Students’ resilience and school efficiency in one of the most unequal countries in the world: empirical evidence from Colombia

Sebastian López-Estrada[a,1], Tommaso Agasisti[b], Víctor Giménez[c], Diego Prior[c]

a Department of Economics. Pontificia Universidad Javeriana Cali (Colombia)
b School of Management. Politecnico di Milano (Italy)
c Department of Business. Universitat Autònoma de Barcelona (Spain)

Abstract

This article analyzes students’ resilience in 7,789 schools in the Colombian educational system and its relationship with educational efficiency between 2014 and 2019. The empirical analysis is carried out in two stages. First, a multilevel model with random intercept and slope is estimated to determine the students categorized as resilient. Then conditional order-m models are used to calculate the efficiency. The results indicate a negative relationship between the resilience of schools and their inefficiency of up to 33% in public schools and 12% in private schools.

Keywords: Educational inequity; students’ resilience; efficiency analyses; schools; conditional order-m

JEL codes: I21; I24.

[1] Corresponding author. Sebastian.lopezestra@javerianacali.edu.co

1. Introduction

Education is a priority and a fundamental right that matters to the government, institutions, and society in general. In this context, the fourth Sustainable Development Goal (SDG 4) aims to "Guarantee inclusive and equitable quality education and promote lifelong learning opportunities for all" within the framework of the United Nations 2030 agenda. This policy prioritizes inclusion, equity, and giving the same opportunities to all students and is designed to align the main government efforts addressed to the most vulnerable and marginalized population, ensuring that everyone is provided with the same access to and quality of education, regardless of their circumstances (UNDP & UNESCO, 2015).

Some studies have focused on those students who, despite coming from relatively disadvantaged socioeconomic and cultural backgrounds, obtain outstanding educational performance (Agasisti et al., 2021; Cordero & Mateos-Romero, 2021; Gabrielli et al., 2021; Vicente et al., 2021). This stream of research takes into account the multiple positive economic and social externalities that result from higher educational levels (Hanushek, 1986; Hanushek & Woessmann, 2008), and advocates the active promotion of resilience in these students as a potential strategy to raise a country's development levels.

In social terms, the concept of resilience originally emerged in the field of psychology (Finn & Rock, 1997; Luthar et al., 2000), and due to the potential of its definition, interest spread to other areas of research, such as the economics of education (Agasisti & Longobardi, 2017; Cordero & Mateos-Romero, 2021). Resilience has generally been defined as the achievement of success in a situation where a person is disadvantaged or facing adversity (Ungar, 2005; Windle, 2011).

In general, educational or academic resilience has been defined as the ability of a student to achieve outstanding academic performance despite their disadvantaged background (OECD, 2011). Such students stand out because they develop behavior that goes against expectations. The literature regards resilience as a sign of hope (Clavel et al., 2021), since it breaks the vicious cycle in which poverty is perpetuated across generations.

The growing body of research into educational resilience has mainly focused on student effectiveness (Ye et al., 2021). It is therefore important to explore whether those schools that have a higher proportion of resilient students also perform efficiently, since a balance between efficiency and effectiveness is of vital importance for educational policies (OECD, 2006).

In order to optimize resource allocation, it is essential to understand the relationship between different variables and the efficiency of the educational system (Agasisti, 2013; Sagarra et al., 2017). In this regard, analyzing the relationship between educational inequity and schools' efficiency is critical for designing coherent educational policies. Reducing inequity among students is desirable while maintaining or improving academic performance; however, this process requires physical, human, and/or financial resources. The tradeoff between the additional use of resources and the better performance of resilient students can be analyzed from a policymaker's point of view, where the efficiency of resource management pairs with the difficulties involved in improving this educational process. Departmental and municipal governments are interested in knowing how to improve this relationship and help to empower their students, since it has a positive effect on higher education, job placement, economic and social growth, and development.

Research on educational efficiency has recently considered problems related to educational inequality (Arbona et al., 2022; Giménez et al., 2017a; Giménez et al., 2017b) or inequity (Cordero et al., 2015; Marchesi, 2006; Sicilia & Simancas, 2023). In this type of analysis, the most commonly used variables are the standard deviation in the results of standardized tests or the number of students who reach minimum standards in these tests. Different behaviors or performance levels are found in educational systems when these types of variables are considered. The current paper is one of the first studies to directly analyze the relationship between educational resilience and efficiency within an educational system, in addition to the analysis by Sicilia and Simancas (2023) for the case of Spain.

This paper defines two specific objectives to analyze resilience in 7,789 schools in the Colombian educational system and its relationship with educational efficiency between 2014 and 2019. First, it analyzes the schools by considering their performance in two aspects: the number of resilient students and the schools' relative efficiency. Second, it identifies the differences in this relationship across sectors (public and private schools) and regions.

Colombia is a representative case of an emerging country with high social and educational inequalities and inequities. The Programme for International Student Assessment (PISA) results for 2018 show lower performance than OECD countries, with only 35% of Colombian students obtaining proficiency level 2 in mathematics. In addition, 14% of the variation in reading results is explained by the socioeconomic conditions of the students, which is 2% higher than the OECD average (OECD, 2018b). Likewise, as highlighted in a relevant OECD (2018a) report, the situation in Colombia is one of the most concerning, since it has the worst performance in closing gaps: on average it would take at least 300 years for children from low-income families to reach the mean.

The empirical analysis of this study is carried out in two stages. First, we estimate a multilevel model with random intercept and slope (Vicente et al., 2021) to define the students categorized as resilient, which takes into account the variance between the different levels of analysis (students within a municipality). Second, we use one of the most robust methods to estimate efficiency, namely conditional order-m models (Cazals et al., 2002), which also reduces the influence of atypical and extreme values.

To analyze the relationship between resilience and efficiency, we construct a database by integrating two sources containing information from 2014[2] to 2019. The first source is the Colombian Institute for the Promotion of Higher Education (ICFES), which provides the results in the standardized exams in Colombia at different levels of analysis. The second source is the National Administrative Department of Statistics (DANE), which provides access to the inventory of physical and personnel resources of each school.

Our main finding is the negative relationship between the inefficiency of schools and the number of resilient students. This negative correlation is strongly heterogeneous among departments and between public and private schools. The innovative contribution of this work to the literature is threefold. First, it contributes to the scarce (Sicilia & Simancas, 2023) literature that analyzes the schools of an educational system based on their resilience and efficiency at the same time. Second, it is one of the first analyses of educational resilience in a developing country. Third, compared to previous

[2] Before 2014, the results of the standardized test for secondary education are not comparable due to methodological changes.

applications, this is the first study to be carried out with data other than those from the Progress in International Reading Literacy Study (PIRLS), Trends in International Mathematics and Science Study (TIMSS) or PISA, thus contributing to the analysis of the phenomenon from an alternative empirical perspective. Indeed, the availability of a detailed administrative dataset allows for a much more complete and robust empirical analysis than existing studies based on international samples.

The paper is divided into five sections. After this introduction (section 1), a literature review is provided in section 2; the methodological approach is described in section 3 and the empirical aspects related to the databases and variables are explained in section 4. Finally, the results are presented in section 5, and conclusions are drawn in section 6.

2. Literature Review

In recent years, many academic studies have focused on improving educational achievement as a proxy variable of quality (Evans et al., 2000). However, this cannot be the only objective; Tsai et al., (2017) highlight that the golden rule in educational policy should be to consider excellence (high performance) and equality (low variability in performance) in the results.

This discussion initially became relevant with the Equality of Educational Opportunities report (Coleman et al., 1966), which revealed the importance of social and economic components as determinants of educational performance at an international level (Hanushek & Woessmann, 2011). Since then, policymakers have endeavored to reduce inequity, understanding it as the differences in educational performance caused by people's social, cultural or economic circumstances. That is, students' educational performance must be a function exclusively of effort and abilities (OECD, 2011), and not focus solely on reducing the difference between students, which is understood as inequality.

Although various ways have been proposed to reduce inequity, the debate has focused on different types of strategies. At the international level Hanushek and Ludger (2006) refer to the choice between selective (for example, Germany, Hungary, Austria) or comprehensive (for example, Japan, Canada, Norway) systems for grouping students in classrooms. Other studies have analyzed early follow-up approaches of students (Dupriez et al., 2008; Hanushek & Woessmann, 2008), grouping of skills and/or performance in the classroom (Hindriks et al., 2010) and individualized support (Ferrer-Esteban, 2016), alluding to the peer effect as a tool for working on inequity (Betts & Shkolnik, 2000). These approaches reflect options to reduce differences in educational performance caused by people's social, cultural, or economic circumstances, which have been mentioned as essential for evaluating educational systems (Sicilia & Simancas, 2023).

In this research, educational resilience is used as a proxy to study inequity in the Colombian educational system. Although this is the first study to analyze educational efficiency and resilience for Colombia jointly, approaches considering efficiency and resilience independently can be found in the literature. While educational efficiency has not been studied as much in Colombia as internationally, some studies in higher education focus on the difference in programs (Melo-Becerra et al., 2017) and the public and private academic sectors (Moreno-Gómez et al., 2019). At the international level, Cordero et al. (2017) analyze the efficiency of the educational system of 36 countries participating in PISA 2012, including Colombia. In addition, Arbona et al. (2023) examine how contributions from the private sector can affect the efficiency of educational institutions at the secondary level. Finally, only one study (Arbona et al.,

2022) has addressed efficiency in conjunction with a problem close to inequity (differences in the standard deviation of student performance), in which the evolution of the public and private sectors is considered for the period 2014–2019.

Educational resilience is a phenomenon where students in a situation of disadvantage or adversity achieve outstanding academic results (Wang & Walberg, 1994). We are not aware of any research on this phenomenon specifically for the case of Colombia, although a series of studies have highlighted the effects of achieving a more equitable educational system at an international level (Agasisti & Longobardi, 2017; Clavel et al., 2021; Cordero & Mateos-Romero, 2021; Gabrielli et al., 2021; OECD, 2011; Vicente et al., 2021), some of which use data from Colombia at the country level as a member of or allied to the OECD (Agasisti et al., 2018; OECD, 2011; Vicente et al., 2021; 2023).

The academic literature has studied educational resilience from two perspectives. The first is the perspective of psychology and sociology, in which notable contributions use mainly qualitative methodologies to explore factors such as character, commitment and self-confidence (Borman & Overman, 2004; Wang, G., & Walberg, 1994). The second perspective mainly focuses on analyzing the composition of resilient student groups and their determinants, comparing their behavior and proportion between countries (Agasisti et al., 2021; Clavel et al., 2021; OECD, 2011; Vicente et al., 2021).

Following the objective of this study, which analyzes the phenomenon of educational resilience from the second point of view, four factors must be taken into account in the conceptual framework (Ye et al., 2021): first, the definition of educational resilience; second, how to measure socioeconomic adversity (composite versus distinct measures of student background); third, how to measure positive academic results (selecting the educational achievement indicator to use as a benchmark); and fourth, thresholds for adversity and academic results, and how to compare students, whether cross-country or within-country.

The first factor in the conceptual framework (definition of resilience), has been studied from different disciplines, many related to behavioral sciences. Although there is no universal definition across the disciplines, all academics base their analyses on the concepts of adversity and positive adaptation (Windle, 2011). From this perspective, when resilience is analyzed in the educational context, the consensus in the literature is that students' conditions and experiences must be considered as a measure of adversity, and a greater probability of success in school should be regarded as a measure of positive adaptation (Wang & Walberg, 1994).

Regarding the second factor, the studies that analyze educational resilience through international large-scale assessment (ILSA) data—such as PISA or TIMSS—consider that the effect of students' background on educational achievement is related not only to material goods but also to their social and cultural circumstances. The most commonly used variables to measure adversity in educational resilience research are the PISA socioeconomic status (SES) index and the TIMSS home educational resources (HER) index, the main difference being that the SES index considers parents' occupation while the HER index does not.

For the third factor, positive adaptation, cognitive outcomes are generally assessed through standardized tests. The main discussion revolves around whether to use only one dimension of the standardized tests (for example mathematics) or tests in different subjects. Broadly, some authors suggest that if a student is resilient in one of the dimensions they will be resilient in the others, although other studies do not find this consistency (OECD, 2011). This debate has motivated work on finding resilient students across different dimensions (Agasisti et al., 2018).

The fourth factor of the conceptual framework considers the thresholds of analysis, and cross-country or within-country comparisons. Both for the variables of adversity (disadvantaged) and those of positive adaptation (high performance in standardized tests) the question is posed as to whether the comparisons should be made in a “fixed” or a “relative” way. The most recent studies in this line of research opt for within-country comparison thresholds (relative), since they are more useful for educational policy in a specific context (OECD, 2011; Vicente et al., 2021).

In their systematic review of the literature on academic resilience, Ye et al. (2021) find that different criteria are used to define both the variables of adversity, as well as those of positive adaptation and the thresholds. In general, the studies can be categorized into four groups: (I) fixed background and fixed outcome thresholds, (II) fixed background and relative outcome thresholds, (III) relative background and fixed outcome thresholds, and (IV) relative background and relative outcome thresholds.

Based on the above, and in line with the objective of the study, this paper shares the characteristics of the third group, which uses a relative background and fixed outcome thresholds. Within the studies that have followed these characteristics, there are differences in the approaches: some authors use direct threshold approaches (García-Crespo et al., 2019; OECD, 2011), others use residual methods to calculate thresholds (Agasisti et al., 2021; Agasisti & Longobardi, 2014, 2017; Cordero & Mateos-Romero, 2021; Vicente et al., 2021) and finally, cross-domain operationalization of educational outcomes are also taken into account (Agasisti et al., 2018).

Research on educational resilience has paid attention to its determinants, in an attempt to shed light on the phenomenon in order to help close the socioeconomic gaps in the educational system. The current literature focuses on three groups of variables: students’ demographics (Agasisti et al., 2021; Gabrielli et al., 2021; Martin & Marsh, 2006), family background (Agasisti & Longobardi, 2014; Clavel et al., 2021; Hill & Tyson, 2009), and school and class factors (Cordero et al., 2015; Tajalli & Cynthia, 2004).

When considering demographic characteristics, the existing studies draw mixed conclusions on the role of students' immigration status (Gabrielli et al., 2021; Gabrielli & Impicciatore, 2021), the language spoken at home (Christensen & Segeritz, 2006), and gender (Agasisti & Longobardi, 2017; Martin & Marsh, 2006). Studies analyzing students' family background focus on the cultural capital of the home (Park, 2008) and the parents' intervention in or commitment to the education of their children (Hill & Tyson, 2009).

Likewise, school- and class-related factors were studied because of their potential to close the gaps in the students' backgrounds. In this case, the most analyzed variables are the teachers' strategies in the classroom (Tajalli & Cynthia, 2004), class size (Heinesen, 2009), peer effects (Agasisti et al., 2016), and school academic climate (Wang et al., 2010).

Regarding the validation of the concept of educational resilience, Ye et al. (2021) highlight three aspects to consider in future works. First, they suggest taking a country-specific approach to measure adversity, since this offers a more pertinent way of informing public policies in a country. Second, different assumptions must be tested (thresholds and ways of measuring positive adaptation) to increase robustness in the results. Third, the results should focus not only on the number of resilient students but also on the composition of the groups by gender, type of school, ethnicity, etc.

The last part of this literature review highlights the fact that although the number of studies into educational resilience is growing, there are still factors to be explored that are relevant and significant for the elaboration of public policies. Specifically, for the purpose of this study the most important factor is the relationship between resilience and educational efficiency. The OECD (2006) emphasizes that there must be a balance between educational efficiency and effectiveness for the development of educational policies. To the best of our knowledge, this is one of the first academic papers that directly address the relationship between efficiency and educational resilience in an empirical analysis, together with the study by Sicilia and Simancas (2023) for the case of Spain.

3. Methodological approach

This section presents the two methodologies used to carry out the empirical analysis. First, it explains how a student is conceptually categorized as resilient, in order to compute the proportion of resilient students by school. Second, we explain the conditional order-m model, which is a robust methodology for calculating the efficiency of schools. In this regard, Daraio and Simar (2005; 2007a; 2007b) recommend using conditional models since they include contextual variables in a single stage, and they are not too sensitive to atypical observations.

a. Defining a student as resilient

The academic literature defines a resilient student as an individual who, despite coming from a disadvantaged socioeconomic background, reaches a relatively high level of academic performance (OECD, 2011). Based on the suggestions of Ye et al. (2021) in their systematic literature review, various factors must be considered when categorizing these students. First, it is necessary to define the indicator or variable to consider a student in a situation of disadvantage or adversity. Second, the criteria to define high performance must be specified. Third, the thresholds and the group with which the comparisons will be made in the process (country, region, department, municipality, and school) should be defined.

The main methodology used in this stage is multilevel or hierarchical regression. This type of regression allows researchers to take advantage of the nesting of the data structure, i.e., students within schools within departments. In this study, we follow Vicente et al.'s (2021) approach, in which the possible correlation between students from the same school and territory is considered, unlike other studies. For students to be classified as resilient, they must fall within the 25th percentile of the Socioeconomic Index of their municipality. They must also be disadvantaged and achieve an overall test score distribution to be above the 75th percentile. This estimate is made considering the socioeconomic background of the students and taking into account the possible variation of this effect in each of the municipalities. The mathematical function is:

$$Globalscore = \alpha_{00} + \beta_1 INSE_{ij} + \varepsilon_{ij} + \delta_{0j} \quad \textbf{(1)}$$

where i represents the students and j represents the municipalities. In addition, the global score of each student is taken into account considering their Socioeconomic Index and the municipalities in a second level. After performing the estimation, two types of error are obtained, the individual (ε_{ij}) and the cluster (δ_{0j} *and* δ_{1j}); δ_{0j} is the random part of the intercept, that is, the initial position of each student due to their belonging to a specific municipality according to their Socioeconomic Index; in turn, δ_{1j} is the random part of the slope, in other words, the effect of the Socioeconomic Index variation within a specific municipality. Note that δ_{0j} and δ_{1j} are errors that include all factors that cannot be explained after controlling the student's Socioeconomic Index in relation to the global score.

Finally, in order to categorize which students are resilient, equation 1 is estimated to add the individual errors with the clusters ($\varepsilon_{ij} + \delta_{0j}$), while controlling the effect of the variation of the Socioeconomic Index in each municipality (δ_{1j}). Then, the 75th percentile of this sum is calculated, and disadvantaged students above this percentile obtained through estimation errors are categorized as resilient. With this approach, resilient students are used as a proxy to study equity and inequity in the educational system. Indeed, these students are overcoming adversity in a specific environment and obtain a result above what is expected given their individual socioeconomic background.

b. Conditional order-m model for calculating the schools' efficiency scores

The main objective of this study is to analyze the educational efficiency of schools and their relationship with resilient students. In general, the academic literature has used various parametric and non-parametric techniques to measure efficiency. Notable non-parametric techniques include Data Envelopment Analysis (DEA) (Charnes et al., 1978) and Free Disposal Hull (Deprins et al., 1984). Both techniques are based on mathematical programming and do not require any assumption about the production function; however, the main difference between them is that the Free Disposal Hull removes the assumption of convexity, which implies that the relative efficiency is calculated exclusively with other real units and not linear combinations on the frontier (see De Witte & López-Torres, 2017).

A non-parametric approach is adopted in this study as multiple outputs can be used (Thieme et al., 2013), which helps to take into account different aspects of the educational process at the same time (quality, capacity, inequity). Within this approach, this study uses conditional order-m models, since unlike the Data Envelopment Analysis and the Free Disposal Hull, it is a more robust way of making efficiency estimates due to the bootstrapping and the inclusion of environment variables in the estimation process (Cazals et al., 2002).

To estimate schools' efficiency, production technology is considered as the students' transformation of a set of inputs $x(x\epsilon R_{+}^{p})$, such as their socioeconomic index, resources they have at school and their own skills, into a set of outputs $y(y\epsilon R_{+}^{q})$, usually measured through standardized tests. Production technology can be established as the set of possible combinations of outputs and inputs:

$$\Psi = \{(x, y)\epsilon R_{+}^{p+q} | x \; can \; produce \; y\} \qquad \textbf{(2)}$$

Following the probabilistic framework presented by Cazals et al. (2002), we develop a conditional model that takes into consideration contextual variables $Z\epsilon R_{+}^{k}$ since they have an impact on school performance and efficiency. The objective is to illustrate how a school operating at a specific level (x, y) can be compared to another school operating under similar contextual conditions (Z = z) using the joint production function $H_{XY|Z}$, where Z represents the set of variables characterizing a particular operational environment. Following Cazals et al. (2002) and Daraio and Simar (2005; 2007a; 2007b), it can be expressed as:

$$H_{XY|Z}(x, y|z) = \Pr(X \leq x, Y \geq y|Z = z) \qquad \textbf{(3)}$$

Furthermore, the equation can be decomposed into two components, namely $S_Y(y|x, z)$, which signifies the survival function of Y, and $F_X(x|z)$, which denotes the cumulative distribution function of X:

$$H_{XY|Z}(x, y|z) = \Pr(X \leq x, Y \geq y|Z = z) \Pr(X \leq x|Z = z) = S_Y(y|x, z)F_X(x|z) \textbf{(4)}$$

Some non-parametric techniques for estimating efficiency, such as DEA, are prone to sensitivity when dealing with extreme or outlier values. To address these issues, a solution is to employ an order-m frontier evaluation process as defined by Cazals (2002). This process allows us to calculate conditional estimators $\lambda(x, y|z)$. Order-m models require the specification of a parameter, denoted as m, which signifies the number of units randomly selected from the sample for comparison. Consequently, smoothing techniques are applied to the contextual variables, resulting in the conditional model that can be expressed through the following integral:

$$\hat{\lambda}_{m,n}(x, y|z) = \int_0^{\infty}[1 - \left(1 - S_{Y,n}(uy|x, z)\right)]^m du \quad \textbf{(5)}$$

Based on equation 5, efficient schools are on the frontier (efficiency score equal to one); on the other hand, inefficient students can be measured using equation 5 when an output orientation is taken: an inefficient student will obtain an efficiency score greater than one, estimating the potential improvement of the outputs. Importantly, the conditional order-m model also allows us to obtain values less than one, which means that the evaluated school is located above the production frontier, that is, this school can be defined as super-efficient.[3] Note that the output orientation is used, since the objective of students in all educational systems is generally to obtain the best results with the given resources.

The relative efficiency estimation process is defined through the Free Disposal Hull model mentioned above, which removes the assumption of convexity for the estimation of the technological set. Then, conditional order-m is introduced, intuitively explaining the main changes with respect to the Free Disposal Hull and the origin of its robustness.

[3] This type of result has implications for the average values of the sample studied since it can bring the averages to values close to one. However, this possibility is mitigated by setting m to obtain 10% super-efficient units (Tauchmann, 2012).

Intuitively this implies that each school can be compared only with other existing schools, and not with convex combinations of them, as in the DEA models. To estimate the efficiency model, Cazals et al. (2002) propose estimating partial frontiers with $m(\geq 1)$ units randomly drawn from the sample. These estimates are repeated B times,[4] obtaining multiple measurements, then the final measurement ($\hat{\lambda}_{m,n}$) is calculated with a simple average.

This estimator allows comparisons to be made with *m* potential units that have a similar input and contextual level. Note that since we do not use the entire sample, it is less sensitive to outliers and extreme values. Therefore, for higher values of *m*, the estimators of order-m tend tothe values of the Free Disposal Hull.

4. Data, variables and descriptive statistics

According to the 1991 Constitution and the 1994 Education Law, education is a right to which all people in Colombia have access. The Colombian educational system up to higher education is divided into four stages: preschool, primary education (5 years), basic secondary education (4 years), and middle education (2 years). Higher education is more complex since there are different programs of varying length and with multiple providers. In total, 8,604,145 students are enrolled. The public sector represents 78% and the private sector 19.6%. In recent years, total enrollment has decreased by approximately 23,000 students: in 2014, total enrollment was 8,627,797 students, and by 2019, it had fallen to 8,604,145. However, sector behavior was not homogeneous; the public sector increased by 1.2% (84,797 students) while the private sector decreased by 6.8% (122,938 students).

The Colombian Institute for the Evaluation of Education (ICFES) is responsible for evaluating education throughout the country. These evaluations are carried out with multiple standardized exams at the national level, although the most important are Saber 3,

[4] The estimates are repeated 200 times, following the trend in this line of research (Thieme et al., 2013).

5, and 9, and Saber 11 for middle and high school education. Saber 11 is a standardized exam that students normally between 16 and 17 years old take at the end of secondary education and before entering technical, technological, or university education. In 2019, private sector students (average of 263) outperformed public sector students (average of 241) by 23 points.

In line with the objective of the analysis, a database is built from two sources of information, the Colombian Institute for the Promotion of Higher Education (ICFES) and the National Administrative Department of Statistics (DANE). The ICFES offers information on standardized tests (for instance Saber 11, Saber Pro), and general characteristics of the students, their families, and the school. DANE offers information on the schools' physical and human resources. Based on the methodological approach proposed in the previous section, this paper uses a dependent and an independent variable for the multilevel regression. To study the relationship between the resilient students and the estimation of efficiency, it uses two outputs, five inputs and two contextual variables.

To estimate resilient students through the multilevel model, students in a disadvantaged situation and those who have outstanding performance must be identified. In this study, students in a disadvantaged situation are selected based on the socioeconomic index calculated by the ICFES, following an item response theory methodology (Demars, 2010). This index is a comprehensive measure of the students' social, economic, and cultural environment, which includes their parents' education level and occupation, and the household income, among other factors.

To define students with outstanding academic performance, the students' global score is used, which is a weighted average of the individual scores of each of the tests that the students take in the exam, divided by the total weight (13) and multiplied by the number of tests (5). A weighted average (three points for mathematics, reading, social studies, and natural sciences and one for the English language) is used on the recommendation of multiple authors (Agasisti et al., 2021; Hauser, 2009), who consider it provides a more general evaluation of the students, and it is also the measure used in Colombian educational policy for accessing higher education.

Two outputs are used to estimate the efficiency model: first, the global score *(y1)* explained above as a measure of quality (Cordero et al., 2016; Tavana et al., 2018) of the students; and second, the students who pass *(y2)* the school year, as a complement to traditional measures to evaluate educational systems. The five selected inputs have frequently been used in the educational efficiency literature (De Witte & López-Torres, 2017). The number of electronic devices *(x1)* includes tablets, desktop computers, and laptops, reflecting the resources available at their school (Agasisti, 2011; Mancebón et al., 2012). Human capital is measured through teaching directors *(x2)* and teachers in classrooms *(x3)*; these variables provide an approximation of the educational and management personnel that educational institutions have for their operation (Haelermans & Ruggiero, 2017; Tran & Villano, 2018).[5] The number of students enrolled (*x4*) is one of the most commonly used inputs in the literature (Podinovski et al., 2014). Finally, the Socioeconomic Index (*x5*), which was also used to categorize resilient students and in the literature on educational efficiency, is one of the main sources of information on the production function (De Witte & López-Torres, 2017). In addition, two categorical environment variables are included to control the context in which they operate: the educational sector *(z1)* and the year of application *(z2)*.

[5] Two variables are added to measure the human capital because, first, there is a difference in the functions, and second, in the public sector budgets are generally allocated according to the number of students in the school, whereas in the private sector this depends on the administrative orientation of the school and its board; these allocations affect the number of teachers employed.

Table 1 shows a summary of the descriptive statistics of the variables used in the empirical section, both to estimate resilient students and to measure efficiency. The variables provided by the ICFES through the Saber 11 exam are generalized for the entire school by dividing the sum of the variable by all the students who took the test per school and multiplying it by the number of students enrolled. Table 1 shows high levels of standard deviation in all the variables; in general, this is due to the heterogeneity in the territory where the schools operate and the functioning of the public and private sectors.

Due to the heterogeneities in educational access and quality among the 32 departments and the educational sectors (public and private) of Colombia, the descriptions and results are presented with this disaggregation. The gaps between educational sectors are worrying, since there is great pressure on household spending to provide access to education for their children, which generates effects on their well-being (OECD, 2016). In addition, there is evidence of the gap generated by the availability of resources between these sectors (Castro Aristizabal, 2019).

Table 1. Descriptive statistics of inputs and outputs

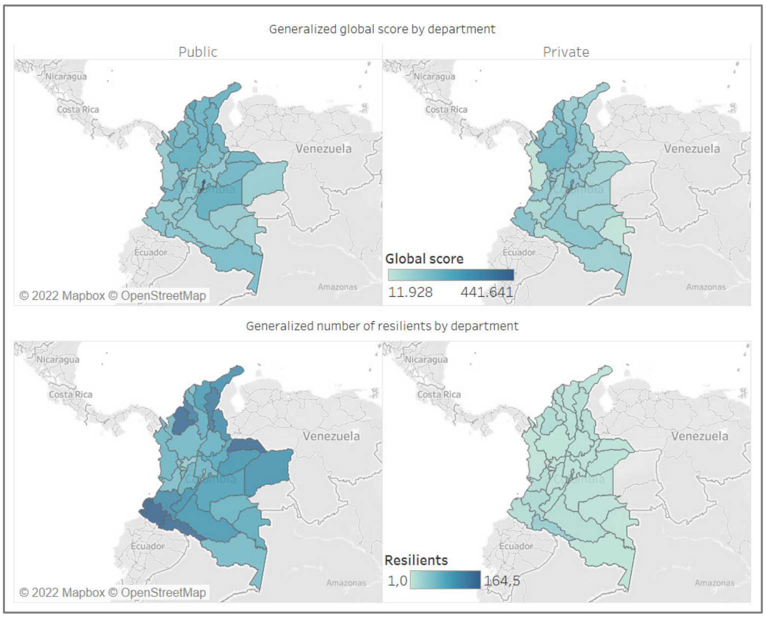
Variables	Description	Average	Q1	Q3	Standard deviation	Average public	Average private	Source
Output								
y1: global score	(Sum of the global score / Number of students Saber 11) * Educational institution enrollment	172,148	58,529	239,942	165,217	185,329	141,887	ICFES
y2: successful students	Number of students who pass the school grade	597	217	820	558	648	477	DANE
Input								
x1: electronic equipment	Number of tablets, desktops, or laptops in use	111	28	131	153	135	54	DANE
x2: teachers in management roles	Number of teachers who carry out management, planning, coordination, administration and orientation tasks	3.7	3.0	4	2.1	3.6	3.9	DANE
x3: teachers	Number of teachers in educational work in classrooms	30	15	38	23	31.2	26.9	DANE
x4: enrollment	Total number of students enrolled in the educational institution	672	238	938	624	746	500	DANE
x5:socioeco-nomic Index	(Sum of the socioeconomic and cultural index/ Number of students Saber 11) * Educational institution enrollment	33,769	11,164	47,336	32,847	35,579	29,614	ICFES
contextual variables[6]		**Category**	**%**					
Sector	Educational sector where the school operates (public or private)	Public	71.96%					DANE
		Private	28.04%					
Year	Current educational year. (2014–2019)	Each year represents approximately 16.5% of the sample						ICFES

Source: the authors.

The global score and the number of resilient students have a significant correlation of 17% in the public sector and 20% in the private sector. The difference in the magnitude of the relationships between variables of the educational sectors should be highlighted, since the private sector has only 28.8% of the schools under analysis and 21.3% of enrolled students.

[6] Categorical variables are used to control for the environment in which schools operate. For this reason, descriptive statistics are not presented.

Figure 1. Global score and resilient students by department and sector



Source: the authors.

Figure 1 shows the global score and the number of resilient students by department and educational sector. Two aspects stand out from this figure. First, there are significant differences between the sectors. The private sector does not have many resilient students per department, compared to the public sector; note that not all the departments have schools in the private sector (Vichada department has no private sector schools in the sample). Second, part of the heterogeneity in the public sector is explained by the concentration in the departments with the largest populations, which logically have a greater number of schools and students. Finally, due to the nature of private sector financing, it is not expected to have many students in a disadvantaged situation, at least in the main cities, where there is a relatively larger educational market.

5. Results

This section presents the results of the equity (number of resilient students) and efficiency estimates, as well as the relationship between these measures. First, the results of the multilevel model are shown considering multiple thresholds, followed by the descriptive results of the conditional order-m model. Finally, the relationship disaggregated by the educational sector of these two measures is analyzed.

The results obtained in the first stage with the multilevel model identify resilient students for the later stages. Models are estimated with three (20%, 25%, and 33%)[7] different thresholds to increase robustness. Table 2 shows the model results as the percentage of resilient students by sector, year, and threshold. On average, between 0.5% and 2.24% of students in the private sector are resilient, whereas in the public sector the range is between 7.36% and 18.48%.

Table 2. Percentage of resilient students by school sector, year and thresholds

Sector	Year	Resilients 20%	Resilients 25%	Resilients 33%	Number of students
Public	2014	7.95%	11.86%	19.76%	331,643
	2015	7.74%	11.72%	19.85%	339,026
	2016	7.57%	11.32%	19.07%	317,061
	2017	7.07%	10.54%	17.62%	328,675
	2018	6.90%	10.27%	17.23%	326,181
	2019	6.96%	10.32%	17.36%	331,143
Private	2014	0.40%	0.71%	1.75%	93,622
	2015	0.33%	0.71%	1.71%	97,367
	2016	0.45%	0.82%	1.95%	79,923
	2017	0.78%	1.36%	2.80%	82,315
	2018	0.77%	1.27%	2.66%	75,605
	2019	0.75%	1.30%	2.59%	93,246
Total		5.94%	8.92%	15.09%	2,495,807

Source: the authors.

The results of the multilevel model show that there is a significant relationship between the Socioeconomic Index and the global score. In addition, the variance participation coefficient justifies the inclusion of the two levels (school and municipality). The

[7] After the estimation in Table 2, the baseline scenario described in this section of the paper considers the threshold of 25%.

correlation between the results when different thresholds are used is significant and high, as can be seen in Annex 1.

The efficiency estimates are made following the methodological proposal described in section 4b. To estimate a conditional order-m model, the value of the parameters *m* and *B* must be determined, which is the size of the partial frontier with which the other schools are going to be compared and the number of times this process is repeated. In this case, it is determined as 11,000 (*m*) and 200 (*B*), since these are the numbers with which there are 10% of super-efficient units in the estimates (Bonaccorsi et al., 2006; Felder & Tauchmann, 2013) per year. An orientation toward output is used, since the general objective of students and educational managers is to maximize performance subject to given resources.

Table 3 offers an overview of the estimates, where levels greater than one show inefficiency or potential efficiency given the inputs, and the results of models are disaggregated by year and educational sector. General interpretations of the results are made with the average efficiency measure (column 3). The average inefficiency level in 2019 is 1.2151; this means that they could increase their test scores and the number of students who pass the exam by 21.51% without using a higher level of inputs. In this regard, public schools have a potential level of efficiency of 22.59%, while for the private sector it is 18.84% in that year. The results show that on average levels of inefficiency are higher in the public sector than in the private sector from 2016 until 2019.

Table 3. Descriptive statistics of the results by sector

Year	Sector	Mean	SD	Min	Q1	Median	Q3	Max
Total	Public	1.2321	0.1199	0.9277	1.1532	1.2272	1.3038	2.0085
	Private	1.2168	0.1343	0.9998	1.1202	1.2001	1.2985	2.0334
	Total	1.2278	0.1243	0.9276	1.1436	1.2204	1.3025	2.0334
2019	Public	1.2259	0.1164	0.9277	1.1511	1.2213	1.2928	2.0085
	Private	1.1884	0.1223	1.0000	1.1040	1.1701	1.2504	1.8325
	Total	1.2151	0.1193	0.9277	1.1357	1.2068	1.2851	2.0085
2018	Public	1.2280	0.1136	0.9336	1.1548	1.2260	1.2996	1.8383
	Private	1.2107	0.1395	1.0000	1.1124	1.1918	1.2929	2.0171
	Total	1.2235	0.1212	0.9336	1.1419	1.2177	1.2984	2.0171
2017	Public	1.2471	0.1270	0.9723	1.1634	1.2405	1.3208	1.9026
	Private	1.2255	0.1253	1.0000	1.1361	1.2190	1.3066	1.7549
	Total	1.2412	0.1269	0.9723	1.1560	1.2346	1.3182	1.9026
2016	Public	1.2439	0.1224	0.9853	1.1636	1.2435	1.3180	1.8216
	Private	1.2208	0.1402	1.0000	1.1176	1.2060	1.3049	1.9343
	Total	1.2375	0.1280	0.9853	1.1517	1.2355	1.3158	1.9343
2015	Public	1.2501	0.1273	0.9695	1.1622	1.2504	1.3339	1.8265
	Private	1.2517	0.1444	0.9999	1.1518	1.2420	1.3409	2.0334
	Total	1.2505	0.1325	0.9695	1.1595	1.2484	1.3352	2.0334
2014	Public	1.1991	0.1036	0.9535	1.1351	1.1968	1.2566	1.9164
	Private	1.2054	0.1250	1.0000	1.1154	1.1904	1.2839	1.9038
	Total	1.2009	0.1103	0.9535	1.1291	1.1953	1.2626	1.9164

Source: the authors

Table 4 shows the efficiency levels by educational sector and municipal category. In Colombia these categories are defined according to variables such as economic activity, financial performance and institutional capacity. The municipalities where the greatest development and economic and institutional capacities are found is a special case, with only nine municipalities, mainly the large departmental capitals. On the other hand, there are 1,178 municipalities in category F, close to 88% of the total. This table highlights two findings. First, the poorer performance of public sector institutions is marked by the vast majority of small municipalities with few institutional capacities, since it is the only category where performance is worse in the public sector (1.2544) than in the private one (1.2305). Second, as the municipal category decreases, the levels of inefficiency increase, rising from 1.1972 to 1.2530 on average.

Table 4. Efficiency results by educational sector and municipal category

Category	Public	Private	Total
Special	1.1986	1.1959	1.1972
A	1.19859	1.22986	1.21156
B	1.18912	1.21956	1.20325
C	1.22168	1.23855	1.22724
D	1.23216	1.2427	1.23523
E	1.2319	1.26969	1.24226
F	1.25442	1.23056	1.25302
Total	1.23196	1.21656	1.22764

Source: the authors.

Table 5 illustrates the levels of correlation between resilient students and efficiency, disaggregated by educational sector and year. There is a negative correlation between the levels of inefficiency (efficiency values greater than the unit) and the number of resilient students. In the public sector there is a negative correlation up to 33 in 2019; this means that the number of resilient students falls as the level of inefficiency increases. When the total sample and the private sector schools are analyzed, the correlations follow the same trend (-19 and -11 in 2019), but with lower magnitudes. When the relationship between resilient students and educational efficiency is analyzed, the same consistency is found by sector and year.

Table 5. Spearman correlation between resilient students and efficiency estimated by sector and year

Year	Public sector	Private sector	Total
2019	-0.3307***	-0.1193***	-0.1988***
2018	-0.2941***	-0.1108***	-0.2012***
2017	-0.3103***	-0.1209***	-0.2199***
2016	-0.2761***	-0.0339	-0.1821***
2015	-0.2519***	-0.1089***	-0.1969***
2014	-0.2281***	-0.0706**	-0.1802***

(***): significant at 1% confidence level.
Source: the authors.

As mentioned above, the differences between educational departments and sectors in Colombia are significant. Annex 2 reports the efficiency results disaggregated by department to show the social gaps between regions. The highest level of average inefficiency by department is evidenced in 2019 in the private sector: Chocó has an inefficiency level of 83.15%. Likewise, in 2018, the difference between the department with the best and worst performance in the public sector is 26.17%, whereas in the private sector the largest gap is identified in 2019 with a difference of 70.90%. Finally, the results show that in all the years at least 33% of the departments (10 out of 32) have worse behavior in the public sector than in the private sector.

We can summarize the results presented in the paper in three main points. First, there is a high correlation when different thresholds are used to estimate resilient students. Second, there is a negative relationship between inefficiency and the number of resilient students in both sectors. Third, there are large differences between educational departments and sectors, in general, with worse performance in the public sector than in the private sector.

6. Concluding remarks and policy implications

This article uses two complementary methodologies to analyze resilient students and their relationship with educational efficiency. First, a multilevel model with random intercept and random slope is estimated with the students' socioeconomic index as the independent variable and the global score as the dependent variable, considering the municipalities at a second level. Afterwards, a conditional order-m model is used to estimate educational efficiency and analyze the relationship. These models are based on the global score of Saber 11 and the number of students who pass the year exams as outputs.

The main conclusion that can be drawn in this study is the negative relationship between educational inefficiency and the number of resilient students. In addition, three further aspects can be highlighted. First, we found that in most of the years analyzed, the public sector performs worse than the private sector in the models estimated. Second, it is highlighted that dominance is found by the behavior of the public to the private sector in 33% of the departments. Third, there are large gaps in efficiency, up to 70%, between educational departments.

Decision makers and policymakers in Colombia should take these findings into account, since part of the educational system is evaluated in a complementary way to the traditional one, with a specific focus on social mobility. It is important to develop specific territorial policies that consider the differences between sectors and departments. A negative relationship is found between the inefficiency of schools and the number of resilient students, that is, as the number of resilient students increases, a lower level of efficiency is found.

These conclusions are not ideal for an educational system, although they do coincide with the low levels of social mobility found by the OECD (2018a). The negative relationship between efficiency and resilience could be associated with problems and costs in educational processes. These costs may be related to the increase in problems of cooperation and coordination of educational processes with students from low socioeconomic levels within the specific environment. In addition, educational institutions and municipalities have limited capacity to efficiently manage inequality within vulnerable contexts.

The problems of coordination and cooperation may be compounded by pedagogical problems in the classroom due to the presence of diverse groups, as educational systems have implemented a comprehensive approach to address the difficulties have in dealing with differences among the students. However, in a country as unequal and inequitable as Colombia, a diverse mix of students is both ideal and necessary, since it helps social mobility. This conclusion opens up an interesting line of research, namely to analyze the tradeoff between the decrease in efficiency and the benefits of social mobility due to resilient students.

On the other hand, we found that on average there is lower efficiency in municipalities with lower capacities. In general, the results show that the public sector helps social mobility significantly more than the private sector does, but the private sector has better levels of efficiency. However, this occurs mainly in municipalities with low institutional capacities. Therefore, the relationship of educational processes within institutions must be analyzed in depth, considering the institutional capacities of the municipalities.

The main implications of the results concern how the allocation of resources helps to improve the efficiency levels of schools. The relationships found between resilient students and school efficiency suggest that, if resources are targeted to improve the efficiency performance of resilient students, there is greater potential for improvement in academic performance for the school, as compared to a situation where efforts are focused on students with average academic behavior.

Educational policymakers should consider several factors when studying educational resilience in developing countries with high social inequity. The environments in some municipalities are challenging: they have experienced violence due to armed conflict, inconsistent access to and quality of basic education, and high or extreme rates of poverty. In such contexts, the efficient use of resources is even more crucial. Educational policies should therefore include robust efficiency measures such as the one we present in this study when determining budget allocation, disaggregated by sector, municipality, and even targeting single educational institutions.

Some lines for future research can be identified. First, the results of Saber 11 are used to represent the whole school; extensions should be made in other stages of the educational cycle. Second, radial distances can be used to analyze how to improve specific outputs as soon as possible, which facilitates a complementary analysis on the allocation of resources to the recipient students (Aparicio et al., 2018). Third, conditional models can be used to recognize and analyze the relevance of specific contextual variables. Fourth, approaches that combine quasi-experimental methodologies should be used to help control endogeneity in the process (i.e. self-selection of students across schools) in order to infer causality in the relationship between schools' efficiency and academic resilience.

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Annex

Annex 1. Correlation matrix of resilient students with different thresholds

Thresholds	20%	25%	33%
20%	1.0000		
25%	0.9688***	1.0000	
33%	0.9058***	0.9586***	1.0000

(***): significant at 1% confidence level.
Source: the authors.

Annex 2. Educational efficiency by department, year and sector

Department	2014		2015		2016		2017		2018		2019	
	Public	Private	Public	Private	Public	Private	Public	Private	Public	Private	Public	Private
Amazonas	1.2922	1.1367	1.3902	1.1799	1.3982	1.1655	1.3336	1.0822	1.4111	1.1257	1.2194	1.1225
Antioquia	1.2031	1.1904	1.2526	1.2292	1.2510	1.1971	1.2387	1.2158	1.2359	1.1919	1.2349	1.1824
Arauca	1.1843	1.2374	1.2193	1.3325	1.2202	1.2488	1.2108	1.2373	1.1692	1.1941	1.1623	1.1684
Atlántico	1.2118	1.2030	1.2409	1.2473	1.2354	1.2331	1.2349	1.2353	1.2231	1.2429	1.2052	1.2069
Bogotá, D.C	1.1328	1.1767	1.1751	1.2301	1.1639	1.1842	1.1731	1.2016	1.1723	1.1845	1.1885	1.1723
Bolívar	1.2341	1.1588	1.2628	1.1925	1.2892	1.1955	1.2992	1.1771	1.2935	1.1410	1.2589	1.1246
Boyacá	1.1868	1.2029	1.2610	1.2288	1.2247	1.1996	1.2223	1.1886	1.1795	1.1877	1.1992	1.1761
Caldas	1.2875	1.2531	1.3444	1.2725	1.3286	1.2572	1.3053	1.2663	1.2707	1.2470	1.2913	1.2184
Caquetá	1.2272	1.1743	1.2731	1.2191	1.3009	1.2768	1.3080	1.2448	1.2775	1.2492	1.2697	1.1312
Casanare	1.1799	1.2435	1.2285	1.3216	1.2019	1.2316	1.2314	1.1958	1.2049	1.2297	1.2070	1.1942
Cauca	1.2156	1.2560	1.2801	1.3248	1.2881	1.3574	1.3024	1.3161	1.2520	1.3070	1.2426	1.2598
Cesar	1.2110	1.2829	1.2455	1.3160	1.2320	1.2979	1.2438	1.2746	1.2285	1.2680	1.1965	1.2366
Chocó	1.3356	1.4203	1.3552	1.4351	1.3954		1.3794	1.7158	1.3856	1.6051	1.3249	1.8315
Córdoba	1.1735	1.2298	1.2194	1.2273	1.2242	1.2244	1.2443	1.2101	1.2185	1.2196	1.2037	1.1746
Cundinamarca	1.1958	1.2068	1.2733	1.2497	1.2486	1.2196	1.2432	1.2217	1.2220	1.2094	1.2300	1.1819
Guaviare	1.2508	1.1540	1.2422	1.3470	1.2923	1.2956	1.3099	1.3259	1.2697	1.3197	1.3222	1.2203
Huila	1.1966	1.2280	1.2554	1.2534	1.2457	1.2208	1.2503	1.2267	1.2164	1.2141	1.2141	1.1917
La Guajira	1.2040	1.2234	1.2395	1.2313	1.2747	1.2271	1.2959	1.2182	1.3095	1.2095	1.2816	1.1769
Magdalena	1.2439	1.2623	1.2669	1.2894	1.2875	1.2989	1.3140	1.2872	1.3027	1.2898	1.2629	1.2457
Meta	1.1879	1.2641	1.1937	1.3138	1.2118	1.2472	1.2189	1.2691	1.2149	1.2766	1.2147	1.2270
Nariño	1.1259	1.2180	1.1895	1.2445	1.1698	1.1947	1.1776	1.2356	1.1494	1.1969	1.1541	1.1619
Norte de Santander	1.1880	1.2043	1.2444	1.2477	1.2099	1.2428	1.2072	1.2438	1.1764	1.2302	1.1798	1.2082
Putumayo	1.2268	1.1257	1.2340	1.2712	1.2432	1.3179	1.2447	1.3003	1.2069	1.3094	1.2333	1.1980
Quindío	1.2095	1.2510	1.2709	1.3071	1.2718	1.1973	1.2653	1.1896	1.2459	1.2045	1.2360	1.1824
Risaralda	1.1903	1.2400	1.2447	1.2633	1.2449	1.2550	1.2439	1.2327	1.2348	1.2296	1.2359	1.1988
Santander	1.1887	1.2236	1.2313	1.2732	1.2061	1.2281	1.2086	1.2118	1.1746	1.2144	1.1835	1.1824
San Andres y Providencia	1.2686	1.1992	1.3635	1.2624	1.3247	1.2208	1.3273	1.2590	1.3549	1.1765	1.3832	1.2347
Sucre	1.1918	1.2053	1.2480	1.2836	1.2540	1.2586	1.2575	1.2111	1.2487	1.2158	1.2361	1.2211
Tolima	1.2142	1.2551	1.3007	1.3053	1.2760	1.2694	1.2949	1.2722	1.2625	1.2588	1.2709	1.2301
Valle del Cauca	1.1982	1.2186	1.2473	1.2857	1.2527	1.2563	1.2599	1.2701	1.2589	1.2293	1.2566	1.2086
Vaupés	1.2326	1.3799	1.2534	1.2985	1.2839	1.3958	1.2855		1.2472		1.2504	
Vichada	1.2113		1.2242		1.2876		1.3037		1.2795		1.2626	
Total	1.1991	1.2054	1.2501	1.2517	1.2439	1.2208	1.2471	1.2255	1.228	1.2107	1.2259	1.1884

Source: the authors.