Designing levels of a video game to promote spatial thinking

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Abstract: In this article we present an educational proposal designed to work on spatial geometry and three-dimensional thinking with students in the last years of Primary Education. The proposal is comprised of several activities that combine the use of a classroom videogame and working with manipulative materials to create a representation of the videogame in the real world. The final goal of the sequence is to design levels that could be incorporated into the video game. This exercise enables students to work on visualisation and positioning in three-dimensional structures and on a problem-solving task.

Keywords: geometry, videogames, manipulative materials, problem solving

1. Introduction

Spatial thinking involves using shapes, locations, paths, and the relation between entities to manipulate, construct, and navigate the physical world (Newcombe & Shipley, 2015). Classroom activities involving spatial geometry should not be neglected, since lack of attention to spatial skills may harm the development of some individuals (Clements & Sarama, 2011). In addition, we cannot disregard the fact that spatial ability plays a critical role in achieving success in academic and intellectual endeavours and in developing expertise in STEM (Wai, Lubinski & Benbow, 2009).

Video games can promote the development of spatial skills of Primary Education students (Connolly, Boyle, MacArthur, Hainey & Boyle,2012). Some video games, such as 3D-puzzle games, offer students problematic situations in three-dimensional environments. This proposal of classroom work follows the line of using commercial video games as a context to promote mathematical learning based on the analysis of the game processes (Frejd and Ärlebäck, 2017; Albarracín et al., 2017) which is shown to be a promising way of approaching teaching. Since video games are characterised by the immediacy of on-screen responses (Dickey, 2005) students may be confronted with them in an unreflective way. Therefore, we propose the use of manipulative materials to focus students' attention on essential visualisation aspects (Risma, Putri & Hartono, 2013).

In this work, we present a sequence of activities to promote spatial thinking in students of the last years of Primary Education (ages 10 to 12). For this purpose, we suggest the combined use of two resources that have proven to be potentially valuable, such as videogames and manipulative resources. In the following sections, we review the potentialities of these two types of resources, introduce the proposed activities, and outline the learning outcomes promoted as a result of the implementation of the classroom activities proposed. The final activity in the sequence is the design of platforms that could potentially be new levels in the videogame. The design of these platforms requires students to combine what they have learnt about both the mechanics of the video game and the three-dimensional elements that make the game interesting for the player. In this way, students realise that the activity of video game design also requires mathematical skills and knowledge.

2. Theoretical framework

In this section we present the theoretical foundations of the didactic proposal presented in the paper. They can be split in three main topics, coordinated in the design of the sequence: spatial thinking, the use of manipulatives to promote spatial thinking, and the use of commercial videogames in Mathematics Education.

2.1. Spatial thinking and the use of manipulatives in Mathematics Education

Spatial thinking involves the assessment of an individual's processes and abilities to perform specific tasks that require seeing or mentally imagining geometric objects in space and understanding the spatial relations among objects, as well as perform specific tasks or geometric transformations with them. Bishop (1988) identifies four processes that are applicable to visualization and working with mental images: (a) generating a mental image from certain given information; (b) exploring the image to observe its position or the presence of parts of its constituent elements; (c) transforming the image using rotations, translations, scaling, or decomposition; and (d) using the image to answer questions.

Ben-Chaim, Lappan and Houang (1989) and Németh (2007) state that spatial thinking is not an innate skill but a competence that must be developed. This ability can only be developed when students participate in learning activities generally related to real-life experiences that arouse their interest. Besides, spatial thinking plays an important role in the development of students' mathematical thinking. For instance, Pittalis and Christou (2010) observed that spatial skills are a strong predictor of students' performance in three-dimensional geometry. The relevance spatial skills in mathematical thinking and its relationship with other learning justifies that spatial thinking should be recognized as a fundamental element of compulsory education and as an integrator and facilitator for solving problems across the curriculum.

Planar representations are the most frequent type of representational mode for representing three-dimensional geometric objects in school mathematics and are sometimes considered during the teaching process as if these planar representations were indeed real objects (Berthelot and Salin, 1998). However, both students and adults have great difficulties in drawing three-dimensional objects and representing parallel and perpendicular lines in space (Ben-Chaim, Lappan, & Houang, 1989). Several studies point out that the representation of a three-dimensional object by means of a flat figure requires considerable conventionalism that is neither trivial nor explicitly taught in school (Duval, 1988; Parzysz, 1988; Pittalis & Christou, 2010). Fujita, Kondo, Kumakura and Kunimune (2017) observed that students who have the most difficulty solving challenging geometrical problems in space are those who do not manipulate 3D shape representations effectively mentally. Later on, Fujita, Kondo, Kumakura, Kunimune and Jones (2020) found that the use of spatial visualisation or property-based analytical reasoning is not enough for problems that require more than one reasoning step. These authors suggest that more opportunities should be offered to primary school pupils in which they could not only exercise their spatial reasoning skills, but also consolidate their geometry-specific knowledge for productive reasoning in geometry.

The literature in the field of Mathematics Education shows that the use of manipulative resources ease learning mathematics by: (a) supporting the development of abstract thinking (Montessori, 1964; Piaget, 1962), (b) stimulating the knowledge from the students' real world (Rittle-Johnson and Koedinger, 2005), (c) providing ways of representation that favour the coding of mathematical concepts (Kormi-Nouri, Nyberg, & Nilsson, 1994), and (d) providing opportunities for the students to discover mathematical concepts through a guided exploration (Piaget & Coltman, 1974).

Recently, Carbonneu, Marley and Selig (2013) carried out a meta-studio based on the results of 55 previous studies on the influence of the use of manipulative materials. Their work led to the evidence that manipulative materials benefit mathematical learning under the appropriate conditions. That is, choosing the suitable materials for each type of content to be worked on and incorporating them into classroom routines consistently over time and not just incidentally. In this work we follow Risma, Putri, and Hartono (2013), who proposed that manipulative activities based on the use of building blocks support the development of students' spatial visualization skills.

2.2. Commercial videogames in Mathematical Education

Videogames offer the opportunity to operate in simulated environments, which means the players can be the focus of the activity. This enables and stimulates the intellectual development of the players by introducing situations in which decision making is essential (Ke, 2009). The presence of objectives and the existence of well-defined rules are also relevant (Charsky, 2010). The most significant aspects of videogames that support teaching and learning processes in class are motivation, the possibility to provide an immediate response to the players' actions and directing the focus of the activity towards the players/students themselves (Prensky, 2001). The fast responses to the player's actions and the different types of feedback allow the player to adapt and improve aspects of their performance to overcome the game's challenges (Dickey, 2005).

One interesting pedagogical strategy is to design and use video games in the classroom aimed at teaching specific mathematical content. On the one hand, there are studies that report positive results for assessing fraction knowledge (Ninaus, Kiili, McMullen & Moeller, 2017) or the processes of solving an equation (Gutiérrez, Arnau & González, 2015). On the other hand, it has been proven that educational videogames are not the games that are most attractive to students, who prefer playing commercial videogames, which are designed to maximise entertainment (Hamlen, 2011). In our work we take a similar perspective to Frejd and Ärlebäck (2017), who used the commercial video game Plague Inc: Evolved (2015) to introduce the use of functions to model essential aspects of the game with secondary school students by promoting mathematical modelling work.

In a previous study (Hernández-Sabaté et al., 2016), we identified several mathematical learning opportunities when playing a strategy-type videogame in real time. The results showed the activity to be valuable in this sense, but in order to transform these opportunities into effective learning outcomes it is necessary to pair the gaming experience with complementary activities. Therefore, in this sequence of activities we use two types of tasks to help students understanding the situations proposed in the video game. On the one hand, several paper-written problem statements are presented to be used as an auxiliary support to the reproduction with manipulative materials of three-dimensional elements of the game. On the other hand, all the work is aimed at placing students in the role of a video game designer. The final activity, in which students design the three-dimensional platforms that could be integrated into the game, should help students to deepen and refine the way they use their spatial thinking abilities in an engaging way (Pretelín-Ricárdez y Sacristán, 2015).

3. Design of the teaching sequence

The didactic objectives of the teaching sequence, related to the development of spatial thinking and problem-solving competence, are the following:

- Identification and verbal description, using specific vocabulary, of the properties of two- and three-dimensional geometric figures: polygons and polyhedra.
- Use and analysis of various forms of representation and construction of three-dimensional figures, with manipulative materials and digital resources.
- Description of locations and movements of objects in space.
- Identification of the relationships between the partial views of a three-dimensional object and the object.

To achieve these didactic objectives, a puzzle-type video game is selected to propose an experience in a three-dimensional environment. The video game chosen is Kula World (1998) and is presented in detail in the following section. It is based on the premise that the game activity with Kula World (1998) promotes spatial reasoning and visualization by requiring interaction with complex three-dimensional objects and proposing non-trivial challenges to the player that depend on aspects such as partial views of an object. In this way, it meets the requirements proposed by Ke (2008) for a video game to be considered a didactic tool. However, we cannot consider that the game experience is enough to achieve the proposed learning objectives, since students may use alternative strategies to develop adequate knowledge to overcome the proposed challenges, such as acting by trial and error. Therefore, the game activity is the beginning of the activity, but the core of the sequence is based on the analysis of the key elements of the game and their properties. Specifically, activities are used to construct three-dimensional figures using manipulative materials by integrating the on-screen views of an object (Gutiérrez, 1996), solving problems related to the location and movement of objects in space and, finally, the creation of platforms designed as levels of the video game. This sequence is designed to take advantage of the essential mathematical aspects promoted by the video game, isolate them and study them separately to achieve the proper understanding that allows taking a different perspective from that of the player, such as that of the level designer.

3.1. The chosen videogame: Kula World

Kula World is a videogame that was launched in 1998, and there are versions of it that can be played on the computer (such as the open code Cubosphere[1]) or on mobile devices. It is a puzzle type videogame in which the player controls a sphere (a beach ball). The latter has to go through a level of one or more three-dimensional platforms, each of these comprised of cubes, in search of the keys to the exit the platform.

Figure 1. A level of Kula World in a three-dimensional platform.

It is important that we describe the game mechanics of Kula World. We understand game mechanics as the actions carried out *inside* the videogame as a result of the actions that take place *outside* of it when the player interacts with the machine. An example of this for a driving simulator videogame would be the action of pressing a specific button on the control for the car to accelerate. In Kula World, the ball rolls across the floor to advance, and we can control its motion in four directions on each plane (forwards, backwards, left and right). When the direction is already defined, we can only move forwards. We can also jump forwards, advancing two of the cubes that make up the platform in a single jump. It is not possible to fall off the platform we are rolling on, since the edge of the platform acts as a wall. It is relevant for the design of the problem sequence that if the ball is located at a point where the platform has a column oriented towards the sky (as in figure 1, opposite the ball and to the left of the picture), when advancing, the whole platform will rotate to situate the ball on the horizontal plane again, restoring gravity to the plane we now see as the ground. And thus, the different platforms can be crossed while the player observes them from different viewpoints as the game proceeds, requiring a visualisation effort to locate the objects to be caught.

[1] https://sourceforge.net/projects/cubosphere/

The game has yet another mechanic that makes it very interesting. When reaching the end of an area of a platform in the shape of a column, as we go forward we can stand on the square that acts as the upper face of the prism. From this position, as shown in the two views of figure 2, we can turn in any of the four possible directions and cross the platform on any of these four sides. This mechanic allows the player to reach objects that are invisible at first and enables them to explore and go through the platforms to complete each of the levels. It also allows to define paths within the platform and distinguish between accessible and non-accessible areas.

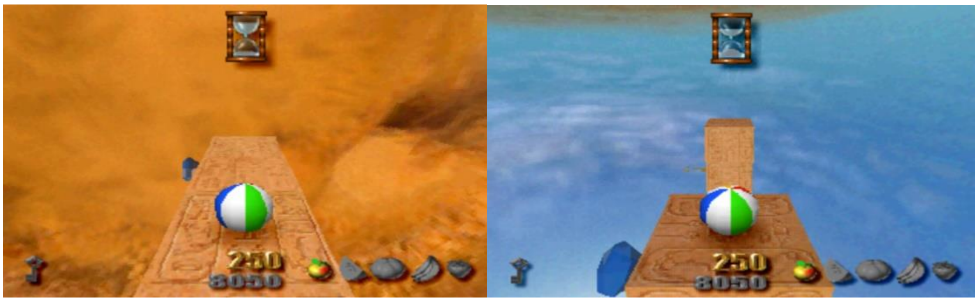

Figure 2. Two views of one end of the platform.

4. The didactical sequence of activities

For the experiences carried out in this videogame, we have worked with students in the fourth year of Primary Education (age 10). However, the scalability of the proposal suggests that it is suitable for students aged 10 to 12. At this learning stage, it is important to work on the identification and construction of figures in space, as well as on the location of objects and movement description in space. Thus, we understand that the videogame used provides an enriching context in which three-dimensional phenomena naturally participate in are natural actors in the game's dynamic and interacting with them is necessary to complete each of the levels.

Playing and making progress in the game provides us with several learning opportunities. However, since Kula World is a commercial game, there are no mechanisms in place to make the student reflect on their learning or to connect concepts. We must therefore complement the gaming experience with activities and problems that highlight the mathematical aspects we wish to address. For this reason, we designed an activity sequence based on the situation offered by the videogame in order to provide contextualised knowledge to the students. Our main aim is for the students to develop their spatial reasoning when moving objects from the screen to reality, and for them to face problem resolution processes regarding paths in a three-dimensional environment, with the same type of restrictions imposed by the videogame with its game mechanics.

4.1. First activity: From the screen to reality

The first step in the classroom work is for the students to tackle the videogame and try to complete several levels. Each one of the levels offers a new challenge that the student has to overcome, from finding a key that is initially not visible, to crossing several disconnected platforms. In the classroom, students can play individually or in pairs, sharing the computer. The option of playing one-player videogame in pairs will promote the discussion between them (Hernández-Sabaté et al., 2015). A session of 20 to 30 minutes is enough for the students to get familiar with the previously detailed game mechanics and to reach levels that are genuinely challenging to them.

After this first stage, students are asked to focus on the platforms found on a certain level and to reproduce them in the real world using manipulative materials such as multilink cubes. The use of manipulative materials is proposed to support the development of abstract thinking and to reflect the learning opportunities promoted by the game by providing a way of representation that favours the coding of the mathematical concepts involved. From the perspective of this didactic proposal, combining virtual environments and manipulative materials is a good opportunity to helps students to consolidate their knowledge when switching from one environment to the other. The videogame presents the students with three-dimensional objects, but it does so from the interaction with a flat screen. Thus, it is initially hard for the students to observe and interpret this virtual three-dimensionality and transfer it to the real world. Figure 3 shows the difference between two platforms built by two different students for the same level of the game. Observing the platform on the right, we can see that the student captured its three-dimensionality, whereas the platform on the lower left-hand side reveals that its creator did not identify the three-dimensionality of the platform, producing a flat visualisation of the same subject.

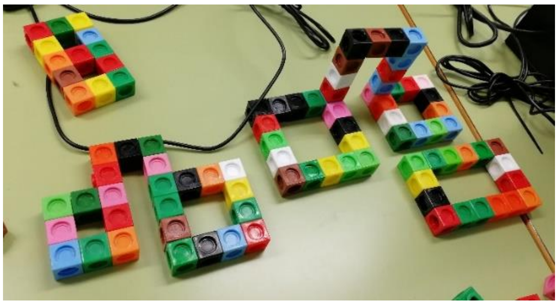

Figure 3. Two different representations for the same gaming platform.

At this stage of the activity, the teacher may ask the students to search for a level of the game with a platform that has a representation complexity suited to each of the students. He/she can also ask them to reproduce several of these platforms. The activity can be complemented by encouraging the students to attempt to explain the geometrical properties of the figure they are constructing among themselves, and then connect these properties with actions and implications of the videogame. These discussions, regardless of the size of the groups, allow the teacher to identify the students' conceptual errors and to monitor their progress.

4.2. Second activity: mathematical problems based on the videogame

The second stage of the activity sequence involves using a list of problems to encourage the students to work on positioning and paths in three-dimensional environments. To achieve this, the game mechanics are used as restrictions on the mobility of objects across the platforms. Essentially, the problems are based on providing the students with a three-dimensional platform built by the teacher with multilink cubes as in figure 4 – or the students can build it following instructions or a reference image – and then making the students differentiate accessible and inaccessible areas of the videogame's platform according to the initial position of the ball.

Each of the problems is comprised of three elements: a three-dimensional figure, a picture of it on paper showing the initial position of the ball, and a problem statement that requires explaining whether it is possible to reach another position following the game's mechanics.

The following is a detailed explanation of one of the problem sheets used. Figure 4 shows one of the platforms given to the students. In the problem statement provided, the students are asked to decide whether the whole platform can be crossed from a given initial position, following Kula World's rules. It is important to note that the platform contains a cube in the top from which it is possible to turn in four directions. Students usually associate this fact with freedom of movement in the game, since this type of structure often allows them to get to areas that seem completely out of reach. However, in the case shown, there are two differentiated zones along which the ball could move without being able to reach the other.

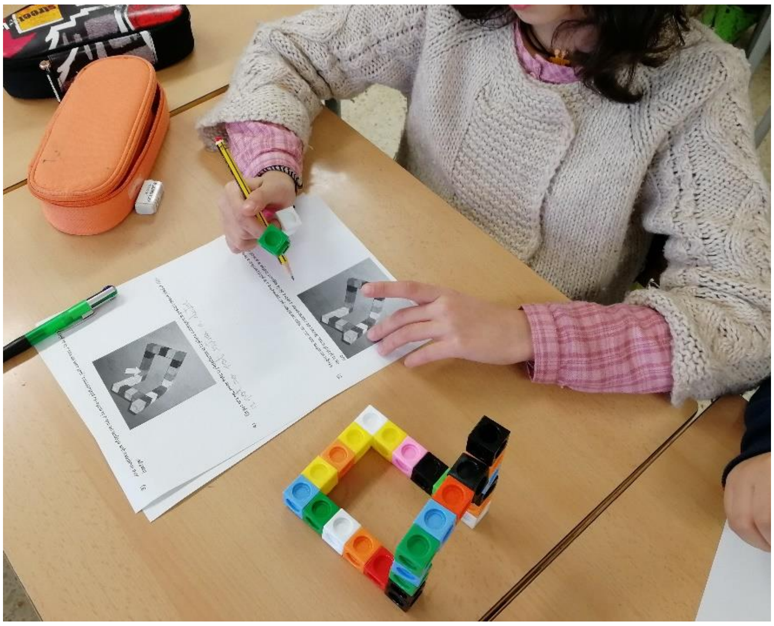

Figure 4. A student solving path problems using a multilink platform.

When the students have identified these two areas of the platform, a second formulation prompts them to modify it slightly to connect all the sides of the platform for all possible paths the ball could follow. This involves working on spatial reasoning and visualisation, taking advantage of the students' wide experience working with these objects when previously playing the videogame itself. In the problem resolution work proposed, it is important that the students share their solutions so that they are obliged to express their reasoning. Firstly, in conversations among them, but also with the teacher. Developing explanations to describe the possible paths across the platform, as well as the ball's motion, are not trivial exercises and reproducing them in writing is a challenge for the students.

4.3. Third activity: designing platforms

Finally, students can be asked to invent and construct their own platforms, as possible levels for the game – a task that they can easily identify with the profession of videogame developer – or to generate new problems for their classmates, searching for cases that are extreme or of interest for discussion. The activity is specified with the demand to students to create a platform using multilink cubes that could be playable if used as a support to design a specific level of the video game. Students are asked to identify the key elements of its construction and to prepare a short presentation to highlight them and share them with the rest of the class. During the presentations there is a further opportunity to discuss aspects of platform design that allow the difficulty of the proposed levels to be calibrated.

Below, we present some of the platforms that students designed to shape the levels of the game. Figure 5 shows a simple platform because it has a flat design. The student who designed it claimed that the platform is large and has many obstacles (marked with cubes arranged on the platform). However, the player shouldn’t really deal with three-dimensionality if he or she were played at this level of the game with a platform as basic as this one.

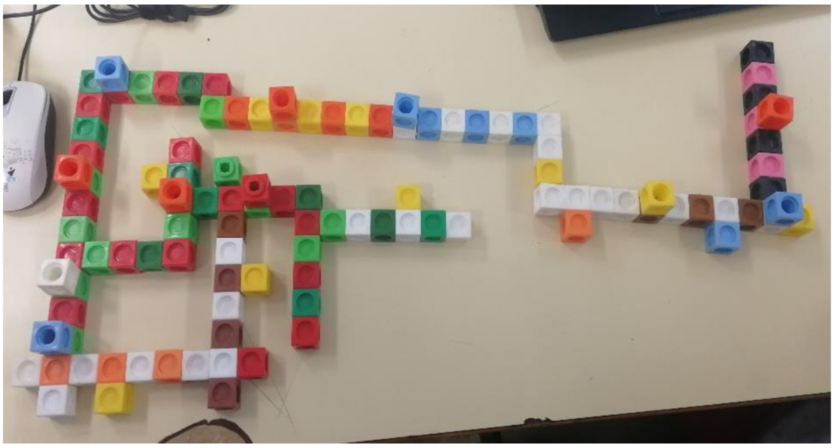

Figure 5. A large but simple platform design

Figure 6 shows a level based on two simple platforms, but it highlights that the student considers the relative position of these two platforms as a way of adding complexity to the game. The student explained in his report that in the initial position of the player, the lower platform cannot be seen and it is necessary to go through the whole of the upper platform to find it and use it later.

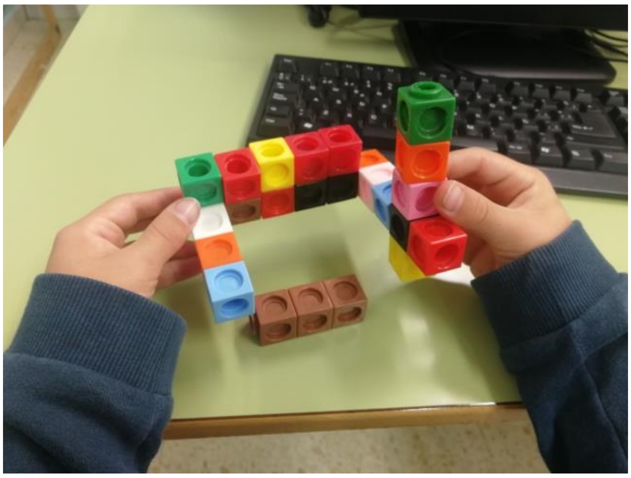

Figure 6. A proposal of game level using two platforms.

The platform shown in figure 7 is much larger than the previous ones and has various elements arranged perpendicularly to each other. This causes the player's vision to constantly change and add difficulty to the proposed level. Also, the cubes that can be seen placed on the columns that make up the platform are elements of the game arranged with the aim of placing obstacles for the player (orange cubes are spikes that can make the ball explode) or bonuses to obtain points. In this way, this student shows how to combine the two essential aspects of level design in the video game: paying attention to the game mechanics to set game objectives and taking into account the three-dimensional structure of the elements that make up the platform.

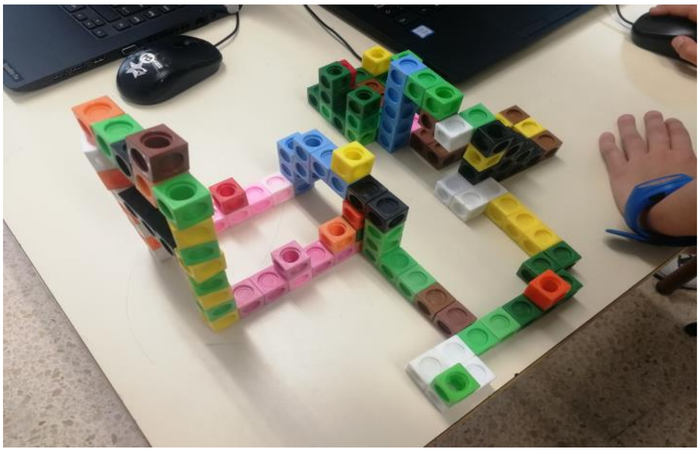

Figure 7. A proposal of game level using two platforms.

The activity of creating platforms as new levels has a creative component and students generate new elements with the intention of making their levels of play more interesting. This practice reveals a connection between the new game mechanics that they propose and the geometric objects and their properties that give them form. Some of the design elements that students incorporate are the following: i) cubes that allow students to move from one side of the platform to the opposite side; ii) cubes that can be moved, so that disconnected platforms can be connected, iii) cubes that make the ball jump, so this cube allows to reach inaccessible areas with the movements of the video game.

5. Final remarks

In this article we propose the combination of different didactical elements that are potentially effective in the classroom. The first element is the use of commercial videogames, taking advantage of their motivating appeal, but also due to the chance they offer to widely experiment in a virtual environment (Dickey, 2005; Ke, 2009). There is currently a huge range of video game genres and sub-genres, many of which do not contain elements that allow them for a clear educational use. Therefore, a key aspect in our proposal is to identify commercial video games that include elements that allow an approach based on mathematical analysis that can be effectively transferred to the mathematics classroom (Frejd & Ärlebäck, 2017). In general, the genres of strategy, simulation and puzzle video games will be the most likely to incorporate mathematical elements in their design. In addition, the chosen game will need to be playable on classroom devices such as computers or tablets, allowing the rules of the game to be understood quickly and each game to last for a short period of time. The second element is the use of manipulative materials to overcome difficulties in interpreting three-dimensional representations on the screen (Pittalis & Christou, 2013) and to allow a form of real three-dimensional representation that makes it possible to clearly identify the geometric concepts to be worked on (Kormi-Nouri, Nyberg, & Nilsson, 1994). Last, but not least, the activity of assuming the role of a video game designer, which is an approach rarely addressed in the literature (Pretelín-Ricárdez y Sacristán, 2015), promotes the connection between a virtual reality and the mathematics that describe it.

In order to engage students in mathematical work on the contents of the commercial video game, it is necessary to complement the game activity with other types of activities in the classroom. From the experience related in this article, we understand that this sequence of activities allows us to effectively exploit the potential of the commercial video game used. In general, the game activity does not require a deep analysis for the player to enjoy the activity. In many cases, players face the challenges of video games in a casual, non-analytical way. We have observed this way of dealing with the challenges of the game in Kula World as well. We therefore consider that it is necessary for students to be confronted with the video game but also with a series of activities that connect with the game activity and, in this case, with the geometric contents that make up the Kula World puzzles. The sequence of activities makes it possible to take advantage of the exploratory component of the game activity in a virtual environment as a context in which problems can be proposed. The video game we used in this experience, Kula World, engages students work on spatial thinking, which is a complex content to work on in the classroom, but which is key to solving three-dimensional geometric problems (Fujita, Kondo, Kumakura & Kunimune, 2017; Fujita, Kondo, Kumakura, Kunimune & Jones, 2020). We understand that Kula World is not the only videogame that offers this connection, so that we consider that there is room for a wide range of didactic innovation in this direction, and also for research on the didactic potentiality of this type of proposals.

Finally, we consider of interest to study in depth the change in the perspective on the use of mathematics that students experience after an activity such as the one presented. In our experiences, we have observed how students take interest in the activity of designing new platforms. This role of video game designers allows them to connect a specific interest, such as playing video games, with a type of knowledge required to develop this activity, such as mathematics in this case. At the same time, the designing levels activity shows that there is scope for creativity in using mathematics by proposing a context that students clearly associate with activities out of school.

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Note

1. https://sourceforge.net/projects/cubosphere/

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