



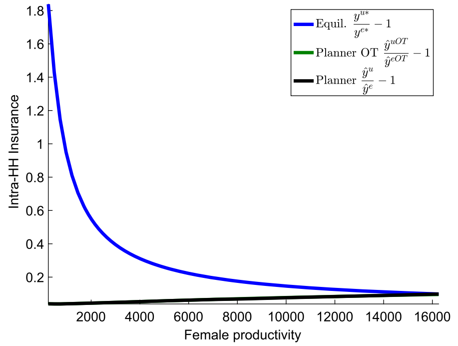
(a) Consumption



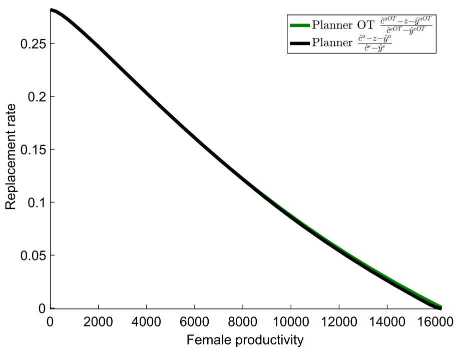
(b) Female's income



(c) Job-finding rate



(d) Female's income difference



(e) Replacement rate

Fig. 3. Planner's and laissez-faire equilibrium allocations. Note: $c^{e*} \equiv w_x^* + y_x^*(w_x^*)$ and $y^{e*} \equiv y_x^*(w_x^*)$, for any productivity x. Likewise, $c^{u*} \equiv z + y_x^*(z)$ and $y^{u*} \equiv y_x^*(z)$ (For interpretation of the colors in the figures, the reader is referred to the web version of this article.)

productivity from just above 28% to 0. The planner's average replacement rate is 21.68%, going down to 20.06% when excluding households with non-employed females.[34]

The difference between the constrained efficient allocation and planner's OT allocation captures how large the costs of private information are. These two allocations are largely indistinguishable in Fig. 3; hence, such costs appear to be fairly small. For example, the percentage consumption difference $\frac{\hat{c}_x^{uOT} - \hat{c}_x^u}{\hat{c}_x^u}$ and income difference $\frac{\hat{y}_x^{uOT} - \hat{y}_x^u}{\hat{y}_x^u}$ are always below 0.02% and 0.2%, respectively. The replacement rate is always higher with observable types, yet the difference is again quantitatively very small.

Finally, the same quantitative exercise is performed with LMP preferences. See the Online Appendix. Recall that full consumption insurance is arranged within the household through the female's labor supply both in equilibrium and the planner's OT allocation. This not only leads to lower replacement rates (just above 12%), but also to an almost flat replacement rate over the household distribution.

Welfare analysis and decentralization of the planner's allocation. To report the welfare gap, we compute the consumption equivalent variation (CEV) at each female's productivity level for the planner's allocation. That is, consider the laissez-faire equilibrium and suppose that consumption is multiplied by a factor $1 + CEV_x$ in all possible states, while keeping participation, labor supply and vacancy creation unchanged. The consumption equivalent gain is determined as the increase in consumption that yields the same expected utility as in the planner's allocation. That is,

$$\hat{\mathcal{U}}_x = \nu(q_x^*) v\Big((1 + \widehat{CEV}_x) c_x^{e*}, \frac{y_x^{e*}}{x}\Big) + (1 - \nu(q_x^*)) v\Big((1 + \widehat{CEV}_x) c_x^{u*}, \frac{y_x^{u*}}{x}\Big)$$

We only report $\widehat{CEV}_x$ below the threshold $x^* = \hat{x}$ because the planner's and the equilibrium allocations are the same for non-participating households. Fig. 4a displays the welfare gains against (the percentiles of) the female's productivity distribution. Beyond the large differences over the distribution of female's productivity, there is a sizable welfare gap between the laissez-faire equilibrium and the planner's allocation. The average $\widehat{CEV}_x$ is 5.85%.

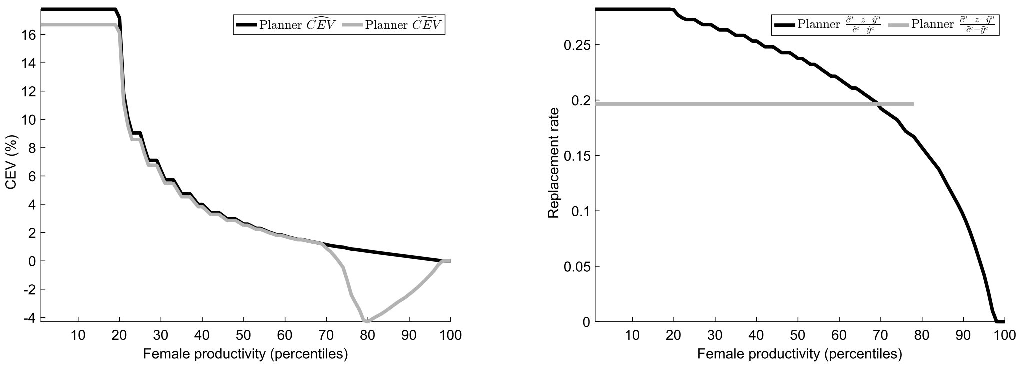
Unlike in the case with observable types, the question of the implementation of the constrained efficient allocation in the market economy has proved theoretically elusive. Despite the lack of theoretical guidance, some non-type-specific public schemes may narrow substantially the welfare gap between the planner's allocation and the laissez-faire equilibrium. For example, consider the following tax function of the sort of Heathcote et al. (2014)

$$T(w, y) = (1 - \tau_0^w (w - z)^{\tau_1^w - 1})(w - z) + (1 - \tau_0^y y^{\tau_1^y - 1}) y$$

together with the following scheme of transfers to jobless males as a function of the spouse's income $b(y) = \tau_0^b + \tau_1^b (1 + y)^{\tau_2^b}$. The policy mix that maximizes social welfare exhibits progressive taxes on males' wages and almost linear and negative on female's income.[35] Transfers strongly decline with female's income, to one third of its value for households with jobless females. The

[34] This 8 percentage points difference in the replacement rate in favor of workers with an unemployed spouse is fairly close e.g. to the 4 to 7 percentage point increase in the replacement rate associated to a dependent in the state of New Jersey.

[35] Because the government's balanced budget constraint is type-specific, there is some, but rather small excess tax revenue at all levels of female's income.



(a) Consumption Equivalent Variation (b) Replacement rate

Fig. 4. Planner's benchmark and alternative allocations. Note: All figures display data under both planner's policies for each percentile of the distribution of female productivity. $\tilde{c}^e$ denotes the consumption of a household with an employed male in the single replacement-rate allocation, and analogously for the other variables.

replacement rate also falls by 22%. For the welfare computation, we proceed analogously, and denote by CEV_x^{tax} the factor that leads households to obtain the expected value in the tax-distorted equilibrium, $\mathcal{U}_x^{tax}$. The average CEV_x^{tax} is 5.21%. Therefore, this tax-and-transfers scheme substantially fills the welfare gap between the laissez-faire equilibrium and the planner's allocation. The remaining gap is primarily explained by a much lower participation rate, which falls (from 97.24%) to 80.54%.

5.3. Comparison with a single replacement-rate policy

There is a large body of literature focused on determining the optimal single replacement rate (see e.g. the research cited in the Introduction). Thus, we now turn to compare the welfare gains obtained in the constrained efficient allocation with those obtained in an alternative allocation in which the planner is further constrained to a constant replacement rate over the distribution of households. For exposition purposes, we will refer to them as the planner's benchmark and alternative allocations. We proceed analogously for the planner's single replacement-rate allocation, and report the corresponding $\widetilde{CEV}_x$ values for households to obtain the expected value in this alternative allocation, $\widetilde{\mathcal{U}}_x$.

Average welfare gains in the alternative allocation are 4.77% for all households with participating males in the equilibrium allocation. Recall that the average gains in the benchmark amount to 5.85%. As shown in Fig. 4a, welfare gains are larger in the planner's benchmark allocation at all female's productivities. It is convenient to distinguish three subsets of the female's productivity distribution. First, welfare gains are significantly larger for households with an unemployed female in the benchmark than in the alternative allocation, 17.79% and 16.70%, respectively. Recall that this subset of households amounts to about 20% of the population. As shown in Fig. 4b, this is due to a much lower replacement rate in the latter allocation for this subset, 28.20% vs. 19.65%, which translates into a 5 percent consumption gap if the two household members are unemployed. A second subset comprises all households with participating males and employed females in the alternative allocation. In this case, the benchmark replacement rate is above the constant one for most households, and differences in welfare gains between the two planner's allocations are not large, but still significant: on average, $\widehat{CEV}_x$ and $\widetilde{CEV}_x$ are 3.68% and 3.20%, respectively. Two subgroups can be distinguished, however. If the benchmark replacement rate is above the constant one, the gap in welfare gains is halved, and the consumption gap for households with an unemployed male steadily converges from 5% to 0. Otherwise, sizable relative welfare losses (over 1.83%) result in the single replacement-rate regime as transfers are too large.[36] Finally, a third subset comprises all households with males not participating in the labor force in the planner's alternative allocation; hence, they are better off participating in the laissez-faire equilibrium. These men (with sufficiently high productive spouses) are expelled from the labor force as the incentive compatibility conditions are more stringent under a single replacement-rate policy. The set of participating households shrinks significantly in the planner's alternative allocation (from 97.24% to 78.39%) because the planner cannot accommodate the transfers to each specific type.

To sum up, the costs of setting a single replacement-rate policy are particularly borne by the households with an unemployed spouse in the bottom 20% of the female's productivity distribution, and by those in the top 30%, most of them expelled from the labor force. The dependency allowance linked to an unemployed spouse at work in some states correctly targets the former costs, while the latter call for a piece-wise replacement rate to raise participation.

6. Conclusions

This paper studies the constrained efficient combination of public and private provision of insurance against unemployment and, hence, consumption risks. In the market economy, insurance is both sought in labor markets by applying to low wage jobs and provided at home through the spouse's labor supply. In the planner's allocation, households with the two members unemployed receive the largest transfers. Indeed, if types are observable, transfers fall and job-finding rates rise over the household distribution. Similar patterns are delivered in our quantitative work. The rising exit rates sharply contrast with the empirical evidence, which can be rationalized as the two insurance margins operating inefficiently in equilibrium in the absence of public provision.

Appendix A

A.1. Proofs of Section 3

Proof of Proposition 3.1. See the Online Appendix for the first two parts of the proposition. We prove here the last bullet.

Consider types $x, x' \in X$ such that $x' < x$. Given that function V_x is strictly increasing and differentiable in w, there exists an inverse function V_x^{-1}, which is also differentiable. Define function

$$\mathcal{V}(s) \equiv \max_y \upsilon\big(V_x^{-1}(s) + y, \frac{y}{x'}\big),$$

which is twice continuously differentiable. Notice that $V_{x'}(w) = \mathcal{V} \circ V_x(w)$. By differentiating with respect to w, we obtain

[36] Consumption for households with unemployed males in this subset flattens out because of the flattening in female's income, which in turn results from the necessity of providing the right incentives for the non-participating males not to pretend to be of some other type in order to claim the planner's transfer.

$$\frac{dV_{x'}}{dw}=\mathcal{V}'(V_x)\frac{dV_x}{dw},\ \frac{d^2V_{x'}}{dw^2}=\mathcal{V}''(V_x)\frac{dV_x}{dw}^2+\mathcal{V}'(V_x)\frac{d^2V_x}{dw^2}$$

Thus, $\mathcal{V}'>1$ because $\frac{dV_x}{dw}$ is decreasing in x given w. Now, using the first derivative, we can rewrite the second expression as

$$\mathcal{V}''(V_x)\frac{dV_x}{dw}^2=\frac{d^2V_{x'}}{dw^2}-\frac{d^2V_x}{dw^2}\frac{\frac{dV_{x'}}{dw}}{\frac{dV_x}{dw}}=\frac{dV_{x'}}{dw}\left(\frac{-\frac{d^2V_x}{dw^2}}{\frac{dV_x}{dw}}-\frac{-\frac{d^2V_{x'}}{dw^2}}{\frac{dV_{x'}}{dw}}\right)$$

Therefore, $\mathcal{V}$ is a concave (convex) function and, hence, $V_{x'}$ is a concave (convex) transformation of V_x if and only if $\frac{-\frac{d^2V_{x'}}{dw^2}}{\frac{dV_{x'}}{dw}}$ is greater (lower) than $\frac{-\frac{d^2V_x}{dw^2}}{\frac{dV_x}{dw}}$. See the Online Appendix for the proof of the concavity of $\mathcal{V}$ if $\upsilon\in\mathcal{F}_1\cup\mathcal{F}_2$. ‖

Proof of Proposition 3.2. See the Online Appendix for the proof of the second bullet, which is straightforward. We are to prove the first result. Consider households with an employed female. The search value of a household of this set is defined by (4). We are to show that it decreases with productivity x. As the domain does not depend on x, it suffices to prove that the value difference $V_x(w)-V_x(z)$ also falls with x for any given wage w. Let $x,x'\in X$ such that $x'<x$, and suppose instead that

$$V_x(w)-V_x(z)\geq V_{x'}(w)-V_{x'}(z)=\mathcal{V}(V_x(w))-\mathcal{V}(V_x(z))$$

where the equality results from $V_{x'}$ being a monotonic transformation of V_x. The mean value theorem along with the monotonicity of function V_x implies that, for some $\omega\in[V_x(z),V_x(w)]$,

$$V_x(w)-V_x(z)\geq\mathcal{V}'(\omega)\big(V_x(w)-V_x(z)\big)\Longrightarrow 1\geq\mathcal{V}'(\omega)$$

This contradicts Proposition 3.1, which states $\mathcal{V}'(V)>1$. In particular, this implies that $S_x^*\leq S_0^*$.

Therefore, $X_p^*=\{x\in X\mid S_x^*\geq k_w\}=\{0\}\cup[\underline{x},x^*]$, where

$$x^*=\begin{cases}\underline{x}-1, & \text{if } S^{e*}(\underline{x})<k_w\\ S^{*-1}(k_w), & \text{if } S^*_{\underline{x}}\geq k_w \text{ and } S^*_{\overline{x})}<k_w\\ \overline{x}, & \text{if } S^*_{\overline{x}}\geq k_w\end{cases}$$

and S^{*-1} is the inverse of S_x^* as a function of x. ‖

Proof of Proposition 3.3. Consider the mass of households with an employed female. Let $V_x'(w)\equiv\frac{dV_x(w)}{dw}$ and similarly for the second derivative. We are to show first that function $\frac{-V_x''}{V_x'}$ being decreasing in x is a necessary and sufficient condition for the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$, also to decline with x. Notice that $\frac{V_x(w)-V_x(z)}{V_x'(w)}=\frac{V_x'(\hat{w})(w-z)}{V_x'(w)}$, for some $\hat{w}\in(z,w)$. Then,

$$\frac{\partial}{\partial x}\left(\frac{V_x(w)-V_x(z)}{V_x'(w)}\right)=(w-z)\frac{V_x'(\hat{w})}{V_x'(w)}\left(\frac{\frac{\partial V_x'(\hat{w})}{\partial x}}{V_x'(\hat{w})}-\frac{\frac{\partial V_x'(w)}{\partial x}}{V_x'(w)}\right)<(>)0$$

$$\Longleftrightarrow\frac{-V_x''(w)}{V_x'(w)}\text{ is decreasing (increasing) in } x$$

Next, we show the dynamics of wages and queue lengths over the space of a female's productivity. Using the constraint to replace q in the objective function of the search problem (4), we can rewrite it as

$$\max_w H(w,x) \equiv \nu(q(w))\big(V_x(w) - V_x(z)\big)$$

The necessary condition with respect to w is

$$\frac{\partial H(w,x)}{\partial w} = 0 \Longleftrightarrow \left(\nu'(q(w))\frac{\partial q}{\partial w}\frac{V_x(w) - V_x(z)}{V_x'(w)} + \nu(q(w))\right)V_x'(w) = 0$$

By evaluating the change of the first derivative with respect to a marginal increase in x at the solution candidate, we obtain

$$\frac{\partial^2 H(w,x)}{\partial w \partial x} = \nu'(q(w))\frac{\partial q}{\partial w}\frac{\partial \frac{V_x(w)-V_x(z)}{V_x'(w)}}{\partial x} > (<)0$$
$$\Longleftrightarrow \frac{-V_x''(w)}{V_x'(w)} \text{ is decreasing (increasing) in } x$$

Thus, the absolute risk aversion of the household's indirect utility function being decreasing in x is a necessary and sufficient condition for wages to increase with x. The direction of the queue lengths dynamics is the same as for wages since the equilibrium zero-profit condition establishes a positive relationship between q and w. ‖

A.2. Proofs of Section 4

Proof of Proposition 4.1. We sketch the proof of the first two results here. See the Online Appendix for further details.

1. Consider the planner's problem. Notice that the equilibrium allocation belongs to its domain as households' equilibrium decisions ensure incentive compatibility, the feasibility constraint becomes the zero-profit condition and the labor-force-participation constraint holds. Thus, because of the continuity of the functional forms and the closedness of the domain, the Weierstrass theorem ensures the existence of a solution to this problem.
Next, consider the following alternative problem, which has the same solution as the planner's. Mainly, this alternative problem only differs from the planner's in that the possibility of misreporting a type from set X_{np} has been eliminated.

$$(P_a)\ \max_{X_p,\mathcal{C}} \int_{X_p} \mathcal{U}_x dG(x) + \int_{X_{np}} \upsilon\big(z + y_x^n, \frac{y_x^n}{x}\big) dG(x)$$
$$\text{s. to } \frac{\kappa}{q_x} = \nu(q_x)\big(\overline{x}_m + y_x^e - c_x^e\big) + (1-\nu(q_x))(z + y_x^u - c_x^u),\ \forall x_f \in X_p$$
$$\nu(q_x)\big(\upsilon\big(c_x^e, \frac{y_x^e}{x}\big) - \upsilon\big(c_x^u, \frac{y_x^u}{x}\big)\big) \ge k_w, \forall x \in X_p$$
$$\mathcal{U}_x \ge \mathcal{U}_x(x'),\ \forall x, x' \in X_p$$
$$\upsilon\big(z + y_x^n, \frac{y_x^n}{x}\big) \ge \mathcal{U}_x(x'),\ \forall x \in X_{np},\ \forall x' \in X_p$$

The necessary condition with respect to y_x^n, for $x \in X_{np}$, is $\frac{\partial \upsilon^n}{\partial c}x + \frac{\partial \upsilon^n}{\partial \ell} = 0$. Thus, the solution assigns value $V_x(z)$ to all households in set X_{np}.
Now, let $\hat{X}_p$ denote the set of participating households in the planner's allocation, and $\hat{X}_{np} \equiv X \setminus \hat{X}_p$. Let households with an unemployed female aside for now. To see the continuity of

$\mathcal{U}_x$ at the boundary of $\hat{X}_p$, suppose instead that it does not converge to $V_x(z)$ for some $x \in \partial \hat{X}_p$, and, without loss of generality, assume that $x \in \hat{X}_p$. That is, $\mathcal{U}_x > V_x(z) + \epsilon$ for some $\epsilon > 0$. Consider a sequence $\{x_n\}_n \subset \hat{X}_{np}$ such that $x_n \to x$. By the continuity of $V_t(z)$ in t, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{U}_x > V_{x_n}(z) + \epsilon/2 \geq \mathcal{U}_{x_n}(x) + \epsilon/2$ for all $n \geq n_0$, where the last inequality follows from incentive compatibility. This contradicts the continuity of $\mathcal{U}_t(x)$ as a function of t.

Next, let $(\hat{q}_x, \hat{c}^e_x, \hat{c}^u_x, \hat{y}^e_x, \hat{y}^u_x)$ denote the planner's solution for any $x \in \hat{X}_p \setminus \{0\}$. We first show that, for differentiable mechanisms, there exists a threshold $\hat{x}$ such that $\hat{X}_p = \{0\} \cup [\underline{x}, \hat{x}]$. Suppose instead that there exists $\tilde{x} \in X$ such that $[\tilde{x} - \delta, \tilde{x}) \subset \hat{X}_{np}$ and $[\tilde{x}, \tilde{x} + \epsilon] \subset \hat{X}_p$ for some $\delta, \epsilon > 0$. Assume $\tilde{x} \in \hat{X}_p$ for simplicity. As shown above, $\mathcal{U}_{\tilde{x}} = V_{\tilde{x}}(z)$. It follows that $\lambda^3_{x',\tilde{x}} = \lambda^4_{x',\tilde{x}} = \lambda^5_{x',\tilde{x}} = \lambda^6_{x',\tilde{x}} = 0$ for all $x' \in X$ because $\mathcal{U}_{x'} \geq V_{x'}(z) > \upsilon\big(c^u_{\tilde{x}}, \frac{y^u_{\tilde{x}}}{x'}\big)$ if $x' \in X_p$ and $V_{x'}(z) > \upsilon\big(c^u_{\tilde{x}}, \frac{y^u_{\tilde{x}}}{x'}\big)$ if $x' \in X_{np}$. That is, no household has incentives to pretend to be $\tilde{x}$. Then, by combining the first order conditions of the Lagrangian with respect to consumption and income in each labor market status at $\tilde{x}$, we obtain

$$\frac{\partial \upsilon}{\partial c}\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\tilde{x} + \frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big) = 0; \;\; \frac{\partial \upsilon}{\partial c}\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\tilde{x} + \frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}_f}}{\tilde{x}_f}\big) = 0$$

Thus, we can write $\upsilon\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big) = V_{\tilde{x}}(z)$ and $\upsilon\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big) = V_{\tilde{x}}(\omega)$, where $\hat{c}^e_{\tilde{x}} = \omega + \hat{y}^e_{\tilde{x}}$. Because of the properties of the indirect utility function $V_{\tilde{x}}$ and the optimal spouse's income stated in Proposition 3.1, it follows that $\hat{c}^e_{\tilde{x}} \geq \hat{c}^u_{\tilde{x}}$ and $\hat{y}^e_{\tilde{x}} \leq \hat{y}^u_{\tilde{x}}$. It also follows from the above necessary conditions that $0 > \frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}_f}\big) \geq \frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)$. Thus, $\frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\hat{y}^e_{\tilde{x}} \geq \frac{\partial \upsilon}{\partial \ell}\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\hat{y}^u_{\tilde{x}}$. Moreover, for all $x \in [\tilde{x}, \tilde{x} + \epsilon]$, the right derivative

$$\begin{aligned}
\frac{d\big(\mathcal{U}_x - V_x(z)\big)}{dx^+} &= -\nu(q_x)\frac{\partial \upsilon}{\partial \ell}\big(c^e_x, \frac{y^e_x}{x}\big)\frac{y^e_x}{x^2} - (1 - \nu(q_x))\frac{\partial \upsilon}{\partial \ell}\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x^2} \\
&\quad + \frac{\partial \upsilon}{\partial \ell}\big(z + y^{u*}_x, \frac{y^{u*}_x}{x}\big)\frac{y^{u*}_x}{x^2} \\
&= \frac{\nu(q_x)}{x}\bigg(\frac{\partial \upsilon}{\partial \ell}\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x} - \frac{\partial \upsilon}{\partial \ell}\big(c^e_x, \frac{y^e_x}{x}\big)\frac{y^e_x}{x}\bigg) - \frac{\partial \upsilon}{\partial \ell}\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x^2} \\
&\quad + \frac{\partial \upsilon}{\partial \ell}\big(z + y^{u*}_x, \frac{y^{u*}_x}{x}\big)\frac{y^{u*}_x}{x^2}
\end{aligned}$$

where $y^{u*}_x \equiv \underset{y}{\operatorname{argmax}}\; \upsilon(z + y, \frac{y}{x})$. The first line results from the fact that incentive compatibility implies $\frac{d\mathcal{U}_x(x')}{dx'}\big|_{x'=x} = 0$ if and only if

$$\nu'(q_x)\dot{q}_x(\upsilon^e - \upsilon^u) + \nu(q_x)\big(\frac{\partial \upsilon^e}{\partial c}\dot{c}^e_x + \frac{\partial \upsilon^e}{\partial \ell}\frac{\dot{y}^e_x}{x}\big) + (1 - \nu(q_x))\big(\frac{\partial \upsilon^u}{\partial c}\dot{c}^u_x + \frac{\partial \upsilon^u}{\partial \ell}\frac{\dot{y}^u_x}{x}\big) = 0$$

where the dot symbol denotes the derivative with respect to x; hence, the derivative $\frac{d\mathcal{U}_x}{dx^+}$ boils down to the first two summands of the first line. The second line is a mere reorganization of the terms. Notice that this derivative evaluated at $x = \tilde{x}_f$ is strictly negative because so is the first term while the sum of the last two terms vanishes. This is a contradiction because $\mathcal{U}_x \geq V_x(z)$ in $\hat{X}_p$.

Now, to see that $\hat{x} \geq x^*$, notice that the labor-force-participation constraint for any x arbitrarily close to, but below x^* delivers $\mathcal{U}_x \geq V_x(z) + S_x^* - k_w > V_x(z)$. This implies that $x \in \hat{X}_p$ and, hence, $x^* \in \hat{X}_p$.

2. Since a female's employment status is observed, the tuple $(\hat{q}_0, \hat{c}_0^e, \hat{c}_0^u)$ is not subject to incentive compatibility. By differentiating the Lagrangian of problem (P_a) with respect to these variables and combining them, we obtain equation (8). Furthermore, the labor-force-participation constraint must be binding as otherwise the labor-force-participation condition would be violated.

   Finally, to show that households with both members unemployed receive the largest transfers, we are to anticipate some results stated in Proposition 4.2. Let $x \in \hat{X}_p \setminus \{0\}$. Notice that the planner's solution $(\hat{q}_x, \hat{c}_x^e, \hat{c}_x^u, \hat{y}_x^e, \hat{y}_x^u)$ must solve the following problem

$$\begin{aligned} &\max\ \mathcal{U}_x \\ &\text{s. to } (RC),\ (\text{ICC}^1),\ (\text{ICC}^2) \\ &\text{given } (\hat{q}_{x'}, \hat{c}_{x'}^e, \hat{c}_{x'}^u, \hat{y}_{x'}^e, \hat{y}_{x'}^u), (\hat{c}_{x''}^n, \hat{y}_{x''}^n), \qquad \forall x' \in \hat{X}_p \setminus \{x\}, x'' \in \hat{X}_{np} \end{aligned}$$

   It is obvious that $(\hat{q}_x, \hat{c}_x^e, \hat{c}_x^u, \hat{y}_x^e, \hat{y}_x^u)$ belongs to the domain of problem (P_x^{OT}) for the centralized economy with observable types defined later in Section 4.2.1, for which condition (ICC^1) is removed. Thus, its value is lower than the one obtained by its OT counterpart, $(\hat{q}_x^{OT}, \hat{c}_x^{eOT}, \hat{c}_x^{uOT}, \hat{y}_x^{eOT}, \hat{y}_x^{uOT})$, in problem (P_x^{OT}). Then, it follows from Proposition 4.2 that

$$V_x(c_x^u - \hat{y}_x^u) \leq V_x(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}) \Longrightarrow \hat{c}_x^u - \hat{y}_x^u \leq \hat{c}_x^{uOT} - \hat{y}_x^{uOT} \leq \hat{c}_0^{uOT} = \hat{c}_0^u. \quad \|$$

Proof of Proposition 4.2. We sketch part of the proof here. See the Online Appendix for further details.

1. To show that there exists a threshold $\hat{x}^{OT}$ such that $\hat{X}_p^{OT} = \{0\} \cup [\underline{x}, \hat{x}^{OT}]$, suppose instead that there exists $\tilde{x} \in X$ such that $[\tilde{x} - \delta, \tilde{x}) \subset \hat{X}_{np}^{OT}$ and $[\tilde{x}, \tilde{x} + \epsilon] \subset \hat{X}_p^{OT}$ for some $\delta, \epsilon > 0$. Assume that $\tilde{x} \in \hat{X}_p^{OT}$ for simplicity. By continuity, $\mathcal{U}_{\tilde{x}} = V_{\tilde{x}}(z)$. As shown below, the necessary conditions with respect to consumption and female's income for each labor market status at $\tilde{x}$ lead to

$$\frac{\partial v}{\partial c}\Big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\Big)\tilde{x} + \frac{\partial v}{\partial \ell}\Big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\Big) = 0; \ \frac{\partial v}{\partial c}\Big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\Big)\tilde{x} + \frac{\partial v}{\partial \ell}\Big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\Big) = 0$$

   Thus, we can write $v\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big) = V_{\tilde{x}}(z)$ and $v\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big) = V_{\tilde{x}}(\omega)$, where $\hat{c}_{\tilde{x}}^{eOT} = \omega + \hat{y}_{\tilde{x}}^{eOT}$. Because of the properties of the indirect utility function $V_{\tilde{x}}$ and the optimal female's income stated in Proposition 3.1, it follows that $\hat{c}_{\tilde{x}}^{eOT} \geq \hat{c}_{\tilde{x}}^{uOT}$ and $\hat{y}_{\tilde{x}}^{eOT} \leq \hat{y}_{\tilde{x}}^{uOT}$. Thus,

$$0 > \frac{\partial v_{\tilde{x}}^{eOT}}{\partial \ell} \geq \frac{\partial v_{\tilde{x}}^{uOT}}{\partial \ell}, \text{ and } \frac{\partial v_{\tilde{x}}^{eOT}}{\partial \ell}\hat{y}_{\tilde{x}}^{eOT} \geq \frac{\partial v_{\tilde{x}}^{uOT}}{\partial \ell}\hat{y}_{\tilde{x}}^{uOT},$$

   where $v_{\tilde{x}}^{iOT} \equiv v\big(\hat{c}_{\tilde{x}}^{iOT}, \frac{\hat{y}_{\tilde{x}}^{iOT}}{\tilde{x}}\big)$, for $i \in \{e, u\}$.

   Moreover, for all $x \in [\tilde{x}, \tilde{x} + \epsilon]$, because of the envelope theorem, the right derivative

$$\frac{d\big(\mathcal{U}_x - V_x(z)\big)}{dx^+} = \frac{\nu(\hat{q}_x^{OT})}{x}\left(\frac{\partial v}{\partial \ell}\Big(\hat{c}_x^{uOT}, \frac{\hat{y}_x^{uOT}}{x}\Big)\frac{\hat{y}_x^{uOT}}{x} - \frac{\partial v}{\partial \ell}\Big(\hat{c}_x^{eOT}, \frac{\hat{y}_x^{eOT}}{x}\Big)\frac{\hat{y}_x^{eOT}}{x}\right)$$

$$-\frac{\partial v}{\partial \ell}\big(\hat{c}_x^{uOT}, \frac{\hat{y}_x^{uOT}}{x}\big)\frac{\hat{y}_x^{uOT}}{x^2} + \frac{\partial v}{\partial \ell}\big(z + y_x^{u*}, \frac{y_x^{u*}}{x}\big)\frac{y_x^{u*}}{x^2}$$

where $y_x^{u*} \equiv \underset{y}{\text{argmax}}\ v(z+y, \frac{y}{x})$. This derivative evaluated at $x = \tilde{x}$ is strictly negative, which is a contradiction because $\mathcal{U}_x \geq V_x(z)$ in $\hat{X}_p^{OT}$.

3. We turn to the pattern over the household distribution.
   (a) Transfers, $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} - z$, decline with x within $\hat{X}_p^{OT}$. Let $x' < x$, and consider the alternative tuple $(\hat{q}_x^{OT}, \hat{c}_x^{eOT} - \hat{y}_x^{eOT} + \tilde{y}_{x'}^e, \hat{c}_x^{uOT} - \hat{y}_x^{uOT} + \tilde{y}_{x'}^u, \tilde{y}_{x'}^e, \tilde{y}_{x'}^u)$, where $\tilde{y}_{x'}^i$ is the solution of the maximization problem associated to the indirect utility function $V_{x'}(\hat{c}_x^{iOT} - \hat{y}_x^{iOT})$, for $i \in \{e, u\}$. This tuple satisfies the feasibility condition of problem $(\text{P}_{x'}^{OT})$. It also satisfies the labor-force-participation constraint because

$$\frac{k_w}{\nu(\hat{q}_x^{OT})} \leq V_x(\hat{c}_x^{eOT} - \hat{y}_x^{eOT}) - V_x(\hat{c}_x^{uOT} - \hat{y}_x^{uOT})$$
$$\leq V_{x'}(\hat{c}_x^{eOT} - \hat{y}_x^{eOT}) - V_{x'}(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}),$$

   where the first inequality results from the necessary condition (10), and the second one follows from Lemma 3.4. Therefore, this tuple belongs to the domain of problem $(\text{P}_{x'}^{OT})$; hence, $V_{x'}(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}) \leq V_{x'}(\hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT})$, and then $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} \leq \hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT}$ because of the monotonicity of function $V_{x'}$.
   (b) Consider the problem

$$\max_{q,s}\ H(q,s;x) \equiv V_x(z+s) + \lambda(x)\left(V_x(\omega(q,s)) - V_x(z+s) - \frac{k_w}{\nu(q)}\right)$$

   where λ is the positive Lagrange multiplier and $\omega = \omega(q,s)$ is implicitly defined by the equation $k = \eta(q)(\bar{x}_m - \omega + s) - sq$. Notice that this is equivalent to problem (P_x^{OT}) where $\omega = c_x^e - y_x^e$ and $s = c_x^u - y_x^u - z$. We showed above that s declines with x. To see that so does q, the derivative of H with respect to q:

$$\frac{\partial H}{\partial q} = \lambda(x)\left(V_x'(\omega(q,s))\frac{\partial \omega(q,s)}{\partial q} + \frac{k_w}{\nu(q)^2}\nu'(q)\right)$$

   Thus, the triple $(\hat{\omega}^{OT}, \hat{s}^{OT}, \hat{q}^{OT})$ must be a solution of the equation $V_x'(\omega(q,s))\frac{\partial\omega(q,s)}{\partial q} + \frac{k_w}{\nu(q)^2}\nu'(q) = 0$, i.e. $\frac{\partial w(q,s)}{\partial q} = -\frac{k_w \nu'(q)}{V_x'(w(q,s))\nu(q)^2} > 0$. When computing the change of the first derivative with respect to a marginal increase in x, we obtain

$$\frac{\partial^2 H}{\partial q \partial x} = \lambda'(x)\left(V_x'(w(q,s))\frac{\partial w(q,s)}{\partial q} + \frac{k_w}{\nu(q)^2}\nu'(q)\right) + \lambda(x)\frac{\partial V_x'(w(q,s))}{\partial x}\frac{\partial w(q,s)}{\partial q}$$

   This cross partial derivative is negative at the solution because 1) the first addend is zero as the necessary condition must hold, and 2) the second one is negative because the cross partial derivative of V is negative as stated in Proposition 3.1. Thus, the planner's solution $\hat{q}_x^{OT}$ falls as female's productivity increases.
   Then, condition (10) together with Proposition 3.1 implies that the income of a female married to an unemployed male increases with x as, $\forall x, x' \in \hat{X}_p^{OT}$ such that $x' < x$,

$$y_{x'}^{uOT}(z + \hat{s}_{x'}^{OT}) < y_x^{uOT}(z + \hat{s}_{x'}^{OT}) < y_x^{uOT}(z + \hat{s}_x^{OT})$$

Finally, since the labor-force-participation constraint is binding in the planner's OT allocation, and $\nu(\hat{q}_x^{OT}) > \nu(\hat{q}_{x'}^{OT})$, it must be the case that $V_x(\hat{\omega}_x^{OT}) - V_x(z + \hat{s}_x^{OT}) < V_{x'}(\hat{\omega}_{x'}^{OT}) - V_{x'}(z + \hat{s}_{x'}^{OT})$. ∥

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2022.105477.

References

Acemoglu, D., Shimer, R., 1999. Efficient unemployment insurance. J. Polit. Econ. 107, 893–928.

Akerlof, G.A., 1978. The economics of "tagging" as applied to the optimal income tax, welfare programs, and manpower planning. Am. Econ. Rev. 68, 8–19.

Anderson, P.M., Meyer, B.D., 1997. Unemployment insurance takeup rates and the after-tax value of benefits. Q. J. Econ. 112, 913–937.

Bacher, A., Grübener, P., Nord, L., 2022. Joint search over the life cycle. Mimeo.

Birinci, S., 2019. Spousal labor supply response to job displacement and implications for optimal transfers. Mimeo.

Blau, F.D., Kahn, L.M., 2007. Changes in the labor supply behavior of married women: 1980-2000. J. Labor Econ. 25, 393–438.

Blundell, R., Costa Dias, M., Meghir, C., Shaw, J., 2016a. Female labor supply, human capital, and welfare reform. Econometrica 84, 1705–1753.

Blundell, R., Pistaferri, L., Saporta-Eksten, I., 2016b. Consumption inequality and family labor supply. Am. Econ. Rev. 106, 387–435.

Braxton, J.C., Phillips, G., Herkenhoff, K., et al., 2019. Can the unemployed borrow? Implications for public insurance. Mimeo.

Browning, M., Crossley, T.F., 2001. Unemployment insurance benefit levels and consumption changes. J. Public Econ. 80, 1–23.

Burdett, K., Mortensen, D., 1978. Labor supply under uncertainty. In: Research in Labor Economics, vol. 2. JAI Press.

Carrillo-Tudela, C., Visschers, L., 2013. Unemployment and endogenous reallocation over the business cycle. Mimeo.

Chetty, R., 2008. Moral hazard versus liquidity and optimal unemployment insurance. J. Polit. Econ. 116, 173–234.

Chiappori, P.-A., Mazzocco, M., 2017. Static and intertemporal household decisions. J. Econ. Lit. 55, 985–1045.

Choi, S., Valladares-Esteban, A., 2020. On households and unemployment insurance. Quant. Econ. 11, 437–469.

Cullen, J.B., Gruber, J., 1996. Spousal Labor Supply as Insurance: Does Unemployment Insurance Crowd Outthe Added Worker Effect? Tech. rep.. National Bureau of Economic Research.

Davoodalhosseini, S.M., 2019. Constrained efficiency with adverse selection and directed search. J. Econ. Theory 183, 568–593.

Devereux, P.J., 2004. Changes in relative wages and family labor supply. J. Hum. Resour. 39, 698–722.

Engen, E.M., Gruber, J., 2001. Unemployment insurance and precautionary saving. J. Monet. Econ. 47, 545–579.

Fernández-Blanco, J., 2013. Labor market equilibrium with rehiring. Int. Econ. Rev. 54, 885–914.

Gayle, G.-L., Shephard, A., 2019. Optimal taxation, marriage, home production, and family labor supply. Econometrica 87, 291–326.

Gollier, C., 2004. The Economics of Risk and Time. MIT Press.

Golosov, M., Maziero, P., Menzio, G., 2013. Taxation and redistribution of residual income inequality. J. Polit. Econ. 121, 1160–1204.

Guler, B., Guvenen, F., Violante, G.L., 2012. Joint-search theory: new opportunities and new frictions. J. Monet. Econ. 59, 352–369.

Haan, P., Prowse, V.L., 2017. Optimal social assistance and unemployment insurance in a life-cycle model of family labor supply and savings. SSRN.

Hall, R.E., Milgrom, P.R., 2008. The limited influence of unemployment on the wage bargain. Am. Econ. Rev. 98, 1653–1674.

Hansen, G.D., İmrohoroğlu, A., 1992. The role of unemployment insurance in an economy with liquidity constraints and moral hazard. J. Polit. Econ., 118–142.

Heathcote, J., Storesletten, K., Violante, G., 2014. Consumption and labor supply with partial insurance: an analytical framework. Am. Econ. Rev. 104, 1–52.
Hornstein, A., Krusell, P., Violante, G., 2005. Unemployment and vacancy fluctuations in the matching model. Econ. Q.-Fed. Reserve Bank Richmond, 19–51.
Huckfeldt, C., 2021. Understanding the scarring effect of recessions. Mimeo, Cornell University.
Hyslop, D.R., 2001. Rising US earnings inequality and family labor supply: the covariance structure of intrafamily earnings. Am. Econ. Rev., 755–777.
Kambourov, G., Manovskii, I., 2009. Occupational specificity of human capital. Int. Econ. Rev. 50, 63–115.
Kleven, H.J., Kreiner, C.T., Saez, E., 2009. The optimal income taxation of couples. Econometrica 77, 537–560.
Kolsrud, J., Landais, C., Nilsson, P., Spinnewijn, J., 2018. The optimal timing of unemployment benefits: theory and evidence from Sweden. Am. Econ. Rev. 108, 985–1033.
Krueger, D., Wu, C., 2018. How Much Consumption Insurance in Bewley Models with Endogenous Family Labor Supply?.
Krusell, P., Mukoyama, T., Şahin, A., 2010. Labour-market matching with precautionary savings and aggregate fluctuations. Rev. Econ. Stud. 77, 1477–1507.
Krusell, P., Mukoyama, T., Rogerson, R., Şahin, A., 2011. A three state model of worker flows in general equilibrium. J. Econ. Theory 146, 1107–1133.
Lentz, R., 2009. Optimal unemployment insurance in an estimated job search model with savings. Rev. Econ. Dyn. 12, 37–57.
Lifschitz, A., Setty, O., Yedid-Levi, Y., 2018. On the Provision of Unemployment Insurance when Workers are Ex-ante Heterogeneous. Tech. rep., Working Paper. Tel Aviv University.
Low, H., Meghir, C., Pistaferri, L., 2010. Wage risk and employment risk over the life cycle. Am. Econ. Rev. 100, 1432–1467.
Mankart, J., Oikonomou, R., 2017. Household search and the aggregate labour market. Rev. Econ. Stud. 84, 1735–1788.
Mincer, J., 1962. Labor force participation of married women: a study of labor supply. In: Aspects of Labor Economics. Princeton University Press, pp. 63–105.
Mirrlees, J.A., 1971. An exploration in the theory of optimum income taxation. Rev. Econ. Stud., 175–208.
Moen, E.R., 1997. Competitive search equilibrium. J. Polit. Econ. 105, 385–411.
Ortigueira, S., Siassi, N., 2013. How important is intra-household risk sharing for savings and labor supply? J. Monet. Econ. 60, 650–666.
Pilossoph, L., Wee, S.L., 2021. Household search and the marital wage premium. Am. Econ. J. Macroecon. 13, 55–109.
Pratt, J.W., 1964. Risk aversion in the small and in the large. Econometrica 32, 122–136.
Shavell, S., Weiss, L., 1979. The optimal payment of unemployment insurance benefits over time. J. Polit. Econ. 87, 1347–1362.
Shephard, A., 2019. Marriage market dynamics, gender, and the age gap. Mimeo.
Wang, H., 2019. Intra-household risk sharing and job search over the business cycle. Rev. Econ. Dyn. 34, 165–182.