Article

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(Not) Addressing issues in electoral campaigns

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Abstract
Two candidates competing for election may raise some issues for debate during the electoral campaign, while avoiding others. We present a model in which the decision to introduce an issue, or to reply to the opponent's position on one that she raised, may change the further list of topics that end up being discussed. Candidates' strategic decisions are driven by their appraisal of their expected vote share at the end of the campaign. Real phenomena observed during campaigns, like the convergence of the parties to address the same issues, or else their diverging choice on which ones to treat, or the relevance of issue ownership can be explained within our stark basic model. Most importantly, our analysis is based on a novel concept of equilibrium that avoids the (often arbitrary) use of predetermined protocols. This allows us to endogenously predict not only the list of topics that will be touched upon by each candidate, but also the order in which they will be addressed.

Keywords
Electoral campaigns; equilibrium sets of continuation campaigns; issues; issue convergence; issue divergence; issue ownership

JEL Classification: D72

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1. Introduction

Contenders in an electoral campaign may decide to be first in raising some issues for controversy, never to address others, or do it only in response to their opponents' initiative. These possibilities are not only theoretical: the campaigning strategies of candidates result in a large variety of campaign configurations, some of which have been typified by the empirical literature on the subject. Sometimes an issue is not addressed by any contender. Sometimes each one of them addresses issues that the other skips: this case is called issue divergence. In the opposite side, when both candidates enter open discussion of the same topics, we talk of issue convergence. Of course, a host of combinations may arise between these two polar cases.

The strategic reasons why candidates make such choices include many factors. In some cases, taking the lead in raising a subject can be advantageous, while in others a wait-and-see attitude may be better. The candidates from a certain party may be perceived by voters as having an advantage on some subject over those in a different party, maybe for historical or ideological reasons: we then can speak about issue ownership. Its actual role can be tested for, and it can vary substantially over time.

A vast literature on electoral campaigns has distinguished among the sort of decisions we just described and has analyzed the underlying reasons for agents to adopt different strategies when deciding what issues to address, when to do it and with what intensity. The empirical literature documents evidence for both issue convergence and divergence (see e.g. Petrocik, 1996; Spiliotes and Vavreck, 2003; Sigelman and Buell, 2004; Green-Pedersen, 2007).

Our article has two quite different purposes than those of preceding works, both rather methodological. Our first purpose is to generate a large variety of different potential campaigns with a minimum amount of apparatus. For this purpose, we propose a stark model of campaign formation, where two candidates can independently determine on what issues to remain silent, which ones to address and in which order, based on their expected vote share in the election. Addressing an issue can be given two interpretations. One is that by addressing an issue a candidate announces her policy on that issue while staying silent on an issue means that the status quo policy on that issue will prevail in case the candidate is elected. Voters are then assumed to base their vote on the belief that candidates will keep their promises once elected. The other interpretation is that addressing an issue makes the candidate's position on that issue salient in the eyes of the voters. Here voters are assumed to base their vote on the salient positions of the candidates while they believe that the candidates will stick to the status quo policy for the issues they have not addressed during the campaign. Admittedly, our model omits variables whose role has been analyzed by the literature on electoral campaigns, and even those that we explicitly consider are treated in a simplified matter. Our objective is not to deny the relevance of additional considerations or complications, but to point at the fact that these are not strictly needed to generate the basic phenomena we want to highlight.

In fact much of the theoretical analysis on agenda formation highlights the role of campaign funding and its strategic use, proposing models where candidates or parties can affect the preferences of voters by modifying the salience of issues through the allocation of funds (see, e.g. Amorós and Puy, 2013; Aragonès et al., 2015; Dragu and Fan, 2016; Ash et al., 2017; Osório, 2018; Denter, 2020; Balart et al., 2022; Yamaguchi, 2022). Issue convergence or divergence can indeed result from such considerations but, as we will show, they also obtain if candidates do not face any budget constraints. Demange and Van der Straeten (2020) study candidates' information revelation in an electoral campaign where voters are imperfectly informed about candidates' platforms in a multidimensional policy space. Yet, different from our model they restrict to the case where voters' utility is separable across issues. Moreover, it turns out that in their model there is no real strategic interaction between the candidates because the optimal strategy for each candidate is independent of the strategies of other candidates.

Our second purpose in this article is to show that it is possible to endogenously determine the timing in which candidates will adopt their equilibrium decisions, sometimes taking the lead and addressing a new issue, sometimes waiting for their opponent to raise a new one and then responding or not. There are only few papers that study a dynamic model of an electoral campaign as we do. Among the notable exceptions is Kamada and Sugaya (2020) who study a game where candidates choose the time when to announce their policies. In their model, there is an exogenous Poisson process that determines the opportunities for policy announcement while in our model the timing of policy announcements is fully endogenous. Chen and Eraslan (2017) also consider a dynamic model of policy announcements by two parties who take turn in government. Again the timing of policy announcements is not fully endogenous because only the incumbent party has agenda-setting power.

In our article, we use a variant of a solution concept proposed by Dutta et al. (2004) that allows to endogenize the order in which candidates will eventually address the different issues. This enables us to incorporate, as part of our results, the dynamics according to which the candidates ponder the tradeoffs between benefiting from a short term advantage to be proactive or to wait on a given issue, versus the risks of inducing the opponent to raise others on which the advantage is in the other side. The use of this flexible solution concept contrasts with the rigidity of extensive form games, that impose a predetermined order of play which need not coincide with our rationality-based equilibrium prediction.

The article stresses the applicability of our model to highlight different aspects of full campaigns, but it can also be re-interpreted as a model of political debates. In that context, it is interesting to note that a commonly used strategy that is often recommended to contenders is to ignore the questions posed by their adversaries or by the moderator and to insist on the same topics again and again. Yet, under proper interpretation of this attitude, we believe that our model is also useful to analyze this case, among others.

Notice that the solution concept has already been applied by Barberà and Gerber (2022) to study a related but quite different problem involving issues for debate. In that article, members of a parliament could propose issues for debate and then vote on a position for these issues to become part of a legal text. Here the issues are proposed for different purposes, and those who select which ones to use in a campaign (the candidates) are not the same as those who decide the payoff relevant outcomes (the voters in the election).

The article proceeds as follows. In Section 2, we present our model and the equilibrium notion. Section 3 discusses the special cases and a variant of our basic model where limited resources affect the form of equilibrium campaigns by putting constraints on the number of issues that contenders can afford to productively address. We also present a simple example to analyze the effect of issue ownership on issue convergence and issue divergence. Section 4 considers the general case of an arbitrary number of issues. Section 5 concludes the article.

2. A protocol-free electoral campaign

We consider two candidates, A and B, who compete in an electoral campaign. There is a finite set of policy issues $\mathcal{K}$ with cardinality $K \geq 1$. Examples for issues are social security, education, environmental protection, immigration, etc. During an electoral campaign, each candidate may decide to address some issues in $\mathcal{K}$ and not others, and to do it according to a given sequence, either raising a new one, responding to the opponent's previous mention of it, or ignoring it. Our choice of equilibrium notion is intended to endogenize the sequence of such choices, rather than imposing it in the form of a protocol.

If candidate i addresses issue k, we denote this by the pair (k, i). At any point in time an electoral campaign then is characterized by a sequence of pairs $(k, i) \in \mathcal{K} \times \{A, B\}$ recording which candidate has addressed which issue and in what order. We call any such sequence a *campaign*.

Formally, a campaign of length $m \in \{1, \ldots, 2K\}$ is a sequence $s = (s_1, \ldots, s_m)$ with $s_l \in \mathcal{K} \times \{A, B\}$ for all $l = 1, \ldots, m$, and $s_l \neq s_{l'}$ for all $l \neq l'$. The empty campaign $\varnothing$, where no candidate has addressed any issue is defined to have length 0. By S^m we denote the set of campaigns of length m, where $0 \leq m \leq 2K$, and by $S = \bigcup_{m=0}^{2K} S^m$ we denote the set of all campaigns.

For a given campaign $s = (s_1, \ldots, s_m) \in S$ and $(k, i) \in \mathcal{K} \times \{A, B\}$ we write, for short, $(k, i) \in s$ whenever $(k, i) = s_l$ for some $l \in \{1, \ldots, m\}$, and $(k, i) \notin s$ whenever $(k, i) \neq s_l$ for all $l = 1, \ldots, m$. For $s \in S^m$, where $0 \leq m \leq 2K$, and $(k, i) \in \mathcal{K} \times \{A, B\}$, $(k, i) \notin s$, $(s, (k, i))$ denotes the campaign $s' \in S^{m+1}$ with $s'_l = s_l$ for $l = 1, \ldots, m$, and $s'_{m+1} = (k, i)$.

For each candidate $i = A, B$, let $z^i \in \{0, 1\}^K$ be such that $z^i_k = 1$ if candidate i has addressed issue $k \in \mathcal{K}$ and $z^i_k = 0$, otherwise. Any pair (z^A, z^B) with $z^i \in \{0, 1\}^K$ for $i = A, B$, indicating what issues have been addressed by the candidates is called a *state*. Whenever convenient we write the vector z^i as a binary number $z^i = z^i_1 \ldots z^i_K$. For example, if $K = 3$, then $z^i = 101$ means that candidate i has addressed issues 1 and 3 but not issue 2.

Each campaign $s \in S$ defines a state $(z^A(s), z^B(s))$, where $z^i(s) \in \{0, 1\}^K$ denotes which issues have been addressed by candidate $i \in \{A, B\}$ at campaign s, that is

$$z^i_k(s) = \begin{cases} 1 & \text{if } (k, i) \in s \\ 0 & \text{if } (k, i) \notin s \end{cases}$$

for $i = A, B$.

We assume that the candidates' vote shares in the election only depend on the issues that have been addressed by the different candidates during the campaign, but not on the order. Let $p(z^A, z^B)$ stand for the vote share of candidate A at state (z^A, z^B). Then, if the electoral campaign ends at campaign $s \in S$, A's vote share at s is

$$P(s) = p(z^A(s), z^B(s)).$$

If addressing an issue reveals a candidate's position on that issue[1] and if voters are rational, then this assumption of purely state-dependent vote shares is naturally satisfied. We will also see that even under this assumption our equilibrium will still select some orders of play and not others, even if they lead to the same state.

We assume that vote shares are different in each state, that is,

$$p(z^A, z^B) \neq p(\hat{z}^A, \hat{z}^B) \text{ for } (z^A, z^B) \neq (\hat{z}^A, \hat{z}^B).$$

We shall now define the notion of continuation campaigns, that is crucial for the definition of our equilibrium concept. Let $m \in \{0, 1, \ldots, 2K\}$ and let $s \in S^m$. We say that s' is a *continuation campaign* at s if $s'_l = s_l$ for all $l = 1, \ldots, m$. Note that by definition s is a continuation campaign at s. By $C(s)$ we denote the set of continuation campaigns at $s \in S$. A *collection of sets of continuation campaigns* is a family of subsets of $C(s)$ for each $s \in S$.

We shall use a concept of equilibrium that adapts a proposal by Dutta et al. (2004). We have adopted this concept because it is very general and it allows candidates to choose the order in which they express themselves and also that in which they address the different issues.

The notion of equilibrium is defined on collections of sets of continuation campaigns, rather than on specific campaigns. It demands from equilibrium collections to satisfy three conditions, which together provide it a sense of consistency and rationality. The first condition (E1) just demands that any equilibrium continuation at a given campaign s involves either stopping at s or one candidate addressing an additional issue and then following some equilibrium path from there. In other words if some candidate initiates a continuation at a given campaign, then the candidates are required to have rational expectations regarding the further course of the campaign. The second condition (E2) is also very mild and a stopping requirement. Candidates stop adding issues to the campaign if and only if they unanimously agree that this would hurt them. The third condition (E3) requires continuation equilibria to satisfy minimum rationality requirements: (i) Any continuation campaign that makes the candidate who initiates it better off than some equilibrium continuation must be an equilibrium continuation itself. (ii) If there are multiple continuation equilibria, then candidates should not build campaigns that are worse for them than any other equilibrium campaign. (iii) If all continuation equilibria are initiated by the same candidate, then this candidate must be better off than by stopping at the given campaign. Definition 2.1 formalizes these ideas.

Definition 2.1 (**Equilibrium continuation campaigns**) A collection of sets of continuation campaigns $(CE(s))_{s \in S}$ is an **equilibrium collection of sets of continuation campaigns** if the following conditions are satisfied for all $s \in S$:

(E1) $CE(s)$ is a nonempty subset of $\bigcup_{(k,i) \notin s} CE((s, (k, i))) \cup \{s\}$.

(E2) $s \in CE(s)$ if and only if

$$P(s) > P(s') \text{ for all } s' \in \bigcup_{k:(k,A)\notin s} CE((s, (k, A)))$$

and

$$1 - P(s) > 1 - P(s') \text{ for all } s' \in \bigcup_{k:(k,B)\notin s} CE((s, (k, B))).$$

(E3) For $s \in S$ say that the campaign $s' = (s, (k, i), \ldots) \in S$ is **rationalizable** (relative to s) if $s' \in CE((s, (k, i)))$ and there exists an $s'' \in CE(s)$ with either $s'' = (s, (h, j), \ldots)$ for some $(h, j) \neq (k, i)$ or $s'' = s$ such that

$$P(s') > P(s''), \text{ if } i = A,$$
$$\text{and} \quad 1 - P(s') > 1 - P(s''), \text{ if } i = B.$$

Then the following must hold:

(i) If $s' \in \bigcup_{(k,i)\notin s} CE((s, (k, i)))$ is rationalizable, then $s' \in CE(s)$.

(ii) If $s' = (s, (k, i), \ldots) \in CE(s)$ and if either $s \in CE(s)$ or $s'' = (s, (h, j), \ldots) \in CE(s)$ for some $(h, j) \neq (k, i)$, then s' is rationalizable.

(iii) If $s' = (s, (k, i), \ldots) \in CE(s)$ and there exists no $s'' \in CE(s)$ with $s'' = (s, (h, j), \ldots)$ and $j \neq i$, then

$$P(s') > P(s), \quad \text{if } i = A,$$
$$\text{and} \quad 1 - P(s') > 1 - P(s), \quad \text{if } i = B.$$

Definition 2.1 reveals that our equilibrium notion is ordinal, that is, changes in the numerical vote shares do not change the equilibrium collection as long as the order of vote shares is preserved. In order to determine what is an equilibrium collection of continuation campaigns, one must proceed to a backward induction analysis which differs from the one involved in the analysis of subgame perfect equilibria because of the lack of a specific protocol, that is, a given order of moves. One must first decide whether the full campaign satisfies the conditions as a continuation of each of the campaigns where only one candidate has kept silent on only one issue. Then, in view of that, the next step is to analyze whether each of the latter may be equilibrium continuations of campaigns in which either a candidate does not address two of the issues, or both candidates fail to address one issue each. After completing the backward induction, we will have one or several possible collections of equilibrium continuations. Multiplicity is possible, but we will show in Section 4.2 that all equilibrium collections are outcome equivalent.

Notice that our equilibrium notion imposes no restriction on the order in which candidates take actions. Hence, it is perfectly possible for the same candidate to decide addressing several issues in a row, while the other candidate stays still. Yet, we cannot treat such a sequence of actions as if it was a single composed action, because in our model each single action by an agent leaves open the possibility for the opponent to intervene, and our solution concept takes in consideration the consequences of such an intervention on the future development of the game. In other terms, we rule out not only the possibility of simultaneous actions by different candidates, but also the simultaneous adoption of several actions by the same one.

In what follows, after identifying those collections of continuation campaigns that satisfy our equilibrium conditions, we will focus attention on campaigns that are part of these equilibrium continuations and are continuations of themselves and of the empty set. This formalizes the notion that the disclosure of positions starts from scratch at the beginning of the campaign and follows a path leading to a campaign s, after which no further disclosures will be in the interest of anyone, given the continuations predicted from further additions.

Definition 2.2 s^* is an **equilibrium campaign** if there exists an equilibrium collection of sets of continuation campaigns $(CE(s))_s$ with $s^* \in CE(\varnothing)$.

3. Special cases and issue ownership

As we stated in the introduction, one of our purposes is to discuss relevant features of the process of campaign formation, and to do it within a model that is stark, and yet powerful enough to generate the phenomena that have been considered most salient by previous analysts. In this section, we discuss three special cases and use them with a double purpose.

One is to present the reader with examples of the workings of our general model and equilibrium notion. The other is to show that, indeed, their analysis reveals the basic phenomena that we shall later extend to the general case.

In our first subsection, we study the case in which only one issue is at stake and we provide a full characterization of its equilibria. One first conclusion from this analysis is that equilibrium campaigns are unique and that all campaigns are equilibrium campaigns for some ordering of the vote shares at the different states. Extensions of these results will be discussed in Section 4.

In the second subsection, we present the two-issue case, and different examples confirming that, again, all possible forms of campaign may arise, including now, among others, different combinations of issue divergence and convergence. Again, the results obtained here will be extended in more general terms.

In the third subsection, we study an intermediate case: the situation where two issues are available for discussion, but each candidate can only address one of them. The reason to propose this case is that it nicely incorporates the idea that, because of budgetary reasons or others, the candidates may be constrained in their choices. In that case, we can again offer a full characterization of equilibrium configurations.

In the fourth and last subsections, we provide a simple model that allows us to discuss the meaning and consequences of issue ownership on the shape of campaigns.

3.1. One issue

As an introductory example, we first discuss the case where the choices of candidates are only to discuss an issue, or to stay silent. Since there is only one issue we shortly write (A), (B), (A, B), (B, A) for the campaigns, where only candidate A, only candidate B, first candidate A and then B, first candidate B and then A have addressed the unique issue.

Recall that every campaign s defines a state $(z^A(s), z^B(s))$, where $z^i(s) \in \{0, 1\}$ for $i = A, B$, and $z^i(s) = 1$ if and only if candidate $i \in \{A, B\}$ has addressed the issue at campaign s.

In the following, we characterize equilibrium campaigns in terms of the vote shares $p(0, 0)$, $p(1, 0)$, $p(0, 1)$, and $p(1, 1)$. For illustration, consider the case where

$$p(0, 0) < p(1, 1) < \min\{p(1, 0), p(0, 1)\}.$$

In this case, (E1) and (E2) imply that

$$CE(A) = \{(A, B)\} \text{ and } CE(B) = \{(B)\}$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (B), (A, B)\}$ by (E1). Since $p(0, 0) < p(1, 1)$, (E2) implies that $\varnothing \notin CE(\varnothing)$. $CE(\varnothing) = \{(B), (A, B)\}$ violates (ii) in (E3) since (B) is not rationalizable if $(A, B) \in CE(\varnothing)$. $CE(\varnothing) = \{(B)\}$ violates (iii) in (E3) since $p(0, 0) < p(0, 1)$. Hence, the only remaining option is that

$$CE(\varnothing) = \{(A, B)\}$$

which indeed satisfies (E1) to (E3). This case is illustrated in Figure 1.

The example demonstrates how a specific order of moves is determined as part of the equilibrium. This equilibrium order of moves only obtains by coincidence if the order is

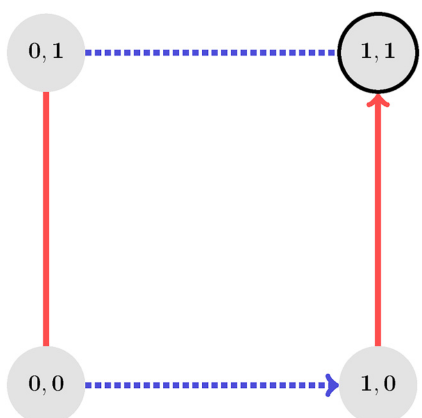

Figure 1. Equilibrium continuations for $p(0, 0) < p(1, 1) < \min\{p(1, 0), p(0, 1)\}$. Edges denote feasible moves between two states. Candidate A moves between adjacent states in the same level of the square (dotted edge) and candidate B moves between adjacent states in different levels of the square (solid edge). Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

imposed exogenously. As a consequence the predicted outcome of the campaign may be substantially different depending on whether the order of moves is endogenous or exogenous. For example, consider the extensive game where first candidate B and then A choose to address the issue or stay silent. Then there is a unique subgame perfect Nash equilibrium where only B addresses the issue, if $p(1, 0) > p(0, 1)$, and where only A addresses the issue, if $p(1, 0) < p(0, 1)$, while according to our equilibrium notion in both cases there is a unique equilibrium campaign where both candidates address the issue.

Table 1 summarizes the necessary and sufficient conditions on the vote shares for all possible equilibrium campaigns.[2] Note that for all orderings of the vote shares the equilibrium campaign is unique. This is true, in particular, for the case where in equilibrium both candidates address the issue. Even though the vote shares of the candidates are independent of the order in which the issue is addressed by them, in equilibrium there is a unique order.

Table 1. Vote shares and equilibrium campaigns for one issue.

CE (∅)	Conditions on vote shares
{∅}	$p(1, 1) < p(0, 0) < p(0, 1)$ and $p(1, 1) < p(1, 0)$
	or $p(1, 0) < p(0, 0) < p(1, 1)$ and $p(0, 1) < p(1, 1)$
	or $p(1, 0) < p(0, 0) < p(0, 1)$ and $p(1, 0) < p(1, 1) < p(0, 1)$
{(*A*)}	$p(0, 0) < p(1, 0) < p(1, 1)$
{(*B*)}	$p(1, 1) < p(0, 1) < p(0, 0)$
{(*A*, *B*)}	$p(0, 0) < p(1, 1) < p(1, 0)$
{(*B*, *A*)}	$p(0, 1) < p(1, 1) < p(0, 0)$

3.2. Two issues

Let there be two issues, that is $K = 2$. Campaigns then can be represented in the 4-hypercube, also called *tesseract*, in Figure 2, where nodes correspond to states and the edges denote feasible moves: There is an edge between two states if and only if there is exactly one candidate and one issue which is addressed by the candidate in one state, but not in the other.

In the following, we will present some examples to illustrate that any outcome can obtain in equilibrium, that is, for any possible state we can find vote shares such that the given state is the unique outcome in equilibrium. In particular, we present examples for issue convergence (both candidates address the same issue) and issue divergence (both candidates address different issues). The general characterization of equilibrium campaigns in terms of properties of the vote shares can be found in Section 4.2 where we consider the general case with an arbitrary number of issues.

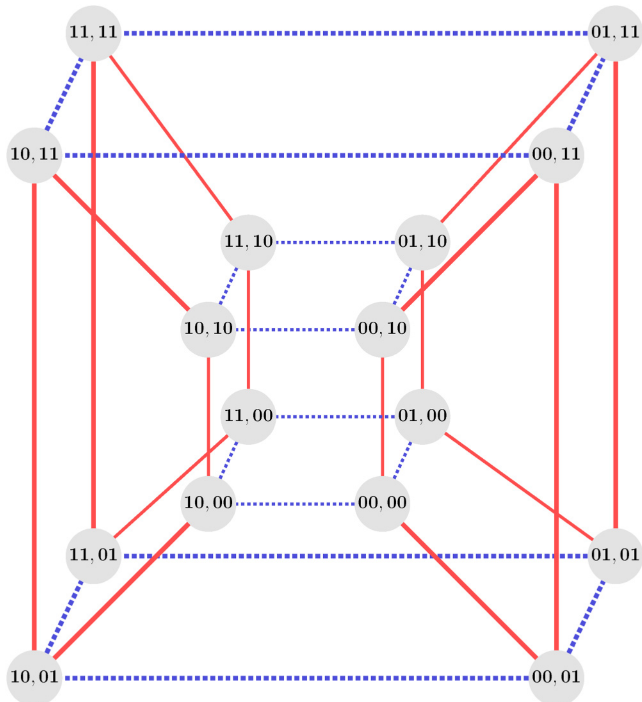

Figure 2. Representation of campaigns in a tesseract. Edges denote feasible moves between two states. Candidate *A* moves between adjacent states in the same level of the tesseract (dotted edge). Candidate *B* moves between adjacent states in different levels of the tesseract (solid edge).

Example 3.1 This is an example for issue convergence. Let candidate *A*'s vote share be given in the following table:

		z^B			
		00	10	01	11
z^A	00	0.5	0.3	0.7	0.65
	10	0.75	0.45	0.9	0.95
	01	0.6	0.4	0.1	0.2
	11	0.8	0.85	0.55	0.35

Figure 3 illustrates the equilibrium continuation campaigns. As we see there is a unique equilibrium campaign, where first candidate *B* and then *A* address issue 1:

$$CE(\varnothing) = \{((1, B), (1, A))\}.$$

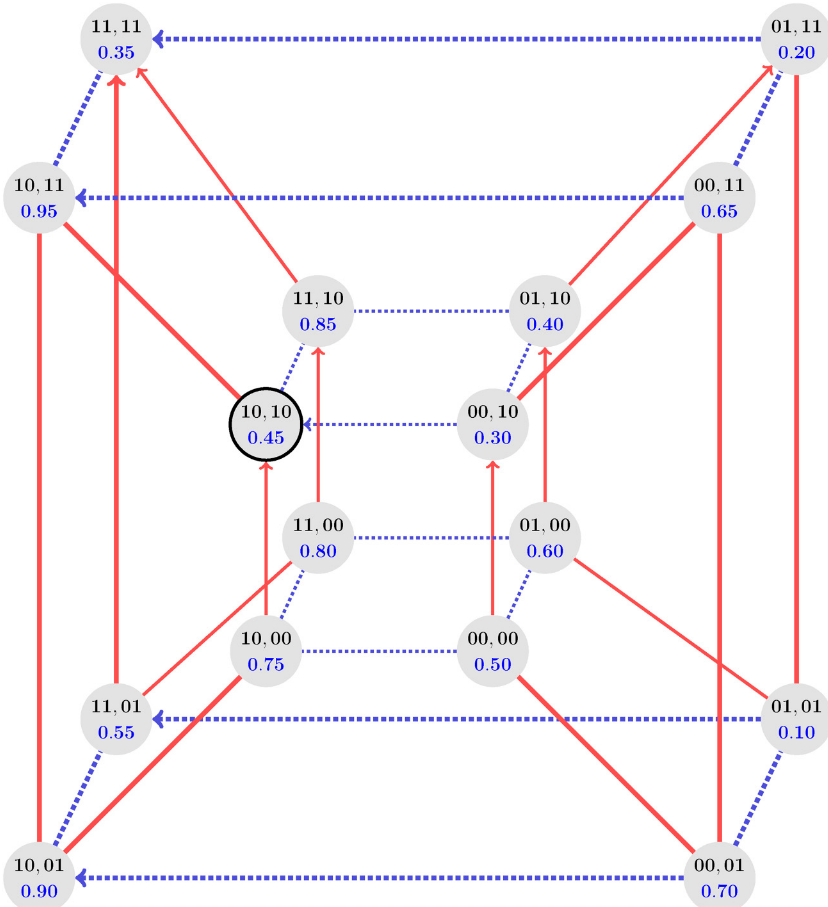

Figure 3. Equilibrium continuations in Example 3.1. The lower number in a node is candidate *A*'s vote share at the given state. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at ∅.

Example 3.2 This is an example for issue divergence. Let candidate *A*'s vote share be given in the following table:

		z^B			
		00	10	01	11
z^A	00	0.5	0.88	0.31	0.73
	10	0.72	0.8	0.58	0.74
	01	0.2	0.67	0.25	0.55
	11	0.51	0.7	0.43	0.62

Figure 4 illustrates the equilibrium continuation campaigns. As we see there is a unique equilibrium campaign, where first candidate A addresses issue 1 and then candidate B addresses issue 2:

$$CE(\varnothing) = \{((1, A), (2, B))\}.$$

Example 3.3 This is an example where no candidate addresses any issue in equilibrium. Let candidate A's vote share be given in the following table:

		z^B			
		00	10	01	11
z^A	00	0.5	0.7	0.68	0.76
	10	0.32	0.28	0.2	0.58
	01	0.3	0.21	0.29	0.59
	11	0.24	0.41	0.42	0.19

Figure 5 illustrates the equilibrium continuation campaigns. Note that there are multiple equilibrium continuations at state $(11, 00)$: If $z(s) = (11, 00)$, then $CE(s) = \{(s, (1, B), (2, B))\}$ and $CE(s) = \{(s, (2, B), (1, B))\}$ are two singleton sets of continuation campaigns which are outcome equivalent. Despite this multiplicity the equilibrium campaign is unique and given by

$$CE(\varnothing) = \{\varnothing\}.$$

Example 3.4 This is an example where both candidates address both issues in equilibrium. Let candidate A's vote share be given in the following table:

		z^B			
		00	10	01	11
z^A	00	0.5	0.2	0.19	0.09
	10	0.72	0.66	0.61	0.49
	01	0.75	0.59	0.7	0.45
	11	0.82	0.79	0.78	0.74

Figure 6 illustrates the equilibrium continuation campaigns. Note that there are several states with multiple equilibrium continuations, where all equilibrium continuations are initiated by the same candidate and lead to the same outcome. This is

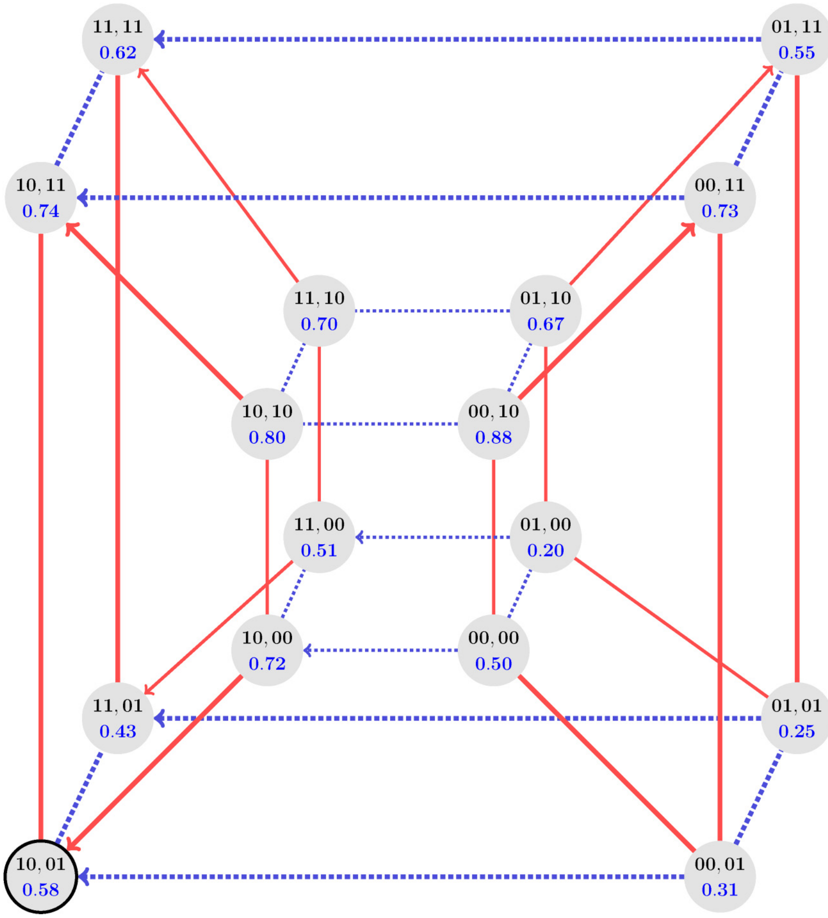

Figure 4. Equilibrium continuations in Example 3.2. The lower number in a node is candidate *A*'s vote share at the given state. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at ∅.

also true for the initial state (00, 00), so we get multiple equilibrium collections and multiple equilibrium campaigns which all give the same outcome, where both candidates have addressed both issues. All equilibrium campaigns are initiated by candidate A which is the candidate who gains from moving to state (11, 11) relative to the initial state (00, 00):

$$
\begin{aligned}
& CE(\varnothing) = \{((1, A), (2, A), (1, B), (2, B))\} \\
\text{or} \quad & CE(\varnothing) = \{((1, A), (2, A), (2, B), (1, B))\} \\
\text{or} \quad & CE(\varnothing) = \{((2, A), (1, B), (1, A), (2, B))\} \\
\text{or} \quad & CE(\varnothing) = \{((2, A), (2, B), (1, A), (1, B))\}.
\end{aligned}
$$

3.3. Candidates can only address one issue out of two

In the following variant of our model, there are still two issues, that is, $K = 2$, but we assume that candidates can address at most one of them. This case is of special interest because it provides a simple way to express that candidates may be constrained by exogenous considerations. As we already mentioned in Section 1, there exist elaborate models that study the consequences of campaigns facing financial constraints. Our results here already allow for different phenomena of interest even in our simple case.

Let $\bar{S} \subset S$ denote the set of campaigns under the restriction that candidates can address at most one issue. The corresponding set of feasible states then is

$$Z = \{(z^A, z^B) | \, z_1^i + z_2^i \leq 1 \text{ for } i = A, B\}$$

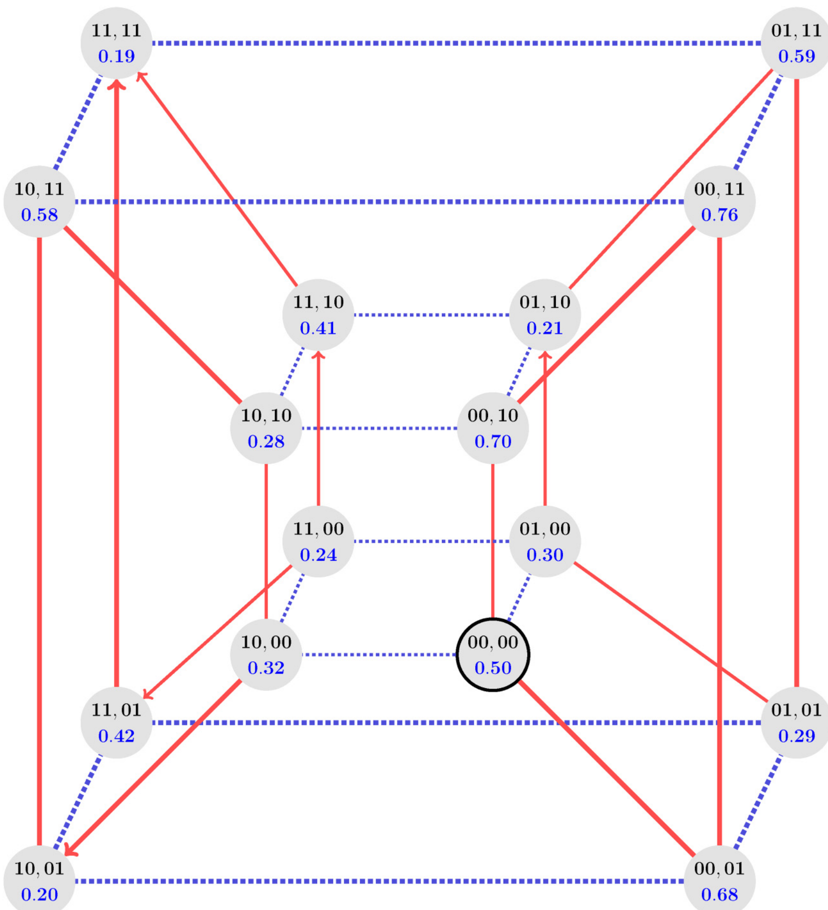

Figure 5. Equilibrium continuations in Example 3.3. The lower number in a node is candidate *A*'s vote share at the given state. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at ∅.

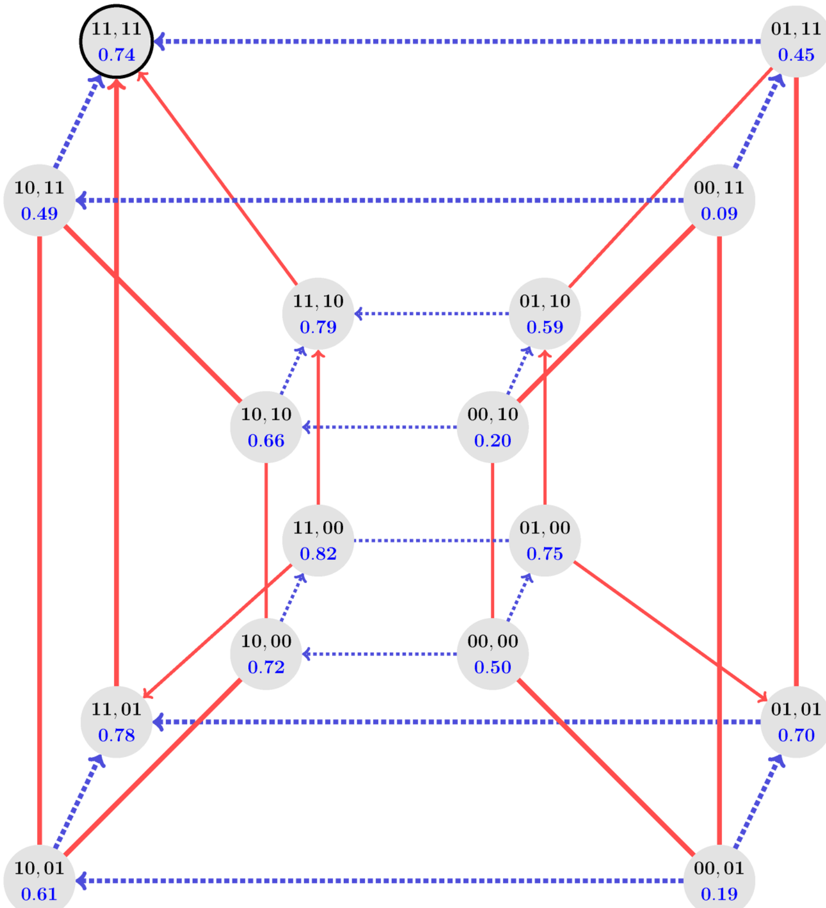

Figure 6. Equilibrium continuations in Example 3.4. The lower number in a node is candidate *A*'s vote share at the given state. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\emptyset$.

and possible campaigns can be represented in a grid, where edges denote the feasible moves between two states (see Figure 7).

The following proposition shows that equilibrium continuations are unique.

Proposition 3.1 There exists a unique equilibrium collection of sets of continuation campaigns $(CE(s))_{s\in\bar{S}}$ and for all $s \in \bar{S}$ there is a unique equilibrium continuation in $CE(s)$, that is, $CE(s) = \{s'\}$ for some $s' \in \bar{S}$.

Since the equilibrium outcome at any campaign s is unique and only depends on the state $z(s)$, that is, on the set of issues that have been addressed by the candidates in s, it

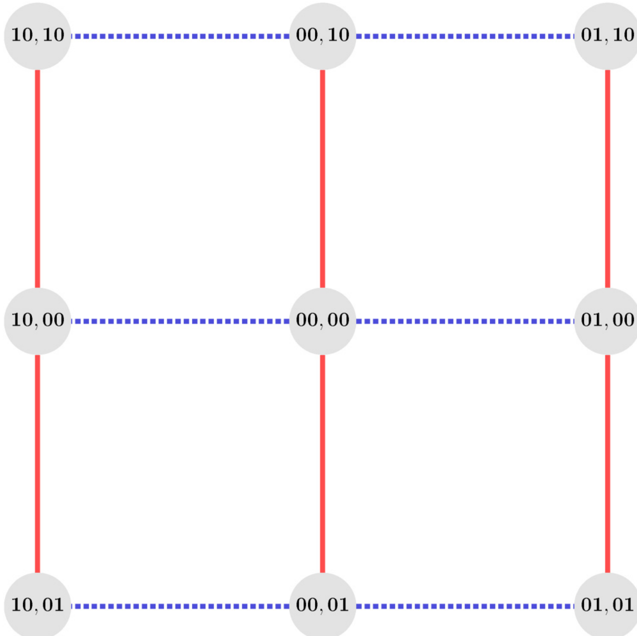

Figure 7. Representation of states in a grid. Edges denote the feasible moves between two states. Candidate *A* moves along the horizontal edges (dotted). Candidate *B* moves along the vertical edges (solid).

makes sense to define

$$p^*(z^A, z^B) \tag{1}$$

as the vote share of candidate A in the unique continuation equilibrium at any campaign s with $z(s) = (z^A, z^B)$. The proof of Proposition 3.1 then provides a full characterization of the unique equilibrium outcome in terms of p^*:

$$\begin{aligned} p^*(z^A, z^B) &= p(z^A, z^B), \text{ if } z_1^i + z_2^i = 1 \text{ for } i = A, B, \\ p^*(z^A, 00) &= \min\{p(z^A, 00), p(z^A, 10), p(z^A, 01)\}, \text{ if } z_1^A + z_2^A = 1, \\ p^*(00, z^B) &= \max\{p(00, z^B), p(10, z^B), p(01, z^B)\} \text{ if } z_1^B + z_2^B = 1. \end{aligned} \tag{2}$$

and

$$p^*(00, 00) = \begin{cases} \min\{p^*(00, 10), p^*(00, 01)\}, & \text{if } p(00, 00) > \min\{p^*(00, 10), p^*(00, 01)\} \\ p(00, 00), & \text{if } \min\{p^*(00, 10), p^*(00, 01)\} > p(00, 00) > \\ & \max\{p^*(10, 00), p^*(01, 00)\} \\ \max\{p^*(10, 00), p^*(01, 00)\}, & \text{if } \max\{p^*(10, 00), p^*(01, 00)\} > p(00, 00) \end{cases} . \tag{3}$$

In particular, it is true that

$$\min\{p^*(00,10), p^*(00,01)\} \geq p^*(00,00) \geq \max\{p^*(10,00), p^*(01,00)\}.$$

Equations (2) and (3) imply that any outcome can obtain for some ordering of the vote shares at the different states. For example, candidate A addressing issue 1 and B addressing issue 2 ('issue divergence') is the unique equilibrium outcome if and only if $p^*(00,00) = p(10,01)$ which by (2) and (3) is equivalent to

$$\max\{p(00,01), p(01,01)\} < p(10,01) < \max\{p(00,10), p(10,10), p(01,10)\}$$
$$\text{and} \quad p(10,01) < p(00,00), \tag{4}$$

or

$$\min\{p(01,00), p(01,10), p(01,01)\} < p(10,01) < \min\{p(10,00), p(10,10)\}$$
$$\text{and} \quad p(00,00) < p(10,01). \tag{5}$$

Also, both candidates addressing issue 1 ('issue convergence') is the unique equilibrium outcome if and only if $p^*(00,00) = p(10,10)$ which by (2) and (3) is equivalent to

$$\max\{p(00,10), p(01,10)\} < p(10,10) < \max\{p(00,01), p(10,01), p(01,01)\}$$
$$\text{and} \quad p(10,10) < p(00,00), \tag{6}$$

or

$$\min\{p(01,00), p(01,10), p(01,01)\} < p(10,10) < \min\{p(10,00), p(10,01)\}$$
$$\text{and} \quad p(00,00) < p(10,10). \tag{7}$$

3.4. Issue ownership

We will now explore the effect of issue ownership on equilibrium outcomes in the limited case we discussed in the last subsection where candidates can address at most one out of two issues. Issue ownership captures the fact that a candidate has an a priori advantage in dealing with some issue and that this is reflected in her vote share (see Petrocik, 1996). Therefore, defining ownership first requires to introduce some notion of competence or reliability. We then illustrate how issue ownership may lead to issue convergence or divergence in more specific terms.

Assume that all voters care about one and only one issue and let α be the share of voters who only care about issue 1 and let $1-\alpha$ be the share of voters who only care about issue 2, where $0 < \alpha < 1$. Moreover, for $k = 1, 2$, let γ_k be the share of voters who consider candidate A more competent or more reliable on issue k than candidate B.

If the candidates address different issues (issue divergence) voters vote for the candidate who has addressed the issue they care about. If the candidates address the same issue

(issue convergence), the voters who care about this issue vote for the candidate they consider more competent on the issue and the voters who do not care about the issue split their vote evenly between the candidates.[3] Moreover, if one candidate does not address any issue and the other candidate addresses issue k, the voters who care about issue k vote for the candidate who addresses this issue and the voters who do not care about the issue split their vote evenly. Finally, if no candidate addresses any issue all votes are split evenly.

Under these assumptions we get the following vote shares of candidate A at the different outcomes of a campaign:

z^A \ z^B	00	10	01
00	0.5	$0.5(1-\alpha)$	0.5α
10	$\alpha + 0.5(1-\alpha)$	$\gamma_1\alpha + 0.5(1-\alpha)$	α
01	$1-\alpha+0.5\alpha$	$1-\alpha$	$\gamma_2(1-\alpha)+0.5\alpha$

Note that the vote shares differ across states if and only if $\alpha \neq 0.5$ and $\gamma_k \neq 0.5$ for $k = 1, 2$, which we will assume in the following. We will now explore under which conditions on the parameters of our voting model the candidates address the same or different issues in equilibrium. In particular, we are interested in the implications of issue ownership, which is modeled by the parameters γ_1 and γ_2. To this end we define candidate A to 'own' issue k if a majority of voters considers A more competent on issue k than B, that is, if $\gamma_k > 0.5$. Similarly, B owns issue k if $\gamma_k < 0.5$.

Consider first the case of issue divergence. W.l.o.g. let candidate A address issue 1 and let candidate B address issue 2 in equilibrium. Then $p^*(00, 00) = p(10, 01)$ which by (4) and (5) is the case if and only if

$$\max\{0.5\alpha, \gamma_2(1-\alpha)+0.5\alpha\} < \alpha < \max\{0.5(1-\alpha), \gamma_1\alpha + 0.5(1-\alpha), 1-\alpha\}$$
$$\text{and} \quad \alpha < 0.5, \tag{8}$$

or

$$\min\{1-\alpha+0.5\alpha, 1-\alpha, \gamma_2(1-\alpha)+0.5\alpha\} < \alpha < \min\{\alpha+0.5(1-\alpha), \gamma_1\alpha+0.5(1-\alpha)\}$$
$$\text{and} \quad 0.5 < \alpha. \tag{9}$$

Equation (8) holds if and only if

$$\frac{\gamma_2}{\gamma_2 + 0.5} < \alpha < 0.5 \tag{10}$$

and (9) holds if and only if

$$0.5 < \alpha < \frac{0.5}{1.5 - \gamma_1}. \tag{11}$$

From (10) and (11), we conclude that a necessary and sufficient condition for issue divergence is that one of the two candidates owns the issue she addresses and that a majority of voters cares about this issue but there is also a sufficiently large share of voters who care about the other issue.[4] The required minimum share of voters is decreasing in the share of voters who consider the candidate who addresses the issue she owns more competent on this issue than the other candidate.

Consider next the case of issue convergence. W.l.o.g. let both candidates address issue 1 in equilibrium. Then $p^*(00, 00) = p(10, 10)$ which by (6) and (7) is the case if and only if

$$\max\{0.5(1-\alpha), 1-\alpha\} < \gamma_1\alpha + 0.5(1-\alpha) < \max\{0.5\alpha, \alpha, \gamma_2(1-\alpha) + 0.5\alpha\} \tag{12}$$

$$\text{and} \quad \gamma_1\alpha + 0.5(1-\alpha) < 0.5, \tag{13}$$

or

$$\min\{1-\alpha+0.5\alpha, 1-\alpha, \gamma_2(1-\alpha)+0.5\alpha\} < \gamma_1\alpha + 0.5(1-\alpha) < \min\{\alpha + 0.5(1-\alpha), \alpha\} \tag{14}$$

$$\text{and} \quad \gamma_1\alpha + 0.5(1-\alpha) > 0.5. \tag{15}$$

Equations (12) and (13) hold if and only if

$$\gamma_1 < 0.5 \quad \text{and} \quad \alpha > \frac{0.5}{0.5 + \gamma_1} \tag{16}$$

and (14) and (15) hold if and only if

$$\gamma_1 > 0.5 \quad \text{and} \quad \alpha > \frac{0.5}{1.5 - \gamma_1}. \tag{17}$$

From (16) and (17), we conclude that a necessary and sufficient condition for issue convergence is that one candidate owns the issue addressed by both candidates and that a sufficiently large share of voters care about this issue. The required minimum share is always larger than 0.5 and increasing in the share of voters who consider the issue owner more competent than the other candidate.

4. The general case

The analysis of special cases in the preceding section has already hinted at two basic facts. Firstly, in our model and under our assumptions, all equilibria are outcome equivalent, and secondly, any campaign may arise as an equilibrium for some adequate combination of vote shares. We shall now prove, in Sections 4.2 and 4.3, that these features of the special cases are, indeed, valid for our general model. Before we do that, in Section

4.1, we address the natural question whether one can provide satisfactory foundations for the vote shares that we use all along.

4.1. Microfoundations for the vote shares

As we have already pointed out, what matters for equilibrium is just the ordering of the vote shares for all states. We will now argue that any ordering of vote shares can be explained as being associated with some profile of voters' preferences over candidate specific announcements on issues during the campaign. This follows from a result by Debord (1987) who proves that for a finite set of alternatives X and for any collection $(m(x, y))_{x \neq y}$ of integers with $m(x, y) = -m(y, x)$ for all $x \neq y$ in X such that $m(x, y)$ is either even for all $x \neq y$ or odd for all $x \neq y$ in X, there exists a set of voters V and strict preference orderings P_i for all voters $i \in V$ such that $m(x, y)$ is the majority margin for x over y for all alternatives $x \neq y$ at the given preference profile, that is, $m(x, y) = |\{i \in V | xP_iy\}| - |\{i \in V | yP_ix\}|$.[5]

Since

$$m(x, y) = 2N\frac{|\{i \in V | xP_iy\}|}{N} - N,$$

where $N = |V|$ it follows that the majority margin for x over y is increasing in the proportion of voters strictly preferring x over y which is the vote share for alternative x. Hence, any given ordering of vote shares can be generated by some preference profile.

Although this general justification seems quite convincing, we can also provide a more concrete microfoundation for the vote shares. Assume that each voter only cares about one single issue and let α_k with $0 < \alpha_k < 1$ be the share of voters who only care about issue k. Then $\sum_{k=1}^{K} \alpha_k = 1$. Voters have ideal positions on the issue they care about and their ideal positions are uniformly distributed in [0, 1]. For every issue $k \in \mathcal{K}$ there is a status quo position $x_k^0 \in [0, 1]$ and every candidate $i \in \{A, B\}$ has a position $\bar{x}_k^i \in [0, 1]$ that will be implemented if the candidate wins the election and has addressed issue k in the campaign. If the winning candidate has not addressed issue k, the status quo x_k^0 is implemented. Voters then vote for the candidate for whom the implemented position on the issue they care about is closest to their ideal position. If no candidate has addressed issue k the voters who care about issue k split their votes evenly among the two candidates.

As before let $z^i \in \{0, 1\}^K$ be such that $z_k^i = 1$ if candidate i has addressed issue k in the campaign and $z_k^i = 0$, otherwise. Among the voters who care for issue k let $p_k(z_k^A, z_k^B)$ be the share of voters who vote for candidate A given $(z_k^A, z_k^B) \in \{0, 1\} \times \{0, 1\}$.[6] The vote share of candidate A at state (z^A, z^B) then is given by

$$p(z^A, z^B) = \sum_{k=1}^{K} \alpha_k p_k(z_k^A, z_k^B).$$

This voting model can also be used to generate separable vote shares, an assumption that facilitates the proof that any state can be an equilibrium outcome (see Corollary 4.1). Vote shares are separable, if for all issues k, addressing issue k always increases or always decreases the candidate's vote share independent of the state. Formally, separability is defined as follows.

Definition 4.1 For $k = 1, \ldots, K$, let $e^k \in \{0, 1\}^K$ be such that $e^k_k = 1$ and $e^k_l = 0$ for all $l \neq k$. The vote shares $p(\cdot, \cdot)$ are **separable**, if for all $z^A, \hat{z}^A, z^B, \hat{z}^B \in \{0, 1\}^K$, and for all k with $z^A_k = \hat{z}^A_k = 0$,

$$p(z^A + e^k, z^B) > p(z^A, z^B) \;\Leftrightarrow\; p(\hat{z}^A + e^k, \hat{z}^B) > p(\hat{z}^A, \hat{z}^B),$$

and for all k with $z^B_k = \hat{z}^B_k = 0$,

$$p(z^A, z^B + e^k) > p(z^A, z^B) \;\Leftrightarrow\; p(\hat{z}^A, \hat{z}^B + e^k) > p(\hat{z}^A, \hat{z}^B).$$

Let $\mathcal{K}^A$ ($\mathcal{K}^B$) be the set of issues such that candidate A (B) can increase her vote share relative to any state (z^A, z^B) by addressing an issue in these sets if no further issues are being addressed, that is,

$$\mathcal{K}^A = \{k \,|\, p(z^A + e^k, z^B) > p(z^A, z^B) \text{ for all } (z^A, z^B) \text{ with } z^A_k = 0\},$$

and

$$\mathcal{K}^B = \{k \,|\, p(z^A, z^B + e^k) < p(z^A, z^B) \text{ for all } (z^A, z^B) \text{ with } z^B_k = 0\}.$$

Note that in the voting model presented above p is separable if p_k is separable for all k and in this case

$$\mathcal{K}^A = \{k \,|\, p_k(1, z^B_k) > p_k(0, z^B_k) \text{ for } z^B_k = 0, 1\},$$

and

$$\mathcal{K}^B = \{k \,|\, p_k(z^A_k, 1) < p_k(z^A_k, 0) \text{ for } z^A_k = 0, 1\}.$$

Moreover, separability of p_k for all k and any $(\mathcal{K}^A, \mathcal{K}^B)$ can be generated by choosing $\bar{x}^A_k, \bar{x}^B_k, x^0_k$ accordingly for all k. For example,

- if $\bar{x}^A_k = 0.3, \bar{x}^B_k = 1, x^0_k = 0.5$, then $k \notin \mathcal{K}^A$ and $k \notin \mathcal{K}^B$,
- if $\bar{x}^A_k = 0.6, \bar{x}^B_k = 0.9, x^0_k = 0.2$, then $k \in \mathcal{K}^A$ and $k \notin \mathcal{K}^B$,
- if $\bar{x}^A_k = 0.9, \bar{x}^B_k = 0.5, x^0_k = 0.4$, then $k \notin \mathcal{K}^A$ and $k \in \mathcal{K}^B$,
- if $\bar{x}^A_k = 0.2, \bar{x}^B_k = 0.85, x^0_k = 0.1$, then $k \in \mathcal{K}^A$ and $k \in \mathcal{K}^B$,

and in all cases p_k is separable.

Let us also remark, finally, that our example in Section 3.4 regarding the role of issue ownership included a particular explanation about the considerations regarding the formation of their preferences, this time partly justified by consideration of the candidates' competence. Although we just presented an example, we think that our modeling decisions are consistent and provide a hint for further development of a more general model.

4.2. A uniqueness result

We first prove that all continuation equilibria are outcome equivalent and that the equilibrium outcome at any campaign s only depends on the state $z(s)$.

Theorem 4.1

(i) There exists an equilibrium collection of sets of continuation campaigns and for all equilibrium collections of sets of continuation campaigns $(CE(s))_{s\in S}$ and for all $s \in S$, there is a unique continuation equilibrium in $CE(s)$.

(ii) All equilibrium collections of sets of continuation campaigns are outcome equivalent, that is, if $(CE(s))_{s\in S}$ and $(\widehat{CE}(s))_{s\in S}$ are two equilibrium collections of sets of continuation campaigns, then for all $s \in S$, $CE(s) = \{s'\}$ and $\widehat{CE}(s) = \{s''\}$ implies that $z(s') = z(s'')$.

(iii) For all $s \in S$ the continuation equilibrium state at s only depends on $z(s)$. That is, if $s, \hat{s} \in S$ are such that $z(s) = z(\hat{s})$ and if $CE(s) = \{s'\}$ and $CE(\hat{s}) = \{\hat{s}'\}$, then $z(s') = z(\hat{s}')$.

Since all equilibrium collections are outcome equivalent and there is a unique equilibrium outcome at any campaign s that only depends on the state $z(s)$ we can again define $p^*(z^A, z^B)$ as the vote share of candidate A in the unique continuation equilibrium at any campaign s with $z(s) = (z^A, z^B)$. The proof of Theorem 4.1 provides a full characterization of the unique equilibrium outcome in terms of p^*:

$$p^*(z^A, z^B) = \begin{cases} \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k), & \text{if } p(z^A, z^B) > \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k) \\ p(z^A, z^B), & \text{if } \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k) > p(z^A, z^B) > \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B) \\ \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B), & \text{if } \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B) > p(z^A, z^B) \end{cases} \tag{18}$$

where the minimum (maximum) over the empty set is defined to be ∞ $(-\infty)$. In particular, it is true that

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq p^*(z^A, z^B) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B).$$

Theorem 4.1 can also be used to determine all equilibrium collections of sets of continuation campaigns. The construction is by backwards induction:

Let s be a full campaign, that is, $z(s) = (1...1, 1...1)$. Then it is obviously true that $CE(s) = \{s\}$. Now consider a campaign s with $z(s) < (1...1, 1...1)$ and assume that $CE(\hat{s})$ has been determined for all $\hat{s}$ with $z(\hat{s}) > z(s)$. Then from (18) there are three cases.

1. If $p^*(z^A, z^B) = p(z^A, z^B)$ the campaign ends at s and $CE(s) = \{s\}$.
2. If $p^*(z^A, z^B) = \min_{k:z_k^B=0} p^*(z^A, z^B + e^k)$, then $CE(s) = \{\hat{s}\}$ for some $\hat{s}$ with
   $\hat{s} = (s, (k, B), \ldots)$, where $CE((s, (k, B))) = \{\hat{s}\}$ and
   $p^*(z^A, z^B + e^k) = \min_{l:z_l^B=0} p^*(z^A, z^B + e^l)$.
3. If $p^*(z^A, z^B) = \max_{k:z_k^A=0} p^*(z^A + e^k, z^B)$, then $CE(s) = \{\hat{s}\}$ for some $\hat{s}$ with
   $\hat{s} = (s, (k, A), \ldots)$, where $CE((s, (k, A))) = \{\hat{s}\}$ and
   $p^*(z^A + e^k, z^B) = \max_{l:z_l^A=0} p^*(z^A + e^l, z^B)$.

4.3. Anything goes

We will now provide sufficient conditions on the vote shares to obtain arbitrary equilibrium outcomes. To this end we consider separable vote shares (see Definition 4.1). The use of this restriction just strengthens the message of our result, since it proves that any state can be an equilibrium outcome even under stringent conditions.

Theorem 4.2 Let the vote shares be separable and let

$$\mathcal{K}^A = \{k | p(z^A + e^k, z^B) > p(z^A, z^B) \text{ for all } (z^A, z^B) \text{ with } z_k^A = 0\},$$

and

$$\mathcal{K}^B = \{k | p(z^A, z^B + e^k) < p(z^A, z^B) \text{ for all } (z^A, z^B) \text{ with } z_k^B = 0\}.$$

For $i = A, B,$ let $\bar{z}^i \in \{0, 1\}^K$ be given by

$$\bar{z}_k^i = \begin{cases} 1, & \text{if } k \in \mathcal{K}^i \\ 0, & \text{else} \end{cases}.$$

For all (z^A, z^B), let $p^*(z^A, z^B)$ be the unique vote share of candidate A in all continuation equilibria at s where $z(s) = (z^A, z^B)$. Then for all (z^A, z^B),

$$p^*(z^A, z^B) = p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big).$$

In particular, $p^*(\mathbf{0}, \mathbf{0}) = p(\bar{z}^A, \bar{z}^B)$.[7]

According to the theorem, the unique vote share in all continuation equilibria at a given campaign with state (z^A, z^B) is the vote share in the state where both candidates have addressed all issues in $\mathcal{K}^A$ and $\mathcal{K}^B$, respectively, plus all issues they have already addressed in (z^A, z^B). Since we have assumed that $p(z^A, z^B) \neq p(\hat{z}^A, \hat{z}^B)$ for $(z^A, z^B) \neq (\hat{z}^A, \hat{z}^B)$ this implies that the unique outcome at any equilibrium campaign is such that the candidates address all issues in $\mathcal{K}^A$ and $\mathcal{K}^B$ and nothing else.

The voting model introduced in Section 4.1, where each voter cares about one issue only, shows that for arbitrary sets of issues $\mathcal{K}^A$, $\mathcal{K}^B$, there exist separable vote shares such that candidate $i \in \{A, B\}$ increases her vote share relative to any state (z^A, z^B) if and only if i addresses an issue in $\mathcal{K}^i$. Hence, we obtain an 'anything goes' result as an immediate corollary to Theorem 4.2.

Corollary 4.1 For any $\mathcal{K}^A, \mathcal{K}^B \subset \mathcal{K}$ there exist vote shares $p(z^A, z^B)$ for all states $(z^A, z^B) \in \{0, 1\}^{2K}$ such that there is a unique equilibrium outcome where candidate A addresses the issues in $\mathcal{K}^A$ and candidate B addresses the issues in $\mathcal{K}^B$.

5. Conclusion

We offer a stark but attractive model to study the phenomena associated with campaign formation and analyze it by using a protocol-free solution concept that adds realism to the study of campaign formation decisions by dispensing with any aprioristic assumptions about a fixed order of play. Solving for equilibria may be demanding if there are many issues, but the backwards induction procedure is no more complex than solving for subgame perfect Nash equilibria in an extensive form game where candidates move according to some exogenously given order. Our analysis highlights the effect of allowing players to decide whether or not to move at any point of the game, and it identifies the order in which rational players will take action on their way to equilibrium outcomes. Even in the case considered here, where campaign payoffs are independent of the order in which issues were included in the agenda, not any path leading to an equilibrium outcome is admissible as part of an equilibrium in our sense. This suggests that strategic campaign designers need to determine not only the set of issues that a candidate should address in the campaign but they also have to determine the order in which the issues are to be addressed, both as a reaction to what the competitor has said before and in anticipation of how the competitor will react to the own announcements.

Our model and results highlight the interaction between issues, thus challenging the possibility of identifying the role of each one of them separately. We show that depending on the vote shares at all states there will be different combinations of silences and voice in equilibrium campaigns. We also provide sufficient conditions for any campaign configuration to arise in equilibrium.

Moreover, we present a general result on the uniqueness of equilibrium outcomes and on properties of the candidates' vote shares in equilibrium. For some relevant special cases we have also provided explicit characterizations of equilibria.

Finally, we show that issue ownership is a useful concept to better understand the shape of equilibrium campaigns, but not the unique determinant of their shape, even in simple contexts. We do not deny the importance in reality of many variables that our model omits. Our concern has been to prove that, in fact, relevant insights can be obtained even before appealing to further qualifications.

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Appendix

Proof of Proposition 3.1: We will prove that there exists a unique equilibrium collection of sets of continuation campaigns $(CE(s))_{s\in\bar{S}}$ and that for all $s \in \bar{S}$ there is a unique equilibrium continuation in $CE(s)$, that is, $CE(s) = \{s'\}$ for some $s' \in \bar{S}$. Since $z(s')$ only depends on $z(s)$ candidate A's vote share in the unique continuation equilibrium at any campaign s only depends on $z(s) = (z^A, z^B)$. We denote this equilibrium vote share by $p^*(z^A, z^B)$.

Let $s \in \bar{S}$ with $z(s) = (z^A, z^B)$. The claim is proved by backwards induction starting with the case where both candidates have addressed one of the issues at s, then considering the case where only one of the candidates has addressed one of the issues at s and finally the case where none of the candidates has addressed one of the issues at s.

1. If $z_1^i + z_2^i = 1$ for $i = A, B$, then (E1) implies that $CE(s) = \{s\}$ for any s with $z(s) = (z^A, z^B)$. Hence, the equilibrium outcome at s is unique and only depends on $z(s)$. This implies that $p^*(z^A, z^B) = p(z^A, z^B)$.
2. Next consider $(z^A, z^B) = (z^A, 00)$ with $z_1^A + z_2^A = 1$ and let s be such that $z(s) = (z^A, 00)$. Then (E1) implies that $CE(s) \subseteq \{s, (s, (1, B)), (s, (2, B))\}$. By (E2) $s \in CE(s)$ if and only if $p(z^A, 00) < \min\{p(z^A, 10), p(z^A, 01)\}$. Moreover, if the latter condition holds, then neither $(s, (1, B))$ nor $(s, (2, B))$ are rationalizable and hence (E3) implies that $CE(s) = \{s\}$ which in turn implies that $p^*(z^A, 00) = p(z^A, 00)$.

   If $p(z^A, 00) > \min\{p(z^A, 10), p(z^A, 01)\}$, then $CE(s) \subseteq \{(s, (1, B)), (s, (2, B))\}$. If $p(z^A, 10) < p(z^A, 01)$, then suppose by way of contradiction that $(s, (2, B)) \in CE(s)$. Then $(s, (1, B))$ is rationalizable and hence (E3) implies that $(s, (1, B)) \in CE(s)$. But then $(s, (2, B))$ is not rationalizable and hence by (E3) $(s, (2, B)) \notin CE(s)$ which is a contradiction. We conclude that $p(z^A, 10) < \min\{p(z^A, 00), p(z^A, 01)\}$ implies that $CE(s) = \{(s, (1, B))\}$ and, therefore, $p^*(z^A, 00) = p(z^A, 10)$.

   In the same way one proves that $p(z^A, 01) < \min\{p(z^A, 00), p(z^A, 10)\}$ implies that $CE(s) = \{(s, (2, B))\}$ and therefore $p^*(z^A, 00) = p(z^A, 01)$.

   Summarizing, if s is such that $z(s) = (z^A, 00)$ with $z_1^A + z_2^A = 1$, then there is a unique continuation equilibrium in $CE(s)$ and

$$p^*(z^A, 00) = \min\{p(z^A, 00), p(z^A, 10), p(z^A, 01)\}. \tag{19}$$

In a similar vein one proves that if s is such that $z(s) = (00, z^B)$ with $z_1^B + z_2^B = 1$, then there is a unique continuation equilibrium in $CE(s)$ and

$$p^*(00, z^B) = \max\{p(00, z^B), p(10, z^B), p(01, z^B)\}. \tag{20}$$

Equations (19) and (20) imply that

$$\begin{aligned} p^*(10, 00) &\leq p(10, 10) \leq p^*(00, 10),\\ p^*(10, 00) &\leq p(10, 01) \leq p^*(00, 01),\\ p^*(01, 00) &\leq p(01, 10) \leq p^*(00, 10),\\ p^*(01, 00) &\leq p(01, 01) \leq p^*(00, 01), \end{aligned}$$

from which we conclude that

$$\max\{p^*(10, 00), p^*(01, 00)\} \leq \min\{p^*(00, 10), p^*(00, 01)\}.$$

3. Finally, consider $(z^A, z^B) = (00, 00)$ and let $s = \varnothing$. From the previous analysis we know that there is a unique continuation equilibrium at $s = ((k, i))$ for $k = 1, 2$, and $i = A, B$. Let $CE((k, i)) = \{s_k^i\}$ for $k = 1, 2$, and $i = A, B$. (E1) then implies that $CE(\varnothing) \subseteq \{\varnothing, s_1^A, s_2^A, s_1^B, s_2^B\}$. By (E2) $\varnothing \in CE(s)$ if and only if

$$\max\{p^*(10, 00), p^*(01, 00)\} < p(00, 00) < \min\{p^*(00, 10), p^*(00, 01)\}.$$

Moreover, if the latter condition holds then s_k^i is not rationalizable for $k = 1, 2$, and $i = A, B$. Hence, (E3) implies that $CE(\varnothing) = \{\varnothing\}$ which in turn implies that $p^*(00, 00) = p(00, 00)$.

If

$$p(00, 00) > \min\{p^*(00, 10), p^*(00, 01)\} \geq \max\{p^*(10, 00), p^*(01, 00)\},$$

then $CE(\varnothing) \subseteq \{s_1^A, s_2^A, s_1^B, s_2^B\}$. W.l.o.g. let $p^*(10, 00) \leq p^*(01, 00)$. Then s_1^A is not rationalizable and (E3) implies that $s_1^A \notin CE(\varnothing)$ which in turn implies that s_2^A is not rationalizable and hence $s_2^A \notin CE(\varnothing)$ by (E3). Therefore, $CE(\varnothing) \subseteq \{s_1^B, s_2^B\}$. Since $z(s_1^B) \neq z(s_2^B)$ it follows that $p^*(00, 10) = p(z(s_1^B)) \neq p(z(s_2^B)) = p^*(00, 01)$.

If $p^*(00, 10) > p^*(00, 01)$, then suppose by way of contradiction that $s_1^B \in CE(\varnothing)$. Then s_2^B is rationalizable and hence (E3) implies that $s_2^B \in CE(\varnothing)$. But then s_1^B is not rationalizable and therefore $s_1^B \notin CE(\varnothing)$ by (E3) which is a contradiction. Therefore, $p^*(00, 10) > p^*(00, 01)$ implies that $CE(\varnothing) = \{s_2^B\}$ and $p^*(00, 00) = p^*(00, 01)$. Similarly, $p^*(00, 10) < p^*(00, 01)$ implies that $CE(\varnothing) = \{s_1^B\}$ and $p^*(00, 00) = p^*(00, 10)$.

In any case we conclude that if

$$p(00, 00) > \min\{p^*(00, 10), p^*(00, 01)\} \geq \max\{p^*(10, 00), p^*(01, 00)\},$$

then there is a unique equilibrium continuation in $CE(\varnothing)$ and

$$p^*(00, 00) = \min\{p^*(00, 10), p^*(00, 01)\}.$$

Finally, if

$$p(00,00) < \max\{p^*(00,10), p^*(00,01)\} \leq \min\{p^*(00,10), p^*(00,01)\},$$

then analogously to the previous case one proves that there is a unique equilibrium continuation in $CE(\emptyset)$ and

$$p^*(00,00) = \max\{p^*(10,00), p^*(01,00)\}.$$

This proves the theorem.

Proof of Theorem 4.1: We will prove that there exists an equilibrium collection of sets of continuation campaigns $(CE(s))_{s\in S}$ and that all equilibrium collections are outcome equivalent and for all $s \in S$ there is a unique equilibrium continuation in $CE(s)$, i.e. $CE(s) = \{s'\}$ for some $s' \in S$. Moreover, we will show that $z(s')$ only depends on $z(s)$ which implies that candidate A's vote share in the unique continuation equilibrium at any campaign s only depends on $z(s) = (z^A, z^B)$. We denote this equilibrium vote share by $p^*(z^A, z^B)$ and prove that it satisfies

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B),$$

where the minimum (maximum) over the empty set is defined to be ∞ $(-\infty)$, and that

$$p^*(z^A, z^B) = \begin{cases} \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k), & \text{if } p(z^A, z^B) > \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k) \\ p(z^A, z^B), & \text{if } \min\limits_{k:z_k^B=0} p^*(z^A, z^B + e^k) > p(z^A, z^B) > \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B) \\ \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B), & \text{if } \max\limits_{k:z_k^A=0} p^*(z^A + e^k, z^B) > p(z^A, z^B) \end{cases}.$$

Let $s \in S$ be a campaign with $z(s) = (z^A, z^B)$. The proof is by induction over the sum of the issues that have not been addressed by the candidates which we denote by L, that is, $L = \#\{(i,k) | z_k^i = 0\}$. Note that $0 \leq L \leq 2K$.

If s is such that $L = 0$, then $(z^A, z^B) = (1\ldots1, 1\ldots1)$ and by (E1) $CE(s) = \{s\}$ which implies $p^*(1\ldots1, 1\ldots1) = p(1\ldots1, 1\ldots1)$. This proves the claim for $L = 0$.

Let $1 \leq M \leq 2K$ and assume that the claim has been proved for all L with $0 \leq L \leq M-1$. Let s be such that $L = M$. From the induction hypothesis, we then know that for all k and i with $z_k^i = 0$ all continuation equilibria in $CE((s,(k,i)))$ are outcome equivalent. Moreover, if l is such that $z_l^A = 0$, then by the induction hypothesis

$$\min_{l':z_{l'}^B=0} p^*(z^A + e^l, z^B + e^{l'}) \geq p^*(z^A + e^l, z^B) \geq \max_{l'\neq l:z_{l'}^A=0} p^*(z^A + e^l + e^{l'}, z^B), \tag{21}$$

and if k is such that $z_k^B = 0$, then

$$\min_{k'\neq k:z_{k'}^B=0} p^*(z^A, z^B + e^k + e^{k'}) \geq p^*(z^A, z^B + e^k) \geq \max_{k':z_{k'}^A=0} p^*(z^A + e^{k'}, z^B + e^k), \tag{22}$$

where the minimum (maximum) over the empty set is defined to be ∞ $(-\infty)$. Equations (21) and

(22) imply that for all k and l such that $z_l^A = 0$ and $z_k^B = 0$,

$$p^*(z^A, z^B + e^k) \geq p^*(z^A + e^l, z^B + e^k) \geq p^*(z^A + e^l, z^B). \tag{23}$$

(23) implies that

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B). \tag{24}$$

From (E1), we know that $CE(s) \subseteq \bigcup_{(k,i):z_k^i=0} CE((s, (k, i))) \cup \{s\}$. By (E2) $s \in CE(s)$ if and only if

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) > p(z^A, z^B) > \max_{k:z_k^A=0} p^*(z^A + e^k, z^B). \tag{25}$$

Assume that (25) is satisfied which implies $s \in CE(s)$. Suppose by way of contradiction that there exist some (k, i) with $z_k^i = 0$ such that $s' \in CE(s) \cap CE((s, (k, i)))$. W.l.o.g. let $i = A$. Let

$$\mathcal{M}^A = \{k | \, z_k^A = 0 \text{ and } \exists \, s' \in CE(s) \cap CE((s, (k, A)))\} \tag{26}$$

and let

$$\hat{k} \in \{k \in \mathcal{M}^A | \, p^*(z^A + e^k, z^B) \leq p^*(z^A + e^l, z^B) \text{ for all } l \in \mathcal{M}^A\}. \tag{27}$$

Then $s' \in CE(s) \cap CE((s, (\hat{k}, A)))$ is not rationalizable which contradicts (E3). Hence, if (25) is satisfied, then $CE(s) = \{s\}$ and

$$p^*(z^A, z^B) = p(z^A, z^B).$$

Assume now that

$$p(z^A, z^B) > \min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B).$$

Then (25) is violated and (E2) implies that $s \notin CE(s)$. Suppose by way of contradiction that $\exists \, s' \in CE(s) \cap CE((s, (k, A)))$ for some k with $z_k^A = 0$. Let $\mathcal{M}^A$ and $\hat{k}$ be defined as in (26) and (27), respectively. Then $s' \in CE(s) \cap CE((s, (\hat{k}, A)))$ is not rationalizable which together with $p^*(z^A + e^{\hat{k}}, z^B) < p(z^A, z^B)$ implies that (E3) is violated. Hence, $CE(s) \cap CE((s, (k, A))) = \varnothing$ for all k with $z_k^A = 0$. (E1) then implies that

$$CE(s) \subseteq \bigcup_{(l,B):z_k^B=0} CE((s, (l, B))).$$

Let

$$\mathcal{M}^B = \{l | \, z_l^B = 0 \text{ and } p^*(z^A, z^B + e^l) \leq p^*(z^A, z^B + e^k) \text{ for all } k \text{ with } z_k^B = 0\}. \tag{28}$$

Suppose by way of contradiction that $CE(s) \cap CE((s, (l, B))) = \varnothing$ for all $l \in \mathcal{M}^B$ and let $s' \in CE((s, (l, B)))$ for some $l \in \mathcal{M}^B$. Then s' is rationalizable and hence (E3) implies that $s' \in CE(s)$ which is a contradiction. Hence, there exists some $l \in \mathcal{M}^B$ and some $s' \in CE(s) \cap CE((s, (l, B)))$.

Suppose by way of contradiction that $CE(s) \cap CE((s, (l, B))) \neq \varnothing$ for some $l \notin \mathcal{M}^B$ with $z_l^B = 0$. Let $\bar{l} \notin \mathcal{M}^B$ be such that there exists some $s' \in CE(s) \cap CE((s, (\bar{l}, B)))$ and $p^*(z^A, z^B + e^{\bar{l}}) \geq p^*(z^A, z^B + e^l)$ for all l with $CE(s) \cap CE((s, (l, B))) \neq \varnothing$. Then s' is not rationalizable and hence (E3) is violated which is a contradiction. Hence, $CE(s) \cap CE((s, (l, B))) = \varnothing$ for all $l \notin \mathcal{M}^B$ with $z_l^B = 0$.

This implies that all continuation equilibria in $CE(s)$ are outcome equivalent and

$$p^*(z^A, z^B) = \min_{k:z_k^B=0} p^*(z^A, z^B + e^k).$$

Finally, assume that

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B) > p(z^A, z^B).$$

Then analogously to the previous case ones shows that all continuation equilibria in $CE(s)$ are outcome equivalent and

$$p^*(z^A, z^B) = \max_{k:z_k^A=0} p^*(z^A + e^k, z^B).$$

This proves the claim for $L = M$ and concludes the proof of the theorem.

Proof of Theorem 4.2: Let (z^A, z^B) be given. The proof is again by induction over $L = \#\{(i,k)|\, z_k^i = 0\}$. Note that $0 \leq L \leq 2K$. For $L = 0$ the claim is trivially true. So let $1 \leq M \leq 2K$ and assume the claim has been proved for all L with $0 \leq L \leq M - 1$.

Let (z^A, z^B) be such that $\#\{(i,k)|\, z_k^i = 0\} = M$. By the induction hypothesis, for all k with $z_k^A = 0$,

$$p^*(z^A + e^k, z^B) = p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big),$$

and for all k with $z_k^B = 0$,

$$p^*(z^A, z^B + e^k) = p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big).$$

Equation (18) then implies the following: If

$$p(z^A, z^B) > \min_{k:z_k^B=0} p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big),$$

then

$$p^*(z^A, z^B) = \min_{k:z_k^B=0} p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big). \tag{29}$$

If

$$\begin{aligned}&\min_{k:z_k^B=0} p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big) > p(z^A, z^B)\\ &> \max_{k:z_k^A=0} p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big),\end{aligned}$$

then

$$p^*(z^A, z^B) = p(z^A, z^B). \tag{30}$$

Finally, if

$$\max_{k:z_k^A=0} p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big) > p(z^A, z^B),$$

then

$$p^*(z^A, z^B) = \max_{k:z_k^A=0} p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big). \tag{31}$$

Let $i \in \{A, B\}$. If there exists k with $z_k^i = 0$, then by definition of $\bar{z}^i$,

$$\bar{z}^i + \max\{z^i + e^k - \bar{z}^i, \mathbf{0}\} = \begin{cases} \bar{z}^i + \max\{z^i - \bar{z}^i, \mathbf{0}\} & \text{if } k \in \mathcal{K}^i \\ \bar{z}^i + \max\{z^i - \bar{z}^i, \mathbf{0}\} + e^k & \text{if } k \notin \mathcal{K}^i \end{cases}.$$

Hence, if $z_k^A = 0$, then

$$\begin{aligned} & p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big) \\ & \le p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big), \end{aligned} \tag{32}$$

with '=' if and only if $k \in \mathcal{K}^A$. This implies

$$\begin{aligned} & \max_{k : z_k^A = 0} p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big) \\ & \le p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big), \end{aligned} \tag{33}$$

with '=' if and only if there exists some $k \in \mathcal{K}^A$ with $z_k^A = 0$.

Similarly, if $z_k^B = 0$, then

$$\begin{aligned} & p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big) \\ & \ge p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big), \end{aligned} \tag{34}$$

with '=' if and only if $k \in \mathcal{K}^B$. This implies

$$\begin{aligned} & \min_{k : z_k^B = 0} p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big) \\ & \ge p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big), \end{aligned} \tag{35}$$

with '=' if and only if there exists some $k \in \mathcal{K}^B$ with $z_k^B = 0$.

Equations (33) and (35) imply that

$$\begin{aligned} & \min_{k : z_k^B = 0} p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\big) \\ & \ge \max_{k : z_k^A = 0} p\big(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big), \end{aligned} \tag{36}$$

with '=' if and only if there exist $k \in \mathcal{K}^A$ and $\hat{k} \in \mathcal{K}^B$ with $z_k^A = z_{\hat{k}}^B = 0$.

Hence, if (z^A, z^B) is such that there exist $k \in \mathcal{K}^A$ and $\hat{k} \in \mathcal{K}^B$ with $z_k^A = z_{\hat{k}}^B = 0$, then (29), (31), (32) and (34) imply that

$$p^*(z^A, z^B) = p\big(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\big),$$

which proves the claim for this case.

Consider next the case where (z^A, z^B) is such that $z_k^A = 1$ for all $k \in \mathcal{K}^A$ and $z_k^B = 0$ for some

$k \in \mathcal{K}^B$. Then $z^A = \bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}$ and (35) implies that

$$\begin{aligned}&\min_{k:z_k^B=0} p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\right)\\ &= p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right)\\ &= p\left(z^A, z^B + \sum_{k\in\mathcal{K}^B : z_k^B=0} e^k\right)\\ &< p(z^A, z^B).\end{aligned}$$

Equation (29) then implies that

$$p^*(z^A, z^B) = p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right),$$

which proves the claim for the case where (z^A, z^B) is such that $z_k^A = 1$ for all $k \in \mathcal{K}^A$ and $z_k^B = 0$ for some $k \in \mathcal{K}^B$.

Next consider the case where (z^A, z^B) is such that $z_k^B = 1$ for all $k \in \mathcal{K}^B$ and $z_k^A = 0$ for some $k \in \mathcal{K}^A$. Then $z^B = \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}$ and (33) implies that

$$\begin{aligned}&\max_{k:z_k^A=0} p\left(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right)\\ &= p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right)\\ &= p\left(z^A + \sum_{k\in\mathcal{K}^A : z_k^A=0} e^k, z^B\right)\\ &> p(z^A, z^B).\end{aligned}$$

Equation (31) then implies that

$$p^*(z^A, z^B) = p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right),$$

which proves the claim for the case where (z^A, z^B) is such that $z_k^B = 1$ for all $k \in \mathcal{K}^B$ and $z_k^A = 0$ for some $k \in \mathcal{K}^A$.

Finally, consider the case where (z^A, z^B) is such that $z_k^A = 1$ for all $k \in \mathcal{K}^A$ and $z_k^B = 1$ for all $k \in \mathcal{K}^B$. Then $z^A = \bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}$ and $z^B = \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}$. Then (33) and (35) imply that

$$\begin{aligned}&\min_{k:z_k^B=0} p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B + e^k - \bar{z}^B, \mathbf{0}\}\right)\\ &> p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right)\\ &= p(z^A, z^B)\\ &> \max_{k:z_k^A=0} p\left(\bar{z}^A + \max\{z^A + e^k - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right).\end{aligned}$$

Equation (30) then implies that

$$p^*(z^A, z^B) = p(z^A, z^B) = p\left(\bar{z}^A + \max\{z^A - \bar{z}^A, \mathbf{0}\}, \bar{z}^B + \max\{z^B - \bar{z}^B, \mathbf{0}\}\right),$$

which proves the claim for the case where (z^A, z^B) is such that $z_k^A = 1$ for all $k \in \mathcal{K}^A$ and $z_k^B = 1$ for all $k \in \mathcal{K}^B$.

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Notes

1. See the microfoundation for the vote shares in Section 4.1.
2. The result can be verified directly or by application of the characterization in the proof of Theorem 4.1 (see (18) in Section 4.2).
3. In case of a finite and odd number of voters, just assume that an odd number of voters abstain.
4. Note that (10) implies that $\gamma_2 < 0.5$, that is, that candidate B owns issue 2, and (11) implies that $\gamma_1 > 0.5$, that is, that candidate A owns issue 1.
5. We thank Felix Brandt for pointing us to Debord (1987). Le Breton (2005, Remark 2.5 and Section 4) provides an alternative proof of the Debord's theorem.
6. $p_k(z_k^A, z_k^B)$ is the share of voters who only care for issue k and whose ideal position on issue k is closer to the policy candidate A is expected to implement (which is $\bar{x}_k^A$ if $z_k^A = 1$ and x_k^0 otherwise) than to the policy candidate B is expected to implement (which is $\bar{x}_k^B$ if $z_k^B = 1$ and x_k^0 otherwise).
7. $\mathbf{0}$ denotes the zero vector in $\mathbb{R}^K$.

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