Trade credit contracts: Design and regulation

Florina Silaghi[a,*], Franck Moraux[b]

[a] *Universitat Autonoma de Barcelona, Campus de Bellatera, 08193, Barcelona, Spain*
[b] *Univ Rennes, CNRS, CREM - UMR6211, F-35000 Rennes, France*

Abstract

This paper provides a theoretical analysis of trade credit within a real options framework. We show that under trade credit the buyer delays the decision to stop production, getting closer to the supply chain optimal stopping decision. Therefore, trade credit may serve as a coordination device. The supplier can optimally choose to offer trade credit for free, since this will guarantee her business for a longer period of time. Optimal trade credit design is analyzed for an integrated supply chain (cooperative solution) and for external procurement (Nash bargaining and Stackelberg solutions). When regulation imposes a limit on trade credit maturity, the wholesale price is reduced, trade credit decreases and internal procurement increases. The model's predictions are in line with recent empirical evidence on the effects of regulation in the retail industry.

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[*] Corresponding author. Tel.: +34 93 581 4302.
Email addresses: florina.silaghi@uab.cat (Florina Silaghi), franck.moraux@univ-rennes1.fr (Franck Moraux)

1. Introduction

Trade credit as a source of financing has been generally neglected by the literature (as opposed to bank or bond financing), although it is the most important source of financing not only for small and medium enterprises, but also for large ones. Cuñat (2007) provides evidence that trade credit accounts for roughly one-fourth of the total assets of a representative firm and abouth half of the short-term debt in two different samples of medium-sized UK firms and small US firms. Similarly, Yang and Birge (2018) claim that as of June 2016, accounts payable (the amount of trade credit owed by buyers to suppliers) were 3.3 times as large as bank loans on the aggregated balance sheet of non-financial US businesses. Moreover, for large public retailers in North America, accounts payable represent roughly one third of their liabilities. In this paper, we contribute to this literature by providing a first theoretical analysis of trade credit within a real options framework, which allows us to shed light on important issues related to the design of trade credit terms, and the effects of regulation. Furthermore, we contribute to the long-standing debate over whether trade credit is an expensive form of financing, or on the contrary a low-cost financing source (Yang and Birge, 2018).

On the one hand, the popular view in the initial literature on trade credit relies on two assumptions. First, trade credit was considered an expensive form of finance (Cuñat, 2007; Petersen and Rajan, 1997).[1] Researchers trying to address the so-called trade credit puzzle attempted to answer the following questions "why does trade credit appear to be so expensive?" and "why is trade financed by suppliers instead of banks" (Cuñat, 2007, p. 492). Several explanations have been offered in the literature (see Seifert et al., 2013 for a review): trade credit is a price discrimination device (Brennan et al., 1988), trade credit can serve as a warranty for product quality (Long et al., 1994), trade credit can be used to signal information to banks in the context of asymmetric information in times of credit rationing (Biais and Gollier, 1997), and/or trade credit can be justified by the special link existing between the supplier and the customer when the product is buyer-specific (Cuñat, 2007). Second, another common assumption is that a large manufacturer finances a relatively small and young financially constrained retailer (Petersen and Rajan, 1997).

On the other hand, recent empirical evidence seems to challenge these two assumptions. First, Giannetti et al. (2011) find that a majority of firms in their sample receive trade credit at low cost, only a minority of firms reporting that their main supplier offers early payment discounts. The median firm in their sample receives trade credit at zero cost. The most common trade credit terms in the sample of Yang and Birge (2018) are "net terms", which are interest-free loans from suppliers to buyers.[2] Secondly, there is recent evidence that many small suppliers finance large financially unconstrained buyers (Fabbri and Klapper, 2016; Klapper et al., 2012; Murfin and Njoroge, 2015).[3] The question that naturally arises is why do small financially constrained suppliers offer trade credit for free to large unconstrained buyers. Why don't they offer a price discount? As pointed out by Giannetti et al. (2011), a big challenge for future research is to answer this question. A possible explanation suggested in the empirical papers of Klapper et al. (2012) and Fabbri and Klapper (2016) is that they offer trade credit as a competitive gesture, in order to attract the buyer. Fabbri

[1] Cuñat (2007) explains the example of a two-part trade credit contract called "2-10 net 30", where if customers pay within 10 days they qualify for a 2% discount, otherwise they can pay up to 30 days after delivery. This discount on early payment implies receiving credit at 2% for 20 days, which is equivalent to a 44% annual interest rate.

[2] For example, under the "net 30" term, buyers need to pay supplier within 30 days of delivery.

[3] Murfin and Njoroge (2015) document that the largest two decile firms by size are net receivers of trade credit in the retail industry.

and Klapper (2016) show that suppliers are more likely to offer trade credit and better credit terms to powerful and important customers. A price reduction is observable by competitors and can trigger a price war, detrimental for the whole industry, while trade credit is less aggressive and more flexible. They also show evidence of complementarity between these two instruments, price discount and trade credit: firms that lowered prices relative to the previous year are more likely to offer trade credit.

Anecdotal evidence from the retail and automotive industry also seems to support this new perspective on trade credit from small suppliers to large buyers and seems to suggest that large buyers abuse their bargaining power in order to extract margins from small suppliers.[4] This has drawn the attention of policy makers and regulators, who have tightened regulation by reducing the maturity of trade credit contracts, in an attempt to protect suppliers. Two such cases have been recently analyzed in the empirical papers of Breza and Liberman (2017) and Barrot (2016). Supermarkets in Chile saw their trade credit maturity reduced from 90 to 30 days, while French trucking firms were prevented from extending to their customers trading terms exceeding 30 days (representing a 15% reduction in payment terms).

In this paper, we contribute to this debate by providing a first theoretical analysis of this issue. Within a real options framework, we model the trade credit between two firms, a manufacturer (supplier, S) and a producer (buyer, B). The supplier sells an intermediate product to the buyer at the wholesale price and incurs a production cost. The buyer produces a final product that is then sold to consumers (see Figure 1). We assume that the retail price at which the buyer sells the final product to a consumer is uncertain, following a geometric Brownian motion. The buyer can optimally choose the time at which to stop the business. We derive the buyer, supplier and supply chain values first under no trade credit, and second under trade credit. In the no trade credit case, the buyer stops too early compared to the supply chain optimal due to double marginalization. In fact, the supply chain can only be coordinated (and the value of the supply chain maximized) if the wholesale price is equal to the production cost. In the trade credit case, we show that the stopping threshold optimally chosen by the buyer is lower than the one without trade credit. This is because extending trade credit acts as a decrease in the effective wholesale price. Hence, the buyer delays the decision to stop when receiving trade credit, getting closer to the supply chain optimal stopping threshold. Thus, trade credit is effectively a coordination device. Moreover, although offering trade credit implies a cost of delay for the supplier (receiving the payment later), it also has the advantage of guaranteeing her business for a longer period of time. Given this trade-off, we show that as long as the trade credit maturity is not too high, the supplier can optimally choose to offer trade credit for free. Our model's implications are in line with recent empirical evidence suggesting trade credit is offered at a low cost (Giannetti et al., 2011).

We consider three different solutions in designing the optimal trade credit terms, i.e., the pair of optimal wholesale price (price charged by the supplier to the buyer) and optimal trade credit maturity (delay granted by the supplier). The first one is the cooperative solution corresponding to a vertically integrated supply chain. We show that for each potential wholesale price there exists an optimal trade credit maturity that maximizes the value of the supply chain. There thus exists an

[4]See John Plender, "The Wal-Martisation of almost everything", The Financial Times, 15 November 2004; John Plender, Martin Simons and Henry Tricks, "Cash benefit: how big supermarkets fund expansion by using suppliers as bankers", The Financial Times, 7 December 2005; "Competition policy in the EU: Big chains enjoy a buyer's market", The Economist, 15 February 2007; "FT interview transcript: Christine Lagarde", The Financial Times, 12 May 2008, among others.

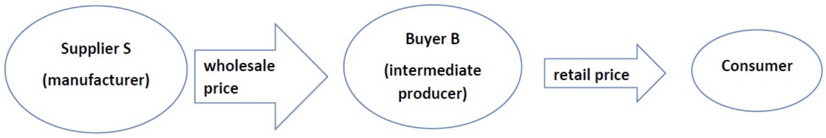


Figure 1: Trade credit from the supplier to the buyer. A supplier (a manufacturer) sells an intermediate good at a wholesale price to the buyer (an intermediate producer). The buyer then produces a final good sold on the market to consumers at the retail price.

infinite number of pairs (wholesale price, trade credit maturity) maximizing the supply chain value. While under no trade credit the supply chain is only coordinated when the wholesale price is equal to production cost, with trade credit it is possible to coordinate the supply chain for any wholesale price, further reinforcing that trade credit is effectively a coordination device. The second solution that we analyze arises in case of external procurement designed by Nash bargaining. In this case, the outcome depends on the status quo of the two parties and their bargaining power. Third, we analyze the Stackelberg solution, with the leader being the buyer and the follower the supplier. This strategy characterizes well recent empirical and anecdotal evidence that large retailers finance themselves through trade credit from their smaller and weaker suppliers.

Finally, we study the impact of regulation, in particular, the effect of a limit on trade credit maturity imposed by regulators. We analyze two extreme scenarios in which price discrimination is forbidden and price discrimination is fully allowed. When price discrimination is forbidden, buyer value decreases following the regulation, causing him to possible switch to internal procurement, thus trade credit decreases. When full price discrimination is possible, the two parties can undo regulation by decreasing the wholesale price. Our model's predictions are in line with recent empirical evidence provided by Breza and Liberman (2017).

Our work makes important contributions to three strands of the literature. First, we extend current research on continuous-time real options models by applying a real options model to trade credit. Although corporate debt has been extensively analyzed within a real options framework, to our knowledge this is the first paper to apply it to trade credit.[5] Within the real options literature, the forward start model of Shackleton and Wojakowski (2007) has been recently employed to explain findings from different areas ranging from duopolistic competition (Pereira and Rodrigues, 2014) to public-private partnerships (Adkins et al., 2019). However, a "forward stop" model, as the one we employ, similar to the deferred American put model of Gerber and Shiu (1993), has not been exploited in the literature. Second, we contribute to the literature on real options in supply chains, which is at the interface between finance, supply chain management and industrial organization (see Chen et al., 2017; Hult et al., 2010; Liu and Wang, 2019; Wang and Tsao, 2006, among others).[6] Although these papers analyze real options within supply chains, they do not use standard real options methodology as we do. Moreover, they investigate ordering policies and competition,

[5] For standard real-options methodology in corporate finance see Dixit and Pindyck (1994). Trigeorgis and Tsekrekos (2018) provide a review of the literature on supply chains using real options methods. Moreover, Billette de Villemeur et al. (2014) analyze investment timing in vertical relationships within a real options framework. However, none of these studies considers trade credit.

[6] Hult et al. (2010) analyze several real options in the supply chain context and show that they operate differently in supply chains than they do in firms. Wang and Tsao (2006) and Chen et al. (2017) study bidirectional option contracts in supply chains. More recently, Liu and Wang (2019) study competing supply chain firms from a real options perspective with supplier capacity investment.

while we focus on the buyer's stopping decision under uncertainty and its impact on the supplier's incentives to provide trade credit. Third, we contribute to the operations management literature on supply chains. Different contract types have been shown to coordinate the supply chain, such as buy back contracts, revenue sharing contracts and quantity flexibility contracts (Cachon, 2003). Recent work in the supply chain management literature has shown under a newsvendor framework that trade credit is a tool for supply chain coordination, as it coordinates the buyer's order quantity decision (see, for example, Lee and Rhee, 2011). Our model offers a new perspective on trade credit as a coordination mechanism for the supply chain, since in our framework trade credit coordinates the strategic business-exit/stopping decision of the buyer. Finally, our work on trade credit design is also relevant for practitioners who welcome normative insights.

The rest of the paper is organized as follows. Section 2 describes the financial setup, first under no trade credit and then under trade credit. Section 3 analyzes the benefits of offering trade credit for free. In Section 4 we provide an analytical approximation approach for trade credt. Section 5 presents the design of trade credit terms under different cooperative and non-cooperative solutions. In section 6 we analyze the social planner's perspective and discuss issues related to regulation. A numerical illustration and comparison of the model's implications with empirical evidence on trade credit is provided in section 7. Finally, section 8 concludes.

2. The framework

A producer, called the buyer (B), produces a final good using as an input a good that it buys from a manufacturer, called the supplier (S). The project under study consists of selling on the market the final good whose (retail) price is denoted by P. The price is given by $P = x$, where the state variable x introduces uncertainty in the model. For simplicity, the quantity of output that the buyer produces per period is normalized to one.[7] Following Dixit and Pindyck (1994), Charalambides and Koussis (2018) and Silaghi and Sarkar (2020), x follows a geometric Brownian motion:

$$dx_t = \mu x_t dt + \sigma x_t dW_t,$$

where $W = (W_t)_t$ is a standard Brownian motion, μ and σ represent the drift and volatility terms respectively. The buyer continuously pays to the supplier the wholesale price, c, per unit time, which is fixed, and the supplier faces a fixed continuous production cost, γ, per unit time, with $c \geq \gamma$. For the time being we will consider c as given. Later on, in Section 5 the wholesale price will be determined either as a cooperative solution in the case of a vertically integrated supply chain, or as the result of a non-cooperative game between the buyer and the supplier (under Nash and then Stackelberg bargaining games). The instantaneous net cash flow for the buyer is therefore $P_t - c$ per unit time, while the one of the seller is $c - \gamma$ per unit time. Each firm is risk neutral, thus each firm maximizes expected profits. Let us denote by B the value of the project from the buyer's perspective and by S the value from the supplier's perspective. V stands for the value of a supply chain that comprises both the buyer and the supplier, so that $V = B + S$.

We will now analyze two cases. First, the simple case in which the timing of delivery and the timing of payment of the goods delivered by the supplier to the buyer coincide. Therefore, there

[7] Similar modeling assumptions have been made for example in Dixit and Pindyck (1994), Chapter 7. Alternatively, one could assume random revenues without distinguishing between prices and quantities as in Charalambides and Koussis (2018).

is no delay in payment, and the only contractual parameter is the wholesale price, c. Second, we will consider the case in which payment occurs some time after the delivery of the goods, thus the supplier is offering trade credit to the buyer. In this case, the two contractual parameters are the wholesale price, c, and the trade credit maturity, denoted by Δ.

2.1. The no trade credit case

In case there is no trade credit, we assume that the buyer can decide to stop the project at any time.[8] Of course he will undertake this decision so as to maximize his profit.[9] Let's denote by τ such an optimal time. Since the project has no special expiration, as standard in real option theory, the optimal decision will occur as soon as the sale price P reaches a certain constant threshold K from above. The optimal time is defined by $\tau = \inf\{t : P_t = K\}$. Then the buyer, supplier and supply chain valuations are given by:

$$\begin{aligned} B^{NT} &= E_0\left(\int_0^\tau (P_s - c)e^{-rs}ds\right) \\ S^{NT} &= E_0\left(\int_0^\tau (c - \gamma)e^{-rs}ds\right) \\ V^{NT} &= E_0\left(\int_0^\tau (P_s - \gamma)e^{-rs}ds\right), \end{aligned}$$

for which we can derive explicit formulae (where r denotes the risk-free interest rate, and for convergence $r > \mu$).

Proposition 1: *Consider a stopping threshold K. The buyer's valuation of the project is*

$$B^{NT}(c; K) = \left[\frac{P_0}{r-\mu} - \frac{c}{r}\right] - \left[\frac{K}{r-\mu} - \frac{c}{r}\right]\left(\frac{P_0}{K}\right)^{-X} \tag{1}$$

The supplier's valuation of the project is

$$S^{NT}(c; K) = \left[\frac{c}{r} - \frac{\gamma}{r}\right] - \left[\frac{c}{r} - \frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X} \tag{2}$$

The supply chain's valuation of the project is

$$V^{NT}(c; K) = \left[\frac{P_0}{r-\mu} - \frac{\gamma}{r}\right] - \left[\frac{K}{r-\mu} - \frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X} := V^{NT}(K) \tag{3}$$

where

$$X = \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2} > 0. \tag{4}$$

[8] In absence of trade credit, the buyer is apparently the only party exposed to the unique uncertainty of our framework, i.e., the business risk, while the supplier receives some strictly positive cash flows over the life of the business. This is not however the case, as the buyer will internalize this uncertainty to design his exit-from-business strategy, which in turn affects the supplier.

[9] Following Cachon (2003), we adopt the convention that the upstream firm (the supplier) is female, and the downstream firm (the buyer) is male.

In case the buyer chooses to stop production optimally, the optimal threshold is

$$K_B^{NT} = c\frac{(r-\mu)}{r}\frac{X}{X+1} \tag{5}$$

and this optimal threshold must be plugged into the above formulae.

Proof: See Appendix.

The above formulae are intuitive given the building blocks of real option theory. For example, the first term in between square brackets in the buyer's value represents the present value of receiving the net cash flows in perpetuity. The second term represents the value of the option to stop production when the price reaches the threshold K. In this case, the buyer foregoes the value of the net cash flows. Finally, $(P_0/K)^{-X}$ is the present value of \$1 obtained in case the threshold is reached. The formulas also highlight the direct influence of the parameters on the pricing functions. For instance, the buyer's valuation of the project is decreasing with respect to the wholesale price, whereas the valuation of the supplier is symmetrically increasing and the supply chain's valuation is insensitive to it. The proposition is also in line with standard results from the real option theory. Since $\frac{(r-\mu)}{r} < 1$ and $\frac{X}{X+1} < 1$, equation (5) implies that the optimal threshold chosen by the buyer to stop production is lower than the Marshallian one, namely c. This optimal stopping threshold is proportional to the cost c and it depends non-linearly on r, μ and σ. It is worth noticing that the condition $P_0 > K_B^{NT}$ requires that the parameters satisfy $[P_0/(r-\mu)]/(c/r) > X/(X+1)$.

The optimal threshold selected by the buyer must be plugged into the formulae for valuing B^{NT}, S^{NT} and V^{NT}. Consequently, although equation (3) shows that the total supply chain value V^{NT} does not *directly* depend on the wholesale price c, it depends on it indirectly through the stopping threshold K_B^{NT} chosen by the buyer. When choosing the optimal stopping threshold, the buyer does not take into account the supplier's profits, nor the interest of the whole supply chain. The following proposition provides the appropriate threshold to consider for maximizing the value of the supply chain.

Proposition 1 bis: *The threshold that maximizes the total supply chain value is*

$$K_{SC}^{NT} = \gamma\frac{(r-\mu)}{r}\frac{X}{X+1}.$$

Proof: Similar to the proof of Proposition 1, since by substituting c by γ in equation (1) we obtain equation (3).

For notation purposes, denote by $K(.)$ the function defined by $K(x) = x\frac{(r-\mu)}{r}\frac{X}{X+1}$. Then we have that $K_B^{NT} = K(c)$ and $K_{SC}^{NT} = K(\gamma)$. This function is increasing in x. Since $c \geq \gamma$, we have that $K_B^{NT} = K(c) \geq K_{SC}^{NT} = K(y)$. The following corollary summarizes this discussion.

Corollary 1: *If $c > \gamma$, the optimal threshold for the buyer K_B^{NT} is strictly larger than the optimal stopping threshold for the supply chain K_{SC}^{NT}. In our stopping time setting, this means that the buyer will decide to stop production earlier than what is desirable for the supply chain.*

Proof: Directly follows from the inequalities presented above.

The fact that the buyer decides to stop production *too early* implies that there is a coordination issue. One way to coordinate the supply chain, or more precisely to make the buyer choose the stopping threshold that maximizes the supply chain value, is to set the wholesale price equal to the production cost, i.e., $c = \gamma$ because in that case we have $K_B^{NT} = K_{SC}^{NT}$. This corresponds to a vertical integration strategy where the buyer decides to integrate the upstream supplier so as to pay only the production cost. Indeed, vertical integration is known to be one of the most common strategies to coordinate supply chains. Alternative supply chain coordination mechanisms have nevertheless been studied in the context of the newsvendor model: two-part tariff, buy back, quantity flexibility and revenue sharing. Cachon (2003) makes an excellent survey of this literature. In the next subsection we introduce trade credit and show that this is also a coordination mechanism in our real option setting.

2.2. Introducing trade credit

We now introduce trade credit. Let the maturity of the trade credit contract be denoted by Δ. Under such a contract, the buyer receives the goods from the supplier, starts production immediately and sells immediately the final product to consumers at time zero, but he does not make any payments to the supplier until time Δ. As a result, the buyer continuously receives from time zero the price of the final goods (sold P per unit), but only starts making a continuous payment c to the supplier at time Δ. Thus, the payments to the supplier are deferred/delayed by a period of time Δ. We assume that the buyer stops production whenever the price reaches a threshold K from above, *after* time Δ.[10] This implies that the buyer's value involves a long position in a deferred perpetual put option or a forward stop perpetuity. At time Δ, the state variable P_Δ can either be below or above the trigger K. For the former case, the buyer will stop production at Δ, that is, he will give up the present value of future cash flows, which is equivalent to the difference between an asset-or-nothing put option on $\frac{P_\Delta}{r-\mu}$ and a cash-or-nothing put option on $\frac{c}{r}e^{-r\Delta}$, both with maturity Δ exercised if $P_\Delta < K$. However, if the buyer does not stop production at time Δ, he still has the option to stop later on, which produces an additional value. We thus have the following two sequences of cash flows illustrated in Figure 2. If $P_\Delta > K$, the buyer will continuously receive the price P until time $\tau > \Delta$ at which the threshold K is reached, and will continuously make the payment c to the supplier until time $\tau + \Delta$. If $P_\Delta \leq K$, the production is stopped at time Δ, thus the buyer receives the price P until time Δ and makes the payment c until time $\Delta + \Delta = 2\Delta$.[11] The buyer's valuation is given by:

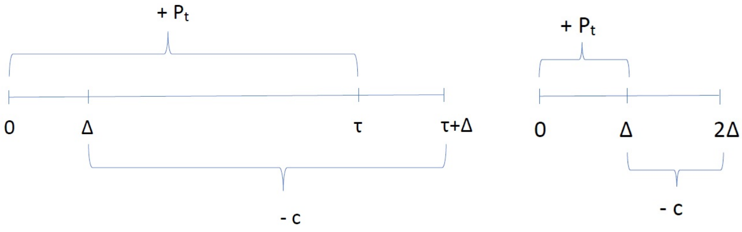
$$\begin{aligned} B^{TC} &= E_0\left(\int_0^\Delta P_s e^{-rs} ds\right) + E_0\left(1_{\{P_\Delta < K\}}\int_\Delta^{2\Delta} -ce^{-rs} ds\right) \\ &+ E_0\left[1_{\{P_\Delta > K\}}\left(\int_\Delta^{\tau} P_s e^{-rs} ds + \int_\Delta^{\tau+\Delta} -ce^{-rs} ds\right)\right] \end{aligned}$$

Rearranging, as $\tau > \Delta$, we have:

$$B^{TC} = E_0\left(\int_0^\Delta P_s e^{-rs} ds\right) - \int_\Delta^{2\Delta} ce^{-rs} ds + E_0\left[1_{\{P_\Delta > K\}}\left(\int_\Delta^{\tau} P_s e^{-rs} ds + \int_{2\Delta}^{2\Delta+(\tau-\Delta)} -ce^{-rs} ds\right)\right]$$

[10]Since trade credit maturity is relatively small, usually up to 180 days, we believe that this assumption is not very restrictive. Allowing the buyer to stop before time Δ would imply a time-dependent optimal stopping threshold very complicated to determine, and therefore, a loss of analytical tractability of the model. Hereafter, one assumes without loss of generality that the stopping time is defined by $\tau = \inf\{t \geq \Delta : P_t = K\}$.

[11]In the first case the length of the relationship is τ, while in the second case it is Δ. In both cases the buyer collects the retail price from consumers from time zero until the stopping time (τ and Δ respectively), and pays the wholesale price to the supplier with a delay, from time Δ, the trade credit delay, until the stopping time plus the trade credit delay ($\tau + \Delta$ and 2Δ respectively). Note that under trade credit the optimal stopping time is defined as $\tau = \inf\{t > \Delta : P_t = K\}$, and thus depends on Δ, but for the sake of simplicity we omit this dependence in our notation.



a) Stopping at τ b) Stopping at Δ

Figure 2: Trade credit and the stopping threshold. Panel a) presents the buyer's stream of continuous cash flows per unit time under trade credit in the case $P_\Delta > K$, that is, the retail price at the trade credit maturity Δ is above the stopping threshold K, when the production is stopped at time $\tau > \Delta$. In this case, the buyer collects the retail price P_t from consumers from time zero until the stopping time τ, and pays the wholesale price c to the supplier from time Δ, the trade credit delay, until the stopping time plus the trade credit delay $(\tau + \Delta)$. Panel b) depicts the case $P_\Delta \leq K$, when the production is stopped at Δ. In this case, the buyer collects the retail price P_t from consumers from time zero until the stopping time Δ, and pays the wholesale price c to the supplier from time Δ, the trade credit delay, until the stopping time plus the trade credit delay $(\Delta + \Delta = 2\Delta)$.

Similarly, the supplier's valuation is given by:

$$S^{TC} = \int_0^{\Delta} -\gamma e^{-rs} ds + \int_{\Delta}^{2\Delta} c e^{-rs} ds + E_0 \left[1_{\{P_\Delta > K\}} \left(\int_{\Delta}^{\tau} -\gamma e^{-rs} ds + \int_{2\Delta}^{2\Delta + (\tau - \Delta)} c e^{-rs} ds \right) \right]$$

The supply chain's valuation is given by:

$$V^{TC} = E_0 \left(\int_0^{\Delta} (P_s - \gamma) e^{-rs} ds \right) + E_0 \left[1_{\{P_\Delta > K\}} \left(\int_{\Delta}^{\tau} (P_s - \gamma) e^{-rs} ds \right) \right]$$

The following proposition provides explicit formulae for the buyer, supplier and supply chain valuations in the trade credit case.

Proposition 2: *Consider a stopping threshold K. The buyer's valuation of the project with trade credit is*

$$\begin{aligned} B^{TC}(c, \Delta; K) &= \left[\frac{P_0}{r - \mu} - e^{-r\Delta} \frac{c}{r} \right] - e^{-r\Delta} \left[\frac{P_0}{r - \mu} e^{\mu\Delta} N(-d_1(\mu, \Delta)) - \frac{c}{r} e^{-r\Delta} N(-d_2(\mu, \Delta)) \right] \\ &\quad - \left[\frac{K}{r - \mu} - e^{-r\Delta} \frac{c}{r} \right] \left(\frac{P_0}{K} \right)^{-X} N\left(d_2\left(\mu - X\sigma^2, \Delta \right) \right), \end{aligned} \tag{6}$$

The supplier's valuation of the project with trade credit is

$$S^{TC}(c, \Delta; K) = \left[e^{-r\Delta} \frac{c}{r} - \frac{\gamma}{r} \right] \left[1 - e^{-r\Delta} N(-d_2(\mu, \Delta)) - \left(\frac{P_0}{K} \right)^{-X} N(d_2\left(\mu - X\sigma^2, \Delta \right)) \right], \tag{7}$$

The supply chain's valuation of the project with trade credit is

$$V^{TC}(c,\Delta;K) = \left[\frac{P_0}{r-\mu}-\frac{\gamma}{r}\right]-e^{-r\Delta}\left[\frac{P_0}{r-\mu}e^{\mu\Delta}N\left(-d_1(\mu,\Delta)\right)-\frac{\gamma}{r}N\left(-d_2(\mu,\Delta)\right)\right]$$
$$-\left[\frac{K}{r-\mu}-\frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X}N\left(d_2\left(\mu-X\sigma^2,\Delta\right)\right):=V^{TC}(\Delta;K) \tag{8}$$

with:

$$d_1(\mu,\Delta) = \frac{\ln(P_0/K)+\left(\mu+1/2\sigma^2\right)\Delta}{\sigma\sqrt{\Delta}}, \qquad d_2(\mu,\Delta)=\frac{\ln(P_0/K)+\left(\mu-1/2\sigma^2\right)\Delta}{\sigma\sqrt{\Delta}},$$
$$d_2\left(\mu-X\sigma^2,\Delta\right) = \frac{\ln(P_0/K)-\sqrt{(\mu-\sigma^2/2)^2+2r\sigma^2}\Delta}{\sigma\sqrt{\Delta}}. \tag{9}$$

Proof: See Appendix.

In the Appendix, we make an extensive use of techniques coming from the field of option pricing. Option theory is also useful to interpret the various components highlighted in the prevous valuation formulae. For instance, the first term in equation (6) represents the value of owning the business forever, i.e., receiving the retail price P perpetually, and paying the wholesale price c with a delay Δ. The second term captures the value of exercising the option to stop at Δ and one may recognize two terms, $N(-d_1(\mu,\Delta))$ and $N(-d_2(\mu,\Delta))$, that appear in the Black and Scholes formula for pricing put options. The last term captures the value of exercising the option to stop at a later stage, after Δ, if the trigger K is not reached at Δ ($P_\Delta > K$). Hence, the above valuation problem appears closely related to the valuation of deferred perpetual American puts studied by Gerber and Shiu (1993) and to the pricing of forward start perpetuities investigated in Shackleton and Wojakowski (2007).

The above formulae critically depend on the threshold to decide to stop production that will be endogenously chosen by the buyer. In particular, for given trade credit terms, the supplier benefits from any decrease in the stopping threshold chosen by the buyer (everything else equal). This result will be particularly useful when analyzing the supplier's trade-off when granting trade credit in the next section. We summarize it in the following proposition.

Proposition 3: *For given trade credit terms (namely Δ), the supplier's valuation of the project with trade credit, $S^{TC}(c,\Delta;K)$, is decreasing in K.*

Proof: See Appendix.

3. Coordination and the trade-credit trade-off

The optimal stopping threshold for the buyer is given by finding K_B^{TC} such that:

$$\left.\frac{\partial B^{TC}(c,\Delta;K)}{\partial K}\right|_{K=K_B^{TC}} = 0 \tag{10}$$

for all P. Of course, the optimal threshold chosen by the buyer will depend on the maturity of the trade credit, i.e., $K_B^{TC}(\Delta)$, and this dependence can be instrumentalized by the supplier. The buyer's valuation of the project with trade credit given by equation (6) appears somewhat intricate and the derivative with respect to the stopping threshold is not trivial. But the following proposition shows that the resulting optimal stopping threshold is simple and intuitive.

Proposition 4: *In presence of trade credit, the optimal stopping threshold for the buyer is:*

$$K_B^{TC}(\Delta) = e^{-r\Delta} \frac{c(r-\mu)}{r} \frac{X}{X+1}. \tag{11}$$

Proof: See Appendix.

It is worth deriving, by a similar approach, the threshold that maximizes the supply chain's valuation of the project with trade credit.

Proposition 4 bis: *In presence of trade credit, the threshold that maximizes the integrated supply chain value is*:

$$K_{SC}^{TC} = \gamma \frac{(r-\mu)}{r} \frac{X}{X+1}.$$

Proof: Similar to the proof of Proposition 4, since by substituting $ce^{-r\Delta}$ by γ in equation (6) we obtain equation (8).

Comparing the optimal threshold chosen by the buyer with the optimal one for the supply chain, we have that $K_B^{TC}(\Delta) \geq K_{SC}^{TC}$, i.e., the buyer stops too early compared to what would be optimal for the supply chain. This is because $ce^{-r\Delta} \geq \gamma$ since the supplier's revenue must cover her production cost for the supplier value to be non-negative (see equation (7)). On the other hand, by comparing the stopping thresholds chosen by the buyer in the two scenarios, with and without trade credit, we now have the following proposition.

Proposition 5: *Comparing the stopping threshold with and without trade credit, we have:*

$$K_B^{TC}(\Delta) \leq K_B^{NT}$$

meaning that the buyer will decide to postpone his decision to stop production. The postponement is directly influenced by the granted delay. By contrast, one has $K_{SC}^{TC} = K_{SC}^{NT} = K_{SC}$. Hence the thresholds to maximize the value of the supply chain are the same.

Proof: straightforward as $K_B^{TC}(\Delta) = e^{-r\Delta} K_B^{NT}$.

Hence, trade credit makes the buyer decide to stop production *later*. Granting trade credit acts here as a reduction of the wholesale price required by the supplier. Indeed, in supply chain management, trade credit and commercial price discounts are sometimes viewed as substitutes. By contrast, in a vertically integrated supply chain, the stopping decision is not influenced by whether the supplier grants trade credit to the buyer or not. This proposition has in turn two important implications.

The first implication of Proposition 5 is that trade credit makes the optimal threshold K_B^{TC} be closer to K_{SC}. It is therefore effectively a coordination device that can align the buyer's interest with the interest of the supply chain. Compared to the no trade credit case, the distance between the stopping threshold chosen by the buyer and the threshold desirable for the supply chain is smaller. In the no trade credit case the ratio between these thresholds is: $\frac{K_B^{NT}}{K_{SC}} = \frac{c}{\gamma}$, while under trade credit this ratio becomes: $\frac{K_B^{TC}(\Delta)}{K_{SC}} = \frac{ce^{-r\Delta}}{\gamma} = \frac{K_B^{NT}}{K_{SC}} e^{-r\Delta} < \frac{K_B^{NT}}{K_{SC}}$. We can thus assess the extent to which the supply chain is more coordinated. This is formalized in the following corollary.

Corollary 2: *Trade credit is a coordination device for the supply chain as it brings the optimal threshold chosen by the buyer closer to the optimal one of the supply chain.*

Proof: Directly follows from the inequalities presented above.

Trade credit has been shown to be a tool for supply chain coordination in the supply chain management literature as well, e.g., Lee and Rhee (2011). While in their framework trade credit coordinates the buyer's order quantity decision, in our model it coordinates his strategic stopping decision, thus we offer a new perspective of trade credit as a coordination device.

The second implication of Proposition 5 is that the supplier can now assess the *direct* gain implied by granting trade credit. The supplier will now trade for a longer period of time with the buyer. The buyer now stops production later with trade credit, so that the period of business increases when granting trade credit. Since $K_B^{TC}(\Delta) < K_B^{NT}$ we obtain (by virtue of Proposition 3) that $S^{TC}(c,\Delta;K_B^{TC}(\Delta)) > S^{TC}(c,\Delta;K_B^{NT})$. We call this gain of extended business from granting trade credit, $S^{TC}(c,\Delta;K^{TC}) - S^{TC}(c,\Delta;K^{NT})$, the value of flexibility.

On the other hand, granting trade credit in general also implies a cost for the supplier since the cash flow and revenue of the supplier will be delayed (and therefore decreased from a present value perspective). In general, for a naive buyer that does not internalize trade credit into his stopping decision, we have that $S^{TC}(c,\Delta,K^{NT}) < S^{NT}(c,K^{NT})$. We call this cost $S^{NT}(c,K^{NT}) - S^{TC}(c,\Delta,K^{NT})$, the cost of delay.[12] Whether granting trade credit is *finally* beneficial for the supplier depends on the trade-off between the cost of delay and the value of flexibility. We can decompose the supplier's valuation of the project with trade credit into these terms in order to illustrate this trade-off:

$$\begin{aligned} S^{TC}\left(c,\Delta;K_B^{TC}(\Delta)\right) &= S^{NT}\left(c;K_B^{NT}\right) - \underbrace{\left[S^{NT}\left(c;K_B^{NT}\right) - S^{TC}\left(c,\Delta;K_B^{NT}\right)\right]}_{\text{cost of delay}} \\ &+ \underbrace{\left[S^{TC}\left(c,\Delta;K_B^{TC}(\Delta)\right) - S^{TC}\left(c,\Delta;K_B^{NT}\right)\right]}_{\text{value of flexibility } \geq 0} \end{aligned}$$

The following proposition provides a sufficient condition for the supplier to find it optimal to extend trade credit for free. To simplify the notation, we will denote $S^{TC}\left(c,\Delta;K_B^{TC}(\Delta)\right)$ by $S^{TC}(c,\Delta)$ hereafter, and $S^{NT}(c;K_B^{NT})$ by $S^{NT}(c)$ (and similarly for the buyer B and total supply chain value V).

Proposition 6: *If* $(c-\gamma)X\left(\frac{P_0}{K_B^{NT}}\right)^{-X} > c\left(1-\left(\frac{P_0}{K_B^{NT}}\right)^{-X}\right)$*, for any cost* c*, there exists a horizon* $\widehat{\Delta} > 0$ *such that the supplier is indifferent between granting trade credit or not*

$$S^{NT}(c) = S^{TC}\left(c,\widehat{\Delta}\right)$$

and such that, for any $\Delta < \widehat{\Delta}$*, trade credit dominates*

$$S^{TC}(c,\Delta) > S^{TC}\left(c,\widehat{\Delta}\right) = S^{NT}(c)\,.$$

Proof: See Appendix.

As we explain in Appendix, this sufficient (not necessary) condition forces the supplier's val-

[12] There exist however, some extreme scenarios under unreasonable parameter values where trade credit is beneficial to the supplier even when the buyer does not internalize trade credit into his stopping decision. This is the case when the stopping threshold is triggered by the selling price during the period with no payment to the supplier. Since we assumed that the buyer only stops after time Δ, he is forced to make business until time Δ and this artificially maintains some business and revenues for the supplier at the expense of the buyer. In this case, the cost of delay is actually a value of delay for the supplier. Nevertheless, combined with the value of flexibility, this only reinforces the fact that the supplier might find it overall beneficial to grant credit for free.

uation of the project to be first increasing with respect to Δ.[13] Figure 3 illustrates the previous proposition in a typical situation that satisfies the above condition and where the supplier's valuation of the project is concave with respect to the delay Δ. Here, the supplier's valuation under trade credit is equal to her valuation in the no trade credit case for $\Delta = 0$ as expected and, thanks to the sufficient condition of Proposition 6, it then strictly increases as Δ enlarges. By definition of $\widehat{\Delta}$, we see that trade credit dominates the no trade credit arrangement when $\Delta < \widehat{\Delta}$. In that region, that is, below $\widehat{\Delta}$, the gains of extended business compensate the cost of delay implied by granting trade credit. It is therefore optimal for the supplier to grant trade credit "for free" (that is, without increasing the price c). In our illustration, the supplier will suffer from offering trade credit only if the trade credit maturity is set larger than $\widehat{\Delta}$. We can see in Figure 3 that for the chosen set of parameters the supplier's valuation of the project is hump-shaped in the trade credit delay Δ, meaning that there exists a unique optimal trade credit delay for the supplier that maximizes her profit. Note that we find such a single hump for all parameters (satisfying the sufficient condition) that we explore, but it has not been possible for us to prove uniqueness of the optimal maturity mathematically.

The above proposition provides an explanation for the puzzling recent empirical findings that challenge the common wisdom that trade credit is an expensive source of finance and that are in line with anecdotal evidence of cheap trade credit (Giannetti et al., 2011).

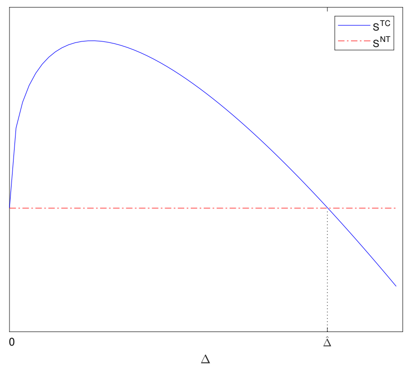


Figure 3: Supplier's valuation with $(S^{TC}(c, \Delta, K^{TC}(\Delta)))$ and without $(S^{NT}(c, K^{NT}))$ trade credit, defined in equations (7) and (2) respectively, as a function of the trade credit maturity Δ. $\hat{\Delta}$ denotes the trade credit maturity for which the supplier is indifferent between offering TC or not.

So far we have derived our results for any trade credit delay Δ. In practice however, trade credit delays are typically short. Regulation most often imposes a largest possible delay $\Delta_{\max}^{\text{reg}}$, that is in general expressed in months (say 6 months or equivalently 180 days). In the next section, we focus on the typical trade credit maturities and explore analytically some approximations of the valuation formulae for $\Delta \in [0, 0.5]$.

[13] Since this is not a necessary condition, in case $(c-\gamma)X\left(P_0/K_B^{NT}\right)^{-X} < c\left(1-\left(P_0K_B^{NT}\right)^{-X}\right)$, the supplier could still find it optimal to grant trade creditİn particular, even if $S^{TC}(c, \Delta; K)$ is first decreasing in the delay, it could still attain S^{NT} back from below at a later horizon. However, we only find this to be the case under unreasonable parameter values.

4. An analytical approximation approach for trade credit

Since trade credit terms are almost always less than 180 days (i.e., half a year) and in general less than 90 days (that is, 0.25 year) (Yang and Birge, 2018), the valuation formulae (6), (7) and (8) can be approximated by simple and intuitive expressions. This will allow us to derive further analytical approximation results useful in the following sections on trade credit design and regulation. If the delay is small, then it is very unlikely that the price has sufficient time to leave the neighbourhood of its contemporaneous level. In other words, if the price level P_0 is sufficiently far from the stopping threshold (which is at maximum K_B^{NT}) given the level of volatility, the probability of stopping at Δ is negligible in such a short period of time and probabilities such as $N\left(-d_1\left(\mu,\Delta\right)\right)$ and $N\left(-d_2\left(\mu,\Delta\right)\right)$ are practically null, whereas $N\left(d_2\left(\mu - X\sigma^2,\Delta\right)\right)$ is practically equal to one. Hence we may find some simple approximations to the valuation formulas. The following proposition summarizes our approach.

Proposition 7: *For trade credit delays $\Delta \in [0, 0.5]$ and P_0 sufficiently high compared to K_B^{NT}, the buyer and supplier values can be approximated by:*

$$B^{TC}\left(c,\Delta;K_B^{TC}\left(\Delta\right)\right) \approx b^{TC}\left(c,\Delta\right) = \left[\frac{P_0}{r-\mu} - \frac{c}{r}e^{-r\Delta}\right] - \left[\frac{K_B^{TC}\left(\Delta\right)}{r-\mu} - \frac{c}{r}e^{-r\Delta}\right]\left(\frac{P_0}{K_B^{TC}\left(\Delta\right)}\right)^{-X} \tag{12}$$

$$S^{TC}\left(c,\Delta;K_B^{TC}\left(\Delta\right)\right) \approx s^{TC}\left(c,\Delta\right) = \left[e^{-r\Delta}\frac{c}{r} - \frac{\gamma}{r}\right]\left(1 - \left(\frac{P_0}{K_B^{TC}\left(\Delta\right)}\right)^{-X}\right) \tag{13}$$

where the optimal stopping threshold for the buyer is $K_B^{TC}\left(\Delta\right)$ given by equation (11). The errors of approximation are bounded by:

$$\frac{B^{TC}\left(c,\Delta;K_B^{TC}\left(\Delta\right)\right) - b^{TC}\left(c,\Delta\right)}{b^{TC}\left(c,\Delta\right)} < \epsilon_B(\Delta), \tag{14}$$

$$\frac{S^{TC}\left(c,\Delta;K_B^{TC}\left(\Delta\right)\right) - s^{TC}\left(c,\Delta\right)}{s^{TC}\left(c,\Delta\right)} < \epsilon_S(\Delta), \tag{15}$$

where ϵ_B and ϵ_S are small and strictly decreasing functions of Δ given in Appendix.

Proof: See Appendix.

We first note that the approximated valuations are very intuitive. Comparing the approximated buyer and supplier values under trade credit with the values under no trade credit given by equations (1) and (2), we can see that extending trade credit acts as a decrease in the wholesale price. Indeed, this is how trade credit is used in practice. While direct price discrimination might be forbidden or lead to a trade war, trade credit is a less aggressive and more flexible instrument (Fabbri and Klapper, 2016). Second, the optimal stopping threshold for the buyer resulting from the approximated value coincides with the one for the exact buyer value.[14] Moreover, Proposition 7 implies that $V^{TC}\left(c,\Delta\right) \approx v^{TC}\left(c,\Delta\right) = b^{TC}\left(c,\Delta\right) + s^{TC}\left(c,\Delta\right).$

In the Appendix we formalize mathematically the condition for P_0 to be sufficiently high compared to K_B^{NT} and we link it to the degree of accuracy of the approximation, i.e., the ap-

[14] Note that the sufficient condition for the supplier to find it optimal to grant trade credit for free under the approximated value also coincides with the sufficient condition for the exact value provided in Proposition 6 (for a proof see Appendix).

proximation error. For example, for an approximation error for the buyer value lower than $N\left(-\widetilde{d}_2\left(\mu - X\sigma^2, \Delta\right)\right) = 0.00003$, we need $\widetilde{d}_2\left(\mu - X\sigma^2, \Delta\right) \geq -N^{-1}(0.00003) = 4$. The latter inequality implies a condition for P_0 to be sufficiently high compared to K_B^{NT} that is directly linked to the approximation error.

From now on, we consider these approximations. In section (7.6), we numerically evaluate the accuracy of these approximations and discuss some cases in which the probability to stop production at Δ is not negligible.

Under our analytical approximation approach we can show analytically that $\frac{\partial b^{TC}(c,\Delta)}{\partial \Delta} > 0$,[15] and hence the buyer benefits from any extension of the trade credit delay. This result has important implications for regulation imposing a limit on the maximum trade credit delay that we investigate in section 6.

5. Designing trade credit terms

The trade credit terms will be designed taking both parties' interests into account, their willingness to cooperate and their respective bargaining power. We will analyze three different possibilities: a cooperative solution corresponding to a vertically integrated supply chain, a non-cooperative Nash bargaining game, and a non-cooperative Stackelberg equilibrium in which the buyer acts as the leader and the supplier as a follower.

5.1. Cooperative solution

One possibility is the cooperative solution permitted by the vertical integration of the supply chain. The two parties join their force and find the pair (c, Δ) that maximizes $v^{TC}(c, \Delta)$, the total supply chain value, that is, the sum $s^{TC}(c, \Delta) + b^{TC}(c, \Delta)$. This situation leads to the following maximization problem:

$$\max_{c,\Delta} v^{TC}(c, \Delta) \tag{16}$$

In general a supply chain is not coordinated, and thus its value is not maximized, because the buyer stops too early compared to the supply chain optimal. Coordination in the supply chain with trade credit can be obtained by setting (c, Δ) such that the buyer decides to optimally stop production at the supply chain optimal stopping trigger. That is, (c, Δ) have to be set such that $K_B^{TC}(c, \Delta) = K_{SC}^{TC}$. This condition implies that the pair (c, Δ) has to be such that $ce^{-r\Delta} = \gamma$. For a given c, we have $\Delta^*(c) = 1/r\ln(c/\gamma)$. There exists an infinite number of pairs (c, Δ) that satisfy this relationship, and that thus maximize the total supply chain value. In particular, $c = \gamma$ and $\Delta = 0$ corresponds to the coordination mechanism in the no trade credit case which is setting the wholesale price equal to the production cost. As in the no trade credit case, the cooperative solution implies that the buyer captures all profits and the supplier value is equal to zero. Comparing to the no trade credit case, we note that the supply chain value is larger under trade credit: $v^{TC}(c, \Delta^*(c)) \geq V^{NT}(c)$.[16] Indeed, the two values are only equal for $c = \gamma$. For any $c > \gamma$ the supply chain in the no trade credit case is not coordinated, thus the total supply chain

[15] To see this note that $\frac{\partial b^{TC}(c,\Delta)}{\partial \Delta} = ce^{-r\Delta}\left[1 - e^{-rX\Delta}\left(\frac{P_0}{K_B^{NT}}\right)^{-X}\right] > 0$.

[16] To see that for any $c > \gamma$ we have that $v^{TC}(c, \Delta^*(c)) > V^{NT}(c)$ first note that when $c = \gamma$, the optimal Δ is $\Delta^*(\gamma) = 0$. Then we have that $v^{TC}(c, \Delta^*(c)) = v^{TC}(\gamma, \Delta^*(\gamma)) = v^{TC}(\gamma, 0) = V^{NT}(\gamma) > V^{NT}(c)$.

value is lower than its maximum possible value. On the contrary, introducing trade credit adds extra flexibility in setting the contractual terms, which makes is possible to coordinate the supply chain for any wholesale price, c. This further shows that trade credit is a coordination device.

Recent empirical evidence shows that buyers prefer external suppliers to internal procurement (pure production arguments: weak economy of scale, financial arguments: conglomerate discount) and only after a change in regulation they internalize procurement to their own subsidiaries (Breza and Liberman, 2017). Therefore, we assume that internal suppliers are less efficient than external suppliers. Formally $\gamma_i > \gamma_e$, where the subindices i and e denote internal and external suppliers, respectively.

5.2. Nash bargaining game

When the buyer trades with an external supplier, the above cooperative solution will hardly be possible. The outcome will instead be the solution of a non-cooperative bargaining game. This outcome depends on the parties' threat points and on their relative bargaining power. The threat points reflect the status quo between the negotiating parties. This status quo represents the utility gained by the parties if the bargaining breaks down (Binmore et al., 1986). The bargaining power of each party will depend on their market power, size and the competition they are facing (Fabbri and Klapper, 2016; Klapper et al., 2012). Let η denote the buyer's bargaining power and $1-\eta$, the one of the supplier. Let Γ_S and Γ_B be the status quo of the supplier and the buyer respectively. In case the bargaining breaks down, the buyer might internalize procurement to its own subsidiaries, while the supplier might lose the business if she does not grant trade credit (her status quo could then be lower than her no trade credit valuation).

The buyer obtains $b^{TC} = \theta v^{TC}$ in negotiation, while the supplier gets $s^{TC} = (1-\theta)v^{TC}$, where θ is a parameter that reflects the sharing rule. The Nash solution is characterized by:

$$\begin{aligned} \theta^* &= \underset{\theta}{\operatorname{argmax}} \{\theta v^{TC}(c,\Delta) - \Gamma_B\}^{\eta}\{(1-\theta)v^{TC}(c,\Delta) - \Gamma_S\}^{1-\eta} \\ \theta^* &= \eta - \frac{\eta\Gamma_S - (1-\eta)\Gamma_B}{v(c,\Delta)} \end{aligned}$$

Thus, the buyer's stake is worth $\theta^* v(c,\Delta) = \theta^*[b(c,\Delta) + s(c,\Delta)]$. But the buyer' stake is also given by $b(c,\Delta)$. Equating the two expressions, $\theta^*[b(c,\Delta) + s(c,\Delta)] = b(c,\Delta)$, we get the Nash equilibrium relationship between the price c and the trade credit maturity Δ:

$$\eta s^{TC}(c,\Delta) - (1-\eta)b^{TC}(c,\Delta) = \eta\Gamma_S - (1-\eta)\Gamma_B \tag{17}$$

This equation defines $ce^{-r\Delta}$ as an implicit function of the bargaining power η. There is an infinite number of pairs (c,Δ) satisfying this relationship. In particular, for $\eta = 1$ and the supplier's status quo $\Gamma_S = 0$, this equation becomes $s^{TC}(c,\Delta) = 0$. From equation 13 we then have that $ce^{-r\Delta} = \gamma$. In this case, the optimal pair (c^*,Δ^*) resulting from the Nash bargaining game coincides with the cooperative solution that maximize the total supply chain value. Indeed, the buyer in this case has all bargaining power and the supplier's status quo is zero. However, in general, for $\eta < 1$ or $\Gamma_S > 0$, a non-cooperative bargaining game will be a second best solution, i.e., in general it leads to a lower total value of the supply chain than a cooperative one.[17] Nevertheless, the Nash

[17]Since $s^{TC}(c,\Delta) > 0$ in this case, this implies that $ce^{-r\Delta} > \gamma$, so that $K_B^{TC}(\Delta) > K_{SC}^{TC}$.

bargaining solution with trade credit improves the supply chain value compared to the no trade credit case. Granting trade credit acts as a decrease in the wholesale price, which implies that the stopping threshold is delayed and gets closer to the optimal one.

Although the Nash bargaining solution is second best, since the external supplier is more efficient compared to the internal supplier, $\gamma_i > \gamma_e$, as we have previously assumed, a supply chain with external procurement under non-cooperative Nash bargaining could achieve higher profits than with internal procurement under a cooperative solution.

5.3. Stackelberg equilibrium

Finally, we analyze the non-cooperative Stackelberg equilibrium in which the buyer acts as the leader and the supplier as the follower.[18] Our choice is motivated by the fact that we are trying to explain the puzzling relationship where large retailers finance themselves off the back of small, weaker supplier evidenced in the recent trade credit empirical literature (Fabbri and Klapper, 2016; Klapper et al., 2012; Murfin and Njoroge, 2015). Although the buyer has the interest of paying the lowest possible price c in every period to the supplier, he has to consider the possibility that a too low price may become unattractive for the supplier, given her status quo, Γ_S.

The optimal pair (c, Δ) is obtained from the following maximization problem:

$$\begin{aligned} & \max_{c,\Delta} b^{TC}(c,\Delta) \\ s.t. \quad & s^{TC}(c,\Delta) \geq \Gamma_S \end{aligned}$$

The binding participation constraint of the supplier defines $ce^{-r\Delta}$ as an implicit function of the supplier's status quo, Γ_S. Note that the Stackelberg solution coincides with the Nash bargaining solution for the polar case $\eta = 1$.

The buyer will procure from the external supplier under trade credit as long as the profits obtained under the Stackelberg equilibrium exceed his status quo, Γ_B. Since empirical evidence suggests that the buyer's status quo is to internalize procurement, we assume that $\Gamma_B = b^{TC}(c_i, \Delta_i)$, the buyer's profits under internal procurement.

6. The effects of regulation

We now turn our attention to the social planner's perspective and to regulation. From the social planner's perspective, the optimal pair (c, Δ) is chosen taking into account both the buyer and the supplier's valuations of the project. In case buyers abuse their bargaining power to extract margins from suppliers, the social planner might want to protect suppliers, by limiting Δ. Indeed, as previously discussed, regulating trade credit maturity is currently the most popular approach. In an interview to the Financial Times, Christine Lagarde was suggesting "reduced payment term" to improve the cash flow of small companies.[19] By imposing an upper limit on the trade credit

[18] This is in contrast with the manufacturer Stackelberg supply chain strategy, where the manufacturer is the leader and the retailer is the follower (see Baron et al., 2016). If the manufacturer were the Stackelberg leader, he would maximize his value subject to the buyer value being larger or equal than his status quo Γ_B. This participation constraint would be binding so that $b^{TC}(c,\Delta) = \Gamma_B$. Note that this case coincides with the Nash bargaining solution for the polar case of $\eta = 0$.

[19] "FT interview transcript: Christine Lagarde", The Financial Times, 12 May 2008.

maturity, Δ_n, the social planner is trying to increase the supplier's bargaining power. Such regulation implicitly assumes that the supplier's valuation of the project is a decreasing function of Δ. However, we have seen that this is not necessarily the case. Therefore, regulation might have undesired consequences. We now discuss the implications of such a measure considering the case of external procurement with weak suppliers (Stackelberg game).

Let $(c_{init}, \Delta_{init})$ be the solution to the Stackelberg game, where $\Delta_{init} > \Delta_n$. Once the regulation is implemented, the buyer will have to solve a constrained maximization problem:

$$\max_{c,\Delta} b^{TC}(c, \Delta)$$
$$s.t. \quad \begin{cases} s^{TC}(c, \Delta) \geq \Gamma_S \\ \Delta \leq \Delta_n \end{cases}$$

Since $\partial b / \partial \Delta > 0$, the constraint on trade credit maturity imposed by regulation will be binding, the buyer now chooses the maximum Δ allowed by regulation, $\Delta = \Delta_n$, decreasing his optimal maturity until the limit allowed by regulation. We consider two extreme cases: 1) price discrimination is forbidden by law as it is the case in the U.S. through the Robinson-Patman Act, so that suppliers cannot offer different prices to different customers, 2) full price discrimination is permitted. Real practice is probably in between the two extremes considered, with some degree of price discrimination taking place.

We consider first the case where price discrimination is not allowed, i.e., suppliers cannot adjust wholesale prices following a change in regulation. Assuming that supplier value is decreasing in Δ, and no change in the initial wholesale price, the effect of such a policy would be to increase the value of the supplier and decrease the value of the buyer. However, if the buyer's value falls below his status quo, $b^{TC}(c_{init}, \Delta_n) < \Gamma_B$, he might prefer to switch to internal procurement. Therefore, trade credit would decrease as a result of regulation.

When full price discrimination is possible, since there exists an infinite number of pairs (c, Δ) that maximize v, parties can undo regulation. To avoid losing the business with the buyer, the supplier will accept a price reduction as a complementary strategy to a decrease in the trade credit maturity (see Fabbri and Klapper, 2016 for evidence of such complementarity). The new price c is chosen by the buyer in the constrained maximization problem such that the supplier remains indifferent at her status quo Γ_S. Therefore, a wholesale price reduction is observed as a result of regulation.

The above discussion is summarized in the following proposition:

Proposition 8: *a) If price discrimination is forbidden, regulation cannot be undone and trade credit usage might decrease.*

- *Consider $s^{TC}(c_{init}, \Delta_{init}) = \Gamma_S$, and then a regulation shock Δ_n. Following the shock we have that $s^{TC}(c_{init}, \Delta_n) > s^{TC}(c_{init}, \Delta_{init})$ and $b^{TC}(c_{init}, \Delta_n) < b^{TC}(c_{init}, \Delta_{init})$. The new buyer value could decrease below the buyer's status quo, Γ_B, causing the buyer to switch to internal production, leading to a decrease in trade credit usage.*

b)If full price discrimination is possible, regulation can be undone through a price decrease.

- *The supply chain will decrease the wholesale price to c_n, so that $s^{TC}(c_n, \Delta_n) = s^{TC}(c_{init}, \Delta_{init}) = \Gamma_S$ and $b^{TC}(c_n, \Delta_n) = b^{TC}(c_{init}, \Delta_{init})$.*

Proof: See Appendix.

Corollary 3: *The model predicts that following a regulation that imposes an upper limit on trade credit maturity, trade credit will decrease and internal procurement will increase.*

Proof: See Appendix.

These predictions are in line with the empirical findings of Breza and Liberman (2017). When analyzing a change in regulation regarding Chilean supermarkets and their suppliers that limits the delay period from 90 to 30 days, they find that trade credit decreases by 11% and vertical integration is more likely, that is, the superstore procures from a wholly owned subsidiary. Nevertheless, they show that this is costly since the superstore is not able to replicate the pre-regulation market equilibrium. Moreover, they show that suppliers also adjust through prices: procurement prices decrease by 3.8% for treated firms relative to control firms in their sample. These findings are consistent with a scenario with a certain degree of price discrimination, in between the two extreme cases analyzed above.

We end this discussion with the following remark. In the discussion above, we first focused on the case in which supplier value is decreasing in Δ, as implicitly assumed by the social planner. However, we have seen that this is not necessarily the case (see Figure 3). If this is not the case, then such a policy, assuming no change in the initial wholesale price, would not only decrease the buyer's value, but also the supplier's value. In this case, the consequences of regulation would be counterproductive.

7. Numerical simulations

We now illustrate our model using numerical simulations. The parameter values are set as follows: $r = 0.1$, $\mu = 0$, $\sigma = 0.1$, $P_0 = 5$, $\gamma = 2$. These values are standard in the real options literature in corporate finance and operational research. For example, $r = 0.1$ and $\sigma = 0.1$ have been used in Hagspiel et al. (2016).

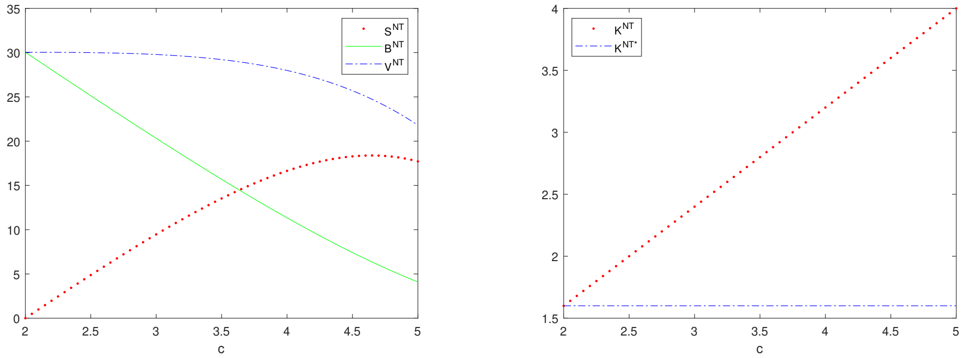
7.1. The no trade credit case

Figure 4 plots in panel a) S^{NT}, B^{NT}, and V^{NT} as a function of the wholesale price $c \geq \gamma$.[20] We can see that the total supply chain value, V^{NT}, decreases with c and is maximized for $c = \gamma$. This is precisely one mechanism to coordinate the supply chain, setting the wholesale price equal to the production cost. In this case the supplier value is equal to zero, $S^{NT} = 0$ (see equation (2)), the buyer capturing all the profits. Therefore, the total supply chain value is simply equal to the buyer's value, $V^{NT} = B^{NT}$, for $c = \gamma = 2$ as can be seen in panel a). Moreover, for $c = \gamma = 2$, the buyer chooses the optimal stopping threshold that maximizes the value of the total supply chain, $K^{NT} = K^{NT}_{SC}$, as shown in panel b), so that the supply chain is coordinated. For other values of the wholesale price the supply chain is not coordinated, and the buyer chooses to stop earlier than optimal, $K^{NT} > K^{NT}_{SC}$, as we can see in panel b) of the same figure.

7.2. The trade credit case

We now consider the trade credit case. Figure 5 panel a) plots the supplier value under both trade credit (S^{TC}) and no trade credit (S^{NT}) as a function of the trade credit period, Δ, for a wholesale price $c = 4.8$. For $\Delta = 0$, the two values coincide as expected. We note that the supplier

[20] The wholesale price c has to be larger than the production cost γ, since otherwise the supplier would not produce. Therefore, the lower bound of c is γ set to a value of 2.



a) Values

b) Stopping thresholds

Figure 4: The no trade credit case. Panel a) presents the buyer ($B^{NT}(c, K^{NT})$), supplier ($S^{NT}(c, K^{NT})$) and total supply chain ($V^{NT}(c, K^{NT}) = B^{NT}(c, K^{NT}) + S^{NT}(c, K^{NT})$) valuations under no trade credit, as defined in equations (1), (2) and (3) respectively, as a function of the wholesale price c. Panel b) depicts the stopping threshold chosen by the buyer K^{NT}, as defined in equation (5), and the optimal stopping threshold that maximizes supply chain value K^{NT*}. In both panels, the lower bound of the wholesale price c is the production cost $\gamma = 8$. For $c = \gamma = 8$ the supply chain is coordinated, the supplier's value is equal to zero ($S^{NT} = 0$), the buyer capturing all profits ($V^{NT} = B^{NT}$) and choosing the optimal stopping threshold ($K^{NT} = K^{NT*}$). The parameter values are set as in the benchmark case: $r = 0.1$, $\mu = 0$, $\sigma = 0.1$, $P_0 = 5$,$\gamma = 2$.

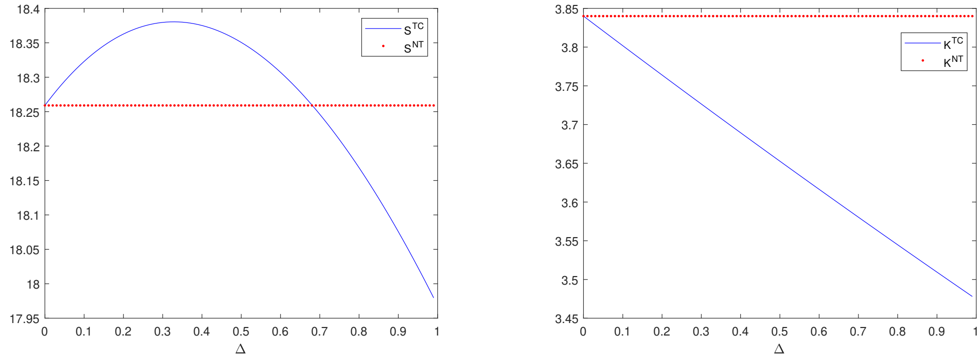
value under trade credit is a hump-shape function of Δ. Although offering trade credit implies a cost of delay for the supplier, it also implies an advantage due to the added value of flexibility. The buyer now stops later than in the case without trade credit, as we can see in panel b). Since the threshold is lower, this shows that trade credit is a coordination technique as we are getting closer to the supply chain optimal threshold K_{SC}. For $\Delta \leq 0.68$, we have that $S^{TC} \geq S^{NT}$, thus the value of flexibility dominates the cost of delay implied by trade credit. In this case it is optimal for the supplier to offer trade credit for free.

7.3. Designing trade credit terms

We analyze the case of external procurement with non-cooperative Nash bargaining. The optimal pair (c, Δ) is determined by the bargaining power of the two parties, η (buyer) and $1 - \eta$ (supplier). This pair exactly splits the supply chain value according to η and satisfies equation (17). In Table 1 we illustrate these pairs for a buyer bargaining power $\eta = 0.25$ in Panel A and for $\eta = 0.75$ in Panel B. We consider Δ between 0 and 180 days, the most common trade credit maturities used in practice, and compute the corresponding optimal wholesale price c that maximizes V and splits it according to the bargaining power of the parties (see equation (17)).[21] The values for the buyer, supplier and total supply chain are also presented.[22] A decrease of 36 days in the trade credit maturity from 72 to 36 days is equivalent to a price discount of 1.05% for $\eta = 0.25$.

[21] In section 5 we derived our results based on the approximated value functions for tractability. Here we present numerical results for the exact value functions.

[22] We assume that the status quo of the two parties is zero, $\Gamma_S = \Gamma_B = 0$.



a) Supplier value   b) Stopping threshold

Figure 5: The trade credit case. Panel a) presents the supplier value with ($S^{TC}(c, \Delta, K^{TC}(\Delta))$) and without ($S^{NT}(c, K^{NT})$) trade credit, as defined in equations (7) and (2) respectively, as a function of the trade credit maturity Δ. Panel b) depicts the stopping threshold with (K^{TC}) and without (K^{NT}) trade credit, as defined in equations (11) and (5) respectively. When the trade credit maturity $\Delta = 0$, the trade credit and no trade credit cases coincide. The parameter values are set as in the benchmark case: $r = 0.1$, $\mu = 0$, $\sigma = 0.1$, $P_0 = 5$, $\gamma = 2$. The wholesale price is set to $c = 4.8 > \gamma = 2$.

7.4. The effects of regulation

Let us now illustrate the effects of regulation numerically, assuming that the trade credit maturity is decreased from 90 to 30 days. This corresponds to the change in regulation affecting Chilean supermarkets analyzed by Breza and Liberman (2017). Thus, we assume that the optimal initial trade credit maturity was set to $\Delta_{init} = 0.25$ years, that is, a maturity of 90 days. Assuming a Stackelberg game with the buyer being the leader and a supplier status quo of $\Gamma_S = 5$, the corresponding optimal wholesale price is $c = 2.5771$, for which buyer value is equal to $B^{TC}(c_{init}, \Delta_{init}) = 24.9972$.[23] Then the initial optimal trade credit terms are $(c_{init}, \Delta_{init}) = (2.5771, 0.25)$ with corresponding initial values of the supplier and buyer are $S^{TC}_{init} = 5$ and $B^{TC}_{init} = 24.9972$ respectively. Assume that now by regulation we have $\Delta_n = 0.0833$ years (30 days). For this reduced trade credit maturity, and assuming no change in the initial wholesale price, the new values of the buyer and supplier are: $B^{TC}(c_{init}, \Delta_n) = 24.5857 < B^{TC}_{init}$ and $S^{TC}(c_{init}, \Delta_n) = 5.4018 > S^{TC}_{init}$.[24] Thus in principle, the regulator seems to achieve its purpose, increasing the value of the supplier at the expense of the buyer. Nevertheless, if the buyer's new reduced value falls below his status quo, the buyer could stop the relationship with the supplier and switch to internal procurement.[25] Therefore, the change in regulation could lead to a decrease in trade credit if price discrimination is forbidden and no adjustment in the wholesale

[23]The optimal pair of trade credit terms maximizes the total supply chain value subject to the supplier being indifferent at his reservation price, $S^{TC}(c_{init}, \Delta_{init}) = \Gamma_S = 5$ (see the maximization problem given in equation (18)). For a given assumed optimal $\Delta = 0.25$, the optimal wholesale price is found. This then allows us to compute the buyer's value.

[24]Note that for these parameter values, supplier's value is decreasing with respect to maturity. Therefore, a decrease in maturity imposed by the regulator leads to an increase in the supplier's value, assuming c is unchanged.

[25]This would be the case for example if the status quo of the buyer were $\Gamma_B = 24.8$, which corresponds to a production cost for internal procurement $\gamma_i = 2.5336 > \gamma_e = 2$, the internal supplier being less efficient than the external one.

Table 1: Design of trade credit terms under Nash bargaining. The table presents for different trade credit maturities Δ (varying from 0 to 180 days, the most common trade credit maturity in practice), the optimal wholesale price c (bounded below by the production cost $\gamma = 8$) maximizing the value of the supply chain V. The buyer bargaining power η corresponding to the different trade credit terms pairs (extracted from equation 17) is presented in column 3. The corresponding values of the supplier (S^{TC}), buyer (B^{TC}) and the total supply chain ($V^{TC} = B^{TC} + S^{TC}$) under trade credit, as well as the total supply chain under no trade credit (V^{NT}), as defined in equations (7), (6), (8) and (3) respectively, are presented in columns 4-7.

Panel A. $\eta = 0.25$					
c	Δ	S^{TC}	B^{TC}	V^{TC}	V^{NT}
4.68	0 years (0 days)	18.3735	6.1245	24.4980	24.4980
4.73	0.1 years (36 days)	18.3735	6.1245	24.4980	24.1500
4.78	0.2 years (72 days)	18.3735	6.1245	24.4980	23.7814
4.83	0.3 years (108 days)	18.3735	6.1245	24.4980	23.3912
4.87	0.4 years (144 days)	18.3735	6.1245	24.4980	22.9780
4.92	0.5 years (180 days)	18.3735	6.1245	24.4980	22.5407
Panel B. $\eta = 0.75$					
c	Δ	S^{TC}	B^{TC}	V^{TC}	V^{NT}
2.78	0 years (0 days)	7.4783	22.4348	29.9131	29.9131
2.81	0.1 years (36 days)	7.4783	22.4348	29.9131	29.9005
2.83	0.2 years (72 days)	7.4783	22.4348	29.9131	29.8869
2.86	0.3 years (108 days)	7.4783	22.4348	29.9131	29.8722
2.89	0.4 years (144 days)	7.4783	22.4348	29.9131	29.8565
2.92	0.5 years (180 days)	7.4783	22.4348	29.9131	29.8395

price is possible following the change in regulation.

On the contrary, if the parties have full flexibility to adjust the wholesale price in response to a limit on trade credit maturity, then, in order not to lose the business with the buyer, the supplier will agree to decrease the wholesale price to compensate him for the decrease in trade credit maturity. Since the buyer is the Stackelberg leader, he will now solve a constrained maximization problem given by equation (18). The new wholesale price is set to maintain the value of the supplier equal at her status quo, $\Gamma_S = 5$, given the new constrained trade credit maturity $\Delta_n = 0.0833$. Since the supplier benefits from the imposed reduction in maturity, a reduction in the wholesale price is needed to maintain the supplier's value constant at the status quo. The new wholesale price will be $c_n = 2.5345 < c_{init}$. A reduction of 60 days in the wholesale price will thus lead to a decrease of 1.65% in the wholesale price. The buyer's new value will be unchanged $B^{TC}(c_n, \Delta_n) = B^{TC}(c_{init}, \Delta_{init}) = 24.9972$.

Comparing to the reduction in wholesale prices documented by Breza and Liberman (2017) for the case of Chilean supermarkets, 3.8%, the reduction in our numerical example is of similar magnitude, although smaller. The limited impact can be due to the fact that we model the supply chain using perpetual cash flows. In practice, trade credit relationships have an average duration of 8 years (Murfin and Njoroge, 2015). As we argue in the conclusions, our model is a first attempt to explain recent empirical findings in trade credit, and we have modeled it in infinite time for tractability. Future research could try to address this limitation, considering trade credit in finite horizon.

7.5. Sensitivity analysis

Finally, we analyze the sensitivity of the model with respect to the parameter values. For this, we will reproduce Figures 4 and 5 presenting the buyer and supplier valuations, as well as the stopping thresholds, with and without trade credit, for different values of each of the parameters of the model: volatility, σ, drift, μ, risk-free rate, r, retail price, P, and production cost, γ. The results are provided in the Appendix.

We present sensitivity results with respect to volatility in Figure B.1 (see Appendix). We can see in panels a) and b) respectively that the value of the supplier under no trade credit is decreasing in volatility, while the one of the buyer is increasing in volatility. The same holds under trade credit in panels c) and d). Buyer value includes an option to stop production which will be more valuable when uncertainty is higher. Thus, buyer value increases with volatility. Moreover, as standard in real options, we observe in panels e) and f) that the larger the volatility, the lower and more distant the stopping threshold. However, under higher volatility there is a higher likelihood that this lower threshold is reached and that the buyer ends the relationship with the supplier. Therefore, a larger volatility results in a lower supplier value. Finally, in panel c) we also observe that the larger the volatility, the lower the threshold $\hat{\Delta}$ at which the supplier is indifferent between granting trade credit or not. In particular, for $\sigma_3 = 0.15$ we have that $S^{TC} < S^{NT}$, for all values of Δ. While for $\sigma_1 = 0.05$ and $\sigma_2 = 0.10$ the sufficient condition for the supplier to find it optimal to offer trade credit for free is satisfied (see Proposition 6), for $\sigma_3 = 0.15$, this condition is no longer satisfied. This implies that for the latter the cost of delay implied by trade credit is larger than its value of flexibility.[26]

In Figure B.2 we present the sensitivity tests with respect to the drift rate μ. We note in panels a)-d) that the values of the buyer and supplier both with and without trade credit increase with the drift. Moreover, a larger drift leads to a lower stopping threshold (panels e) and f)). Intuitively, a larger rate of growth for the retail price implies higher revenues for the buyer, increasing his value. Moreover, it also leads to a lower likelihood of reaching the stopping threshold, which implies that the supplier enjoys a longer business relationship with the buyer, thus increasing supplier value. We observe in panel c) that S^{TC} is hump-shaped only for $\mu = 0$, while decreasing in Δ for higher drift rates. The sufficient condition for the supplier to find it optimal to offer trade credit for free provided in Proposition 6 is not satisfied under the latter parameter values. Hence, the larger the drift, the fewer incentives the supplier has to grant trade credit. Indeed, since a larger drift rate already results in an extended business with the buyer, the additional gain of extended business that could be obtained through granting trade credit is not as valuable.

Regarding the risk-free rate, we observe the opposite behavior in Figure B.3: the values of the buyer and supplier are decreasing in r, while the stopping thresholds are increasing in r. An increase in the retail price, P_0, naturally increases the values of the buyer and the supplier (Figure B.4), while the stopping thresholds do not depend on P_0 (thus figures are not provided). Finally, the lower the production cost γ, the larger the supplier's value, and the lower the stopping threshold that maximizes the supply chain (Figure B.5).

[26]Note that this does not necessarily mean that the supplier will not provide trade credit. Indeed, if she does not provide credit she might lose the business with the buyer. Therefore, her status quo Γ_S could be lower than S^{NT}, so that she still finds it optimal to provide some credit.

7.6. Discussion of results

In this section we evaluate the accuracy of the approximation made in equations (12-13) and discuss the case in which there is a positive probability of stopping at Δ and its implications on our results. Table 2 presents the actual and the approximated supplier values, S^{TC} given by equations (7) and (13) respectively for different combinations of parameter values. These results confirm the near perfect accuracy of our results especially for a trade credit maturity of 0.25, that is, 90 days. The accuracy slightly decreases when the likelihood of stopping at Δ is larger, e.g., for very low P_0 close to the stopping threshold, for high volatility, or for very large credit durations of 180 days.

Table 2: Comparison of approximated and actual supplier values. The table presents for different combinations of volatilies σ, final good prices P_0 and trade credit maturities Δ, the actual and the approximated supplier values, S^{TC} and s^TC given by equations (7) and (13) respectively. The rest of the parameter values are set as in the benchmark case: $\mu = 0$, $r = 0.1$, $\gamma = 2$, $c = 4$.

σ	P	Δ	S^{TC}	s^{TC}
0.1	5	0.25	16.1262	16.1262
0.1	5	0.5	15.5699	15.5699
0.1	3.5	0.25	7.0075	6.9915
0.1	3.5	0.5	7.7770	7.7233
0.5	5	0.25	9.1134	9.1133
0.5	5	0.5	9.4754	9.4754
0.5	3.5	0.25	7.5127	7.5127
0.5	3.5	0.5	7.2791	7.2743
0.8	5	0.25	7.0628	7.0628
0.8	5	0.5	6.7765	6.7757
0.8	3.5	0.25	5.9484	5.9483
0.8	3.5	0.5	5.7309	5.7243

A first implication of a positive probability of stopping at Δ on our results is that the cost of delay implied by trade credit can become a value of delay, as previously explained. For a high probability of reaching the stopping threshold within the first delta-long period (such as P_0 very low and close to K or extremely high volatility σ), although the supplier receives the payment delayed, she can lock in a minimum period of business, as the buyer does not stop the relationship before Δ. In that case, trade credit is strictly preferred by the supplier since 1) it guarantees a minimum period of business Δ (value of delay), and 2) it implies an extended period of business through the decrease in the stopping threshold (value of flexibility). A positive probability of stopping at Δ also has an implication on our results regarding the design of trade credit and regulation. In this case, the two credit terms, maturity Δ and wholesale price c, are no longer perfect complements, since Δ plays a role in the value functions independently of c, as can be seen in equations (6-7). This implies that in the case of regulation, even with full price discrimination allowed, parties cannot perfectly undo regulation. Assuming that the parties initially chose the optimal pair of credit terms that maximizes supply chain value and splits it according to their bargaining power, following a decrease in Δ imposed by regulation the parties will decrease c so as to mantain the supplier indifferent at his status quo. However, the new pair (c_n, Δ_n) leads to a lower supply chain value. Since supplier value is maintained constant, this implies a decrease in buyer value with a possible decrease in trade credit if buyer value falls below its status quo.

8. Conclusions

In this paper we provide a real options analysis of trade credit design. Our model offers a first theoretical explanation for the recent empirical findings regarding trade credit from small suppliers to large buyers and the effects of regulation. It illustrates the impact of granting trade credit with a focus on the terms of trade credit, without excluding other important factors that might affect the results. We show that suppliers might find it optimal to offer trade credit at zero cost. Moreover, trade credit acts as a coordination device for the supply chain. Regarding regulation, we show that following regulation that imposes a limit on trade credit maturity, wholesale prices are reduced, trade credit decreases and internal procurement increases. Our implications are in line with recent empirical evidence.

There are several implicit assumptions behind our framework. First, in deriving our results we have assumed a perpetual demand. In practice however, the average length of trade credit contracts is of 8 years (Murfin and Njoroge, 2015). An interesting avenue for future research would be to model trade credit within a finite time framework. This could potentially lead to obtaining larger impacts of regulation on the values of the buyer and the supplier. Second, we have assumed that the supplier's cash flows and production costs are deterministic. Future research might consider introducing two sources of risk, one for the supplier and one for the buyer, although at the cost of losing tractability. Third, we have abstracted from default risk. We know nevertheless that financial risk is an important element of trade credit contracts. Klapper et al. (2012) suggested that the financial risk matters and that this issue may impact the trade credit design. For trade credit relationships with large buyers and small suppliers, the buyer might take into account the supplier's default risk when setting the contractual terms. This is left for future research.

From a policy perspective, our model highlights that offering trade credit has not only costs, but also benefits. In particular, trade credit can be favorable for the firm extending it since it enables and guarantees trade. Thus, policy makers considering a change in regulation affecting trade credit should consider both its benefits and costs in their analysis.

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