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Revenue-sharing and volume flexibility in the supply chain

Nicos Koussis [a], Florina Silaghi [b,1,*]

[a] Frederick University Cyprus, Department of Business Administration, 7, Y. Frederickou Str. Pallouriotisa, Nicosia, 1036, Cyprus
[b] Universitat Autònoma de Barcelona, Campus de Bellatera, Barcelona 08193, Spain

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ABSTRACT

We model capacity choice and utilization of a manufacturer selling products through a retailer under a revenue sharing contract. We derive the optimal revenue sharing contract which internalizes the impact on capacity, utilization choice and the final downstream price of the product and provide a valuation of the retailer and supplier under uncertainty in a multiperiod setting. In extensions of this framework, we analyze constraints on minimum delivered quantities and also build a finite-time numerical method that considers an abandonment option for the supplier and hold-up problems for the retailer, as well as the supplier's option to expand capacity. Our model predicts higher revenue sharing ratios charged by retailers when suppliers operate in more volatile upstream markets, when the product is a necessity rather than a luxury good and when retailers can impose minimum delivered quantities. On the contrary, we find that suppliers will be able to obtain a higher revenue share when operating in industries with high fixed costs and for contracts of shorter horizon since hold-up problems for retailers increase and the retailer has to provide more incentives to suppliers not to abandon operations. The option to expand capacity benefits more significantly the supplier compared to the retailer. Finally, we consider the decisions of a vertically integrated firm showing the gains from vertical integration and demonstrating that the optimum vertically coordinated production can be achieved in the decentralized multiperiod setup through a combination of a fee per product and revenue sharing.

1. Introduction

Revenue sharing contracts are quite common in practice. Their popularity has spiked in recent years with the increase in sales via online marketplaces. Revenue sharing occurs in many different types of industries such as the airline industry (Fu and Zhang, 2010), video rental (Altug and van Ryzin, 2014; Cachon and Lariviere, 2005; Giannoccaro and Pontrandolfo, 2004), newspapers (Gerchak and Khmelnitsky, 2003), electronics, e.g., Apple App Store, Google Play, or online market places such as Amazon.com (Bart et al., 2021). They are also used in franchising in sectors such as hotels, fast foods and automobile renters (Lal, 1990; Mathewson and Winter 1985).

Several big retailers such as Wal Mart, Target or Ahold USA offer revenue sharing contracts to their suppliers through which the suppliers can rent shelf space and sell their products directly (Lee and Chu, 2005; Zhao et al., 2020). Moreover, in online marketplaces such as Amazon.com the retailer (Amazon) must determine revenue sharing ratios for different products sold on their platform. Google and Apple need to decide how much to charge app developers that sell applications through their platforms. How should these firms determine their revenue sharing offers to the suppliers to account for the future uncertainty in demand of products and the response of suppliers in deciding their installed capacity? And how should these contracts also account for volume flexibility, i.e., the possibility that suppliers may adjust the volume of production each period based on uncertain demand? For example, a supplier that is offered a low revenue share by a retailer may decide to install low capacity that may reduce the profitability of a retailer. In addition, a supplier may adjust the volume of quantities produced downwards in response to higher energy prices, rationalizing the range of products offered focusing on those offering higher profit

* Corresponding author.Universitat Autònoma de Barcelona, Campus de Bellatera, Barcelona, 08193, Spain.
E-mail addresses: bus.kn@fit.ac.cy (N. Koussis), florina.silaghi@uab.cat (F. Silaghi).

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margins or simply by reducing production of a product in response to consumers shift in demand.[2] In this paper, we propose a framework which accounts for the above features and thus solves an important real business problem faced by retailer firms in practice in determining optimal revenue sharing contracts. Our analysis helps understand the factors determining the various revenue sharing contracts observed in online marketplaces (e.g., revenue sharing ratios by Amazon range from 8% for computers to 15% for books or 20% for gift cards) and also how Google and Apple determine the share charged to their app developers that sell applications through their platforms (ranging between 15% and 30%).

Thanks to their popularity in the real world, revenue sharing contracts have received considerable attention in the academic literature. Nevertheless, most of the studies on revenue sharing propose a static framework with a one-period model and a fixed quantity to be ordered, neglecting the ability of the firms to adjust the volume of operations depending on market conditions. Moreover, with the exception of a few studies (e.g., Cachon and Lariviere, 2001; Wang and Gerchak, 2003), most studies focus on production quantities only, disregarding capacities. On the other hand, volume flexibility has been analyzed only in the context of a single firm within the real options literature (e.g., Hagspiel et al., 2016; Ritchken and Wu, 2021; Sarkar, 2009, 2018), thus neglecting the effects that volume flexibility might have on other firms in the supply chain. To fill these gaps in the literature, we analyze a decentralized supply chain where the retailer firm offers a revenue sharing contract to a supplier firm that faces capacity and volume flexibility choices. Overall, the significance of our work is that we bridge the revenue sharing literature (e.g., Cachon and Lariviere, 2005; Gianoccarro and Potrandolfo, 2004) with the real options literature on production flexibility by proposing a unified real options framework to analyze a revenue sharing contract within a decentralized supply chain under supplier volume flexibility. The innovation of our paper lies in that it extends these settings to multiple periods under uncertain demand and incorporates capacity choice and volume flexibility of supplier firms. In addition, in extensions of the basic framework we account for the common practice that retailers may impose minimum delivered quantities, study finite horizon contracts, the option of the supplier to expand capacity at an optimal time in the future and hold-up problems that may arise for retailers when a supplier stops the supply of products and abandons operations. Finally, we contribute to the revenue sharing literature by offering a comparison of channel performance of our decentralized pure revenue sharing contract with a vertically integrated supply chain in terms of capacity installed, produced quantities, prices and supply chain value within our multi-period under uncertainty setting with volume flexibility. We also show how a two-parameters revenue sharing contract with a share of revenue and a fee per product can achieve the efficiency gains of a vertically integrated supply chain.

To determine the optimal revenue share of the retailer we solve a Stackelberg (leader-follower) game in which the retailer chooses its revenue sharing terms taking into account supplier's reaction in terms of capacity choice and utilization rate and their repercussions on downstream product prices. Our analysis quantifies a novel trade-off involved in the retailer's choice of the optimal revenue sharing ratio in the presence of capacity choice and volume flexibility where besides the direct positive impact of a higher revenue share on retailer value the retailer has to balance out the negative impact on supplier's installed capacity and produced quantities. We demonstrate that these trade-offs result in an optimal revenue sharing ratio which maximizes retailer's value. Our model predicts higher revenue sharing ratios charged by retailers when suppliers operate in more volatile upstream markets where production flexibility is more valuable, or when the product is a necessity rather than a luxury good. On the contrary, we expect suppliers to be able to extract higher revenue sharing ratios when operating in industries characterized by high operational leverage (important fixed costs). To our knowledge, these are all novel predictions of our model.

We also extend the model to account for the common practice that retailers may impose various constraints on the supplier's production decision such as minimum customer demand fill rates or service levels (Fry et al., 2001; Wang et al., 2004). This implies that the supplier would need to produce and deliver a minimum quantity of goods to meet a certain fill rate or service level. We quantify the negative impact on supplier's value of these constraints and show that when the constraints become binding the supplier needs to install higher capacity and engage in higher utilization of quantities in production. These effects benefit the retailer despite the dampening effect that higher quantities have on the prices at which the goods are sold.

Moreover, we investigate the channel performance of our decentralized pure revenue sharing contract by comparing the produced quantities and prices in the downstream market with those of a vertically integrated supply chain. We analyze the supply chain gain, defined as the percentage difference in total supply chain value between a centralized channel and a decentralized channel, and how it varies with model parameters such as volatility of demand, demand elasticity, retailer's share of the costs, etc. We show that an uncoordinated supply chain results in lower capacity and utilization rates and higher downstream prices compared to the vertically integrated firm, in line with a double marginalization problem. The percentage supply chain gain from vertically integrating production ranges between 11% and 22% for the parameters considered. The gains from vertical integration are relatively higher when demand uncertainty is higher, i.e., for environments where production flexibility is more valuable. We therefore expect a higher likelihood of vertical mergers taking place when suppliers operate in volatile upstream markets with production flexibility.

We also build a finite-time numerical method whose solution converges to the solution of our perpetual time analytical model for a sufficiently large horizon. We find that shorter duration contracts lead to lower firm values as well as lower capacity installed and utilization, while the revenue sharing ratio does not vary with maturity. We then consider an abandonment option of the supplier leading to hold-up problems for the retailer, as well as the supplier's option to expand capacity. We show that when the supplier has the option to abandon, the retailer may offer higher revenue ratios to the supplier firm to prevent a hold-up problem, and this results in higher capacity and utilization. The hold-up problem reduces retailer's ability of using minimum delivered quantities to extract value from the supplier firm and becomes more severe for shorter-term contracts. Regarding the option to expand capacity, we show that it improves retailer and supplier values and leads to a more conservative choice of initial capacity for the supplier firm.

Finally, we extend our pure revenue sharing contract to a two-parameters revenue sharing contract that includes, besides the share of supplier's revenues captured by the retailer, a fee per product paid by the supplier to the retailer (e.g., as a fee for using the retailer's platform for selling products). Under this contract, the optimal capacity choice and utilization rate of the supplier coincide with the optimal choice of a coordinated supply chain, so that coordination is achieved. In designing this contract, we ensure that a win-win condition holds, i.e., both parties obtain a higher profit under the coordinating contract than under a pure revenue sharing contract, by tuning the contract parameters. However, a certain degree of cooperation between the parties would be needed to design such a contract. Our results thus extend Cachon and Lariviere (2005) and Gianoccarro and Potrandolfo (2004)'s results to a multi-period setting under uncertainty with capacity choice and volume flexibility within a two-echelon supply chain.

The rest of this paper is organized as follows. Section 2 presents the related literature. Section 3 describes the framework and the

[2] See recent example in UK: https://www.theguardian.com/business/2022/apr/27/uk-retail-sales-slump-as-soaring-energy-prices-hit-households-cbi.

mathematical solution for the decentralized pure revenue sharing contract, as well as for the vertically integrated supply chain. Section 4 provides numerical sensitivity and our main results. Section 5 extends the framework to consider the case of minimum delivered quantities imposed by the retailer. Section 6 presents the finite-time numerical method solution and analyzes the option to abandon, hold-up problems and the option to expand capacity. Section 7 analyzes a two-parameters revenue sharing contract that coordinates the supply chain. Finally, Section 8 provides the managerial implications of our results and concludes.

2. Related literature

Our paper is related to two strands of the literature. First, we contribute to the literature on revenue sharing contracts. A recent survey by Bart et al. (2021) summarizes the operating research literature on revenue sharing contracts. An important strand of this literature analyzes the channel performance of revenue sharing contracts in comparison with other types of contracts (Dana and Spier, 2001; Gerchak and Wang, 2004). Other studies investigate issues such as horizontal competition (Chakraborty et al., 2015; Kong et al., 2013; Krishnan and Winter 2011; Wang and Shin, 2015; Yao et al., 2008), risk/loss-averse supply chains (Zhang et al., 2015), asymmetric information (Gerchak and Khmelnitsky, 2003; Xiao and Xu, 2018) or effort and cost sharing (Bhaskaran and Krishnan, 2009). Recent advances in this literature study revenue sharing contracts for supply chains of virtual products (Avinadav et al., 2015a and 2015b; Tan and Carrillo, 2017), behavioral laboratory experiments (Katok and Wu, 2009), sustainable supply chains (Govindan and Popiuc, 2014; Hsueh, 2014) and carbon emissions (Yang and Chen, 2018). Other recent developments include Zhang et al. (2022) who characterize coordinating contracts that result in Pareto-optimal actions in a setting with risk averse agents and Ha et al. (2022) who study how the channel structure impacts a retailer's incentive in exerting service effort.

The revenue sharing literature has analyzed two main types of contracts: a wholesale-price contract with revenue sharing (Cachon and Lariviere, 2005) and a consignment contract with revenue sharing (Wang et al., 2004) which omits the wholesale price component. Our paper extends these settings to multiple periods under uncertain demand and incorporates capacity choice and volume flexibility of supplier firms. Relatedly, Tsay (1999) proposes quantity flexible contracts with commitment of the retailer to buy and the supplier to provide certain quantities, however their context is single period and they do not study revenue sharing (see also recent work by Li et al., 2021). In extensions of our basic framework we study a number of other features including finite horizon contracts and options to expand capacity, the common practice that retailers may impose minimum delivered quantities and hold-up problems in the supply chain. Similarly, Wang et al. (2021) study minimum order quantities, nevertheless in their setting these are imposed by the supplier and they have a different setup with multiple suppliers and buyback contracts. Interestingly however, in their analysis they show that suppliers can use minimum order quantities to extract more value from retailers (see also Tsay, 1999 and Li et al., 2021 for related results). We contribute with new insights by showing that the opposite may be true when retailers impose such minimum quantity requirements on delivered quantities from suppliers. This is particularly important for large retailers (e.g., WalMart) which have significant market power over supplier firms. Xu et al. (2020) provide a comprehensive literature review of disruption risks in the supply chain which are usually caused by natural causes (e.g. earthquakes, hurricanes, and floods), due to human factors (e.g. fires, strikes, and terrorism), or legal disruptions (e.g. environmental laws). They provide examples of the significant economic impact this would cause to firms when such interruptions occur. Our analysis measures the economic impact of disruptions focusing on hold-up problems for retailers caused by the potential insolvency of suppliers.

Secondly, we contribute to the growing literature that applies the real options approach to operations research (for a review see Trigeorgis and Tsekrekos, 2018). A significant part of this literature has focused on the optimal timing of capacity decisions. Early work in this area includes Dangl (1999) for single stage capacity choice of a monopolist and recent extensions include Chronopoulos et al. (2017) who extend this setting to stepwise investment in capacity and Lavrutich (2017) who considers capacity choice with duopoly competition. Another important focus of study of this literature has been the analysis of volume flexibility which has been investigated in the context of a single firm within the real options literature (Hagspiel et al., 2016; Ritchken and Wu, 2021; Sarkar, 2009, 2018). Our work is closely related to Hagspiel et al. (2016) who analyze optimal capacity choice and production flexibility, to Sarkar (2018) who identifies a firm's optimal degree of operating leverage (DOL) under investment and production flexibility, and to Ritchken and Wu (2021) who introduce corporate debt and analyze the impact of production flexibility on leverage and capital structure. De Giovanni and Massabò (2018) incorporate both downside and upside volume flexibility in this context. Our work extends this strand of the literature in a supply chain context by adding the retailer firm and focusing on the capacity and volume flexibility choices of upstream firms. Thus, while the previous literature has focused on studying these issues for single firms or has considered competition among firms in the same industry, our focus is on the interactions between the retailer and supplier within the supply-chain through revenue sharing contracts that incorporate real options of capacity choice and volume flexibility of the supplier firm. Our framework thus provides a connection of the revenue sharing literature with the real options literature on capacity choice and production flexibility in a supply chain setting.

3. The model

3.1. The model setup

The price of the good sold in the downstream market in period t is p_t per unit of goods sold and given by the iso-elastic inverse demand function:

$$p_t = x_t q_t^{\varepsilon} \tag{1}$$

where $-1 < \varepsilon < 0$ is a measure of price sensitivity and x_t represents the demand shock. The elasticity of demand which is usually defined as the percentage sensitivity of quantity demanded to price changes is thus $\left(\frac{1}{|\varepsilon|}\right)$. Thus, a higher $|\varepsilon|$ implies a more inelastic demand. Since $|\varepsilon| < 1$ this implies that our focus is on $\frac{1}{|\varepsilon|} > 1$, i.e., an elastic demand where an increase in prices by 1% causes a more than 1% decrease in quantity. In line with previous literature, demand is assumed elastic since if demand were inelastic profits would tend to infinity as the quantities tend to zero. The same iso-elastic demand was used in Aguerrevere (2009), Dixit and Pindyck (1994), Dobbs (2004) and Silaghi and Sarkar (2021). For a review of the implications of different forms of demand functions on firms' capacity choice see Huberts et al. (2015). The demand shock x_t affecting the price per unit at which the goods can be sold in the downstream market follows a Geometric Brownian motion (GBM hereafter):

$$\frac{dx_t}{x_t} = \mu dt + \sigma dZ \tag{2}$$

where μ is the expected rate of change, σ is the volatility and dZ is a standard increment of a Weiner process. The demand shock x_t can be interpreted as the relative strength of the demand in the downstream market. We assume risk-neutrality, with r denoting the risk-free interest rate, and that $r > \mu$ such that there is a rate of return shortfall similar to a convenience yield $\delta = r - \mu$. A higher δ (while keeping r constant) captures a lower rate of growth of the good's demand. We assume that the supplier selects the optimal capacity Q at $t = 0$. In order to select its capacity, the supplier needs to incur a one-time investment cost of kQ^{η}, where Q is the capacity of the goods (i.e., maximum units of goods that can be produced per unit time), Q^{η} is the amount of capital required to produce at that capacity (with $\eta > 1$), and the cost of capital is $\$k$ per unit. For simplicity, we assume that the optimal capacity is installed at an exogenous given threshold x_0. In later sections of the paper we study a supplier with a choice of initial capacity and an option to expand capacity at an optimal timing using our finite-time numerical model. The supplier firm faces both fixed costs of production c, as well as variable costs v. Due to variable costs, following the capacity choice the supplier selects the level of utilization of capacity q_t by maximizing its profits (see analysis that follows on determining the optimal q_t). Following Hagspiel et al. (2016) (see p.97), we assume that the decision relating the volume of production q_t is contemporaneous to the realization of demand shock x_t, i.e., performed once the firm has knowledge of the demand realization. With this note in mind, we drop the use of time subscripts from the subsequent analysis. The firm can either produce below full capacity, $q < Q$, in which case the level of production q varies with the demand shock x or at full capacity, with $q = Q$. This type of flexibility is important in many settings including, among others, car manufacturing (Hagspiel et al., 2016). The manufacturer produces the goods and sells them to the market through a retailer under a revenue sharing contract. We do not incorporate abandonment timing in the main analytical framework since the supplier can adjust the volume of production to limit losses when demand is not favorable. Thus, adding the optimal timing of stopping production will likely not have any major impact unless fixed costs are significant. We nevertheless provide the impact of an exit/abandon option in the presence of fixed costs in our finite-time numerical analysis.

The retailer incurs variable costs c_R per unit sold. We model the decision making of the two firms as a Stackelberg (leader-follower) game in which the retailer acting as the Stackelberg leader first chooses how much to charge the supplier by selecting its share of the revenue α obtained from each unit sold in the downstream market. This is the only choice variable for the retailer in the problem. In turn, the choice of α affects the capacity and utilization of capacity (i.e., production) decisions of the supplier and the price of goods in the downstream markets.[3]

3.2. The model solution

Since the retailer obtains a fraction α from the value of each unit sold this means that $(1-\alpha)$ remains to the supplier. The profits per dt interval for the supplier are then as follows: $\pi_S = ((1-\alpha)p - v)q - c = (1-\alpha)xq^{\varepsilon+1} - vq - c$. Maximizing the profits with respect to q results in the optimal level of $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$. It can be seen that $\frac{dq}{dx} > 0$, $\frac{dq}{dv} < 0$, $\frac{dq}{d\alpha} < 0$, while $\frac{dq}{d\varepsilon}$ is indeterminate. The analytic expressions for all derivatives are shown in Appendix B. These effects are intuitive. For example, a higher level of demand (x) will result in a higher utilization of capacity, while a higher level of variable costs results in a lower capacity utilization. In addition, when the share of revenues of the retailer increases this creates an incentive for the supplier firm to reduce the quantities delivered. Careful inspection of the expression $\frac{dq}{d\varepsilon}$ reveals that $\frac{dq}{d\varepsilon} > 0$ when the share of revenue demanded by the retailer is relatively low and/or the relative of the price to cost ratio (x/v) is high (indicating suppliers with relatively high profit margins). On the contrary, when the retailer extracts a high revenue share and/or the supplier has little profit margins then the supplier reacts by reducing quantities produced when demand becomes more elastic (i.e., $\frac{dq}{d\varepsilon} < 0$). As we will show later on, these same factors appear to influence whether the supplier is more profitable in more elastic or inelastic markets.

The corresponding price is $p = xq^{\varepsilon} = \frac{v}{(1-\alpha)(\varepsilon+1)}$. Note that the final downstream price adds a constant mark-up $\frac{1}{(1-\alpha)(\varepsilon+1)} > 1$ over the variable cost v (where $\varepsilon > -1$ is needed to ensure a positive mark-up). This mark-up increases as the share of revenue of the retailer increases thus highlighting the double marginalization effect in place. This mark-up also increases as the price sensitivity ε (in absolute terms) increases (i.e., as the demand becomes more inelastic). We find that $\frac{dp}{dv} > 0$, $\frac{dp}{d\alpha} > 0$, $\frac{dp}{d\varepsilon} < 0$. Substituting prices and quantities into supplier profit we obtain $\pi_S = A\,x^{-1/\varepsilon} - c$, where $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$. In line with economic intuition, the supplier's profits depend on parameters as follows: $\frac{d\pi_S}{dx} > 0$, $\frac{d\pi_S}{dv} < 0$, $\frac{d\pi_S}{d\alpha} < 0$. However, the impact of elasticity, $\frac{d\pi_S}{d\varepsilon}$ is indeterminate and depends on the share of revenues claimed by the retailer and the relative level of x relative to v. More thorough analysis of $\frac{d\pi_S}{d\varepsilon}$ shows that when the share claimed by the retailer is high and x relative to v is low, then $\frac{d\pi_S}{d\varepsilon} < 0$, but when the share claimed by the retailer is low and when x is sufficiently higher than v, then $\frac{d\pi_S}{d\varepsilon} > 0$. Thus, when the supplier operates with relatively high profit margins then its profitability is further enhanced by a more elastic demand (and vice versa).

We observe that q increases with x $\left(\frac{dq}{dx} > 0\right)$, however it cannot increase beyond Q which is the maximum capacity level. Assuming that the maximum capacity level is reached at $x = \overline{x}$ then using the optimal quantities we find that $Q = \left(\frac{\overline{x}(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$, which implies that the maximum capacity is reached at $\overline{x} = \frac{v}{(1-\alpha)(\varepsilon+1)Q^{\varepsilon}}$. Note that the threshold where full capacity is reached depends on the variable cost of production v, the installed capacity Q, the retailer's share of the price α and the elasticity of demand ε, as follows: $\frac{d\overline{x}}{dv} > 0$, $\frac{d\overline{x}}{dQ} > 0$, $\frac{d\overline{x}}{d\alpha} > 0$ and $\frac{d\overline{x}}{d\varepsilon}$ is indeterminate. Intuitively, a higher variable cost of production v, higher installed capacity Q, and higher retailer's share of the price α results in the supplier postponing production at full capacity. A more elastic demand results in an acceleration of the supplier firm entering into full scale operations when the installed capacity is small, while when the installed capacity is large a more elastic demand results in the supplier postponing switching to full capacity.

There are two operating regions depending on whether $x < \overline{x}$ or $x \geq \overline{x}$ as follows:

Region 1: $x < \overline{x}$: $p = \frac{v}{(1-\alpha)(\varepsilon+1)}$, $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ and $\pi_S = A\,x^{-1/\varepsilon} - c$, with $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$.

Region 2: $x \geq \overline{x}$: $p = xQ^{\varepsilon}$, $q = Q$ and $\pi_S = (1-\alpha)xQ^{\varepsilon+1} - vQ - c$.

Since we assume that there is no working capital (e.g., inventory or credit) the profits are equivalent to cash flows. Following standard arguments in the real options literature (see Dixit and Pindyck, 1994) the supplier firm value $S_i(x)$ satisfies the following differential equations depending on the region of operation:

$$rS_i(x) = (r-\delta)xS_i^{'}(x) + \frac{\sigma^2}{2}x^2S_i^{''}(x) + \pi_{Si},\ i = 1, 2. \tag{3}$$

where the last term denotes the cash flows received per dt.

The following proposition presents the supplier value in both regions.

Proposition 1. (Value of the supplier firm)

[3] One could also consider a model with retailer capacity and utilization choice in which the revenue sharing ratio is decided by the supplier acting as the Stackelberg leader. Earlier versions of the paper modeled such a framework. Such a setting would apply for example to franchising where the share of revenues claimed by a large multinational franchise firm (e.g., McDonalds) which acts as a supplier of a product affects the capacity of a local franchisee (e.g., by determining its store size or number of stores opened). The supplier would face a similar trade off in setting the optimal revenue sharing ratio in line with insights in Altug and van Ryzin (2014).

The supplier value is given by:

Region 1, $x<\bar{x}$:

$$S_1(x)=\frac{A}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon}-\frac{c}{r}+\Omega_1x^{\beta_1} \quad (4)$$

Region 2, $x\geq\bar{x}$:

$$S_2(x)=\frac{(1-\alpha)xQ^{\varepsilon+1}}{\delta}-\frac{c+vQ}{r}+\Omega_2x^{\beta_2} \quad (5)$$

$$\text{where } \beta_1=\frac{1}{2}-\frac{(r-\delta)}{\sigma^2}+\sqrt{\left(\frac{(r-\delta)}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}>1 \quad (6a)$$

$$\beta_2=\frac{1}{2}-\frac{(r-\delta)}{\sigma^2}-\sqrt{\left(\frac{(r-\delta)}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}<0 \quad (6b)$$

and Ω_1 and Ω_2 are determined from the following boundary conditions:

$$S_1(\bar{x})=S_2(\bar{x}) \text{ (Value}-\text{matching)} \quad (7)$$

$$S_1^{'}(\bar{x})=S_2^{'}(\bar{x}) \text{ (Smooth}-\text{pasting)} \quad (8)$$

Proof. The particular solutions in equations (4) and (5) are obtained by applying the differential equation in (3) the particular solution $S_i(x)=A_0+A_1x+A_2x^{-\frac{1}{\varepsilon}}$. Ω_1 and Ω_2 are obtained by applying (7) and (8) respectively using equations (4) and (5) (see Appendix A for the detailed expressions).

The term $\Omega_1x^{\beta_1}$ captures the adjustment in value when the supplier moves to full capacity in region 2, while the term $\Omega_2x^{\beta_2}$ captures the option to reduce the utilization of capacity below full capacity at level $q<Q$.

At time zero the value of the supplier firm is given by:

$$S_1^{Net}(x)=max_Q\{S_1(x)-kQ^{\eta}\}, if\ x<\bar{x} \quad (9)$$

else, if $x\geq\bar{x}$, the value of the supplier firm is given by:

$$S_2^{Net}(x)=max_Q\ \{S_2(x)-kQ^{\eta}\}\ for\ x\geq\bar{x} \quad (10)$$

Since $\bar{x}$ depends on Q, to find the optimal capacity we run various levels of capacity based on a dense grid of Q values where we apply (9) or (10) depending on the region being $x<\bar{x}$ or $x\geq\bar{x}$. Then the maximum value among supplier values among all grid levels defines the optimal capacity, as well as the operating region since it determines $\bar{x}$ where the firm operates.

We next move to the retailer. When the supplier is in region 1 (below full capacity), the retailer firm has the following profits per period $\pi_R=(\alpha p-c_R)q=\alpha xq^{\varepsilon+1}-c_Rq$. The optimal quantity level is given by the supplier's optimization which resulted in $q=\left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ (for $x<\bar{x}$) and the corresponding price is $p=xq^{\varepsilon}=\frac{v}{(1-\alpha)(\varepsilon+1)}$. Thus, the profit per dt for the retailer is: $\pi_R=Bx^{-1/\varepsilon}$ where:

$$B=\left(\frac{\alpha v}{(1-\alpha)(\varepsilon+1)}-c_R\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$$

The comparative statics for the retailer are as follows: $\frac{d\pi_R}{dx}$ is indeterminate, $\frac{d\pi_R}{dc_R}<0$, $\frac{d\pi_R}{d\alpha}$ is indeterminate, $\frac{d\pi_R}{dv}$ is indeterminate and $\frac{d\pi_R}{d\varepsilon}$ is indeterminate. Retailer's profits increase with the demand shock as long as the retailer's operating cost c_R is relatively low to allow for positive margins and they decrease with retailer operating costs. The overall effect of the upstream variable costs on retailer's profit depends on which of the following two effects dominate. On the one hand, increasing variable costs reduces quantities produced which has a negative impact on retailer profit. On the other hand, it also increases the price which positively affects retailer profit. Increasing retailer's revenue share has similar opposite indirect effects on retailer profits, in addition to a direct positive effect. In Appendix B, we show that $\frac{d\pi_R}{dv}<0$ and $\frac{d\pi_R}{d\alpha}<0$ when c_R is relatively small. Intuitively this implies that the effect of reduced quantities on the profitability of the retailer is more important than price increases when the retailer has high profit margins (implied by lower c_R). Finally, the effect of elasticity depends on parameter values, as in the case of the supplier.

For $x>\bar{x}$, the supplier produces at full capacity, $q=Q$ and the corresponding price is $p=xQ^{\varepsilon}$, and $\pi_R=(\alpha p-c_R)Q=\alpha xQ^{\varepsilon+1}-c_RQ$.

The retailer value satisfies the following differential equation:

$$rR_i(x)=(r-\delta)xR_i^{'}(x)+\frac{\sigma^2}{2}x^2R_i^{''(x)}+\pi_{Ri}, i=1,2 \quad (11)$$

The following proposition derives the value of the retailer.

Proposition 2. (Value of the retailer firm)

Region 1, $x<\bar{x}$:

$$R_1(x)=\frac{B}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon}+\Omega_1^Rx^{\beta_1} \quad (12)$$

Region 2, $x\geq\bar{x}$:

$$R_2(x)=\frac{\alpha xQ^{\varepsilon+1}}{\delta}-\frac{c_RQ}{r}+\Omega_2^Rx^{\beta_2} \quad (13)$$

where the solutions for Ω_1^R and Ω_2^R are determined from equations

$$R_1(\bar{x})=R_2(\bar{x}) \text{ (Value}-\text{matching)} \quad (14)$$

$$R_1^{'}(\bar{x})=R_2^{'}(\bar{x}) \text{ (Smooth}-\text{pasting)} \quad (15)$$

Proof. The particular solutions in equations (12) and (13) are obtained by applying the differential equation in (11) the particular solution $R_i(x)=A_0+A_1x+A_2x^{-\frac{1}{\varepsilon}}$. Ω_1^R and Ω_2^R are obtained by applying (14) and (15) respectively using equations (12) and (13) (see appendix A).

Note that the condition in equation (15) is a continuity (not an optimality) condition since the retailer value depends on the optimal choice of capacity of the supplier as described in equations (9) and (10) which also define the optimal threshold $\bar{x}$.

Finally, for comparison, we calculate the value of the firm if there is vertical integration. The vertically integrated profit per period in region 1 (unconstrained) is as follows: $\pi_V=(p-v-c_R)q-c=xq^{\varepsilon+1}-(v+c_R)q-c$. Maximizing the profits with respect to q results in the optimal level of $q_V=\left(\frac{x(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$ and the corresponding price is $p_V=xq_V^{\varepsilon}=\frac{v+c_R}{(\varepsilon+1)}$. Substituting this into profit we obtain $\pi_V=A_Vx^{-1/\varepsilon}-c$ where $A_V=-\left(\frac{(v+c_R)\varepsilon}{\varepsilon+1}\right)\left(\frac{(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$. A comparison of the vertically integrated firm with the non-coordinated profits of the supplier shows that the share of revenues of the retailer does not affect the optimally produced quantities nor the downstream prices. However, now the downstream operating cost c_R enters the picture; the higher the cost c_R, the lower the produced quantities and the higher the price in the downstream market.

Assuming that the maximum capacity level is reached at $x=\bar{x}_V$ then using the optimal quantities we find that $Q_V=\left(\frac{\bar{x}_V(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$ which implies that the maximum capacity is reached at $\bar{x}_V=\frac{v+c_R}{(\varepsilon+1)Q_V^{\varepsilon}}$. Note that the threshold where full capacity is reached depends on the variable cost of production v, the retailer's variable cost c_R, the price sensitivity of demand ε, the installed capacity Q_V as follows: $\frac{d\bar{x}_V}{dv}>0$, $\frac{d\bar{x}_V}{dc_R}>0$, $\frac{d\bar{x}_V}{d\varepsilon}$ is indeterminate and $\frac{d\bar{x}_V}{dQ_V}>0$.

The value of the vertically integrated firm satisfies the following differential equation:

$$rV_i(x) = (r-\delta)xV_i^{'}(x) + \frac{\sigma^2}{2}x^2V_i^{''}(x) + \pi_{Vi}, i = 1, 2 \tag{16}$$

The following proposition derives the value of the vertically integrated firm.

Proposition 3. (The value of the vertically integrated firm)

Region 1, $x < \bar{x}_V$:

$$V_1(x) = \frac{A_V}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon} - \frac{c}{r} + \Psi_1 x^{\beta_1} \tag{17}$$

Region 2, $x \geq \bar{x}_V$:

$$V_2(x) = \frac{xQ_V{}^{\varepsilon+1}}{\delta} - \frac{c + (v + c_R)Q_V}{r} + \Psi_2 x^{\beta_2} \tag{18}$$

where β_1 and β_2 are given by (6a) and (6b) and Ψ_1 and Ψ_2 are determined from the following boundary conditions:

$$V_1(\bar{x}_V) = V_2(\bar{x}_V) \text{ (Value − matching)} \tag{19}$$

$$V_1^{'}(\bar{x}_V) = V_2^{'}(\bar{x}_V) \text{ (Smooth − pasting)} \tag{20}$$

Proof. The particular solutions in equations (17) and (18) are obtained by applying the differential equation in (16) the particular solutions $V_i(x) = A_0 + A_1x + A_2x^{-\frac{1}{\varepsilon}}$. Ψ_1 and Ψ_2 are obtained by applying (19) and (20) respectively using equations (17) and (18) (see Appendix A).

At time zero the value of the vertically integrated firm is given by:

$$V_1^{Net}(x) = max_{Q_V}\{V_1(x) - kQ_V{}^{\eta}\}, if\ x < \bar{x}_V \tag{21}$$

else, if $x \geq \bar{x}$, the value of the vertically integrated firm is given by:

$$V_2^{Net}(x) = max_{Q_V}\ \{V_2(x) - kQ_V{}^{\eta}\}, for\ x \geq \bar{x}_V \tag{22}$$

Since $\bar{x}_V$ depends on Q_V , to find the optimal capacity we run various levels of capacity based on a dense grid of Q_V values and check whether $x < \bar{x}$ in which case apply (21), else we apply (22). Then the maximum value among firm values among all grid levels defines the optimal capacity, as well as the operating region (since it determines $\bar{x}_V$) where the firm operates.

3.3. Interactions between supplier and retailer firm

In this section we provide a quantification of the trade-offs involved for the retailer firm when choosing the share of revenue taking into consideration the interactions with the supplier firm. Formally, one can take the derivative of the retailer function in Proposition 2 which depends on the present value of the profits (captured by the particular solution), as well as on the flexibility of the supplier to switch between operating regions. These later effects are incorporated in $\Omega_1^R x^{\beta_1}$ and $\Omega_2^R x^{\beta_2}$ in Proposition 2, however due to the non-linearities involved, the expression of the derivative with respect to α is complicated. Instead, we try to gauge the trade-offs by breaking down the direct revenue effects and the effect of switching between regions below.

In the region where the supplier operates unconstrained, the retailer's profits are:

$$\pi_R = (\alpha p - c_R)q \tag{23}$$

By using total differentiation of the retailer profits we obtain the following:

$$\frac{d\pi_R}{d\alpha} = \underbrace{pq}_{>0} + \alpha q\underbrace{\frac{dp}{d\alpha}}_{>0} + (\alpha p - c_R)\underbrace{\frac{dq}{d\alpha}}_{<0} \tag{24}$$

Equation (24) shows that the retailer faces the following trade-offs when deciding to increase its share of revenues α.[4] On the positive side, increasing α increases the revenue gained if quantities and prices are held fixed (first term) and increases the revenues due to higher prices (second term). On the negative side however, increasing α has an adverse effect on profits due to lower quantities produced and delivered by the supplier (third term). Similar trade-offs hold when firms are in region 2 (supplier operates at full capacity) and equations (23) and (24) hold, albeit q is replaced for Q and only numerical comparative statics (not analytical) are available.

In addition to the above direct effects on profits there are also some non-linear effects that complicate the retailer's decision. These effects relate to the switching options that the supplier has between partial and full utilization of capacity and the level of its capacity choice. Although it is not possible to identify fully these non-linearities, we note some insights. First, we found earlier that $\frac{d\bar{x}}{d\alpha} > 0$ which implies that the higher the revenue share claimed by the retailer the longer the delay of the supplier switching to full capacity. In addition, $\frac{dQ}{d\alpha} < 0$ which shows that capacity is reduced when the retailer claims a larger revenue share. The combination of the above direct and indirect trade-offs determines the choice of the optimal revenue share offer that the retailer makes to the supplier firm.

4. Numerical analysis

We next provide sensitivity results and the implications of the model relating to the optimal revenue sharing offer of the retailer and the capacity and utilization of capacity of the supplier, as well as the prices of goods in the downstream markets. We assume the following base case parameters: $x = 10, \sigma = 0.2, v = 1, c = 0, c_R = 1, \varepsilon = -0.7, k = 3, \eta = 2, r = 0.05, \delta = 0.03$. Our base parameters used for r, δ and σ are in line with other real options models (e.g., Mauer and Sarkar, 2005; Hackbarth and Mauer, 2011). η is the same as in Nishihara et al. (2019). A positive k alongside η determines an optimal capacity level for the supplier. The elasticity parameter ε is similar to Aguerrevere (2009) and Dobbs (2004).[5] We initially set $c = 0$ to avoid cases of negative profits when the volume of production is zero (we analyze $c > 0$ in our sensitivity analysis). The relative level between x and v is set to retain positive values for both the supplier and retailer firms at various revenue sharing levels. Throughout the analysis we run a dense grid search for optimal capacity choice with increments of Q of 0.01. Similarly, for the share of revenues of the retailer we run a dense grid search with increments of α of 0.01.

4.1. Baseline results

Fig. 1 shows retailer values as a function of the retailer claimed share of revenues α. The figure highlights our first important result regarding the existence of an optimal revenue sharing ratio, which is summarized as follows.

Result 1. There is an optimal sharing level α that maximizes the value of the retailer. The optimal sharing level α balances: a) the direct positive impact of a higher α on retailer revenues, b) the negative impact of a

[4] Note that taking the derivative of the retailer's profits with respect to α is equivalent to taking the derivative of the particular solution of the retailer with respect to α. Indeed, we have that $\pi_R = Bx^{-1/\varepsilon}$, while the particular solution for the retailer in region 1 is $\frac{B}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-\frac{1}{\varepsilon}}$

[5] Aguerrevere (2009) uses $\varepsilon = -0.625$ and Dobbs (2004) an $\varepsilon = -0.5$. Note that the choice of ε values are restricted so that $r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right) > 0$ so that the particular solution in region 1 of Proposition 1 remains positive. Hagspiel et al. (2016) and Sarkar (2009) use a linear demand function and they also need to impose some constraints to maintain positive values on particular solutions. Specifically, they need to assume a high r and small μ and σ to maintain positive values for the particular solutions.

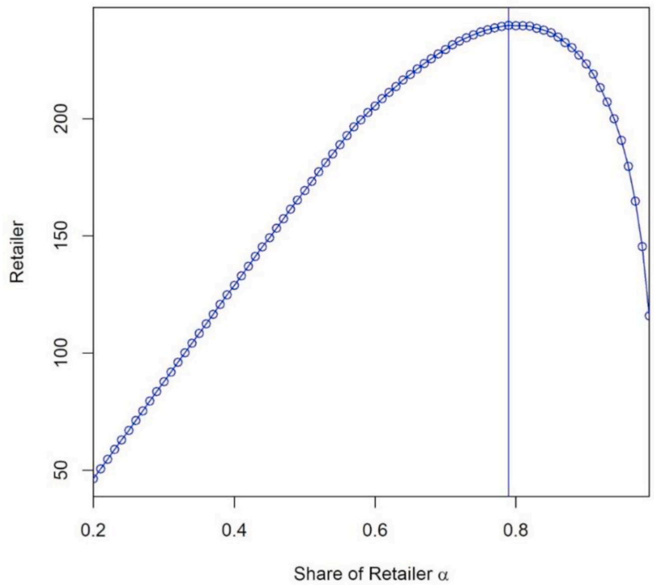

Fig. 1. Optimal share of retailer]. Notes: Parameters used $r = 0.05$, $\delta = 0.03$, $\sigma = 0.2$, $\nu = 1$, $c = 0$, $c_R = 1$, $x = 10$, $k = 3$, $\eta = 2$, $\varepsilon_B = -0.7$.

higher α on supplier's optimal capacity and production, c) the positive impact on prices due to lower quantities produced, and d) a negative effect caused by a delay in the supplier firm moving to full capacity.

Our result is related to previous work on quantity flexible contracts. For example, in Tsay (1999) a retailer operating in a single period context provides a planning of forecasted quantities to be purchased which may not be materialized when uncertainty unfolds unfavorably (see also recent work by Li et al., 2021). They show that in such a case the risks are shifted on the supplier which will internalize this and consequently adjust its capacity of production downwards. Relatedly, in our context the supplier bears the cost of installing capacity when in fact capacity may not be fully utilized in future production. However, in contrast to this line of work which proposes quantity flexible contracts with commitment of the retailer to buy and the supplier to provide certain quantities, we focus on revenue sharing contracts. In particular, we analyze how the retailer internalizes the impact of a revenue share contract on the supplier's installed capacity given that the supplier controls the volume of production in a multiperiod context with uncertain demand.

In order to better understand the forces involved in determining Result 1, i.e., the optimal level α, Table 1 Panel A shows how the main variables of the model change as the share of revenues claimed by the retailer (α) changes. The bold line shows the optimal pricing (revenue sharing) choice for the retailer firm. We observe that when the share of revenues of the retailer is low, the supplier selects a high capacity level and a high utilization rate. Actually, when $\alpha \leq 0.3$ the supplier starts operations at full capacity (Region 2). As α increases, the supplier reduces both the optimal capacity level and the optimal production level. The low quantities produced result in an increase in the price of the final good sold.

For the retailer, an increase in α thus implies the following trade-offs. On the one hand, there is a direct positive effect, since the retailer is capturing a larger fraction of the revenues. Moreover, we have an additional indirect positive effect due to the resulting increase in the price, which increases per unit profit. On the other hand, we have an indirect negative effect since quantities sold decrease, which decreases net revenues. In addition, the supplier postpones entering into full capacity for higher α (notice that $\overline{x}$ increases with α). For small increases in α the positive effect dominates. However, at relatively large values of α the negative impact of low quantities dominates the positive effect of a higher per unit profit. Hence, we obtain an optimal level of α. Although the optimal revenue sharing ratio that we find in the benchmark case is relatively high compared to percentages observed in practice, when incorporating supplier's option to abandon and finite maturity horizon we obtain revenue sharing ratios as low as 20% in line with practice (see Section 6).

An interesting managerial question is to investigate how likely it is for a supplier firm to hit the full capacity threshold over a certain period T. To investigate this, we simulate 5000 firms. For the GBM dynamics we use the following to generate sample paths $x_t = x_{t-1}\, exp\, (\mu\, dt + \sigma\sqrt{dt}\, Z_t)$ where $Z_t \sim N(0, 1)$ and $x(0) = x_0$ is the initial price using a $dt = 1$ (yearly) for a period $T = 20$ years. We then count all cases where $x_t > \overline{x}$ over this period. Our analysis revealed that only 1.48% of supplier firms will hit the full capacity boundary over that period showing that a manager should expect that most of time the firm will remain under full capacity for quite long periods of time.

4.2. Fixed revenue sharing contract

To understand how different economic conditions affect supplier's optimal selection of capacity and its utilization rate for a given revenue sharing contract, we first run sensitivity results with respect to model parameters for a given α. We use a value of $\alpha = 0.4$, instead of 0.79, the optimal α value for the benchmark parameter values, because it allows us to better illustrate the entire model including both regions, below and at full capacity.

Fig. 2 shows the effect of volatility which highlights some interesting real options effects relating to operational flexibility. A higher volatility

Table 1
Sensitivity with respect to the share of revenues of the retailer (α) Panel A: Non-cooperative values Panel B: Vertical integration.

α	$\overline{x}$	Region	$S^{Net}(x)$	R	Q	q	(q/Q)	Price (p)
0.2	45.03	2	288.648	42.25	3.18	3.18	1.00	4.45
0.3	47.93	2	243.28	89.80	2.88	2.88	1.00	4.77
0.4	51.65	1	199.56	133.60	2.56	2.32	0.90	5.56
0.5	56.38	1	157.73	172.78	2.24	1.78	0.80	6.67
0.6	62.75	1	118.12	205.54	1.89	1.30	0.69	8.33
0.7	77.78	1	81.213	229.48	1.52	0.86	0.57	11.11
0.79	111.11	**1**	**50.89**	**239.84**	**1.17**	**0.52**	**0.44**	**15.87**
0.8	116.67	1	47.732	239.68	1.12	0.48	0.43	16.67
0.9	233.33	1	19.095	223.42	0.67	0.18	0.27	33.33

$\overline{x}_V$	Region	$V^{Net}(x)$	Q_V	q_V	(q_V/Q_V)	Price (p_V)	Gain
14.75	1	335.87	3.11	1.78	0.57	6.67	15.5%

Notes: We assume the following base case parameters: $x = 10$, $\sigma = 0.2$, $\nu = 1$, $c = 0$, $c_R = 1$, $\varepsilon = -0.7$, $k = 3$, $\eta = 2$, $r = 0.05$, $\delta = 0.03$. In Panel A, we show values varying α (share of supplier in revenues). Panel B shows the optimal values under vertical integration. Gain is calculated as $(V^{Net}(x) - (S^{Net}(x) + R))/(S^{Net}(x) + R)$. Q increments of 0.01.

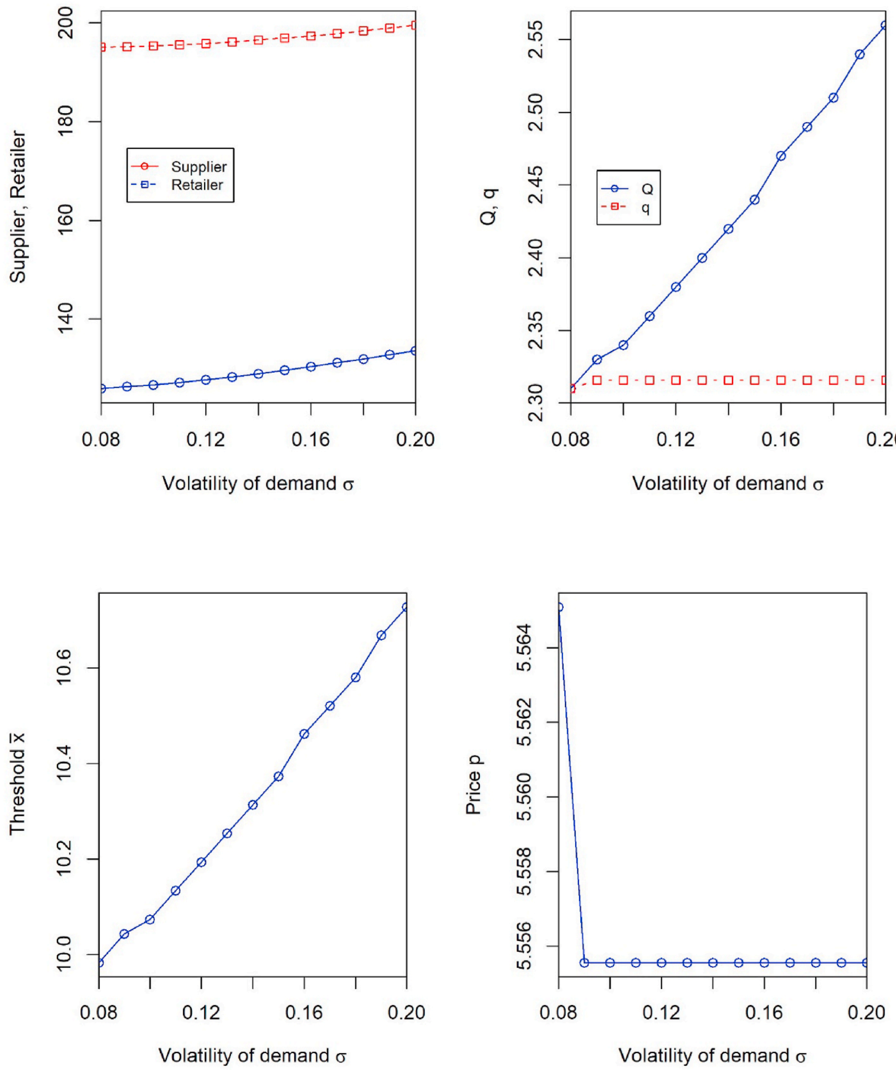

Fig. 2. Sensitivity with respect to volatility σ Notes: Parameters used $r = 0.05$, $\delta = 0.03$, $v = 1$, $c = 0$, $c_R = 1$, $x = 10$, $k = 3$, $\eta = 2$, $\varepsilon_B = -0.7$, $\alpha = 0.4$.

creates a more valuable operational flexibility option for the supplier firm in varying the level of production. A higher volatility thus increases the optimal capacity choice and despite the delay in moving to full capacity ($\overline{x}$ increases), the values of both the retailer and supplier improve. This is beneficial for both the retailer and the supplier since both count on the option to switch to full capacity when demand is favorable which is more valuable when volatility is high (while on the downside both firms are protected from losses due to volume flexibility, i.e., the ability to reduce production). At low enough volatility the firm starts at full capacity (see case of low volatility equal to only 8%) but operates at the lowest capacity and highest price levels. An increase in volatility from this low level of volatility increases capacity, however, the firm does not utilize all capacity and hence the firm moves to region 1 (below full capacity). Note that the downward jump in the price from the 8% volatility to 10% volatility reflects the change in capacity and utilization from a region of lower capacity and production (utilization) q to a higher capacity and higher production q. Further increases in volatility do not change q and p (firm continues in region 1) despite the higher installed capacity. A larger volatility thus makes it more likely for the firm to move from region 2 (full capacity) to region 1 (this is despite the increase in the threshold $\overline{x}$).

Although surprising at first sight, the aforementioned effect of volatility stems from the importance of supplier production flexibility captured in our setting and is in line with previous literature (Hagspiel et al., 2016). Under more volatile demand environments the supplier firm installs more capacity in order to be able to react in future favorable scenarios. Our analysis can help explain, for example, why Tesla installed a significant capacity for the production of electric cars even when demand for electric cars remained highly uncertain (see Randall, 2021). We summarize the following important result.

Result 2a. (Effect of downstream volatility for fixed contracts). The retailer and supplier values increase with σ. The supplier's capacity Q and utilization increases with σ (flattens out for high σ).

Next, we also highlight the effect of elasticity which has important implications for different types of products (e.g., luxury vs. necessities). A higher absolute value of ε implying a more inelastic demand, i.e., a lower $\frac{1}{|\varepsilon|}$ (e.g., implying the product becomes more of a necessity) has a significant negative impact on both capacity and utilization (see Fig. 3). Thus, at higher absolute value of ε the price increases due to the lower quantities produced. The overall impact on retailer and supplier value depends on which of the two effects (lower quantities or higher price) dominates and we generally have a U-shape effect on values: for low absolute ε (more elastic demand) retailer and supplier values decrease since the impact on quantities is relatively more important than the impact on prices. However, this reverses for more inelastic demand levels (higher absolute ε).

We summarize a second result concerning the effect of elasticity of

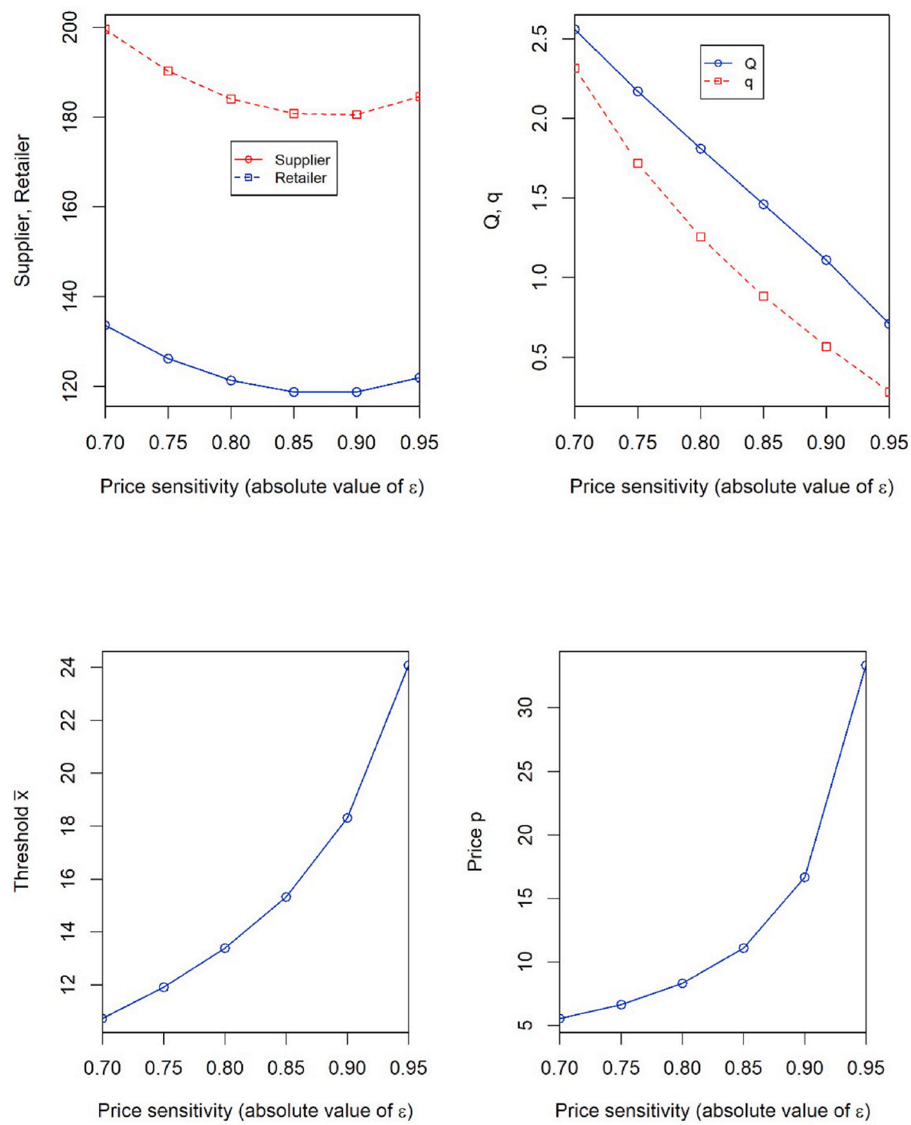

Fig. 3. Sensitivity with respect to price elasticity Notes: Parameters used $r = 0.05$, $\delta = 0.03$, $\sigma = 0.2$, $v = 1$, $c = 0$, $c_R = 1$, $x = 10$, $k = 3$, $\eta = 2$, $\alpha = 0.4$.

demand.

Result 2b. (Effect of elasticity of demand for fixed contracts). The retailer and supplier have a U-shape with respect to absolute value of ε (more inelastic demand). The supplier's capacity Q and utilization decreases with respect to the absolute value of ε.

Some other parameters have very intuitive effects and are not shown for brevity. All sensitivities are available upon request. We discuss briefly some findings which are in line with economic intuition. For example, higher initial demand level x or lower level of variable costs v result in an improvement in the value of the retailer and supplier firm and an increase in capacity (Q) and utilization (q). A higher cost for installing capacity k reduces the level of capacity of the supplier and has an adverse effect on both the retailer and supplier firm. A higher fixed cost of production for the supplier c does not change the supplier's capacity or utilization and thus has no effect on the threshold $\bar{x}$, the price of goods sold in the downstream market, nor on retailer value. However, higher fixed costs reduce the supplier firm value because even when reducing the volume of production to zero in unfavorable demand states, the supplier still needs to incur a fixed operational cost. In Section 6 we consider the supplier having the option to truncate the downside losses with an abandonment option and potential hold up problems for the retailer firm. A higher δ which implies a reduction in demand growth adversely affects capacity levels (Q). This has a negative effect on both the retailer and the supplier values. A higher c_R reduces only the retailer value and has no other impact on supplier's policy or values. Finally, a higher r acts in the opposite direction of δ since it effectively implies a higher drift in demand. A higher η acts in the same direction as k since it implies higher costs of installing capacity.

4.3. Optimal revenue sharing contract

In this section we allow the retailer to optimize its pricing policy by optimally selecting α. That is, the retailer firm anticipates supplier's capacity and utilization decisions, and hence we solve a Stackelberg leader-follower type of game by optimizing retailer's claim of revenue share (see Section 3.3). There are counterbalancing forces in place when the retailer decides to change its share α in response to a parameter change since this causes a reaction in the supplier's capacity and utilization. Due to these counterbalancing forces, there is relatively small variation in optimal α. For example, when σ increases from 10% to 50% the optimal α only increases by 3% (from 79% to 82%). We find that the above effects for σ are magnified for higher v. Table 2 illustrates the effect of volatility which highlights some interesting trade-offs that need

Table 2
Optimal revenue sharing contract for different levels of volatility of demand.

σ	α	$\bar{x}$	Region	S^{Net}	R	Q	(q/Q)	Price (p)
0.1	0.71	40.12	1	39.06	128.07	0.37	0.137	80.46
0.2	0.75	78.43	1	37.45	151.15	0.78	0.053	93.33
0.3	0.77	101.45	1	39.82	174.89	1	0.037	101.45
0.4	0.79	123.28	1	41.23	198.97	1.16	0.028	111.11
0.5	0.8	140.19	1	43.83	220.86	1.3	0.023	116.67

Notes: We assume the following base case parameters: $x = 10$, $v = 7$, $c = 0$, $c_R = 1$, $\varepsilon = -0.7$, $k = 3$, $\eta = 2$, $r = 0.05$, $\delta = 0.03$. Q increments of 0.01.

to be considered by the retailer firm when adjusting α at different volatility levels. We demonstrate the results for a higher v compared to the base case since the effects are more pronounced and thus easier to illustrate. Similar directional effects hold for our base case parameters as can be seen in reported results in Appendix C.

For a given revenue share, a higher σ increases the capacity of the supplier which improves retailer value even if its share of revenues remains unchanged (see earlier discussion relating to Fig. 2). Thus, the retailer's decision of increasing α involves a trade-off. While increasing α has a direct positive effect on revenues, it also has a negative impact on produced quantities due to lower capacity and utilization of the supplier. This negative impact is however also mitigated by the positive effect on prices of lower quantities produced. Despite the counterbalancing forces, as shown in the results of Table 2, the retailer will generally find preferable to increase its optimal claimed share for higher volatility. For example, when volatility increases from 10% to 20% the share of the retailer increases from 71% to 75%. Had the retailer not increased its share the supplier would choose an optimal capacity $Q = 0.87$, a utilization rate of 5.8% and the resulting price in the market would be 80.46. The retailer would then have a value of 150.36. Instead, we observe that it is optimal to increase its share from 71% to 75%, which despite the slight decrease in capacity and utilization (Q drops to 0.78 and utilization to 5.3%) results in a higher retailer value of 151.15 since there is a higher resulting equilibrium price of 93.33.

The above trade-offs characterize how the retailer chooses optimal α when there is a variation in other model parameters. For example, at a higher demand level x the supplier would increase optimal capacity and utilization if α remained fixed. Thus, the retailer would face a similar dilemma in increasing α since it would have a counterbalancing effect on optimal capacity and utilization. We have found that for some parameters these counterbalancing forces create no discernible variation in α, while for others the effects show a clearer direction. We provide the following result which summarizes the effects of model parameters on the optimal share of revenues where a clearer directional pattern could be determined. All sensitivity results are shown in Appendix C.

Result 3. (Optimal revenue sharing contract). The optimal share of revenues α increases with σ, c_R, the absolute value of ε , r, and η and decreases with v and δ. The optimal revenue sharing contract exhibits low variation to other model parameters.

Result 3 has important implications for the design of optimal contracts for retailer firms. It suggests when there should be a significant adjustment in the claim of optimal share of revenues in response to different market conditions. For example, the results generally suggest that retailers would require a higher optimal share for necessities (more inelastic) compared to luxury (more elastic) products, while they will tend to require a lower share of revenues when the variable costs of production of the supplier are high or when the demand growth is smaller. Importantly, the result also shows that there is no significant variation in optimal α for other parameters and thus a one-for-all contract offered by the retailer to supplier firms may not be far from an optimal choice for the retailer in these situations. Thus, in cases where the retailer faces significant costs of discerning information (e.g., about the demand level in the supplier's market), offering the same contract to all supplier firms will not be far from the optimal choice. This is in line with anecdotal evidence. For example, Amazon charges the same fee to multiple products belonging to different categories.

4.4. *Gains from vertical integration*

We now analyze the gains from vertical integration. In Table 1, Panel B we provide the solution with vertical integration for the base case parameters. We observe that vertical integration results in an improvement in capacity (Q) relative to the optimal solution with non-coordinated production of $\alpha = 0.79$. The gain in overall value of integrating production relative to the sum of retailer and supplier values with no coordination is 15.5%. We also note that the vertically integrated firm produces higher quantities, and this results in a reduction in the price of the good offered in the market. We summarize the following main result.

Result 4a. (Coordinated vs. non-coordinated production). Relative to the non-coordinated production, integration of production results in an improvement in capacity and quantities produced, a gain in overall value relative to the sum of retailer and supplier values and a lower price of the good offered in the market.

Result 4a is in line with the double marginalization problem of disintegrating production where positive mark-ups are added in different stages of production resulting in higher prices (see Spengler, 1950 and Tirole, 1988, ch.4). Tsay (1999) has shown (see Proposition 4b) that with non-coordinated production the supplier's installed capacity is lower than the one under coordination (see also Li et al., 2021, p.4 for a similar result). However, our framework allows for further insights regarding the sensitivity of these gains with respect to model parameters within a context of revenue sharing and multiperiod production flexibility that also provides managerial insights with respect to environments where vertically integrating production will be most beneficial.

Due to the counterbalancing effect on supplier's capacity and utilization that the retailer needs to consider for adjusting optimal α in response to changes in parameter values (discussed in the previous section), there is a small variation in gains from vertical integration. The gains from vertical integration for alternative model parameters for which we have obtained clearer directional effects are summarized in Result 4b (see Appendix C for all sensitivity results).

Result 4b. (Gains from vertical integration). The gains for vertical integration are higher with higher σ, v and r and lower δ, c_R and lower absolute value of ε. The gains are almost invariant to other model parameters.

We note that there are generally higher gains from vertical integration when volatility is high, i.e., when operational flexibility is more valuable. However, due to counterbalancing forces involved in the choice of the retailer firm regarding α the gains are not substantially different and range from about 15% for low volatility ($\sigma = 0.1$) to about 19% for very volatile demand ($\sigma = 0.4$) (see Appendix C). Since gains from vertically integrating production do not vary substantially with volatility this implies that the managerial decision to vertically integrate production would not be particularly beneficial especially when one considers integration costs. Also, a higher volatility is not generally beneficial for end consumers in a decentralized setting. This is because despite the higher capacity installed when operational flexibility is present, a possible lower utilization rate may actually result in prices being higher for end consumers.

5. Minimum quantities delivered imposed by the retailer

We now consider the case in which the retailer may impose various constraints on the supplier's production decision such as minimum customer demand fill rates or service levels (Fry et al., 2001; Wang et al., 2004). Therefore, the supplier would need to produce and deliver a minimum quantity of goods Q_{min} to meet a certain fill rate or service level. Just like before, the supplier also has capacity constraints with a maximum capacity Q which is reached at $\overline{x} = \frac{v}{(1-\alpha)(\varepsilon+1)Q^{\varepsilon}}$. Assuming that the minimum quantity level is reached at $x = \overline{x}_{min}$ then using the minimum quantity $Q_{min} = \left(\frac{\overline{x}_{min}(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ implies that the minimum capacity is reached at $\overline{x}_{min} = \frac{v}{(1-\alpha)(\varepsilon+1)Q_{min}{}^{\varepsilon}}$. In principle one could add also an abandonment threshold for the supplier which when reached it stops production altogether. Instead, we analyze the case of abandonment option and potential hold-up issues in the finite-time numerical model of the next section. Note also that we are assuming $Q_{min} < Q$ so that $\overline{x}_{min} < \overline{x}$. Nevertheless, in extreme cases the minimum quantity required by the buyer could be so high that it might affect the supplier's optimal capacity choice such that $q = Q = Q_{min}$.

There are now three operating regions as follows:

Region 1: $x < \overline{x}_{min}$, $p = xQ_{min}{}^{\varepsilon}$, $q = Q_{min}$ and $\pi_S = (1-\alpha)xQ_{min}{}^{\varepsilon+1} - vQ_{min} - c$.

Region 2: $\overline{x}_{min} < x < \overline{x}$, $p = \frac{v}{(1-\alpha)(\varepsilon+1)}$, $q = \left(\frac{x(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ and $\pi_S = A\,x^{-1/\varepsilon} - c$, with $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$.

Region 3: $x \geq \overline{x}$, $p = xQ^{\varepsilon}$, $q = Q$ and $\pi_S = (1-\alpha)xQ^{\varepsilon+1} - vQ - c$.

The supplier firm value $S_i(x)$ satisfies the following differential equation within regions:

$$rS_i(x) = (r-\delta)xS_i{}'(x) + \frac{\sigma^2}{2}x^2S_i{}''(x) + \pi_{Si}, i = 1,2,3 \tag{25}$$

where the last term denotes the cash flows received under that region.

Proposition 4. (Value of the supplier and optimal capacity choice with minimum delivered quantities)

The supplier value is given by:

Region 1, $x < \overline{x}_{min}$:

$$S_1(x) = \frac{(1-a)xQ_{min}{}^{\varepsilon+1}}{\delta} - \frac{c+vQ_{min}}{r} + \Omega_1 x^{\beta_1} \tag{26}$$

Region 2, $\overline{x}_{min} < x < \overline{x}$:

$$S_2(x) = \frac{A}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon} - \frac{c}{r} + \Omega_2 x^{\beta_1} + \Omega_3 x^{\beta_2} \tag{27}$$

Region 3, $x \geq \overline{x}$:

$$S_3(x) = \frac{(1-a)xQ^{\varepsilon+1}}{\delta} - \frac{c+vQ}{r} + \Omega_4 x^{\beta_2} \tag{28}$$

and Ω_1, Ω_2, Ω_3, Ω_4 are determined from the following boundary conditions:

$$S_1(\overline{x}_{min}) = S_2(\overline{x}_{min}) \text{ (Value} - \text{matching region 1 and 2)} \tag{29}$$

$$S_1'(\overline{x}_{min}) = S_2'(\overline{x}_{min}) \text{ (Smooth} - \text{pasting region 1 and 2)} \tag{30}$$

$$S_2(\overline{x}) = S_3(\overline{x}) \text{ (Value} - \text{matching region 2 and 3)} \tag{31}$$

$$S_2'(\overline{x}) = S_3'(\overline{x}) \text{ (Smooth} - \text{pasting region 2 and 3)} \tag{32}$$

Proof. Similar to the proof of Proposition 1.

At time zero the value of the supplier firm is given by:

$$S_1^{Net}(x) = max_Q\ \{S_1(x) - kQ^{\eta}\},\ if\ x < \overline{x}_{min} \tag{33}$$

$$S_2^{Net}(x) = max_Q\ \{S_2(x) - kQ^{\eta}\},\ for\ \overline{x}_{min} < x < \overline{x} \tag{34}$$

$$S_3^{Net}(x) = max_Q\ \{S_3(x) - kQ^{\eta}\},\ x \geq \overline{x} \tag{35}$$

Since $\overline{x}$ and $\overline{x}_{min}$ depend on Q and Q_{min} to find the optimal capacity we run various levels of capacity based on a dense grid and check which of the three regions apply to calculate the net value of the supplier. Then the maximum value among supplier values among all grid levels defines the optimal capacity, as well as the operating region where the firm starts to operate.

The value of the retailer is derived in a similar way. The following proposition derives the value of the retailer.

Proposition 5. (Value of the retailer firm with minimum delivered quantities)

Region 1, $x < \overline{x}_{min}$:

$$R_1(x) = \frac{axQ_{min}{}^{\varepsilon+1}}{\delta} - \frac{c_RQ_{min}}{r} + \Omega_1^R x^{\beta_1} \tag{36}$$

Region 2, $\overline{x}_{min} < x < \overline{x}$:

$$R_2(x) = \frac{B}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon} + \Omega_2^R x^{\beta_1} + \Omega_3^R x^{\beta_2} \tag{37}$$

Region 3, $x \geq \overline{x}$:

$$R_3(x) = \frac{axQ^{\varepsilon+1}}{\delta} - \frac{c_RQ}{r} + \Omega_4^R x^{\beta_2} \tag{38}$$

where the solutions for Ω_1^R, Ω_2^R, Ω_3^R and Ω_4^R are determined from the following boundary conditions:

$$R_1(\overline{x}_{min}) = R_2(\overline{x}_{min}) \text{ (Value} - \text{matching region 1 and 2)} \tag{39}$$

$$R_1'(\overline{x}_{min}) = R_2'(\overline{x}_{min}) \text{ (Smooth} - \text{pasting region 1 and 2)} \tag{40}$$

$$R_2(\overline{x}) = R_3(\overline{x}) \text{ (Value} - \text{matching region 2 and 3)} \tag{41}$$

$$R_2'(\overline{x}) = R_3'(\overline{x}) \text{ (Smooth} - \text{pasting region 2 and 3)} \tag{42}$$

Proof. Similar to the proof of Proposition 2.

In Table 3 we investigate numerically the effect of minimum delivered quantities imposed by the retailer. For $Q_{min} = 0$, we obtain our base case framework in which there were no minimum requirements for the quantities delivered. Indeed, comparing the values obtained for $Q_{min} = 0$ in Table 3 with those in bold in Table 1 corresponding to an optimal α of 0.79, we can see that they coincide.

As the minimum delivered quantity increases supplier value decreases while the retailer value increases. For $Q_{min} = 0.5$ the constraint is not binding at the current level of x, the supplier chooses both a capacity and a utilization level above the minimum delivered quantity. The quantity produced is between the minimum delivered quantity and the capacity level, i.e., $Q_{min} < q < Q$ (region 2). Although not binding at $t = 0$, the constraint on minimum quantities $Q_{min} = 0.5$ may become binding subsequently if demand drops below $\overline{x}_{min}$. Thus, we observe a slight decrease in the value of the supplier due to the imposed constraint. As the minimum delivered quantity increases further to $Q_{min} = 1$ the constraint becomes binding at $t = 0$ and the supplier produces the minimum delivered quantity, which is below full capacity. Therefore, the firm is in region 1, $q = Q_{min} < Q$. For even higher values of the minimum delivered quantity such as 1.5 or 2, the constraint affects not only the quantities produced by the supplier, but also its capacity. In this case, the supplier firm decides to set up an optimal capacity level equal

Table 3
Various level of minimum delivered quantities with fixed revenue share ratio α

Q_{min}	Region	$\bar{x}_{min}$	x	$\bar{x}$	S^{Net}	R	Q	q	Price (p)	α
0	2	0	10	17.72	50.89	239.84	1.17	0.52	15.87	0.79
0.5	2	9.77	10	17.72	50.00	242.82	1.17	0.52	15.87	0.79
1	1	15.87	10	17.72	47.18	250.26	1.17	1.00	10	0.79
1.5	1	21.08	10	21.09	42.30	267.39	1.50	1.50	7.53	0.79
2	1	25.79	10	25.79	34.18	284.20	2.00	2.00	6.16	0.79

Notes: We assume the following base case parameters: $x = 10$, $\sigma = 0.2$, $v = 1$, $c = 0$, $c_R = 1$, $\varepsilon = -0.7$, $k = 3$, $\eta = 2$, $r = 0.05$, $\delta = 0.03$. The share of revenues α is fixed as in the base case at 0.79. Q is optimally chosen (increments of search 0.01).

to the minimum delivered quantity, $q = Q = Q_{min}$.

In sum, the minimum delivered quantity provides a tool for the retailer to improve its value over the supplier. We summarize the following result.

Result 5. (Minimum delivered quantity). When the retailer has the ability to impose minimum delivered quantities, then it can reduce the supplier's ability to choose capacity and its production flexibility and extract more value from the supplier firm.

Our analysis provides new insights with respect to the use of minimum order quantities as a tool used by supply chain members to extract value from partners in the network. Wang et al. (2021) (see also Tsay, 1999, Proposition 4b) show that suppliers can use minimum order quantities to extract more value from retailers, while our analysis focuses on how retailers may use this to extract value from their suppliers. In the next section we investigate the impact of an abandonment option for the supplier firm and potential hold-up problems that mitigate the power of the retailer firm in extracting value from the supplier firm by imposing minimum delivered quantities.

6. Finite horizon

6.1. Abandonment option and hold-up problem for retailer

The perpetual horizon assumption allowed for analytic solutions. However, the optimal sharing contract may change when the supplier expects a finite horizon supply of goods to the retailer. In addition, it is worth investigating how the optimal revenue sharing contract and retailer and supplier values change when the supplier decides to interrupt (abandon) the production of goods creating a hold-up problem for the retailer. These issues may become important when the supplier faces fixed costs of production and minimum delivered quantities since the retailer may suffer a hold-up cost of re-establishing a contact with other potential suppliers when the supplier abandons operations. In order to address the aforementioned issues, we formulate the problem in a finite horizon using a binomial tree approach.

We build a binomial tree describing the evolution of the demand shock starting from $x = x_0$. Since the demand shock x follows a GBM, we use a standard formulation of the lattice parameters for the up and down jumps and the up and down probabilities (see Cox et al., 1979) which requires that $u = exp\ (\sigma\sqrt{dt})$, $d = \frac{1}{u}$, $p_u = \frac{exp\ ((r-\delta)dt - d}{u-d}$, where $dt = \frac{T}{N}$ with T denoting the finite operational horizon of the supplier firm and N denoting the number of steps used in the binomial tree. We keep track of the following information at each node of the binomial tree: demand prices (p), retailer value (R), supplier value (S), supplier's optimal volume of production (q) and supplier's decision (D) between operation (O) and abandonment of operations (A). All variables notation used in earlier section is kept the same for consistency. In addition, in this section we allow for a cost c_p incurred by the retailer if the supplier abandons operations which intends to capture hold-up costs. These hold-up costs may for example capture search costs for establishing a new agreement with other potential suppliers when the supplier stops the supply of goods. In sum, the numerical approach of this section allows for finite horizon contracts, abandonment of the contract by the supplier firm and potential hold-up costs for the retailer when the contract is interrupted by the supplier firm.

Like in models of capital structure (e.g., Leland, 1994), abandonment is endogenously chosen by the supplier to maximize its firm value. We start in the last period T so that supplier value is calculated as follows:

$$S_T = max\ [[((1-\alpha)p_T - v)q_T - c]dt, 0]$$

where $p_T = x_T q^{\varepsilon}$ and $x_T = x_0\ u^{N+1-i} d^{i-1}$ with $i = 1, 2, \ldots N$ the possible states of the demand shock (from the highest to the lowest values). The optimal quantity is selected to maximize the profits of the period as follows: $q_T = \left(\frac{(1-\alpha)x_T(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$. If the capacity is not binding q_T holds, otherwise we replace quantity with $q_T = Q$. If $S_T > 0$ then retailer value is $R_T = (ap_T - c_R)q_T\ dt$ otherwise if $S_T = 0$ (i.e., supplier abandons operations) then $R_T = -c_p$ and $D_T = "A"$. Note that the retailer incurs a possible cost if the supplier abandons operations which can be set to zero for comparison with the base case model of the earlier section.

To solve the problem, we move backwards in periods prior to last $t < T$ where the supplier value is calculated as follows:

$$S_t = max\ [[((1-\alpha)p_t - v)q - c]dt + \widetilde{S}_t, 0]$$

where $p_t = x_t q^{\varepsilon}$ and $x_t = x_0\ u^{(n-j)} d^{(j-1)}$ with $n = N-1, N-2, \ldots .1$ describing the tree steps going backwards from T-1 to period 0. The optimal produced quantity maximizes period profits is given by $q_t = \left(\frac{(1-\alpha)x_t(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ unless $q_t > Q$ in which case $q_t = Q$. In addition, $\widetilde{S}_t = [p_u S_{t+dt,u} + p_d S_{t+dt,d}]exp\ (-rdt)$ describes the expected present value of the supplier's value by weighing the supplier value on the next lattice step if demand shock goes up (u) or down (d). The supplier operates when $S_t > 0$ in which case $R_t = (ap_t - c_R)q\ dt$, otherwise if $S_t = 0$ the supplier abandons operations and $R_T = -c_p$ and $D_T = "A"$. At $t = 0$ we note that the supplier incurs capacity costs hence its net value at $t = 0$ is $S_0^{Net} = S_0 - kQ^{\eta}$.

To optimize the capacity choice of the supplier we run a dense grid search for different Q choices and select the Q that maximizes the supplier net value. Similarly, we run a numerical search to optimize the retailer's choice of the optimal revenue share α given the supplier's optimal capacity and operational choices.

Table 4 shows the numerical simulations by varying the operational horizon of the supplier using the same assumptions as in the analytical model, i.e., no abandonment (fixed costs are zero) and no cost incurred at abandonment by the retailer firm. First, we note that there is a significant difference in the value of the retailer and supplier values and capacity and utilization choices for shorter horizons. However, the optimal revenue share remains rather stable irrespective of horizon. As expected, the numerical model solution converges very close to the

Table 4
Finite horizon model: sensitivity with respect to the operational horizon of the supplier firm.

T	α	S^{Net}	R	Q	q	(q/Q)	Price (p)	Region
5	0.8	5.910	29.366	0.30	0.30	1.00	23.23	2
10	0.8	10.581	53.529	0.40	0.40	1.00	18.99	2
15	0.8	14.845	76.125	0.50	0.48	0.96	16.67	1
20	0.8	18.702	96.684	0.60	0.48	0.80	16.67	1
50	0.8	34.800	178.140	0.90	0.48	0.54	16.67	1
100	0.8	45.018	228.729	1.10	0.48	0.44	16.67	1
200	0.8	48.118	241.356	1.10	0.48	0.44	16.67	1

Notes: We assume the following base case parameters: $x = 10$, $\sigma = 0.2$, $v = 1$, $c = 0$, $c_R = 1$, $\varepsilon = -0.7$, $k = 3$, $\eta = 2$, $r = 0.05$, $\delta = 0.03$. The share of revenues α and capacity choice Q are optimized based on increments of 0.1. The solutions are based on the binomial model solution with $dt = T/N = 1$ year.

Table 5
Finite horizon model: sensitivity with respect to the fixed costs, hold-up costs, minimum delivered quantities and expansion option.

		α	S^{Net}	R	Q	q	(q/Q)	Price (p)	Region
A. Fixed cost sensitivity									
	c = 0	0.8	48.12	241.36	1.1	0.48	0.44	16.67	1
	c = 5	0.6	32.21	165.32	1.7	1.30	0.76	8.33	1
	c = 10	0.5	6.30	113.93	1.9	1.78	0.94	6.67	1
B. Hold-up costs of the retailer									
c = 5	$c_p = 5$	0.6	32.21	163.40	1.7	1.30	0.76	8.33	1
	$c_p = 15$	0.5	67.23	160.44	2.2	1.78	0.81	6.67	1
C. Minimum delivered quantities									
c = 5	$Q_{min} = 2$	0.6	30.79	171.64	2	2.00	1.00	6.16	2
	$Q_{min} = 3$	0.6	20.13	184.47	3	3.00	1.00	4.63	2
D. Expansion option									
c = 5	e = 1.5	0.6	37.52	172.14	1.6	1.30	0.81	8.33	1
	e = 3	0.6	40.15	175.99	1.6	1.30	0.81	8.33	1

Notes: We assume the following base case parameters: $x = 10$, $\sigma = 0.2$, $v = 1$, $c_R = 1$, $\varepsilon = -0.7$, $k = 3$, $\eta = 2$, $r = 0.05$, $\delta = 0.03$, $T = 200$. The share of revenues α and capacity choice Q are optimized based on increments of 0.1. The solutions are based on the binomial model solution with $dt = T/N = 1$ year.

analytic solution when the horizon is sufficiently large to approximate the perpetual horizon ($T = 200$). This is reassuring that we can use the model to further investigate the impact of supplier abandonment option and hold-up costs.

In Table 5, panel A we investigate the impact of the abandonment option by varying the fixed costs of the supplier. With higher fixed costs the supplier may stop production if demand drops sufficiently low. Thus, with higher fixed costs incurred by the supplier we find that the retailer offers better terms involving a higher revenue share to the supplier to avoid a hold-up. We show in unreported results that this result is amplified for shorter horizons, e.g., when $T = 15$ the retailer may need to offer as much as 80% of revenues when fixed costs of the supplier are high. The higher revenue share gained by the supplier leads to higher installed capacity, higher utilization and lower downstream prices when the supplier faces higher fixed costs. In Panel B, we investigate the case where the retailer faces higher hold-up costs (e.g., in order to establish a relationship with another supplier when the supplier abandons operations). In the presence of this hold-up risk we find that the retailer may offer improved revenue sharing terms to the supplier which gives further incentives for the supplier to increase capacity and utilization. In Panel C, we investigate the impact of minimum delivered quantities in the presence of an abandonment option. With minimum delivered quantities set by the retailer the optimal revenue share does not appear to change and the retailer is able to extract more value while the supplier is hurt. However, compared to the previous section where there was no abandonment option, we notice that the option to abandon limits the losses incurred by the supplier and reduces the value increase for the retailer. Based on aforementioned analysis we summarize the following result.

Result 6. (Hold-up effects). In the presence of an abandonment option of the supplier the retailer faces a hold-up problem and may offer higher revenue incentives to the supplier firm which result in higher capacity and utilization. The hold-up problem reduces retailer's ability of using minimum delivered quantities to extract value from the supplier firm.

Our analysis extends earlier work (see a review by Xu et al., 2020) on disruption risks in the supply chain which has usually focused on risks outside the control of the network (e.g., natural causes, worker strikes or changes in environmental laws). We provide the framework to measure the economic impact of possible disruptions focusing on hold-up problems that can occur due to insolvency of the supplier. Our context can be extended to measure the impact of other sources of disruption risk in the supply chain.

6.2. *Expansion of capacity option*

Now assume that the supplier has the option to expand capacity Q by $e > 1$ to become $Q_e = eQ$ at cost $X = k(Q_e - Q)^\eta$. The option has maturity $T_1 < T$ but can be exercised at an optimal time before T_1.

We start from the last period assuming that the option to expand has been exercised so we have that the supplier's value with expanded capacity S_T^e is given by:

$$S_T^e = max\left[\left[\left((1-\alpha)p_T^e - v\right)q_T^e - c\right]dt, 0\right]$$

The optimal quantity is selected to maximize the profits of the period as follows: $q_T^e = \left(\frac{(1-\alpha)x_T(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ unless the capacity is binding in which case $q_T^e = Q_e$ and $p_T^e = x_T\left(q_T^e\right)^\varepsilon$ and $x_T = x_0\ \ u^{N+1-i}d^{i-1}$ with $i = 1, 2, \ldots N$ denoting the possible states of the demand shock. If $S_T^e > 0$ then retailer value is $R_T^e = (ap_T^e - c_R)q_T^e\, dt$ otherwise if $S_T^e = 0$ and $R_T^e = -c_p$ and $D_T = ''A''$. Applying the same procedure as in the no growth case, we then proceed backwards to $t = T_1$ and calculate all values under the expanded capacity case for any $T_1 \leq t < T$ by adding the cash flows of the period, calculating the expected continuation by weighing with up and down probabilities and making the necessary adjustments depending on whether the supplier firm operates or not under the new expanded capacity level.

At the maturity of the growth option T_1 we investigate whether the growth option is worth being exercised as follows: $S^e_{T_1} = max\,[S^e_{T_1} - X, S_{T_1}]$. If the option is exercised then $D_{T_1}="E"$, else the firm remains with the decision followed under the no growth path in which case we write $D_{T_1}="W"$ (wait). Note that $D_{T_1}="E"$ implies that the firm operates under the expanded capacity with $q^e_{T_1} = \left(\frac{(1-\alpha)x_{T_1}(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ unless the capacity is binding in which case $q^e_{T_1} = Q_e$. In case $D_{T_1}="W"$ the firm operates under the no-growth policy quantities $q_{T_1} = \left(\frac{(1-\alpha)x_{T_1}(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ if capacity is not binding, otherwise we replace quantity with $q_T = Q$. Case $D_{T_1}="W"$ also captures the possibility that the firm abandons if $S_{T_1} = 0$.

At any other period $t < T_1$ we investigate whether the option is exercised early or whether the supplier firm waits for one more period by calculating $S^e_t = max\left[\left[((1-\alpha)p^e_t - v)q^e_t - c\right]dt + \widetilde{S}^e_t - X, \left[((1-\alpha)p_t - v)q - c\right]dt + \widetilde{S}_t, 0\right]$ where $\widetilde{S}^e_t$ denotes the expected continuation value if the firm does not expand at t.

At $t = 0$ we ensure that initial capacity cost is paid so have $S^{Net}_0 = S^e_t - kQ^\eta$. Note that at $t = 0$ the supplier needs to incur the base (initial) capacity cost anyway in order to be able to at least operate under a no growth capacity level.

To optimize the capacity choice of the supplier we run a dense grid search for different Q choices picking the one that maximizes the supplier net value accounting for the supplier's option to expand. We run a numerical search to optimize the retailer's choice of the optimal revenue share α given the supplier's optimal capacity and operational choices which include the option to expand.

Table 5 shows that the option to expand improves supplier's value and may make the supplier somewhat more conservative in its choice of initial capacity. For the range of expansion factors considered the value improvement for the supplier is substantial, ranging between 16% and 24%. We find that the retailer also benefits from the potential to increase capacity, albeit the improvement in value is less and ranges between 4% and 6% for the parameters considered.

Interestingly, we find that the retailer's optimal revenue share is not very sensitive to the supplier's option to expand, rather the retailer maintains its revenue sharing proposition and gains from supplier's expansion when this is materialized.

Result 7. (Expansion of capacity). The option to expand improves retailer and supplier values and leads to a more conservative choice of initial capacity for the supplier firm. The retailer's optimal revenue share is not very sensitive to the expansion factor of revenues.

The real options literature has studied the option to expand capacity in the context of a single firm. (e.g., Dangl, 1999; Hagspiel et al., 2016; Chronopoulos et al., 2017). In contrast, we study the revenue-sharing implications of the option to expand capacity in the supply chain. Our analysis shows how a supplier's option to expand capacity affects the revenue sharing contract proposed by the retailer and how it also affects the initial and subsequent capacity levels of the supplier.

7. Coordination in the supply chain

Previous literature on revenue sharing has proposed two-parameters revenue sharing contracts consisting of a wholesale price and a revenue sharing ratio as a coordination mechanism in the supply chain (Cachon and Lariviere, 2005; Giannoccaro and Potrandolfo, 2004, among others). Such contracts are shown to coordinate the supply chain, that is, to reach the maximum supply chain profit attainable under vertical integration.

In a similar fashion, our model can be extended to incorporate a fee paid by the supplier to the retailer for each product sold through the retailer. This type of per product fees which are combined with revenue sharing is used by many online platforms such as Amazon.[6] Such a revenue sharing contract can be shown to also achieve coordination in the supply chain in our framework under supplier production flexibility. In the rest of this section, we derive the contract that can coordinate the supply chain similarly to previous literature.

We modify the benchmark model by assuming that the retailer receives not only a fraction α of the supplier revenues, but also a fee w per unit. Regarding notation, we will use the upper index c to denote retailer and supplier values under the coordinated case.

The profits per dt interval for the supplier are then as follows: $\pi^c_S = ((1-\alpha)p - w - v)q - c = (1-\alpha)xq^{\varepsilon+1} - (w+v)q - c$. Maximizing the profits with respect to q results in the optimal level of $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{w+v}\right)^{-1/\varepsilon}$.

The retailer firm has the following profits per period $\pi^c_R = (\alpha p + w - c_R)q = \alpha x q^{\varepsilon+1} + (w - c_R)q$.

Summing the profits of the retailer and the supplier we obtain the vertically integrated profit per period: $\pi_V = (p - v - c_R)q - c = xq^{\varepsilon+1} - (v + c_R)q - c$. Maximizing the profits with respect to q results in the optimal level of $q_V = \left(\frac{x(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$.

Hence, to achieve channel coordination, we need that the optimal quantity chosen by the supplier q corresponds to the quantity that optimizes the SC total profit q_V. This happens if the supplier offers the retailer a fee per product equal to:

$$w = (1-\alpha)(v + c_R) - v = c_R - \alpha(v + c_R) < c_R \tag{43}$$

Therefore, under the two-parameters revenues sharing contract, the supplier offers the retailer a fee lower than the retailer's marginal cost c_R, but in exchange the retailer receives a fraction α of the supplier's revenue.

From equation (43) we can see that $w + v = (1-\alpha)(v + c_R)$, that is, the total cost of the supplier consisting of its own variable costs plus the fee paid to the retailer represents a fraction $1-\alpha$ of the total variable costs of the whole supply chain. Thus, the supplier shares both the revenues and variable costs of the supply chain in the same proportion. Since under the coordination contract the supplier will choose the same quantity as the supply chain, $q = q_V$ and $p = p_V$, we can express the supplier's profits as a function of the total supply chain profit: $\pi^c_S = (1-\alpha)\pi_V - \alpha c$. Indeed, substituting $w + v = (1-\alpha)(v + c_R)$ into the supplier's profit $\pi^c_S = ((1-\alpha)p - w - v)q - c$, we get $\pi^c_S = (1-\alpha)pq - (1-\alpha)(v + c_R)q - c = (1-\alpha)(p - v - c_R)q - c = (1-\alpha)\pi_V - \alpha c$.

Thus, the revenue sharing contract makes the supplier's profit function an affine transformation of the supply chain's profit function.

Assuming that the maximum capacity level is reached at $x = \overline{x}$ then using the optimal quantities we find that $Q = \left(\frac{\overline{x}(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$, which implies that the maximum capacity is reached at $\overline{x} = \overline{x}_V = \frac{v+c_R}{(\varepsilon+1)Q^\varepsilon}$.

We have two operating regions for the supplier depending on whether $x < \overline{x}$ or $x \geq \overline{x}$ as follows:

Region 1: $x < \overline{x}$: $p = p_V = \frac{v+c_R}{\varepsilon+1}$, $q = q_V = \left(\frac{x(\varepsilon+1)}{v+c_R}\right)^{-1/\varepsilon}$ and $\pi^c_S = (1-\alpha)A_V\, x^{-1/\varepsilon} - c$.

Region 2: $x \geq \overline{x}$: $p = xQ^\varepsilon$, $q = Q$ and $\pi^c_S = (1-\alpha)xQ^{\varepsilon+1} - (1-\alpha)(v + c_R)Q - c$.

Similar to the benchmark model, the supplier firm value $S^c_i(x)$ satisfies the following differential equations depending on the region of operation:

$$rS^c_i(x) = (r-\delta)xS^{c'}_i(x) + \frac{\sigma^2}{2}x^2 S^{c''}_i(x) + \pi^c_{Si}, i = 1, 2. \tag{44}$$

[6] See https://sell.amazon.com/pricing.

Proposition 6. (Value of the supplier firm under a coordinating contract).

The supplier value function is given by:
Region 1, $x < \bar{x}$:

$$S_1^c(x) = \frac{(1-\alpha)A_V}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon} - \frac{c}{r} + \Omega_1^c x^{\beta_1} \tag{45}$$

Region 2, $x \geq \bar{x}$:

$$S_2^c(x) = \frac{(1-\alpha)xQ^{\varepsilon+1}}{\delta} - \frac{c + (1-\alpha)(v+c_R)Q}{r} + \Omega_2^c x^{\beta_2} \tag{46}$$

and Ω_1^c and Ω_2^c are determined from the following boundary conditions:

$$S_1^c(\bar{x}) = S_2^c(\bar{x}) \text{ (Value} - \textit{matching}) \tag{47}$$

$$S_1^{c'}(\bar{x}) = S_2^{c'}(\bar{x}) \text{ (Smooth} - \text{pasting}) \tag{48}$$

Proof. The particular solutions in equations (45) and (46) are obtained by applying the differential equation in (44) the particular solution $S_i^c(x) = A_0 + A_1 x + A_2 x^{-\frac{1}{\varepsilon}}$. Ω_1^c and Ω_2^c are obtained by applying (47) and (48) respectively using equations (45) and (46) (see Appendix D).

As with profit functions, we can also express the value functions of the supplier as a function of the supply chain value, and it can be shown that a similar relationship holds as in the following corollary.

Corollary 1. The supplier's value function is an affine transformation of the supply chain's value function under a coordinating contract:

$$S_1^c(x) = (1-\alpha)V_1(x) - \frac{\alpha c}{r} \tag{49}$$

$$S_2^c(x) = (1-\alpha)V_2(x) - \frac{\alpha c}{r} \tag{50}$$

Proof. It follows directly from $\Omega_1^c = (1-\alpha)\Psi_1$ and $\Omega_2^c = (1-\alpha)\Psi_2$ (see Appendix D).

Therefore, the supplier will also choose the same optimal capacity as the vertically integrated supply chain, $Q = Q_V$.

In a similar fashion, it can be shown for the retailer that:

$$R_1^c(x) = \alpha V_1(x) + \frac{\alpha c}{r} \textit{ and } R_2^c(x) = \alpha V_2(x) + \frac{\alpha c}{r}$$

Hence, if the revenue sharing contract satisfies the condition given in equation (43) for the fee per product, then it will achieve channel coordination regardless of the value of α, which should however belong to the interval (0,1).

Nevertheless, this revenue sharing contract that coordinates the supply chain will only be accepted by the two parties if they both obtain higher values under this contract compared to the uncoordinated case. Hence, the value of α has to satisfy a win-win condition (Giannoccaro and Potrandolfo, 2004): $\pi_S^c \geq \pi_S$ and $\pi_R^c \geq \pi_R$.

The first inequality is equivalent to $(1-\alpha)\pi_V - \alpha c \geq \pi_S$, which implies:

$$\alpha \leq \frac{\pi_V - \pi_S}{\pi_V + c} \tag{51}$$

The second inequality is equivalent to $\alpha(\pi_V + c) \geq \pi_R$, which implies:

$$\alpha \geq \frac{\pi_R}{\pi_V + c} \tag{52}$$

Combining equations (44) and (45) we obtain:

$$\alpha \in \left[\frac{\pi_R}{\pi_V + c}, \frac{\pi_V - \pi_S}{\pi_V + c}\right] \tag{53}$$

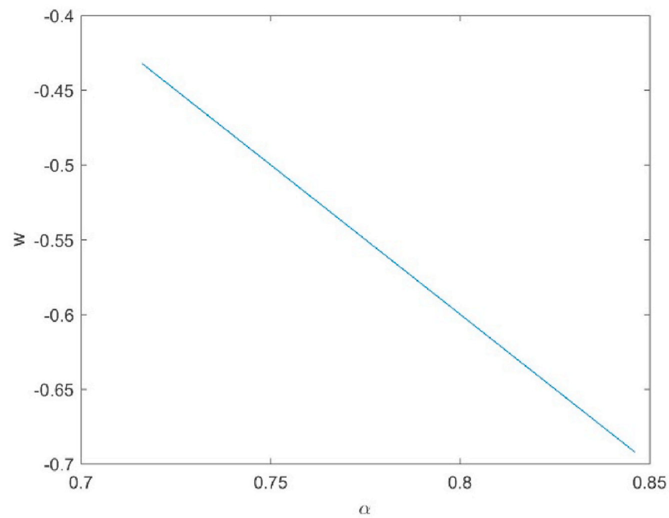

Fig. 4. A two-parameters coordinating contract (fee per product and revenue sharing)Notes: Parameters used $r = 0.05$, $\delta = 0.03$, $\sigma = 0.2$, $v = 1$, $c = 0$, $c_R = 1$, $x = 10$, $k = 3$, $\eta = 2$, $\varepsilon_B = -0.7$.

A revenue-sharing contract (w, α) that satisfies equations (43) and (53) will thus not only coordinate the supply chain, but also be preferred by both the retailer and the supplier. We illustrate this case numerically in Fig. 4. We can see that the α that coordinates the supply chain lies in the interval (0.72, 0.86), with a corresponding fee per product ranging between (-0.71, -0.43). Note that the coordinating fee is actually negative. Indeed, from equation (43) it follows that the fee will be negative, i.e., $w < 0$, whenever $\alpha > \frac{c_R}{v+c_R}$. For our parameter values $\frac{c_R}{v+c_R} = 0.5$, thus it follows that the fee will be negative whenever the retailer captures more than half of the revenues. This is in line with the results of Cachon and Lariviere (2005). A negative fee implies that the retailer is actually subsidizing the supplier. Intuitively, if the supplier's share of the channel's cost is high, then the supplier has already a low profit margin before the retailer takes a slice of revenue. If the retailer wants to claim a large share of revenues, it must subsidize the supplier. As Cachon and Lariviere (2005) argue, if we want to rule out a negative fee, then a positive cost for the supplier establishes a floor on supplier profit under coordinating contracts.

The ultimate contract design, the actual contract parameters chosen by the two parties, will depend on the relative bargaining power of the supply chain parties. However, as pointed out by Giannoccaro and Potrandolfo (2004), it is important to stress that the implementation of this contract requires a certain degree of cooperation among the supply chain parties during the contract design phase.

8. Conclusion

In this paper we bridge the revenue sharing literature with the real options literature on production flexibility. We propose a unified real options framework to analyze a revenue sharing contract within a decentralized supply chain under supplier production flexibility and demand uncertainty. We find that the double marginalization problem is exacerbated under supplier production flexibility. Indeed, a pure revenue sharing contract exhibits losses of around 11%–22% for the parameters considered compared to a vertically integrated supply chain, with lowest losses when the supplier firm operates at full capacity.

We contribute to the literature by analyzing a multiperiod setting under uncertainty and supplier production flexibility and by providing a valuation of both the retailer and supplier firm. In addition, we show how to incorporate minimum delivered quantity constraints which capture many real-world retailer requirements. We quantify the impact of minimum delivered quantity constraints on supplier's capacity choices, capacity utilization and prices in the downstream markets. Furthermore, we also build a finite-time numerical model which considers supplier's option to abandon as well as its option to expand capacity and their effect on the revenue sharing contract. Moreover, we extend prior work by showing that a coordinating contract which combines a fee per product and revenue sharing exists in a multiperiod setting under uncertainty and production flexibility. However, such a contract would require a certain degree of cooperation between the retailer and supplier.

Our findings provide several practical and managerial implications. First of all, we provide guidance to managers of retailers on the design of optimal revenue sharing contracts. When setting its claim of supplier revenues, the retailer should account for the negative impact this will cause due to lower installed capacity of the supplier, a reduction in utilization and a delay in the supplier firm entering full capacity mode. Managers of retail firms should also consider a mitigating factor of this negative impact on revenues which is the higher downstream prices. Moreover, our results suggest that retailers should require a higher optimal share for necessities (more inelastic) compared to luxury (more elastic) products, while they should require a lower share of revenues when the variable costs of production of the supplier are high or when the demand growth is smaller. On the contrary, a one-for-all contract might not be far from the optimal choice for a retailer facing significant costs of discerning information (e.g., about the demand level in the supplier's market or supplier's cost of capital). This is in line for example with Amazon's practice of charging the same fee to multiple products belonging to different categories. Our analysis also highlights that minimum delivered quantities is an important tool for the retailer to extract part of the value from the supplier since it allows the retailer to retain a certain level of quantities sold and thus guarantee a certain level of revenues. Even if not binding initially, a minimum quantity constraint imposed may become binding subsequently as uncertainty unfolds. Nevertheless, if the constraint becomes too restrictive the supplier may interrupt the supply. Indeed, hold-up problems limit the retailer's ability to use minimum delivered quantities imposed on supplier firms.

Secondly, from the supplier's side, our analysis suggests that managers of supplier firms should pay considerable attention to their options to abandon operations and to expand their initial capacity. The supplier's option to abandon operations may allow the supplier to extract better terms from the retailer, i.e., a higher revenue sharing ratio, since abandonment can cause hold-up problems for the retailer. Such hold-up problems are expected to have stronger effects for shorter horizon contracts. Additionally, when setting its initial capacity, the supplier should take into account the option to expand its capacity in the future, as this flexibility can improve both its value and the one of the retailer.

Finally, our analysis suggests that retailers offering a subsidy to suppliers on the fee per product provides room to the supplier to increase its supplied quantities and bring them closer to the optimal level that could be achieved under coordination. This allows the retailer to claim a larger revenue share which will also be acceptable by the supplier firm (since the subsidy and increased capacity and volume leads to an improvement of its value). To our knowledge, subsidies on the fee per product provided to suppliers is not a common practice and is thus something which large retailers (e.g., WalMart or online platforms such as Amazon) who have ample cash resources could consider, thus enabling cash constrained small suppliers to expand offered quantities at a mutual benefit.

Our setting has overlooked several issues which could be addressed in future research. First, we focus on a single retailer and single supplier. It would be interesting to investigate how competition in either the upstream or downstream markets affects the design of the revenue sharing contract under supplier production flexibility. In addition, we have focused on a two-stage supply chain with supplier production flexibility so future work could enrich the framework to consider a three-stage supply chain.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix

Appendix A. Valuation benchmark model

We provide the expressions for Ω_1, Ω_2 determined from the boundary conditions given in equations (7) and (8):

$$\Omega_1 = \frac{1}{(\beta_1-\beta_2)\overline{x}^{\beta_1}}\left[\left(\frac{1}{\varepsilon}+\beta_2\right)\frac{A}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}^{-1/\varepsilon}+\frac{(1-\alpha)\overline{x}Q^{\varepsilon+1}}{\delta}(1-\beta_2)+\beta_2\frac{vQ}{r}\right] \tag{A1a}$$

$$\Omega_2 = \frac{1}{\overline{x}^{\beta_2}}\left[\frac{A}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}^{-1/\varepsilon}-\frac{c}{r}-\frac{(1-\alpha)\overline{x}Q^{\varepsilon+1}}{\delta}+\frac{c+vQ}{r}+\Omega_1\overline{x}^{\beta_1}\right] \tag{A1b}$$

Ω_1^R and Ω_2^R in equations (12) and (13) are given as follows:

$$\Omega_1^R = \frac{1}{(\beta_1-\beta_2)\overline{x}^{\beta_1}}\left[\left(\frac{1}{\varepsilon}+\beta_2\right)\frac{B}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}^{-1/\varepsilon}+\frac{\alpha\overline{x}Q^{\varepsilon+1}}{\delta}(1-\beta_2)+\beta_2\frac{c_RQ}{r}\right] \tag{A2a}$$

$$\Omega_2^R = \frac{1}{\overline{x}^{\beta_2}}\left[\frac{B}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}^{-1/\varepsilon}-\frac{\alpha\overline{x}Q^{\varepsilon+1}}{\delta}+\frac{c_RQ}{r}+\Omega_1^R\overline{x}^{\beta_1}\right] \tag{A2b}$$

Ψ_1 and Ψ_2 in equations (17) and (18) are given by:

$$\Psi_1 = \frac{1}{(\beta_1 - \beta_2)\overline{x}_V^{\beta_1}}\left[\left(\frac{1}{\varepsilon}+\beta_2\right)\frac{A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}_V^{-1/\varepsilon}+\frac{\overline{x}_V Q_V^{\varepsilon+1}}{\delta}(1-\beta_2)+\beta_2\frac{(v+c_R)Q_V}{r}\right] \tag{A3a}$$

$$\Psi_2 = \frac{1}{\overline{x}_V^{\beta_2}}\left[\frac{A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}_V^{-1/\varepsilon}-\frac{c}{r}-\frac{\overline{x}_V Q_V^{\varepsilon+1}}{\delta}+\frac{c+(v+c_R)Q_V}{r}+\Psi_1\overline{x}_V^{\beta_1}\right] \tag{A3b}$$

Appendix B. Sensitivities of profits, quantities, prices and switching thresholds with parameters

Quantities

$$\frac{\partial q}{\partial x} = -\frac{1}{\varepsilon x}\left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon} > 0$$

$$\frac{\partial q}{\partial v} = \frac{1/\varepsilon}{(\varepsilon+1)^{\left(\frac{1}{\varepsilon}\right)}\left(\frac{(1-\alpha)x}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}v} < 0$$

$$\frac{\partial q}{\partial \alpha} = \frac{1/\varepsilon}{(\varepsilon+1)^{\left(\frac{1}{\varepsilon}\right)}(1-\alpha)\left(\frac{x(1-\alpha)}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}} < 0$$

$\frac{\partial q}{\partial \varepsilon} = \frac{\frac{ln\left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)}{\varepsilon^2}-\frac{1}{\varepsilon(\varepsilon+1)}}{\left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{1/\varepsilon}}$. The sign is indeterminate. However, $\frac{\partial q}{\partial \varepsilon} < 0$ for high α or/and low $\frac{x}{v}$ and vice versa.

Prices

$$\frac{\partial p}{\partial v} = \frac{1}{(1-\alpha)(\varepsilon+1)} > 0$$

$$\frac{\partial p}{\partial \alpha} = \frac{v}{(\varepsilon+1)(\alpha-1)^2} > 0$$

$$\frac{\partial p}{\partial \varepsilon} = \frac{v}{(\alpha-1)(\varepsilon+1)^2} < 0$$

Switching threshold

$$\frac{\partial \overline{x}}{\partial v} = \frac{1}{(1-\alpha)(\varepsilon+1)Q^{\varepsilon}} > 0.$$

$$\frac{\partial \overline{x}}{\partial Q} = -\frac{\varepsilon v Q^{-(\varepsilon+1)}}{(\varepsilon+1)(1-\alpha)} > 0$$

$$\frac{\partial \overline{x}}{\partial \alpha} = \frac{v}{(\varepsilon+1)Q^{\varepsilon}(\alpha-1)^2} > 0$$

$\frac{\partial \overline{x}}{\partial \varepsilon} = -\frac{ln(Q)v}{(1-\alpha)Q^{\varepsilon}(\varepsilon+1)} - \frac{v}{(1-\alpha)Q^{\varepsilon}(\varepsilon+1)^2}$. The sign is indeterminate. However, for $Q > 1$ $\frac{\partial \overline{x}}{\partial \varepsilon} < 0$.

Supplier profits in Region 1

$$\frac{\partial \pi_S}{\partial x} = \frac{v x^{-\left(\frac{1}{\varepsilon}+1\right)}}{(\varepsilon+1)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{1/\varepsilon}} > 0.$$

$$\frac{\partial \pi_S}{\partial v} = -\frac{1}{x^{\left(\frac{1}{\varepsilon}\right)}\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}} < 0$$

$$\frac{\partial \pi_S}{\partial \alpha} = -\frac{1}{x^{\left(\frac{1}{\varepsilon}\right)}\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{\frac{1}{\varepsilon}+1}} < 0$$

$\frac{\partial \pi_S}{\partial \varepsilon} = -\frac{v\left(ln\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)+ln\ (x)\right)}{x^{\left(\frac{1}{\varepsilon}\right)}\varepsilon(\varepsilon+1)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{1/\varepsilon}}$. The sign is indeterminate. However, for high enough α and/or low $\frac{x}{v}$ we get $\frac{\partial \pi}{\partial \varepsilon} < 0$ and vice-versa.

Retailer's profits in Region 1

$\frac{\partial \pi_R}{\partial x} = -\frac{\left(\frac{\alpha v}{(1-\alpha)(\varepsilon+1)} - C_R\right)x^{-\left(\frac{1}{\varepsilon}-1\right)}}{\varepsilon\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{1/\varepsilon}}$. The sign is indeterminate. However, it is expected that $\frac{\partial \pi_R}{\partial x} > 0$ when c_R is small ($c_R \to 0$).

$$\frac{\partial \pi_R}{\partial C_R} = -\frac{1}{\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{\frac{1}{\varepsilon}}x^{\left(\frac{1}{\varepsilon}\right)}} < 0$$

$\frac{\partial \pi_R}{\partial v} = -\frac{x^{\left(\frac{1}{\varepsilon}\right)}(\alpha v+(\alpha-1)c_R)}{(\alpha-1)\varepsilon\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}v}$. The sign is indeterminate. It is expected that $\frac{\partial \pi_R}{\partial v} < 0$ when c_R is small ($c_R \to 0$) or when its value relative to v is small. $\frac{\partial \pi_R}{\partial \alpha} = \frac{x^{\left(-\frac{1}{\varepsilon}\right)}\left((v+c_R\varepsilon+c_R)\alpha+\varepsilon v-c_R\varepsilon-c_R\right)}{\varepsilon(\varepsilon+1)\left(\frac{(\varepsilon+1)(1-\alpha)}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}(\alpha-1)^2}$. The sign is indeterminate. It is expected that $\frac{\partial \pi_R}{\partial \alpha} < 0$ when c_R is small ($c_R \to 0$).

$$\frac{\partial \pi_R}{\partial \varepsilon} = \frac{-x^{\left(\frac{1}{\varepsilon}\right)}\left((\alpha-1)c_R\varepsilon + \alpha v + (\alpha-1)c_R\right)ln\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right) + \left((1-\alpha)c_R\ ln(x) - av + (1-\alpha)c_R\right)\varepsilon + \left((1-\alpha)c_R - \alpha v\right)ln\ (x)}{(\alpha-1)\varepsilon^2(\varepsilon+1)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{\left(\frac{1}{\varepsilon}\right)}}$$

Optimal switching threshold

$$\frac{\partial \overline{x}_V}{\partial v} = \frac{1}{{Q_V}^{\varepsilon}(\varepsilon+1)} > 0$$

$$\frac{\partial \overline{x}_V}{\partial c_R} = \frac{1}{{Q_V}^{\varepsilon}(\varepsilon+1)} > 0$$

$\frac{\partial \overline{x}_V}{\partial \varepsilon} = \frac{-(v+c_R)(\ln(Q_V)\varepsilon + \ln(Q_V) + 1)}{{Q_V}^{\varepsilon}(\varepsilon+1)^2}$. The sign is indeterminate $\frac{\partial \overline{x}_V}{\partial \varepsilon} < 0$ when $Q_V > 1$

$$\frac{\partial \overline{x}_V}{\partial Q_V} = -\frac{\varepsilon(v+c_R){Q_V}^{-(\varepsilon+1)}}{\varepsilon+1} > 0$$

Appendix C. Supplementary material for Results 3 and 4b

This appendix supports Result 3 (Optimal revenue sharing contract sensitivities) and Result 4b (gains from vertical integration). Parameters used are $r = 0.05$, $\delta = 0.03$, $\sigma = 0.2$. $v = 1$, $\varepsilon = -0.7$, $c = 0$, $c_R = 1$, $x = 10$, $k = 3$, $\eta = 2$. We use Q increments of 0.01 for optimizing Q.

σ	α	$\overline{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.1	0.79	15.65	1	48.48	231.48	0.98	0.53	15.87	2.69	0.66	6.67	0.15
0.2	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
0.3	0.81	20.62	1	47.62	252.39	1.26	0.36	17.54	3.52	0.51	6.67	0.17
0.4	0.82	22.85	1	47.29	264.96	1.35	0.31	18.52	3.87	0.46	6.67	0.19
0.5	0.82	24.25	1	49.83	276.83	1.47	0.28	18.52	4.15	0.43	6.67	0.18
c_R	α	$\overline{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.2	0.78	17.31	1	54.10	249.46	1.21	0.46	15.15	3.58	1.00	4.10	0.22
0.4	0.79	17.72	1	50.89	246.99	1.17	0.44	15.87	3.43	0.87	4.67	0.21
0.6	0.79	17.72	1	50.89	244.61	1.17	0.44	15.87	3.30	0.74	5.33	0.19
0.8	0.79	17.72	1	50.89	242.22	1.17	0.44	15.87	3.20	0.65	6.00	0.17
1	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
1.2	0.81	18.51	1	44.61	237.57	1.08	0.41	17.54	3.03	0.51	7.33	0.16
1.4	0.82	19.03	1	41.55	235.48	1.04	0.40	18.52	2.95	0.47	8.00	0.16
1.6	0.82	19.03	1	41.55	233.48	1.04	0.40	18.52	2.89	0.42	8.67	0.15
1.8	0.82	19.03	1	41.55	231.49	1.04	0.40	18.52	2.83	0.39	9.33	0.13
2	0.82	19.03	1	41.55	229.50	1.04	0.40	18.52	2.78	0.36	10.00	0.12
2.2	0.82	19.03	1	41.55	227.51	1.04	0.40	18.52	2.73	0.33	10.67	0.11
2.4	0.83	19.47	1	38.53	225.54	0.99	0.39	19.61	2.68	0.31	11.33	0.12
r	α	$\overline{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.05	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
0.06	0.8	19.05	1	49.87	249.23	1.21	0.40	16.67	3.31	0.54	6.67	0.16
0.07	0.81	20.39	1	48.38	257.28	1.24	0.36	17.54	3.49	0.51	6.67	0.17
0.08	0.81	21.08	1	49.87	263.88	1.3	0.34	17.54	3.65	0.49	6.67	0.17
0.09	0.81	21.65	1	51.14	269.40	1.35	0.33	17.54	3.78	0.47	6.67	0.17

(continued on next page)

(*continued*)

σ	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.1	0.82	22.85	1	48.78	274.46	1.35	0.31	18.52	3.89	0.46	6.67	0.18
0.11	0.82	23.32	1	49.70	278.70	1.39	0.30	18.52	3.99	0.45	6.67	0.18
ν	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.25	0.85	6.75	2	42.84	281.99	1.32	1.00	8.23	3.54	0.99	4.17	0.13
0.5	0.82	10.89	1	48.27	265.06	1.26	0.89	9.26	3.36	0.80	5.00	0.13
0.75	0.81	14.69	1	47.65	251.19	1.17	0.58	13.16	3.22	0.67	5.83	0.15
1	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
1.25	0.79	21.21	1	48.23	230.43	1.1	0.34	19.84	3.01	0.50	7.50	0.17
1.5	0.78	23.99	1	49.00	222.15	1.08	0.29	22.73	2.92	0.44	8.33	0.17
1.75	0.79	27.78	1	44.25	215.07	1	0.23	27.78	2.84	0.40	9.17	0.20
2	0.78	30.30	1	45.44	209.00	1	0.21	30.30	2.78	0.36	10.00	0.20
2.25	0.78	33.37	1	44.00	203.51	0.97	0.18	34.09	2.72	0.33	10.83	0.21
2.5	0.77	35.47	1	45.36	198.46	0.97	0.16	36.23	2.66	0.30	11.67	0.20
2.75	0.77	38.17	1	44.17	193.82	0.94	0.15	39.86	2.61	0.28	12.50	0.21
3	0.77	41.01	1	43.10	189.77	0.92	0.13	43.48	2.57	0.26	13.33	0.22
3.25	0.76	42.90	1	44.60	186.04	0.93	0.12	45.14	2.52	0.24	14.17	0.21
3.5	0.76	45.51	1	43.66	182.54	0.91	0.11	48.61	2.48	0.23	15.00	0.21
δ	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0.01	0.83	39.91	1	172.56	1025.90	2.76	0.14	19.61	8.07	0.22	6.67	0.20
0.02	0.81	24.59	1	78.07	412.74	1.62	0.28	17.54	4.54	0.39	6.67	0.18
0.03	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
0.04	0.79	14.28	1	34.24	162.93	0.86	0.60	15.87	2.33	0.77	6.67	0.15
0.05	0.8	12.33	1	23.64	120.95	0.65	0.74	16.67	1.87	0.95	6.67	0.16
0.06	0.79	10.58	1	19.76	95.06	0.56	0.92	15.87	1.58	1.00	7.26	0.15
η	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
1.1	0.75	26.72	1	67.56	267.03	2.7	0.25	13.33	11.87	0.15	6.67	0.16
1.2	0.76	24.27	1	63.08	261.20	2.22	0.28	13.89	9.19	0.19	6.67	0.17
1.3	0.76	22.48	1	62.51	256.62	1.99	0.31	13.89	7.42	0.24	6.67	0.16
1.4	0.77	21.27	1	58.56	252.83	1.73	0.34	14.49	6.18	0.29	6.67	0.17
1.5	0.78	20.41	1	54.84	249.57	1.53	0.36	15.15	5.29	0.34	6.67	0.17
1.6	0.79	19.89	1	51.31	246.99	1.38	0.37	15.87	4.62	0.39	6.67	0.18
1.7	0.79	19.18	1	51.17	244.75	1.31	0.39	15.87	4.10	0.44	6.67	0.17
1.8	0.79	18.56	1	51.05	242.72	1.25	0.41	15.87	3.70	0.48	6.67	0.16
1.9	0.79	18.14	1	50.96	241.30	1.21	0.43	15.87	3.37	0.53	6.67	0.16
2	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
2.1	0.8	17.70	1	47.69	238.50	1.09	0.44	16.67	2.89	0.62	6.67	0.16
2.2	0.8	17.48	1	47.66	237.69	1.07	0.45	16.67	2.70	0.66	6.67	0.15
2.3	0.82	18.26	1	41.54	236.91	0.98	0.42	18.52	2.54	0.70	6.67	0.17
2.4	0.8	17.02	1	47.63	236.02	1.03	0.47	16.67	2.41	0.74	6.67	0.14
2.5	0.82	17.87	1	41.57	235.56	0.95	0.44	18.52	2.29	0.78	6.67	0.16
k	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
1	0.8	26.02	1	51.78	261.84	1.89	0.26	16.67	5.23	0.34	6.67	0.16
2	0.8	20.67	1	49.25	248.05	1.36	0.35	16.67	3.77	0.47	6.67	0.16
3	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
4	0.8	16.43	1	46.63	233.83	0.98	0.49	16.67	2.71	0.66	6.67	0.16
5	0.81	15.66	1	42.80	229.21	0.85	0.53	17.54	2.44	0.73	6.67	0.18
6	0.81	14.74	1	42.14	225.38	0.78	0.57	17.54	2.24	0.80	6.67	0.18
6	0.81	14.74	1	42.14	225.38	0.78	0.57	17.54	2.24	0.80	6.67	0.18
7	0.8	13.63	1	44.45	221.85	0.75	0.64	16.67	2.08	0.86	6.67	0.16
8	0.8	13.11	1	43.92	219.35	0.71	0.68	16.67	1.95	0.92	6.67	0.16
x	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
5	0.8	12.59	1	19.09	96.56	0.67	0.27	16.67	1.85	0.36	6.67	0.16
10	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
15	0.81	22.97	1	75.94	406.56	1.47	0.54	17.54	4.21	0.76	6.67	0.18
20	0.81	26.68	1	110.50	589.32	1.82	0.66	17.54	5.23	0.92	6.67	0.18
25	0.82	30.82	1	137.62	784.82	2.07	0.74	18.52	6.18	1.00	6.99	0.19
30	0.81	33.04	1	186.80	991.09	2.47	0.87	17.54	7.08	1.00	7.62	0.17
35	0.81	35.80	1	227.78	1205.49	2.77	0.97	17.54	7.94	1.00	8.21	0.17
40	0.82	39.40	2	252.22	1427.78	2.94	1.00	18.80	8.76	1.00	8.76	0.19
45	0.82	41.89	2	293.26	1656.68	3.21	1.00	19.89	9.55	1.00	9.27	0.19
50	0.81	43.17	2	359.38	1891.50	3.62	1.00	20.32	10.31	1.00	9.77	0.17
c	α	$\bar{x}$	Region	$S^{Net}(x)$	R	Q	(q/Q)	Price (p)	Q_v	(q_v/Q_v)	Price vertical (p_v)	Gain
0	0.79	17.72	1	50.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.15
0.5	0.79	17.72	1	40.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.16
1	0.79	17.72	1	30.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.16
1.5	0.79	17.72	1	20.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.17
2	0.79	17.72	1	10.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.17
2.5	0.79	17.72	1	0.89	239.84	1.17	0.44	15.87	3.11	0.57	6.67	0.18

Appendix D. Coordination

We provide the expressions for Ω_1^c, Ω_2^c determined from the boundary conditions given in equations (47) and (48). Knowing that under the coordinating contract we have that $\overline{x} = \overline{x}_V$, $q = q_V$ and $p = p_V$, we obtain:

$$\Omega_1^c = \frac{1}{(\beta_1 - \beta_2)\overline{x}_V^{\beta_1}}(1-\alpha)\left[\left(\frac{1}{\varepsilon}+\beta_2\right)\frac{A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}_V^{-\frac{1}{\varepsilon}}+\frac{\overline{x}_V Q^{\varepsilon+1}}{\delta}(1-\beta_2)+\beta_2\frac{(v+c_R)Q}{r}\right] \tag{A4a}$$

$$\Omega_2^c = (1-\alpha)\frac{1}{\overline{x}_V^{\beta_2}}\left[\frac{A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}\overline{x}_V^{-\frac{1}{\varepsilon}}-\frac{c}{r}-\frac{\overline{x}_V Q^{\varepsilon+1}}{\delta}+\frac{c+(v+c_R)Q}{r}+\Psi_1\overline{x}_V^{\beta_1}\right] \tag{A4b}$$

To continue the proof, we will first assume that $Q = Q_V$ and in the end prove that this is indeed the case. Assuming $Q = Q_V$ we get that $\Omega_1^c = (1-\alpha)\Psi_1$ and $\Omega_2^c = (1-\alpha)\Psi_2$. Substituting these expressions into the supplier values given by equations (45) and (46) we obtain the relationship between supplier value and supply chain value given in equations (49) and (50). From these relationships it directly follows that the supplier will choose the same optimal capacity as the supply chain, i.e., $Q = Q_V$, as initially assumed.

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