Vertical integration: A real options approach

Nicos Koussis[1], Florina Silaghi[2*]

Abstract

We develop a theoretical real options model and explore the trade-off between vertical integration and external procurement. In contrast to transaction cost theory, we show that higher volatility in the downstream market reduces the likelihood to switch to internal production. We also analyze the decision to acquire the supplier and provide novel predictions on the acquisition likelihood and premium contributing to studies relating to vertical M&As.

JEL classification: G34, G32

Keywords: vertical integration; real options; optimal timing; acquisition; supply chain

---

Declaration of interest: none

1. Frederick University Cyprus. Department of Business Administration, 7, Y. Frederickou Str. Pallouriotisa, Nicosia 1036, Cyprus. Email: bus.kn@fit.ac.cy

2. Universitat Autònoma de Barcelona, Campus de Bellatera, Barcelona 08193, Spain. Email: florina.silaghi@uab.cat

*Corresponding author

Florina Silaghi, Serra Húnter Fellow, gratefully acknowledges financial support from the Ramón Areces Foundation and from the Spanish Ministry of Science and Innovation through Project PID2020-114460GB-C31.

.

1. Introduction

We provide a real options-based framework for backward integration where we capture many of the advantages of backward integration such as eliminating the adverse effect of double marginalization or reducing the production cost, as well as its main disadvantage, the large investment to set up the factory (Li and Chen, 2018). To the best of our knowledge, this is the first paper to frame the decision to internalize production (vertical integration) in a real options setting.

We show that when downstream demand uncertainty is higher, backward integration is delayed. The real option value of waiting that we highlight is supported by empirical evidence in Tong and Li (2011). On the contrary, transaction cost theory (see Williamson, 1971) predicts that backward integration is more likely when the environment is more uncertain. We thus provide a possible explanation for the mixed empirical evidence on the relationship between uncertainty and vertical integration which points out the need to distinguish between different types of uncertainty (Sutcliffe and Zaheer, 1998).[1]

We also analyze the maximum price that the buyer would be willing to pay to acquire the supplier and compare it with the minimum price that the supplier would accept, thus providing a zone for possible agreement between the buyer and the supplier on the acquisition price and novel predictions on acquisition premia in vertical acquisitions.

Our work makes a twofold contribution to a growing strand of the literature applying real options to operations research (see Trigeorgis and Tsekrekos, 2018 for a review). First, building on well-known models from the capital structure literature (Hackbarth and Mauer, 2011; Leland, 1994), we are the first to develop a real option setting to model a buyer firm's decision to internalize production (backward integration). Second, we fill a gap in the literature since most existing models use real options to study horizontal mergers (Pires and Pereira, 2020; Wang et al., 2015), whereas we focus on *vertical* mergers.

[1] Another prominent theoretical view of the boundary of the firm is property rights theory (for a review see Lafontaine and Slade, 2007).

Finally, we contribute to the economics literature analyzing the firm boundaries (e.g. see Lafontaine and Slade, 2007).

2. Model assumptions

The price per unit x at which the goods can be sold in the market follows a Geometric Brownian motion:

$$\frac{dx}{x} = \mu dt + \sigma dZ \qquad (1)$$

where μ is the expected rate of change, σ is the volatility and dZ is a standard increment of a Weiner process. We assume risk-neutrality, with r denoting the risk-free interest rate, and that $r > \mu$ such that there is a rate of return shortfall similar to a convenience yield $\delta = r - \mu$. A higher δ (while keeping r constant) captures a lower rate of growth of the good's demand in the buyer's market.

A supplier continuously provides a quantity of input goods Q to a single buyer, which are paid in cash.[2] For simplicity we assume that the buyer is a reseller of the goods.[3] The cost of production for the supplier is C_S per unit sold. The supplier charges $p_S > C_S$ per unit of provided goods. On the other hand, the inexperienced buyer internalizing production of the input good will produce at a cost $C_I = C_S + u, u > 0$. The buyer also runs some other fixed operating costs C_B.

The buyer has the option to switch to internal production at an optimal threshold x_I by incurring a one-time installation cost for input production K_i. Default prior to switching to internal production occurs at an endogenously determined threshold optimally selected by the buyer, x_B^0. In the event of default, the supplier receives zero. After switching to internal production, the buyer firm optimally chooses the stopping time (default threshold) at x_B^1. In section 2.3 we also analyze the decision to acquire the supplier by paying a fair price for the value of the supplier that also covers its cost for installing the input capacity.

2.1. Internal production

[2] A single buyer client is a realistic assumption in situations where the buyer is a large firm, e.g., a supermarket.

[3] The framework can be easily extended to markets where *Q* is used for the production of another good by introducing a production function for the buyer firm that uses *Q* as an input.

Following the payment for the fixed cost for the input plant, K_i, the buyer value $B_1(x)$ is obtained as follows (see appendix for details):

$$B_1(x) = \left(\frac{xQ}{\delta} - \frac{C_B}{r} - \frac{C_I Q}{r}\right) - \left(\left(\frac{x_B^1\, Q}{\delta} - \frac{C_B}{r} - \frac{C_I Q}{r}\right)\right)\left(\frac{x}{x_B^1}\right)^{\beta_2}, \tag{2}$$

where the exponent β_2 is the negative solution of the fundamental quadratic and x_B^1 is found by applying the smooth-pasting condition $\frac{\partial B_1}{\partial x}|_{x=x_B^1} = 0$ resulting in the following solution:

$$x_B^1 = \frac{-\beta_2}{(1-\beta_2)}\frac{\delta}{Q}\left(\frac{C_B + C_I Q}{r}\right) \tag{3}$$

2.2. External procurement with option to switch to internal production

Before switching to internal production, we face a double boundary problem since the buyer firm may stop production at x_B^0 before reaching the switching to internal production threshold x_I. We follow Hackbarth and Mauer (2011) and define the value of basic claims that simplifies the exposition of the solution.

Define $H(x)$ to be the basic claim which pays 1 dollar when the switching threshold to internal production is reached first and zero when the stopping threshold x_B^0 is triggered first. $L(x)$ is the basic claim which pays 1 dollar when the stopping threshold x_B^0 is reached first and zero when the switching threshold x_I is triggered first. The solutions for these basic claims are provided in the Appendix. These claims proxy for the probability of switching to internal production and for the probability of stopping production, respectively.

Before switching, the buyer firm value is then (see appendix):

$$B_0(x) = B_0^P(x) + H(x)[B_1(x_I) - B_0^P(x_I) - K_i] + L(x)[0 - B_0^P(x_B^0)], \tag{4}$$

where K_i is the setup cost for building the inputs plant and $B_0^P(x) = \left(\frac{xQ}{\delta} - \frac{C_B + p_S Q}{r}\right)$ is the particular solution (see appendix). The optimal thresholds x_I and x_B^0 are obtained by solving the following optimization (smooth-pasting) conditions:

$$\frac{\partial B_0}{\partial x}|_{x=x_I} \quad = \frac{\partial B_1}{\partial x}|_{x=x_I} \tag{5a}$$

$$\frac{\partial B_0}{\partial x}|_{x=x_B^0} \; = 0 \tag{5b}$$

Similarly, the solution for the value of the supplier at t = 0 can thus be expressed as follows:

$$S_0(x) = S_0^P(x) + H(x)[0 - S_0^P(x_I)] + L(x)[0 - S_0^P(x_B^0)] \tag{6}$$

where $S_0^P(x) = \frac{(p_S - C_S)Q}{r}$ is the particular solution (see appendix).

2.3. Acquiring the supplier firm

As an alternative to internalizing production ("make" decision), the buyer can acquire the supplier and obtain the input good at cost C_S. The value of the buyer if it acquires the supplier firm and incorporates it under its own network $N_0(x)$ would then be[4]:

$$N_0(x) = \left(\frac{xQ}{\delta} - \frac{C_B}{r} - \frac{C_S Q}{r}\right) - \left(\frac{x_B^N Q}{\delta} - \frac{C_B}{r} - \frac{C_S Q}{r}\right)\left(\frac{x}{x_B^N}\right)^{\beta_2} \tag{7}$$

where

$$x_B^N = \frac{-\beta_2}{(1-\beta_2)}\frac{\delta}{Q}\left(\frac{C_B + C_S Q}{r}\right) \tag{8}$$

Note that the network default threshold is lower than the one if the buyer produced internally (see equation 2) since $C_I = C_S + u, u > 0.$ Equation (7) is "gross" since it is does not consider the price that needs to be paid for the acquisition.

The minimum price that the supplier is willing to accept to be acquired is $P_S = S_0(x) + K_i$. This is the value that the supplier firm would get under external procurement and includes the cost it has expended to install input capacity.

On the other hand, the maximum price K_S^* that the buyer firm is willing to pay to acquire the supplier, i.e., the price which makes him indifferent between acquiring the supplier or working under external procurement with the option to make his own input, is obtained by solving the following equation:

$$K_S^* = N_0(x) - B_0(x) \tag{9}$$

[4] The solution follows a similar approach as the one shown in Appendix A and is not shown for brevity.

We identify the maximum premium that the target firm can gain by taking the difference between K_S^* and P_S. The difference between K_S^* and P_S defines the zone of possible agreement for an acquisition of the supplier by the buyer firm (see also Moon et al., 2011 for a related concept of implicit zone of possible agreement (IZOPA)).

3. Numerical simulations

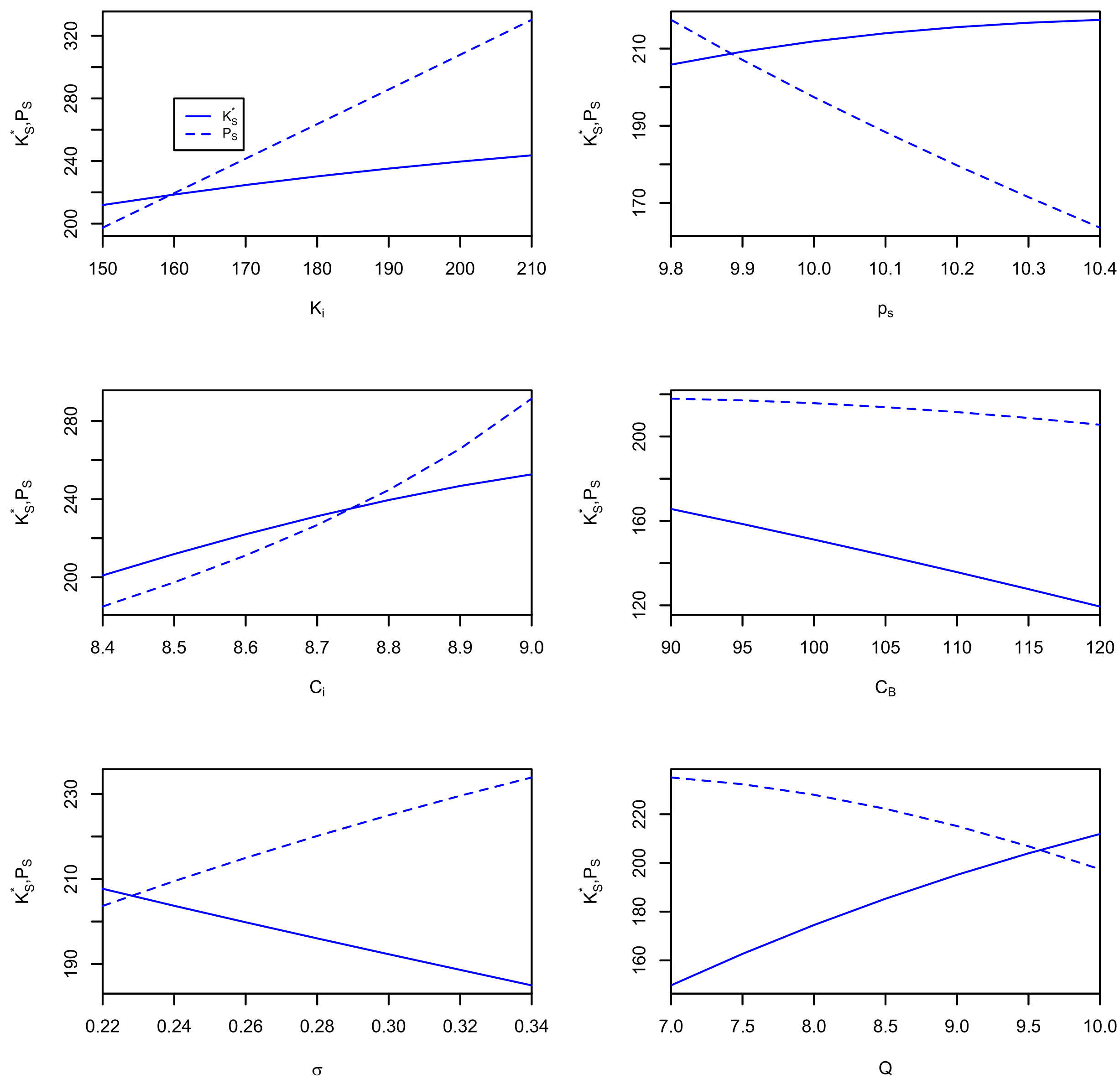
We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $C_B = 50$, $C_S = 8$, $P_S = 10$, $Q = 10$, $\sigma = 0.2$, $x = 10$, $u = 0.5$ and input investment cost $K_i = 150$. Table 1 provides the base case results and sensitivity results with respect to model parameters for backward integration (section 2.2). We derive novel predictions regarding the decision for internalizing production. In line with real option intuition, a higher volatility in demand delays vertical integration. A higher price charged by the supplier, on the contrary, accelerates vertical integration. Relatedly, Zormpas (2021) shows that a higher input price leads to a delay in investment in the downstream market due to the double marginalization effect (Billette de Villemeur et al., 2014).

Table 1. Sensitivity to model parameters for backward integration

Case	**Buyer**	**Supplier**	x_B^0	x_I	x_B^1	H(x)	L(x)	**Vertical integration**
Base case	834.78	47.41	5.43	11.96	4.96	0.70	0.18	
Higher investment cost $K_i = 200$	807.03	107.88	5.48	15.92	4.96	0.42	0.31	-
Higher price $p_s = 10.4$	829.22	13.58	5.51	10.42	4.96	0.92	0.05	+
Higher cost inefficiency u = 1	793.98	141.46	5.50	19.63	5.15	0.30	0.35	-
Higher fixed cost buyer $C_B = 100$	307.95	65.78	7.27	16.19	6.80	0.32	0.51	-
Higher volatility σ = 0.30	1151.96	75.00	4.00	15.36	3.63	0.51	0.31	-
Higher capacity Q = 11	995.91	25.42	5.25	10.84	4.79	0.85	0.09	+
Higher interest rate r = 0.07	1318.93	10.96	4.52	10.38	4.10	0.94	0.02	Hump-shaped
Higher convenience yield $\delta = 0.04$	291.32	73.50	6.82	15.84	6.21	0.32	0.49	-

Notes: All parameters are according to the base case situation unless otherwise specified. The last column provides a summary with the sign of the effect on the probability to switch to internal production as a function of the model parameter.

Figure 1. Minimum acquisition price accepted by supplier (P_s) and maximum acquisition price accepted by buyer (K_s^*)



All parameters are according to the base case situation unless otherwise specified.

Figure 1 shows sensitivity results for the maximum acquisition price the buyer will pay, K_S^* (equation 9). We highlight some novel findings, with some results being very intuitive while other being subtler. For example, when the cost of installing production increases K_S^*increases since the buyer is now willing to expend a higher price to acquire the supplier in order to avoid having to install its own input capacity.

Similarly, when the price charged by the supplier becomes higher or the marginal cost to internal production is higher, the buyer firm is willing to pay a higher price to acquire the supplier and thus avoid the higher cost incurred under external procurement or internal production respectively. The effect of volatility is subtler since a higher volatility increases the buyer's option value of installing its own input production and thus lowers the willingness of the buyer firm to pay a high acquisition price. A higher fixed cost of production decreases both the value of the buyer firm with the option to internalize production, B_0, and the buyer value if choosing to acquire the supplier, N_0. Since the value in case of acquisition decreases at a faster rate, our results show that the maximum acquisition price decreases with higher fixed cost of production of the buyer (intuitively, the buyer has less profits to make from acquiring the supplier when fixed costs are higher). Finally, a higher capacity (volume) increases both values B_0 and N_0, with N_0 increasing at a faster rate. Intuitively, the buyer firm will be more willing to offer a higher price to acquire the supplier firm since there are more gains to be made at a higher scale of operations.

The figure also includes the minimum price that the supplier is willing to accept to be acquired, P_S. The directional effect of a parameter on P_S depends on the effect the parameter has on the anticipated period the supplier can be expected to trade with the buyer under external procurement (without being acquired). For example, when volatility in the downstream market becomes higher, the buyer is expected to delay installing its own capacity for the input. Rationally, the supplier is now in a position to request a higher price to be willing to accept an acquisition offer from the buyer.

When $P_S < K_S^*$, the buyer firm has the opportunity to acquire the supplier immediately. This represents the zone of possible agreement between both. In this case the maximum acquisition premium can be calculated as $K_S^* - P_S$. Table 2 summarizes our predictions for key parameters regarding the maximum acquisition premia ($K_S^* - P_S$). The directional effect of the maximum acquisition premia depends on the directional effects of each of the two components, the maximum price offered by the buyer (K_S^*) and the minimum price that the supplier is willing to accept to be acquired (P_S). The effects on each component along a brief intuition is provided in the last column of Table 2.

Table 2. Sensitivity to key model parameters for acquisition

Parameter	**Acquisition premium ($K_S^* - P_S$)**	**Intuition**
Investment cost K_i	-	Both P_S and K_S^* increase with K_i, P_S increasing at higher rate since K_i is incorporated in supplier's requested price P_S at t = 0.
Price p_s	+	P_S is reduced since higher p_s reduces trade period for supplier. K_S^* increases since buyer is willing to pay higher acquisition price to avoid higher input prices.
Cost inefficiency u	-	P_S increases since supplier benefits from longer period of trade. K_S^* increases since buyer is willing to pay a higher price to acquire supplier and avoid inefficiencies. K_S^* increases at lower rate due to buyer's option to time internalization of production.
Fixed cost buyer C_B	-	P_S decreases since trade period with supplier is reduced as the likelihood of buyer default increases. K_S^* decreases since buyer has less to benefit from acquisition of supplier. K_S^* reduced at higher rate due to option to time internalizing production.
Volatility σ	-	P_S increases since supplier enjoys longer horizon of trade with buyer. K_S^* decreases since option value of internalizing production gains importance.
Capacity Q	+	P_S decreases since buyer has higher incentive to internalize production and limits supplier horizon of trade. K_S^* increases since buyer has more to benefit from acquisition due to higher produced quantities.

Kedia et al. (2011) find that vertical deals have higher returns in noncompetitive markets. This is in line with our prediction regarding higher prices charged by the supplier, i.e., less competitive upstream markets, resulting in a higher acquisition premium.[5] Moreover, they find no effect of uncertainty on the

[5]Kedia et al. (2011) focus on cases where both the buyer and supplier have market power since their noisy proxies of market power pick up large gains to vertical integration (see p.848). Similarly, both firms in our model have market power. The supplier exercises market power by applying a mark-up to marginal costs, charging a price $p_s > C_s$. A higher price p_s charged by the supplier (while keeping C_s constant) implies a higher Lerner index, $L = (p_s - C_s)/p_s$ (see Billette de Villemeur et al., 2014), resulting in a less competitive upstream market. The buyer firm also has market power

value generated from vertical integration. Our study suggests a negative effect of volatility on the acquisition premium as follows. When the volatility in downstream market becomes higher it increases buyer's option value to build its own production and hence reduces the maximum price the buyer is willing to pay to acquire the supplier. On the other hand, it delays exercising the buyer's option to install its own capacity and hence encourages the supplier to request a higher price since his position is strengthened by a longer expected duration of trade with the buyer. The overall effect is a reduction in the expected merger premium. The above-mentioned no effect of uncertainty found in Kedia et al. (2011) may be thus driven by opposing forces between the predictions of transaction cost theory and the real options effects highlighted in our study, calling for further empirical tests to distinguish between theories.

4. Conclusions

We provide a real options framework for the decision to operate under external procurement or internalize production and derive the value of the buyer and supplier firm and predictions on acquisition premia in vertical acquisitions. Future theoretical developments could consider social welfare implications of firm organizational structure choice, competition in upstream and downstream markets, and strategic considerations in the supply chain (e.g. preemption).

References

Billette de Villemeur, E., Ruble, R., & Versaevel, B. (2014). Investment timing and vertical relationships. International Journal of Industrial Organization, 33, 110–123.

Dixit, A. K., & Pindyck, R. S. 1994. Investment under uncertainty. Princeton university press.

Hackbarth, D., & Mauer, D. C. (2011). Optimal priority structure, capital structure, and investment. The Review of Financial Studies, 25(3), 747-796.

Kedia, S., Ravid, S.A. & Pons, V., (2011). When do vertical mergers create value? Financial Management, 40(4), pp.845-877.

Lafontaine, F., & Slade, M. (2007). Vertical integration and firm boundaries: The evidence. Journal of Economic Literature, 45(3), 629-685.

since it can time the installation of its own input production capacity and interrupt its relation with the supplier (similarly, in Billette de Villemeur et al., 2014, the buyer chooses the optimal investment timing).

Li, W. & Chen, J., (2018). Backward integration strategy in a retailer Stackelberg supply chain. Omega, 75, 118-130.

Leland, H. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. The Journal of Finance, 49 (4), 1213-1252.

Moon, Y., Yao, T., & Park, S. (2011b). Price negotiation under uncertainty. International Journal of Production Economics, 134 (2), 413-423.

Pires, M., & Pereira, P. J. (2020). Leverage, premium and timing in corporate acquisitions. Economics Letters, 188, 108933.

Sutcliffe, K. and Zaheer, A. (1998). Uncertainty in the transaction environment: An empirical test. Strategic Management Journal, 19, 1-23.

Tong, T. W., & Li, Y. (2011). Real options and investment mode: Evidence from corporate venture capital and acquisition. Organization Science, 22(3), 659-674.

Trigeorgis, L., & Tsekrekos, A. E. (2018). Real options in operations research: A review. European Journal of Operational Research, 270(1), 1-24.

Wang, X., Wu, C., & Xu, W. (2015). When to buy or sell in supply chains with the presence of mergers. International Journal of Production Economics, 163, 137-145.

Williamson, O. E. (1971). The vertical integration of production: Market failure considerations. American Economic Review, 61(2), 112–23.

Zormpas, D. (2021). Jointly Held Investment Options and Vertical Relationships. Review of Industrial Organization 58, 513-530