FPAT – A Framework for Facilitating the Teaching and Learning Using Fermi Problem Originating in Mathematics Education Research

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Enrico Fermi is remembered for many contributions to theoretical and experimental physics, but from an educational point of view, he also importantly contributed to popularize the use of the kind of questions we today call *Fermi problems* (also known as *Fermi questions*[1]). Fermi problems (FPs) are back-of-envelope problems that arose from the need to make order of magnitude calculations and Fermi himself used them in his research and physics classes. The procedure proposed by Fermi was to decompose the original problem into simpler sub-problems, and to solve these by making reasonable estimates and educated guesses, to reach a solution to the original question. In the literature this way of working is known as the *Fermi (estimates) method*.

Although originating in physics, the use of FPs have spread to different subjects and contexts with multiple purposes as well as to various educational levels.[2] However, educational research focusing on FPs have been relatively scarce. In this article we will highlight some of the recent research developments on the use of FPs in Mathematics education. Our aim is to showcase a framework for facilitating the use of FPs with students, and also to call for further research focusing on the educational use of FPs within as well across subject disciplines.

Fermi Problems in Physics Education

We have no ambition to provide a comprehensive review of the literature on FPs in physics education (for a general review see[2]), but rather to highlight some of the research findings. For example, studying the correlation between first year university students learning the Fermi estimates method and their examination results, Robinson showed that students successfully can develop this problem-solving competence to effectively solve FPs, but that this however does not come easy for all students.[3] Barahmeh et al. showed this also in the context of teaching 9th grade students physics and illustrate how the students' skills in making measurement predictions, taking measurements, and making productive use of measurements improved[4]. Both Robinson and Barahmeh et al. stress the value of the Fermi method for allowing students to think freely and without solemnly no relying on predetermined formulas for solving the tasks, and for promoting discussions of different solutions. Cordry on the other hand used the Fermi estimation method to raise critical environmental issues concerning the world population and the limited resources of planet earth. Students were generally surprised by the results of engaging in this problem—especially when they realize that the present global human population exceeds what they just calculated is feasibly sustainable.[5] The situation that arose allowed for a discussion about limitations of the method, such as that some quantities are difficult to estimate and that the values of other quantities are inferred by old habits or are taken for granted. Similar results of raising critical awareness about ones' beliefs and understandings are presented by Morgan, who used FPs based on the Drake equation and a pre-/post-test strategy to show the change of students' opinions and beliefs about the existence of extraterrestrial life.[6] Together these studies point to the potential and use of FPs to contrast and discuss different solutions and opinions, and to impact beliefs on other important issues, such as climate change.

Fermi Problems in Mathematics Education

There has been a recent line of research in mathematical education exploring the possibilities of using FPs in the classroom in the context of mathematical modelling. Mathematical modelling activities are in many respects similar to the practices of scientific research, but the focus is on the mathematical nature of the models used and developed and not on the phenomenon studied per se. However, many of the findings from this research are arguably transferable to the classrooms of other sciences if adapted and interpreted adequately with respect to the discipline in question. Importantly, FPs can be used for different pedagogical purposes at practically all educational levels. For early primary students, they can be used as a basis for classroom projects such as devising a strategy to compare the populations of a city and a town.[7] With younger students, it might be necessary for the teacher provide (part of) an outline of the necessary estimates needed. For older primary students however, Peter-Koop found that students are fully capable of generating their own mathematical models when for example estimating the number of cars in a motorway traffic jam. Indeed, the students solved the problem in a myriad of ways while developing for them new mathematical knowledge in order to work out their solutions.[8] A positive effect on the development of primary students' general modelling skills have also been shown, as FPs facilitate awareness of the different phases of problem solving in context as well as the development of the modelling sub-competences of simplifying, mathematising, interpreting and explaining real phenomena.[9]

At the lower secondary level, some students can develop considerably more complex models than primary students, [10] whereas upper secondary students generally can use their more extensive knowledge of the real world to generate even more sophisticated models.[11] One of the difficulty students are found to have is however determining key variables of a problem. Here, engaging student in collaborative work or in carefully designed sequences of problems using different real contexts have proven to promote the development and adaptation of increasingly more complex models.[12] In the latter approach, the key has been to base the sequence on a particular concept such as density for example, and propose several problems where density as an idea is central, and let the students develop their own models. In addition, working with sequences of problems, secondary students develop their flexibility as problem solvers and their ability to choose and use appropriate strategies in different problem contexts.[13]

At the college level, FPs can have a more specific role and support students in developing more specific modelling competences. However, even for university students the modelling needed to solve a FP is complex, requiring them to regulate their modelling processes by monitoring how their immediate goals or subgoals align and relate to the problem statement.[14] In addition, FPs also have been seen to promote students in validating their problem solving processes and results in modelling activities.[15]

Recent Developments: Fermi Problem Activity Diagram

Focusing on the research in mathematics education and the other STEM disciplines ,our review of the literature on FPs[1] has allowed us to identify four types of mathematical activities that are used to achieve the numerical values needed of quantities to be able to provide a solution and answer to the problem in question. These four activities are: 1) *guesstimation* (providing a fast answer as in traditional FP solving); 2) *measurement* and *experimentation* (of relevant values, in the lab or the field and using tools); 3) *looking for data* (in official databases or online newspapers); and 4) *statistical data collection* (using surveys and questionaries etc.).

A *Fermi problem Activity Template (FPAT)* is a characterization that focuses on the structure of the solution of a FP in terms of the four different types of activities outlined above.[16] These templates show the sub-problems and variables needed to be considered, the type of activity that allows each partial result to be achieved and the operations that combine these to arrive at the final result. In the FPAT each activity is represented by a specific symbol/shape according to the following scheme: guesstimation (ellipse), experimentation (trapezoid), looking for data (rectangle), statistical data collection (hexagon). Figure 1 presents a FPAT that describes a possible way to estimate the number of toilet paper rolls needed in a school in a school year.

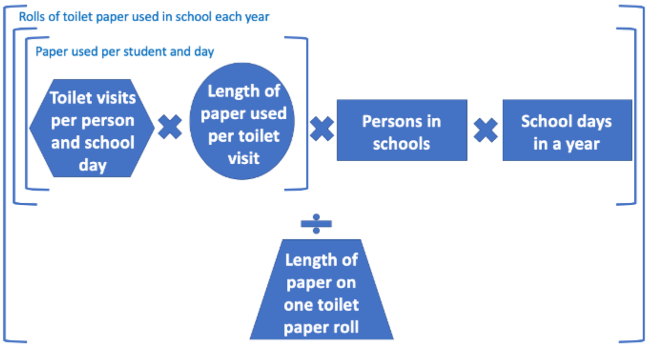

Figure 1: A FPAT for the Toilet Paper Roll Task

To determine how many toilet visits a person make each day, a statistical survey or data collection could be conducted (hexagon). Due to the potential delicate nature of the amount of toilet paper usage, guesstimation could be used to estimate the average use of toilet paper per toilet visit (ellipse). To find the length of the paper on a toilet paper roll, one can engage in trying to calculate the total length using geometrical arguments or, as suggested in Figure 1, conduct some investigation involving either unrolling the paper roll and measure the length or some weighing process (trapezoid). The last two pieces of data needed to complete the calculation, the number school days in a year and the number people in the school, are suggested to be looked up (rectangles).

The FPAT graphical representation of the four potentially involved activities (*guesstimation*, *experimentation*, *looking for data*, and *statistical data collection*) provide a quick overview of the envisioned and potential solutions to a FP, as well as a way to visualize and represent an a posteriori analysis of students' work. Hence, a FPAT characterization of a task can prepare the teacher on what to expect in term of what different problem solving routes the students might take and what to prepare in class. In this way the FPAT reduces some of the challenging aspects associated with the openness of using FPs in class.

However, given the differences in epistemologies, learning goals and practices of teaching and learning mathematics and the one hand, and physics on the other, what would a FPAT in a physics educational setting look like? For example, in physics the activities *experimentation* and *statistical data collection* are fundamentally intertwined and hence it is probable not as productive to separate the two from a physics education perspective as it is in mathematics education. In addition, in physics well-established physical laws and defined quantities play a central role which potentially provide at least a partial pre-structuring of some of the aspects involved in solving FPs in physics educational settings. We illustrate this latter point using an example of a FP combining two FPs discussed in Author[11] and Weinstein[17] respectively: *How much power does an average student use climbing the stairs to the observation deck in the Empire State building?*

This FP is pre-structured by the definition of power (*P*) understood and measured in watts or joules per second (1 W = 1 J/s), which can be determined by estimating the change in potential energy during a given amount of time; if the mass *m* kg is displaced vertically h meters in t seconds, then the power used is $P = \frac{mgh}{t}$ (g being the acceleration due to earth's gravity). Hence, the corresponding FPAT for this problem have the structure illustrated in Figure 2 below:

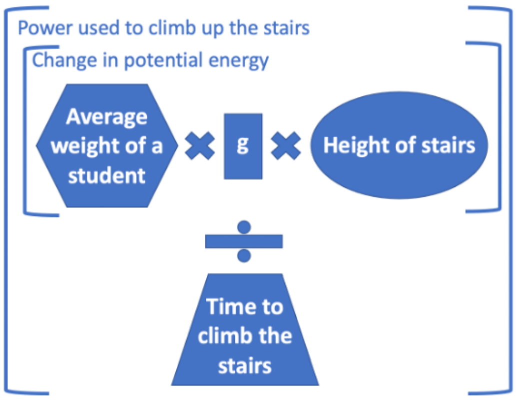

Figure 2: The FPAT describing the structure of the Empire State Building power task

To solve this FP students in a class can engage in measuring their own weights to collect data in order to establish a representative measure of central tendency (mean, median or mode for example) of an average student's weight using some statistical analysis (hexagon). To determine the height of the stairs, the distance between the ground floor and the observatory floor, a guesstimation approach could be used by having students share and discuss their own personal experiences with tall buildings (ellipse). Finally, to find the estimated time it takes to climb all the stairs, the students can make some experiment using the stairs in the school (trapezoid) and then using proportional reasoning to scale up to the estimated height of the stairs in the Empire State Building. In addition, if an effort is made to keep track of the uncertainty and variation in the derived estimated values (using measures of spread for the weight and a fix percentage error margin for the height and the time for example), an error and propagation of error analysis based on the structure of the FPAT could be calculated resulting in an interval capturing reasonable answers to the FP.

In this example, an upper bound can be established using the official data from the Empire State Building Run-Up in which runners covers the 1,576 steps and 86 floors up to the (lower) observation deck situated 320 meters above the ground floor. Using the record time for men (9 minutes and 33 seconds) and women (11 minutes and 23 seconds),[18] together with an estimate for a runners' weight, an upper bound can be calculated.

Final Remarks

With this contribution we intend to show that there is a plethora of research that shows the potential of FPs as tools in the classroom that transcend the initial and traditional use of Fermi. However, many aspects of the use of FP in educational settings are still unexplored, both from an in-discipline as well as a cross-discipline context. We are convinced of the potential and multiple benefits of using FP for in classrooms, and that developing theoretical as well as practical tools facilitating the use and further uptake of FPs are fruitful endeavors. In a globalized world where large and hard to grasp numbers figure in the public discourse and decision-making that directly affect us all, we believe that FPs can play a fundamental role in promoting the skills needed to understand the world better, by focusing on creative ways of thinking, developing critical thinking and supporting decision-making. In particular, we think that future research productively could focus on empirical studies investigating similarities and differences related the use of FPs in different disciplines. Connected to the FPAT framework, interesting questions to look into are for example *How are similarities and differences manifested in the FPAT framework in teaching and learning of mathematics compared with the teaching and learning of physics*? And *What characterize a FPAT representation of students' work in introductory classes compared to more advanced classes?*

References

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[1] Fermi Questions is the name of the section of The Physics Teacher journal in which this type of problems are presentad and discussed.

[2] Jonas Ärlebäck and Lluís Albarracín, "The use and potential of Fermi problems in the STEM disciplines to support the development of twenty-first century competencies," ZDM Math. Educ. 51, 979–990 (2019).

[3] Andrew W. Robinson "Don't just stand there—teach Fermi problems!" *Phys. Educ.* **43**, 83–87 (2008).

[4] Haytham M. Barahmeh, Adwan M. B. Hamad, and Nabeel M. Barahmeh, "The effect of Fermi questions in the development of science processes skills in physics among Jordanian ninth graders," *J. of Educ. Pract.* **8**, 186–194 (2017).

[5] Sean M. Cordry, "Thermodynamics and human population," *Phys. Teach.* **48**, 403-407 (2010).

[6] David L. Morgan, "Measuring the effect of an astrobiology course on student optimism regarding extraterrestrial life," *Int. J. Astrobiol.* **16**, 293–295 (2017).

[7] Authors XXXX

[8] Andrea Peter-Koop, "Teaching and Understanding Mathematical Modelling through Fermi-Problems," *Tasks in primary mathematics teacher education* (Dordrecht, Springer, 2009). pp. 131-146.

[9] Nora Haberzettl, Stephanie Klett, and Stanislaw Schukajlow, "Mathematik rund um die Schule—Modellieren mit Fermi-Aufgaben," *Neue Materialien für einen realitätsbezogenen Mathematikunterricht 5. Ein ISTRON-Band für die Grundschule, (*Wiesbaden, Springer Spectrum, 2018*).* pp. 31–41.

[10] Irene Ferrando and Lluís Albarracín, "Students from grade 2 to grade 10 solving a Fermi problem: analysis of emerging models," Math. Educ. Res. J. 33, 61–78 (2021).

[11] Jonas Ärlebäck, "On the use of Realistic Fermi problems for introducing mathematical modelling in school," Math. Enthus. 6, 331–364 (2009).

[12] Lluís Albarracín and Núria Gorgorió, "Students estimating large quantities: From simple strategies to the population density model," Eurasia J. Math. Sci. Technol. Educ. 14, 1–15 (2018).

[13] Irene Ferrando, and Carlos Segura, "Fomento de la flexibilidad matemática a través de una secuencia de tareas de modelización" *Avances de Investigación en Educación Matemática*, **17**, 84-97. (2020)

[14] Jennifer A. Czocher, "Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking" *Math. Think. Learn.* **18**, 77-106 (2016).

[15] Jennifer A. Czocher, "How does validating activity contribute to the modeling process?," *Educ. Stud. Math.* **99**, 137-159 (2018).
[16] Lluís Albarracín and Jonas Ärlebäck, "Characterising mathematical activities promoted by Fermi problems," Learn. Math. 39, 10–13 (2019).
[17] Weinstein, L. (2012). *Guesstimation 2.0: solving today's problems on the back of a napkin*. Princeton, NJ: Princeton University Press.
[18] https://www.esbnyc.com/empire-state-building-run