

# Assessing potential improvement in education: Unravelling pupils' and schools' inefficiencies using a multi-level metafrontier approach<sup>☆</sup>

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## ABSTRACT

How relevant is a better use of schooling resources to improve learning outcomes? Though the optimal management of public funding is always desirable, achievement improvement might also depend on student and school level factors beyond efficiency. Using data from public primary schools in Uruguay, we discuss the role of the school efficiency gains in the potential attainment improvement of pupils. Our empirical approach resorts to a multilevel metafrontier model based on non-parametric methods to identify the sources of pupils' under-attainment. The results reveal that pure school inefficiencies *per se* are hardly relevant to enhance students' performance while the effect of school resources is still sizable. We also find that school-level efficiency gaps affect the most disadvantaged students and school-types within the public system. The research highlights the importance of correctly assessing the range and roots of potential improvement from schooling efficiency policies, particularly in contexts characterized by scarcity of resources.

## 1. Introduction

The need to enhance the performance and learning opportunities of students has made scholars and policy makers increasingly interested in rising the efficiency and productivity of the current school funding (Bogetoft et al., 2015; Hanushek, 2003; Woessman, 2008). This argument has been recently put forward by the Uruguayan authorities in order to deal with the poor education outcomes in the country. Their concerns on schooling achievement relate to the virtually stagnant performance of students and the high socio-economic inequality in their learning attainment. For instance, primary school pupils are well positioned in the Latin American context, but they have not improved their results over the last 15 years (INEEd, 2015a). Indeed, though regarded as highly egalitarian compared to the region, the country exhibits a very strong association between socioeconomic conditions and children's education performance due to the socio-spatial segregation of schools (INEEd, 2019).

In the recent "Educational Development Plan" (2020–2024), the government interpreted that solving most of these problems entailed correcting expenditure inefficiencies. Interestingly, this rationale coexists with a limited resource allocation to the domestic education

system. Particularly, primary education spending reaches 1.1% of GDP. This level is below the 1.5% for OCDE and for many Latin American countries (Santiago et al., 2016; DINEM, 2020). This paper proposes to look into the efficiency policy argument in the context of tight budget constraints.

According to Debreu (1951) and Farrell (1957), the technical efficiency of one productive unit alludes to the comparison between its potential and actual output given its available inputs. When applied to the education system, efficiency identifies the maximum academic performance students are believed to be capable given the invested resources. However, efficiency assessments in education are more difficult to interpret than in other areas because pupils' performance is not the result of a standardized production process. Since the seminal contribution of the Coleman Report (Coleman et al., 1966), evidence shows that educational outcomes depend on the inherent ability of students, their family and personal background as well as on the schooling resources, the school context, the educational authorities' decisions on teaching and learning. Moreover, each of these aspects feed into one another. Therefore, just as the attained performance is a composite of pupil and school related effects, efficiency gains in student outcomes also depend both on pupils' and school's efforts. Ultimately, only the

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latter are directly under the discretion of educational authorities.

This paper argues that policies oriented to increase educational efficiency should previously gauge the actual range of students' under-attainment and the share attributable specifically to schools. On these grounds, we estimate the scope of potential improvement in pupils' performance to identify whether they depend on individuals or schools.

This study belongs to a limited body of works that measure student and school efficiency effects by using production frontier methodologies. Nowadays, numerous studies apply these techniques to estimate education efficiency and productivity (De Witte and López-Torres, 2017; Johnes, 2020). Indeed, some of them are focused on productivity gains under severe schooling budget restrictions (López Torrez and Prior, 2022; Aparicio et al., 2018). However, just a few papers have assessed the role of school efficiency on pupils' outcomes. Silva-Portela and Thanassoulis (2001) and Thanassoulis and Silva-Portela (2002) were the first to build local and global frontiers to that end. Based on the hierarchical structure of education data (i.e., pupils attend different schools), they measured the pupil's effort to improve her attainment as the distance of that individual to a local frontier formed by students within her own school. Likewise, the school contribution to academic improvement emerged from the distance of that individual to another frontier composed by the best practice pupils among all the schools analyzed. These ideas are closely related with the "metafrontier" framework proposed by Battese and Rao (2002), Battese et al. (2004) and O'Donnell et al. (2008).

As mentioned, in the field of education, just a bunch of studies have reproduced this methodological proposal (De Witte et al., 2010; Silva-Portela and Camanho, 2010; Mancebo and Alonso, 2012; Silva-Portela et al., 2013). Particularly, Thieme et al. (2013) include a multilevel decomposition of frontiers to sequentially address how different inputs –related to pupils and to schools–account for the achievement shortfall. More recently, Cordero et al. (2016) follow the same decomposition to discuss the role of the funding system (public or private) on the students' performance.

In this paper, we extend the multilevel metafrontier approach proposed by Thieme et al. (2013). First, we include one decomposition related to the schools' resource endowment and another to the environmental conditions in which they operate. This procedure allows showing how much these inputs explain of the total shortfall in pupils' attainment net of the effect of any student background variable. While the focus on resources highlights the role of the funding handled by education authorities, the context conditions accounts for the influence of the socio-spatial segregation of schools. Just the remaining share of school level inefficiencies would depend on the school own performance and might be attributed to their underlying unexploited managerial capacity. Next, we enhance the previous analysis by differentiating school effects according to school types within the public system and the socioeconomic and score achievement of pupils. This secondary study allows illustrating whether education institutions are compensating or reproducing initial educational inequities.

Our quantitative approach uses data on primary school pupils of the 6th grade who attend public schools. The information has been taken from the most recent national assessment (ARISTAS) implemented in 2017. Notably, to address the environment effect, we use a novel combination of data on the crime and violence at the school surroundings and the learning motivations of students. Based on them, we untangle the role of schools from that of adverse neighborhoods and poor social incentives of students.

This research also provides new evidence for an unexplored subject

in Uruguay. Previous analyses based on frontier methodologies are limited in the country and refer to secondary education (Deutsch et al., 2013; Sicilia, 2014; Santin and Sicilia, 2015; Azar et al., 2018; Aparicio et al., 2020). Besides, existing studies on primary schools use parametric methodologies and focus on the effects of primary education on the subsequent secondary schooling performance (Cardozo et al., 2017; De Melo and Machado, 2018; Lado, 2019).

Overall, the contribution of this paper is threefold. From a methodological perspective, it follows a rarely applied empirical approach within the education frontier literature and proposes some extensions to consider different socio-economic environments. Regarding policy design, it provides practitioners and policymaker a roadmap to carry out a fair evaluation of the performance of students and schools. Indeed, it provides evidence to evaluate the suitability of the argument about the limited effect on schooling performance of providing more funding –typical of developed economies–on developing settings (Benos and Karagiannis, 2016; Hanushek, 2006; Mancebón and Muñiz, 2003).<sup>1</sup> Finally, at the local level, it is the first study that applies frontier techniques to inform policy choices based on efficiency arguments.

The paper is organized as follows. Section 2 presents the key features of the educational system under analysis. Section 3 describes the methodological approach. Section 4 describes the data and the empirical strategy. Results are discussed in section 5. Finally, section 6 covers the concluding remarks.

## 2. Public primary education system in Uruguay

In Uruguay, compulsory education starts when children are aged 4 and finishes at upper secondary education when they are 17. The system is highly centralized: one governing council (*Consejo Directivo Central, CODICEN*) takes all decisions and co-ordinates the activities of four education directions in charge of each of the schooling levels (pre-primary and primary, secondary, technical and teaching training). These directions manage the teaching and non-teaching staff and the financial resources in the public sphere. They also provide the guidelines for pedagogical practices at public schools and directives to monitor the curricula at both public and private institutions.

Since the mid-1950s, the Uruguayan primary schooling system has had a universal coverage for the children between 6 and 11 years old, no matter their socioeconomic origins (INEED, 2018b). The public sector prevails over the private: pupils enrolled at public schools are above 83% of the total. This percentage surpasses the regional average (79%) and it is above the ratio in Argentina or Chile (74% and 37%, respectively), countries similar to Uruguay in terms of economic development (UNESCO Institute for Statistics, 2022).

In the public system, schools share the same teaching curricula, institutional and organizing rules. However, they may have different modalities according to the length of the school day, the extra-school activities they supply and therefore, the material and personal resources they receive. Thus, school hours are 4 at *Regular*, *Practice* and *Learning* schools.<sup>2</sup> Among them, *Practice* schools receive teacher students who have to complete their training program to obtain a professional degree. There, the teaching staff is particularly highly qualified. *Learning* schools are situated in the most disadvantaged socioeconomic contexts where repetition and drop-out rates are the highest of the system (Mancebo and Alonso, 2012). For this reason, together with the regular classroom instruction, these schools implement education programs to

<sup>1</sup> It is worth noting that even for the richest economies, recent research has provided evidence on the positive effect of expanding school resources (Lafortune et al., 2018; Kirabo et al., 2016; Nusche et al., 2016).

<sup>2</sup> "Learning" translates the Spanish term "Aprender" which is also an acronym for "Priority Attention in Contexts featured by Structural Drawbacks" ("Atención Prioritaria en Entornos con Dificultades Estructurales"). These schools are known as "Aprender", so we will call them "Learning".

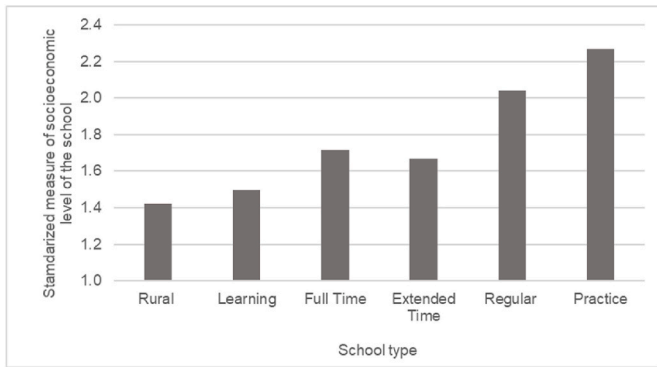


Fig. 1. Average socioeconomic status of primary schools based on 6th grade students based on ARISTAS (2017).

deal with specific pedagogical, psychological or family needs aiming to lift student achievement.

Another school-type comprises the *Extended-Time* schools, where pupils stay for 3 additional hours compared to *Regular* schools because they are offered some extra-curricular activities (arts, sports, computing). These activities are, usually, funded by parents. Finally, *Full-Time* schools have the longest instruction day (7 and a half hours) and at rural areas the school day lasts 5 h. Across these different schools, public primary enrollment is divided as follows: *Learning* and *Regular* schools account for 31% of the total, each; *Full-Time* and *Practice* schools explain 17% each and *Extended-Time* schools cover the remaining 3%. *Rural* schools make up 5% of the total (ANEP, 2019).

Due to their different conditions, public education spending at *Full-Time* and *Learning* schools is higher than in the rest. However, teachers do not receive particularly elevated salaries. At *Learning* schools, they are paid a small compensation to make up for the problematic context: the amount represents less than 10% of the base salary. At *Full-Time* schools, the pay for teachers who work a double shift at the same institution is below the double of the wage paid at a *Regular* school (INEED, 2016).

Families can freely choose the public school for their children. If demand exceeds the school capacity, the student's admission is determined according to whether her family lives at the school's surroundings, any of the parents or guardians work in the area or she has siblings already enrolled at the school. *Full-Time* schools also consider the child's household income and the employment status of her mother (Santiago et al., 2016). As a result of these rules, students typically attend their neighborhood school. Therefore, the distribution of students is highly homogenous within institutions, but differs among them because of the socioeconomic conditions of the district in which they are placed. In fact, Uruguay holds one of the highest segregation indexes in the Latin American context related to the school location (Murillo and Martínez, 2017).

Fig. 1 shows the differences in the socioeconomic contexts of each public school-type based on the average socioeconomic status of their 6th grade students (that is, those at the end of the schooling cycle). The measure ranges between 1 and 4, going from very unfavorable to very favorable socioeconomic contexts (INEED, 2018a). *Rural* and *Learning* schools are situated in the most unfavorable settings, while *Regular* and, particularly, *Practice* schools are in more advantaged environments. In the middle, students at *Full-Time* and *Extended-Time* schools belong to varied socioeconomic contexts.

As already mentioned, public education expenditure is low in Uruguay. This is particularly clear in the wages of mid-career teachers, which are below those in other Latin American countries (Santiago et al., 2016). Despite these limitations, the public system provides universal access since the age of 4 (with coverage rates above the regional average) and learning achievements exceed the Latin American records

(INEED, 2015b). The same happens in terms of teaching qualification, school infrastructure and technology use in the classroom.<sup>3</sup>

However, primary school pupils have not improved their performance over the last 15 years. Besides, the country has exhibited one of the widest regional performance gaps between the 10th and 90th percentile of socioeconomic background and one of the most important urban-rural difference in mathematics (UNESCO, 2008). Accordingly, primary students report the highest academic achievement in favorable settings while the opposite is true in unfavorable contexts, a finding consistent with the international literature (Sirin, 2005; Jerrim et al., 2019). These results appear both for reading and mathematics (Figure A-1 in the Appendix depicts a substantial positive association between socioeconomic level and performance for the public schooling system).

The general picture shows that departing from a disadvantageous position in terms of public funding, Uruguayan public schools have historically provided universal and high-quality access to education in the Latin American context. Despite the quest for a better resource use is always desirable, the output-input imbalance reflected in the previous data, suggests that efficiency concerns might be quite overstated in the country. In the next section, we present our empirical approach to discuss this issue.

### 3. Methodology

Our theoretical approach is based on the Educational Production Function proposed by Levin (1974) and Hanushek (1979), which indicates the maximum quantity of schooling output feasible to be produced from a given quantity of inputs:

$$y_i = f(x_i) \quad (1)$$

where  $y \in R_+^q$  stands for the educational output vector for student  $i$  (usually represented by test scores in a standardized assessment) and  $x \in R_+^p$  identifies the vector of inputs including the student's own abilities, parental and socioeconomic background and school resources. This function can be estimated allowing for the existence of inefficient behaviors in students and schools due to multiple factors, such as poor students' and teacher's motivation, teaching and class organization issues or school management. Then, the production function to be estimated would be:

$$y_i = f(x_i) \cdot u_i \quad (2)$$

where  $u_i$  denotes the individual technical efficiency level, understood as the ability with which student  $i$  can reach the maximum potential achievement given her resource endowment (Farrell, 1957). Whenever  $u_i = 1$ , the evaluated student is fully efficient, and she is on the production frontier. For values of  $u_i > 1$ , the student is inefficient. In practice, the true production frontier  $f(\cdot)$  is not observed and should be estimated from the relative best practices in the sample. To this aim, in this paper we apply the Free Disposal Hull (FDH) method proposed by Deprins et al. (1984). The technique builds the production frontier from the performance of real production units (i.e., students or schools in the educational context).<sup>4</sup> Then, the efficiency component  $u_i$  (equation (2))

<sup>3</sup> All Uruguayan primary schooling teachers are certified while the percentage ranges between 77% and 82% in the region. Together with Chile, local school infrastructure has the lowest levels of inequalities and damage in the Latin American context. Besides, due to a pioneer regional plan dated in 2007, each Uruguayan child entering public education accesses to a computer through the so-called "Ceibal Plan" (UNESCO, 2020).

<sup>4</sup> Different from the Data Envelopment Analysis (DEA) technique, the FDH does not include the linear combination of students as potential best practices. In other words, the FDH benefits from the advantages of non-convex non-parametric models (Kerstens et al., 2019).

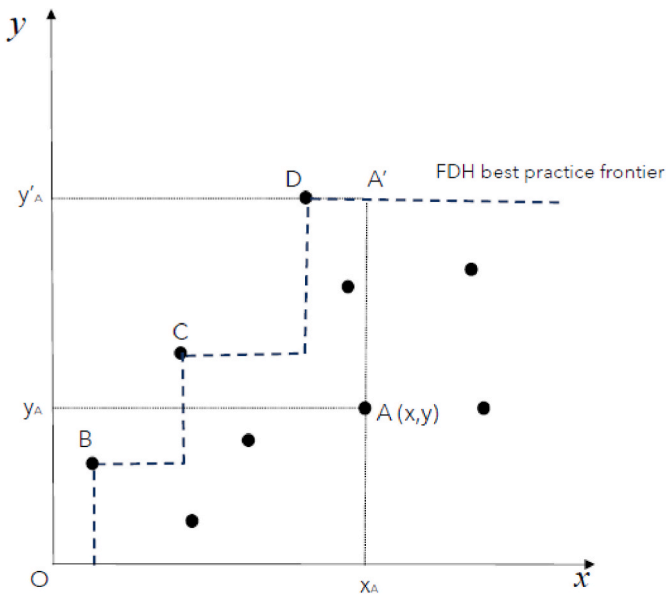


Fig. 2. Free Disposal Hull model illustration.

is estimated using the output oriented FDH index  $\hat{\lambda}_{FDH}$  obtained by solving the following linear optimization problem:

$$\hat{\lambda}_{FDH} = \max \left\{ \lambda \mid \lambda y \leq \sum_{i=1}^N \gamma_i y_i; x \geq \sum_{i=1}^N \gamma_i x_i; \sum_{i=1}^N \gamma_i = 1; \gamma_i \in \{0, 1\}; i = 1, \dots, n \right\} \quad (3)$$

where  $y_i$  and  $x_i$  correspond to the output and input vectors of student  $i$ , respectively,  $\hat{\lambda}_{FDH} \geq 1$  is the estimated efficiency score, and  $\gamma$  is a vector that contains the virtual weights, provided by the model solution and indicating the reference students (best practices) for each individual under analysis. When  $\hat{\lambda}_{FDH} = 1$  the evaluated student belongs to the frontier (is fully efficient), whereas  $\hat{\lambda}_{FDH} > 1$  indicates that the student is inefficient. In the context of our study, this implies that student  $i$  is obtaining a lower level of achievement than it would be expected from best practices (the frontier), given her level of inputs. The value of  $\hat{\lambda}_{FDH} > 1$  indicates the equi-proportional expansion over all outputs needed to reach the frontier. Therefore, the higher the score value  $\hat{\lambda}_{FDH}$ , the greater the inefficiency level.

We illustrate these ideas in Fig. 2 for a single-input single-output case. In this example, students B, C and D are the best practices (reference units), and the rest of pupils are identified as inefficient. For student A, the output-oriented inefficiency measure can be computed as the ratio between the potential maximum output she can achieve relative to her actual achievement (i.e., the distance  $Oy'_A / Oy_A$ ). It shows how much outputs can be expanded given the current inputs, that is how much a student's education achievement might be improved under her set of individual and school characteristics.

To improve the robustness of the FDH estimates, we follow a robust order- $m$  estimation introduced by Cazals et al. (2002).<sup>5</sup> The authors propose to build up a partial frontier that envelops only  $m \geq 1$  observations randomly drawn with replacement from the original empirical sample of  $y_i$  such that  $x_i \leq x$ . By estimating equation (3)  $B$  times, they obtain  $B$  multiple measures of  $\hat{\lambda}_{FDH}$  ( $\hat{\lambda}_{mi}^1, \dots, \hat{\lambda}_{mi}^B$ ). The order- $m$  efficiency

<sup>5</sup> Due to its deterministic nature, the FDH is very sensitive to outliers and measurement errors in data. Besides, statistical inference is not possible (Simar and Wilson, 2000). It also exhibits dimensionality problems due to its slow convergence rate (Kneip et al., 1998).

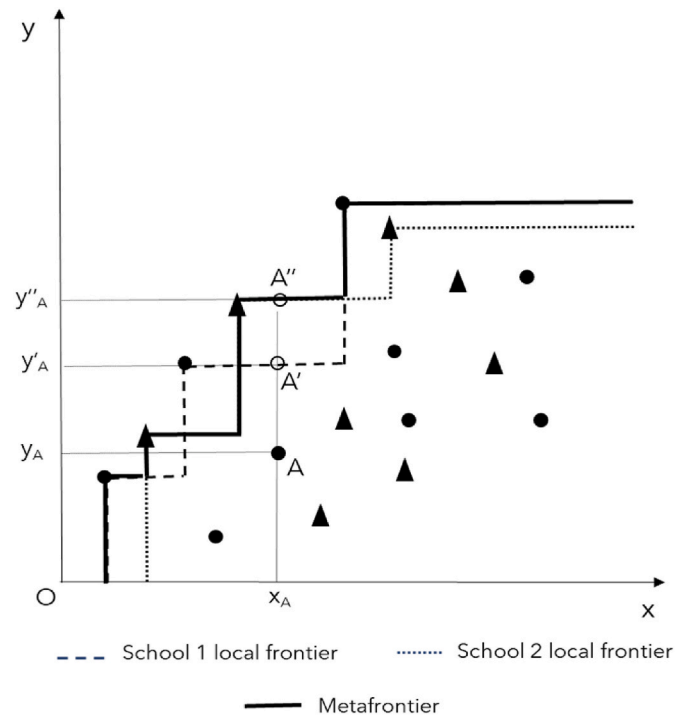


Fig. 3. The metafrontier approach.

measure  $\hat{\lambda}_m$  is then computed as the simple average across the  $B$  efficiency scores. For acceptable  $m$  values, the efficiency scores will be higher than unity, which indicates that students are inefficient, as outputs can be increased without modifying the level of inputs. However, in some cases we can observe  $\hat{\lambda} < 1$ , meaning that the evaluated individual shows a higher output level than the average  $m$  observations in its reference sample (Daraio and Simar, 2007). These are called super-efficient individuals and do not emerge in the traditional nonparametric framework where by construction  $\hat{\lambda} \geq 1$ . Following Daraio and Simar (2007), we set the value of  $m$  at the minimal level required to make the proportion of super-efficient students vary only marginally with the size of  $m$ . In our application this value is equal to 40.

### 3.1. Metafrontier approach

The key to decompose the effect of school and student on the efficiency measure is to recognize the hierarchical structure of educational data (Silva-Portela and Thanassoulis, 2001; Thanassoulis and Silva-Portela, 2002). As pupils are nested within schools, it is possible to resort to the non-parametric literature which developed the concept of "metafrontiers" to deal with a hierarchical dataset (Battese and Rao, 2002; Battese et al., 2004; O'Donnell et al., 2008). We illustrate the metafrontier idea in Fig. 3. We represent two schools in the single-input single-output framework: school 1 where students are represented by dots and school 2 where students are represented by triangles. The wider dotted line represents the local frontier for school 1, the thinner dotted line represents the local frontier of school 2 and the solid line represent the metafrontier (defined by the pooled sample).

In line with Thanassoulis and Silva-Portela (2002), the *student-within-all-schools efficiency* or the overall efficiency (OE) is computed as the distance between the student and the metafrontier. The latter is derived from the robust FDH model according to equation (3) and including all students in the sample. Alternatively, the distance between the student and the local frontier (built from the pupils at the school she attends) is used to estimate the *student-within-school efficiency* (STE), which

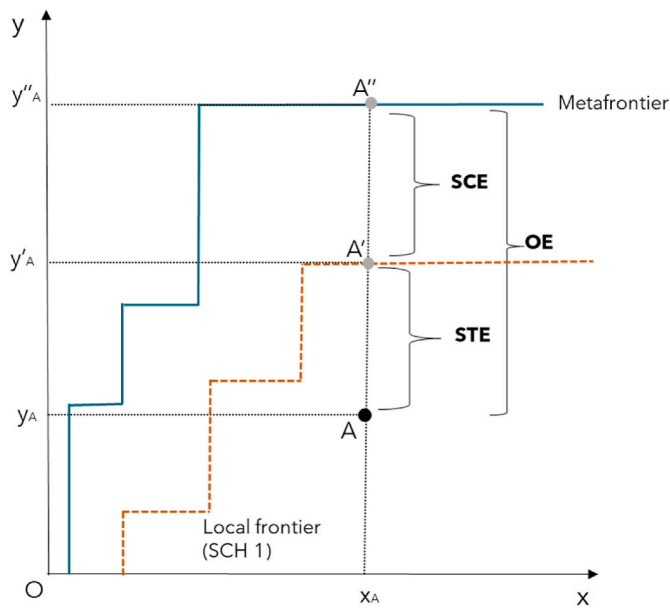


Fig. 4. Metafrontier illustration following Thanassoulis and Silva-Portela (2002) decomposition (level 1).

depends on her own ability to achieve her potential academic achievement given her socioeconomic background. The STE of each student is also estimated using the robust FDH model (equation (3)) but including only those students at the school she attends. Finally, the distance separating the local and the metafrontier can be interpreted as the *school-within-all-schools efficiency* (or school effect, SCE), which is residually computed from the difference between the OE and STE components. We illustrate these concepts, which describe our “level 1 decomposition” in Fig. 4.<sup>6</sup>

Consider student A attending school 1, who achieves an output equal to  $y_A$  given her socioeconomic background (input)  $x_A$ . The *student-within-all-schools efficiency* (OE) corresponds to the distance  $OE_A = Oy'_A / Oy_A$  and provides a measure of the additional output that pupil A could expect if she and school 1 were able to reach the level of the best performing students in the sample. That is, if she were able to reach the performance of student A'' in the metafrontier. Then, the *student-within-school efficiency* can be computed as the distance  $STE_A = Oy'_A / Oy_A$  and represents the extent of the output the student A is not achieving because she is not able to turn her potential into the same output levels reached by the best students attending the same school (student A' in this case). This component might be attributed to the student's own efficiency. Finally, the school efficiency component can be represented by the ratio  $SCE_A = Oy''_A / Oy'_A$  and reflects the output that pupil A is not able to achieve due to a school failure to turn the potential of their pupils on the same level that best practice schools are doing (De Witte et al., 2010). This component represents the technological gap between school 1 and the metafrontier. To sum up, in our first model (model 1 hereafter) the overall pupil efficiency (OE) can be decomposed into a student (STE) and a school efficiency effect (SCE) as:  $OE = STE \times SCE$ .<sup>7</sup>

<sup>6</sup> We follow Thieme et al. (2013) in the notation of the different efficiency effects.

<sup>7</sup> Operationally, we directly estimate the OE and STE components, and the school effect SCE can be residually obtained by the ratio  $OE/STE$ .

### 3.2. Multi-level metafrontier approach

Following the extension to the metafrontier approach introduced by Thieme et al. (2013), we recognize that the schools within the sample may not operate under optimal conditions. This issue is especially relevant in the case of Uruguay, where the current government is considering implementing policies oriented to increase efficiency. This public policy may lead to detrimental results in terms of educational performance if the school efficiency effect (SCE) is driven, for example, by an inadequate resource endowment or harsh environment conditions. Thus, to fairly capture the real level of managerial efficiency specific to each school (net of other components), we need to disentangle these two effects from the SCE component. To do so, we introduce additional variables in our level 1 decomposition to compute a “net” school level efficiency effect.

Regarding schools' resources, we adopt a wide concept which embraces two types: one, the socioeconomic and cultural profiles of peers at the school level –because they influence the student's performance (Coleman et al., 1966).<sup>8</sup> On the other side, we include the physical and human resources available at the school. In developed countries schools share a minimum and often homogenous level of school resources. However, this is not the case in developing countries, where the resource endowment heterogeneity across schools might have a significant influence in the students' performance (Mancebón and Muñiz, 2003).

In relation to the school's environment, we consider that schools are shaped by the different surroundings, cultures and processes created by their communities and contexts. Particularly, in the most disadvantaged environments, learning processes might get more chaotic and disruptive and children might be more prone to fail than in more stable ones (Grosskopf-Hayes et al., 2001).

Empirically, to take these variables into consideration we follow the same procedure described in section 3.1 but introducing the new variables in level 1. We apply a two-step procedure to identify each of the two effects. First, we compute one additional metafrontier (between the local and the global frontier) which includes school-level resources as a new input (level 2). In this way, we decompose the school efficiency (SCE) into a **resource endowment effect** (REE) and the new residual school efficiency effect (SCE2). We illustrate these concepts in Fig. 5. The REE is represented by the ratio  $REE_A = Oy'''_A / Oy'_A$  and reflects the gap between the availability of resources in school 1 compared with those schools with the highest resource endowment. In the case of Uruguay, this component is totally out of the school's control since resource's allocation in public schools is conducted by the national educational authorities. Then, the new decomposition of the overall pupil efficiency is  $OE = STE \times REE \times SCE2$ .

Finally, to isolate the **environment endowment effect** (EEE) from the school effect (SCE2), we estimate another metafrontier but now including the environment characteristics attributable to the context in which the school operates (level 3). We illustrate this idea in Fig. 6. The EEE component is now computed by the ratio  $EEE_A = Oy''''_A / Oy'''_A$ , and indicates the technological gap produced by the different environmental conditions between school 1 and others operating in the most favorable environments. As with the REE, this component is out of the school's control since they cannot move to a different district or influence the environment where they operate (at least in the short term).

The residual component between the SCE2 and the EEE, corresponds to the **net school effect** (SCE3) and represents the actual magnitude of the school managerial inefficiency. As a result, based on the sequential consideration of individual and school-level inputs, we now expand the decomposition of the overall efficiency into four different effects:  $OE =$

<sup>8</sup> Using a nonparametric frontier approach, Lagravinese et al. (2020) have recently highlighted the crucial effect of the cultural and socioeconomic individual conditions on education attainments *vis-à-vis* the characteristics of the education systems themselves.

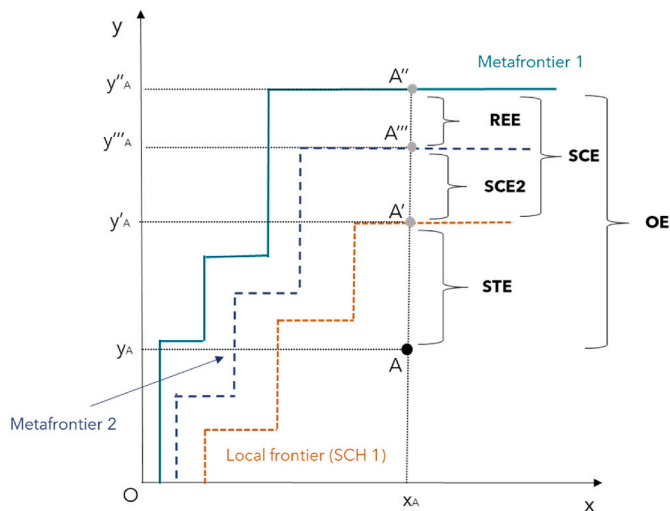


**Fig. 5.** Multi-level metafrontier illustration (level 2: decomposition of student, resource endowment and school effects). **Note:** Metafrontier 1 assumes that school 1 has the optimal endowment of resources to help its students to obtain the maximum educational output. Metafrontier 2 considers the specific level of resources in the school.

STE  $\times$  REE  $\times$  EEE  $\times$  SCE3.

On applying a multilevel approach, we recognize that resource and environment endowments are beyond the school's control (uncontrollable factors) and that schools do not usually operate with the most favorable conditions. We assume that omitting these influences, could lead to overestimate the actual school managerial capacity.<sup>9</sup>

#### 4. Data and variables

This study uses data on primary school pupils from a new dataset called ARISTAS compiled in 2017 by the National Institute for Educational Evaluation (INEEd). The dataset contains the test scores in the fields of reading and mathematics for a representative sample of students. One of the advantages of this national assessment is that the test scores reflect whether students have learnt what they were expected to, according to the goals set by the official curricula not by international standards. ARISTAS also collects data on the socioeconomic and cultural characteristics of the students and their families, the children's socio-emotional skills and a range of school's factors (public or private ownership, resources, relationship between the teaching staff, the families and the community). This information stems from different questionnaires answered by pupils, parents, teachers and school principals.<sup>10</sup>

Our analysis focuses on children in their 6th grade (at the age of 11). We have considered that these older pupils are probably more responsible to decide their level of effort and engagement with the learning process than those in their 3er grade, also surveyed by ARISTAS. To

<sup>9</sup> An alternative approach to account for these non-controllable factors is the conditional frontier model proposed by [Daraio and Simar \(2005\)](#), which also enables to control for the heterogeneous environment where schools operate ([Cordero et al., 2018](#)). The main difference is that the multilevel metafrontier approach allows grouping several factors in two relevant levels (schools' resources and environment). This leads to dimension the role of each level in a simpler and less computational time-demanding procedure than the conditional approach.

<sup>10</sup> Information refers to 247 private and public schools (13.4% and 86.6% of the total, respectively) and 7000 students (the precise number varies around that figure, according to the range of pupils who effectively answered the different questionnaires).

perform fair comparisons, we do not include pupils who have special needs or face particular physical or mental difficulties.

We take students attending public urban schools. The focus on the public system allows us to deal with the challenges and shortcomings faced by the most important education provider in the country. With this choice, we also avoid potential selection biases related to the different rules under which private and public schools operate and the self-selection of pupils from more favorable socioeconomic backgrounds into private schools ([Cordero et al., 2015](#); [Mergoni and De Witte, 2021](#)). Rural schools are excluded because they have a very reduced scale and considerably different teaching and learning practices respect to the urban regions.

Our final sample comprises 4120 students and 144 schools from Montevideo (the capital city) and urban areas of the rest of the country. To ensure within school variability, we constrain our assessment to those with at least 15 students. These schools share common academic directives and organizational rules, so they comprise a suitable set of units to carry out a comparative efficiency assessment.<sup>11</sup> [Table 1](#) summarizes the number of schools and students and their distribution by school-type.

Following the efficiency literature, educational outcomes (the outputs) are represented by pupils' test scores in reading and mathematics ([De Witte and López-Torres, 2017](#)). In ARISTAS, the test records are standardized to a mean of 300 and a standard deviation of 50 points for each of the two competencies.

For the student level input, we use a measure of the individual socioeconomic and cultural characteristics, regarded as one of the most important predictors of the educational achievement ([Coleman et al., 1966](#); [Hanushek and Luque, 2003](#)). This indicator is provided by ARISTAS and combines two sets of information: one is based on the household composition, home possessions, parental education and the number of household income recipients. The other refers to the household's cultural characteristics such as reading habits, use of home spaces with educational objectives and parental involvement in children's education ([INEEd, 2018a](#)). As the original index presented negative values, we have rescaled it so that the minimum value is equal to one.

Additionally, we consider two school level inputs. The first one is the "school resource endowment" which includes two components. One, describes the school's facilities and infrastructure conditions. It is based on the school principal's perceptions about the state of the following items: gymnasium, library, laboratory equipment, sewage, bathrooms, window glasses, electrical connections, tubing, walls, ceilings, curtain walls. The item values were inverted to express an increasing availability. The other component is the effect of peers. This is captured by the average socioeconomic and cultural background of pupils enrolled at the same school (an index computed by ARISTAS). We assume that a student's performance is affected by her classmates and this also conditions the scope of the educational achievement the school can attain ([Coleman et al., 1966](#); [Hanushek, 2003](#)).<sup>12</sup> As in the individual case, we rescaled the variable to use it in the model.

The second school level input refers to the environmental dimension. Children learn and internalize the social norms, expectations and roles of their community ([Sampson, 1997](#)). Particularly, a violent environment negatively affects academic performance and individual behaviors ([Graham et al., 2006](#); [Jarillo et al., 2016](#)). In these cases, schools should devote more teaching time to make up for the emotional contention or to cope with discipline problems, so they are more prone to be regarded as "inefficient" ([Conroy and Arguea, 2008](#)). Our "environmental endowment" input is composed by two indexes. The first one reflects the range

<sup>11</sup> In terms of production economics, this homogeneity means that all schools share the same technology. Thus, we can estimate a unique common productive frontier for all of them.

<sup>12</sup> See [De Witte and López-Torres \(2017\)](#) for a review of studies that used school facilities and peer effects as school level inputs.

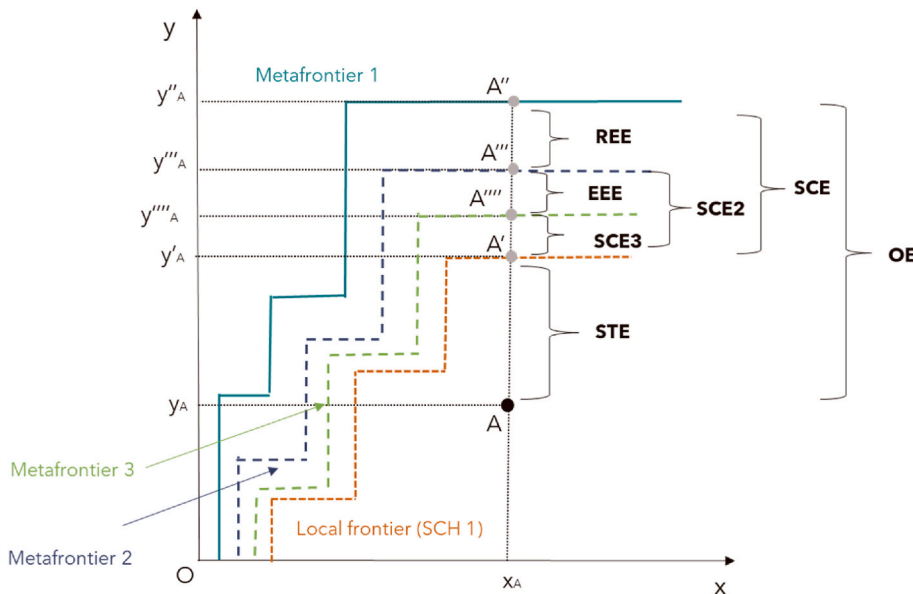


**Fig. 6.** Multi-level metafrontier illustration (level 3: decomposition of student, resource endowment, environment endowment and school effects). **Note:** Metafrontier 1 assumes that school 1 has the optimal endowment of resources to help its students to obtain the maximum educational output. Metafrontier 2 considers the specific level of resources in each school but assumes all schools operate within the optimal environment. Metafrontier 3 considers the specific environmental conditions school 1.

**Table 1**  
6th grade students by school-type in the sample.

| School-type | Number of Students | %    | Number of Schools | %    |
|-------------|--------------------|------|-------------------|------|
| Learning    | 816                | 19.8 | 27                | 18.8 |
| Practice    | 798                | 19.4 | 24                | 16.7 |
| Full Time   | 1359               | 33.0 | 54                | 37.5 |
| Regular     | 1147               | 27.8 | 39                | 27.1 |
| Total       | 4120               | 100  | 144               | 100  |

Note: Extended Time schools, though urban, are not included because their representation was just marginal in the original ARISTAS database.  
Source: own computation based on ARISTAS database.

of crime and violence in the school neighborhood. We estimate it using a Principal Components Analysis (PCA) on a set of school principal perceptions about the probability of violent episodes in the school's surroundings collected in ARISTAS (e.g., public disorders, violent arguments among neighbors or armed robberies).<sup>13</sup> The other index describes the presence of classroom difficulties in interactions, pupil's interest in learning and family's commitment with the education process. We derived it from a PCA on 11 variables provided by the database that represent the school principal opinion about the mentioned attitudes. We rescaled this measure on learning motivations so that more inputs would lead to maintain or increase outputs (monotonicity assumption).

Table 2 contains the descriptive statistics for the selected variables. All inputs fulfil the isotonicity assumption, typical of an efficiency analysis, i.e., increasing inputs results in equal or higher outputs (Coelli et al., 2005). Table A-1 in the Appendix shows the positive and statistically significant correlation between both types of variable.

In Table 3, we present the output and input variables at each school-type. *Learning* and *Full-Time* schools present lower test performances than *Regular* and *Practice* schools (see Section 2). However, it is worth noting that top and bottom achievers are found across all school-types (INEEd, 2018a). Regarding inputs, the peer effect naturally improves as schools are situated in better socioeconomic contexts. However, educational resources seem to follow a similar trend though there seems

**Table 2**  
Descriptive statistics for output and input variables.

|                                   | Obs. | Mean  | Std. Dev. | Min   | Max   |
|-----------------------------------|------|-------|-----------|-------|-------|
| <b>Outputs</b>                    |      |       |           |       |       |
| Mathematics scores                | 4120 | 296.5 | 47.4      | 138.8 | 553.2 |
| Reading scores                    | 4120 | 299.0 | 48.0      | 121.7 | 468.6 |
| <b>Inputs</b>                     |      |       |           |       |       |
| <i>Student level:</i>             |      |       |           |       |       |
| Socioeconomic and cultural status | 4120 | 2.8   | 0.8       | 1.0   | 5.3   |
| <i>School level:</i>              |      |       |           |       |       |
| <i>Resource endowmen</i>          |      |       |           |       |       |
| Educational resources             | 4120 | 6.3   | 2.3       | 1.0   | 10.0  |
| Peer effects                      | 4120 | 1.9   | 0.4       | 1.0   | 3.3   |
| <i>Environmental endowment</i>    |      |       |           |       |       |
| Learning motivations              | 4120 | 5.9   | 2.0       | 1.0   | 10.5  |
| Crime and violence                | 4120 | 4.4   | 1.9       | 1.0   | 7.5   |

Source: own computation based on ARISTAS database.

to be a minimum level shared by all schools (the mean value of the index across school-types is above 5 in a scale that reaches up to 10). Finally, the two inputs on environment endowment exhibit the worst condition at *Learning* and *Full-Time* schools.

This initial distribution of outputs and inputs confirm a long-lasting bias in the Uruguayan schooling system: those students and schools in the lowest end of the socioeconomic distribution are also the most disadvantaged in terms of learning conditions. In the following section, we explore whether the performance of students and schools represent the maximum they are able to achieve from the current resource allocation.

## 5. Results

### 5.1. Student and school effect decomposition

Table 4 reports our estimates for the efficiency scores and their decomposition (Equation (3)). Scores higher than unity indicate the level of inefficiency. The overall score is 1.192 meaning that the mathematics and reading achievement may improve 19.2%, on average, if all pupils perform as efficiently as their best practice peers do. If we exclude super-efficient pupils (whose efficiency ratios are below 1), the average score (inefficiency) raises up to 1.218. Besides, 25% of the

<sup>13</sup> ARISTAS provides a composite index measuring violence and crime. However, it was built to capture increasing violence rates. For that reason, we preferred to create a particular indicator in line with the isotonicity condition.

**Table 3**

Descriptive statistics for output and input variables by school type.

| School type                       | <i>Learning</i> |           | <i>Practice</i> |           | <i>Full Time</i> |           | <i>Regular</i> |           |
|-----------------------------------|-----------------|-----------|-----------------|-----------|------------------|-----------|----------------|-----------|
|                                   | Mean            | Std. Dev. | Mean            | Std. Dev. | Mean             | Std. Dev. | Mean           | Std. Dev. |
| <b>Outputs</b>                    |                 |           |                 |           |                  |           |                |           |
| Mathematics scores                | 282             | 42.7      | 309             | 48.4      | 293              | 48.1      | 303            | 45.9      |
| Reading scores                    | 285             | 46.4      | 311             | 47.3      | 294              | 47.2      | 306            | 47.4      |
| <b>Inputs</b>                     |                 |           |                 |           |                  |           |                |           |
| <i>Student level:</i>             |                 |           |                 |           |                  |           |                |           |
| Socioeconomic and cultural status | 2.4             | 0.6       | 3.1             | 0.8       | 2.60             | 0.7       | 2.94           | 0.7       |
| <i>School level:</i>              |                 |           |                 |           |                  |           |                |           |
| <i>Resource Endowment</i>         |                 |           |                 |           |                  |           |                |           |
| Educational resources             | 5.5             | 2.1       | 6.5             | 2.4       | 7.3              | 1.8       | 5.7            | 2.5       |
| Peer effects                      | 1.5             | 0.2       | 2.3             | 0.4       | 1.7              | 0.4       | 2.0            | 0.4       |
| <i>Environmental Endowment</i>    |                 |           |                 |           |                  |           |                |           |
| Learning motivations              | 5.3             | 1.9       | 7.3             | 1.2       | 5.4              | 1.8       | 6.0            | 2.1       |
| Crime and violence                | 2.9             | 1.7       | 6.0             | 1.4       | 4.0              | 1.7       | 4.9            | 1.6       |

Source: own computation based on ARISTAS database.

pupils who attain the best ratios (1st quartile) are almost 10% short of the potential score they might have obtained. Instead, at the other tail of the distribution, students in the 3rd quartile are almost three times shorter than their peers in the 1st quartile.

The main root of education underachievement corresponds to the student component (STE), which is on average 12% while the mean school inefficiency (SCE) is 6,4%. Therefore, the extent of the output the students do not obtain due to their own inefficiency is notably higher than that attached to the school they attend. This finding is consistent with previous research works (Cordero et al., 2016; Thieme et al., 2013; Silva-Portela and Thanassoulis, 2002).

In level 2, we decompose the school effect (SCE) into a resource endowment effect (REE) and a residual school effect (SCE2). Now, this later drops to 1.037 on average. Moreover, the REE represents 42% of the original average school effect (1.027 vs 1.064). This means that almost half of the initial pupil underachievement attributed to school inefficiencies is due to restrictions stemming from the availability of school facilities and the students' characteristics.

Finally, in the third step (level 3), we decompose SCE2 into the environmental endowment effect (EEE) and a net school efficiency (SCE3). Data in Table 4 show that EEE averages 1.011, which only represents 17% of the original school effect (1.064). In the end, the share of the overall efficiency that really depends on the school managerial competences (SCE3) has diminished 60% (1.026 vs. 1.064). Consequently, our results show that net school inefficiencies are hardly relevant to guide pupils' performance improvement. Had schools behaved more efficiently, they could only have contributed to make students gain an average of 2.6% additional test scores (compared with the 12% under the student's responsibility).

Fig. 7 pictures the kernel density estimation corresponding to the three school level effects. The components SCE, SCE2 and SCE3 respectively shift to the left, so their mean values gradually tend to

accumulate closer to 1 (less inefficiency). This movement proves that our empirical approach, effectively, allows improving the diagnosis about the actual scope of potential improvement in the education system which hinges upon schools. Once we include information about resources and environment endowment, the concerns about the role of school inefficiencies in the public primary system seem rather overestimated.

Our findings also show the real importance of an efficiency improvement in relation to the current academic performance. ARISTAS classifies student's proficiency at mathematics and reading into levels. As a rule, levels 1 to 3 comprise basic accomplishments while levels 4 to 6 (5 in the case of mathematics) describe moderate to full expertise in the corresponding area of knowledge (INEEd, 2018a).

In Table 5, we present the actual distribution of students over the proficiency standards and the potential change in the distribution whenever total inefficiencies are corrected (OE component). The results point out to the improvement that, as discussed, would mainly depend on pupils' efforts. Presently, 74% of students are situated in the first three levels at mathematics (the lowest). The figure would fall to 16% whenever total inefficiencies are removed (at the student and school level). In the case of reading, that originally concentrates 46% of students in the three basic levels, the elimination of overall inefficiencies would lead to gather only 3% of students in the basic proficiency levels.

## 5.2. Heterogeneous efficiency effects

In this section, we examine the heterogeneity of the average efficiency effects. This perspective aims to uncover whether the school contribution to enhance the overall educational attainment depends on student's characteristics or (and) varies across school-types. The approach is rooted on the idea of Silva-Portela and Thanassoulis (2002), who discuss whether schools are "differentially effective" with certain

**Table 4**

Decomposition of overall efficiency effect in three levels.

| Efficiency decomposition        | Mean  | Std. Dev. | Min   | Max   | 1st quartile | Median | 3rd quartile | Mean (inefficient) |
|---------------------------------|-------|-----------|-------|-------|--------------|--------|--------------|--------------------|
| Overall efficiency (OE)         | 1.192 | 0.166     | 0.708 | 2.023 | 1.09         | 1.18   | 1.30         | 1.218              |
| Student efficiency (STE)        | 1.120 | 0.142     | 0.927 | 1.894 | 1.00         | 1.09   | 1.20         | 1.120              |
| <i>Level 1</i>                  |       |           |       |       |              |        |              |                    |
| School efficiency (SCE)         | 1.064 | 0.193     | 0.532 | 1.867 | 0.95         | 1.07   | 1.20         | 1.087              |
| <i>Level 2</i>                  |       |           |       |       |              |        |              |                    |
| Resource endowment effect (REE) | 1.027 | 0.033     | 0.867 | 1.421 | 1.01         | 1.02   | 1.03         | 1.028              |
| School efficiency (SCE2)        | 1.037 | 0.187     | 0.529 | 1.802 | 0.92         | 1.04   | 1.17         | 1.058              |
| <i>Level 3</i>                  |       |           |       |       |              |        |              |                    |
| Env. endowment effect (EEE)     | 1.011 | 0.042     | 0.862 | 1.378 | 0.99         | 1.00   | 1.01         | 1.012              |
| Resource endowment effect (REE) | 1.027 | 0.033     | 0.867 | 1.421 | 1.01         | 1.02   | 1.03         | 1.028              |
| Net school efficiency (SCE3)    | 1.026 | 0.184     | 0.531 | 1.866 | 0.92         | 1.03   | 1.16         | 1.045              |

Source: own computation based on ARISTAS database.

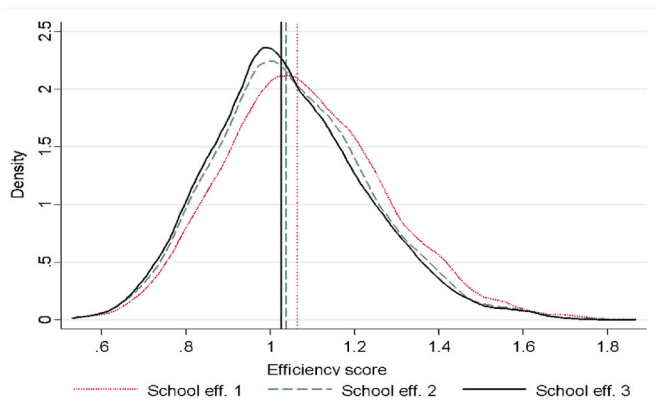


Fig. 7. Kernel density estimation of school efficiency components. **Note:** vertical lines indicate the mean value of each component.

students.

### 5.2.1. Effects by student characteristics

Fig. 8 depicts the distribution of the student and the net school efficiency effects (STE and SCE3, respectively) according to the socioeconomic background of pupils classified into quartiles. Interestingly, we observe slight divergences in the range of the potential improvement which depends on the student own behavior (left panel). The median and mean efficiency value and the proportion of super-efficient students are similarly distributed across all quartiles. Note that these results are in contrast with the positive correlation between test scores and pupils' socioeconomic status (Figure A-1). That is, while test scores are highly dependent on individual socioeconomic differences, the range of potential improvement in education outcomes (i.e., under-attainment due to inefficiencies) does not seem to be more problematic for more disadvantaged pupils.

Additionally, the median SCE3 (right panel) is very close to 1 with slight differences for children at the poorest and richest condition. The boxes also show a wide range of values under 1, implying that for several students, the school shows a better output level than the rest of its reference set (i.e., the evaluated school is super-efficient). Apart from this variability, schools do not seem to be differentially effective with certain groups of favored or disadvantaged students in terms of their socioeconomic situation.

In Fig. 9, we show the density functions of the efficiency components REE (left panel) and EEE (right panel), according to the socioeconomic condition of students. In both cases, the lowest concentration of values

around 1 (full efficiency) correspond to students of the 1st quartile. Accordingly, in the public system the school inefficiencies related to both kinds of inputs have a stronger influence on students from the most unfavorable contexts.

Some new patterns arise when we look at the efficiency components for students according to their reading test scores.<sup>14</sup> Again, STE is rather uniform for the different quartiles (Fig. 10-left panel). Even the range of super-efficient pupils is quite homogenous. However, we do find variations for SCE3 (Fig. 10-right panel). There seems to be a negative association between these net school inefficiencies and the performance of students, so that schools are less efficient with their worst performing pupils. For them, the median potential educational improvement which depends on the school responsibility reaches 20%. Conversely, schools seem to be increasingly supportive as pupils attain higher test marks: the median school effect for students in the 2nd quartile is 1.07 and it is equal to 1 in the 3rd and 4th quartile.

A look at the density functions of REE and EEE by quartiles of reading scores also shows that the largest potential efficiency gains correspond to those pupils with the lowest achievement (Figures A-2 in the Appendix). As a result, this group of students bears both the highest net and input-related school inefficiencies.

The efficiency decomposition also provides insights on the differing performance by gender.<sup>15</sup> The average overall inefficiencies are slightly higher for boys than for girls (1.20 and 1.18, respectively), but the STE component averages around 12% for both groups (indeed, the differences are not statistically significant).<sup>16</sup> In turn, the net school inefficiency is more relevant for boys: for them, the mean value of that component is 4.1% and it is just 1.3% for girls.

This perspective of analysis underlines that school inefficiencies do not equally affect the performance of all the pupils. In other words, schools are differentially effective with students because they face a significant limitation to improve the achievement of those children who have low academic skills. For them, school efficiency constraints are visible both in terms of resources and environment conditions. And, as a matter of fact, this limitation does not seem to be directly connected to the socioeconomic situation of pupils. Complementary, the effect of these school-level inefficiencies seems to be more important for the attainment of boys in comparison to girls.

### 5.2.2. School-type patterns

Potential efficiency gains might be more relevant for certain school-types. To dig into this issue, we start from the pupil level data and go into the estimated decompositions of the overall efficiency scores, but this time we consider the school-type the students attend.

As noted in Section 2, *Learning* schools operate in the most disadvantaged contexts. *Full-Time* and *Regular* schools are more diverse while *Practice* schools are generally situated in favored settings. Table 6 reports our efficiency estimates across these school-types. Complementarily, Table 7 includes the results of the Mann-Whitney tests to explore whether the differences among school-types are statistically significant.

Our results show that the highest mean (and median) overall inefficiency corresponds to *Learning* schools. Inefficiencies gradually decrease for *Full-Time*, *Regular* and finally, *Practice* schools. *Full-Time* schools also exhibit the highest heterogeneity. These differences are statistically significant at a 1% level. The distribution of the STE is roughly the same across school-types: therefore, no matter the type of institution, the maximum students' effort would lead them to improve their

Table 5

Observed and estimated improvement in ARISTAS' test performance by proficiency level.

| ARISTAS' proficiency level | Actual distribution (%) | Distribution (%) after gains (overall eff.) |
|----------------------------|-------------------------|---------------------------------------------|
| Mathematics                |                         |                                             |
| 1                          | 5.8                     | 0.1                                         |
| 2                          | 30.3                    | 1.8                                         |
| 3                          | 37.7                    | 14.3                                        |
| 4                          | 21.5                    | 68.5                                        |
| 5                          | 4.8                     | 15.3                                        |
| Total                      | 100                     | 100                                         |
| Reading                    |                         |                                             |
| 1                          | 4.0                     | 0.2                                         |
| 2                          | 15.6                    | 0.3                                         |
| 3                          | 26.5                    | 2.6                                         |
| 4                          | 27.1                    | 13                                          |
| 5                          | 16.7                    | 36.9                                        |
| 6                          | 10.1                    | 47                                          |
| Total                      | 100                     | 100                                         |

Source: own computation based on ARISTAS data and efficiency estimates.

<sup>14</sup> For visual purposes the example refers to reading scores as they present a higher variability. However, the analysis applied to math scores replicates the main conclusions.

<sup>15</sup> Detailed results are in Table A-2 in the Appendix.

<sup>16</sup> Differences between values for boys and girls are assessed according to the non-parametric Mann-Whitney test. The null hypothesis of the test is that the medians of the variables are the same for each of the groups under analysis.

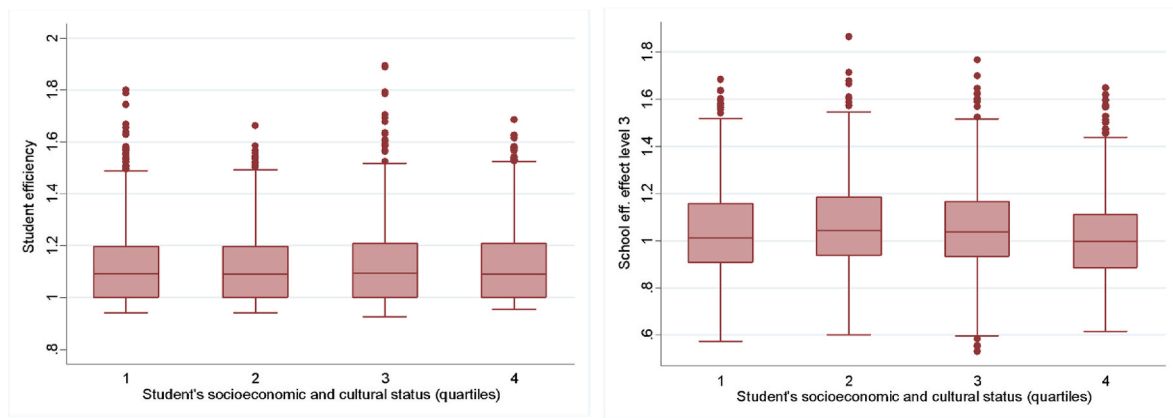


Fig. 8. Student efficiency effect (STE)-left panel- and net school efficiency effect (SCE3)-right panel-by quartiles of pupils' socioeconomic condition.

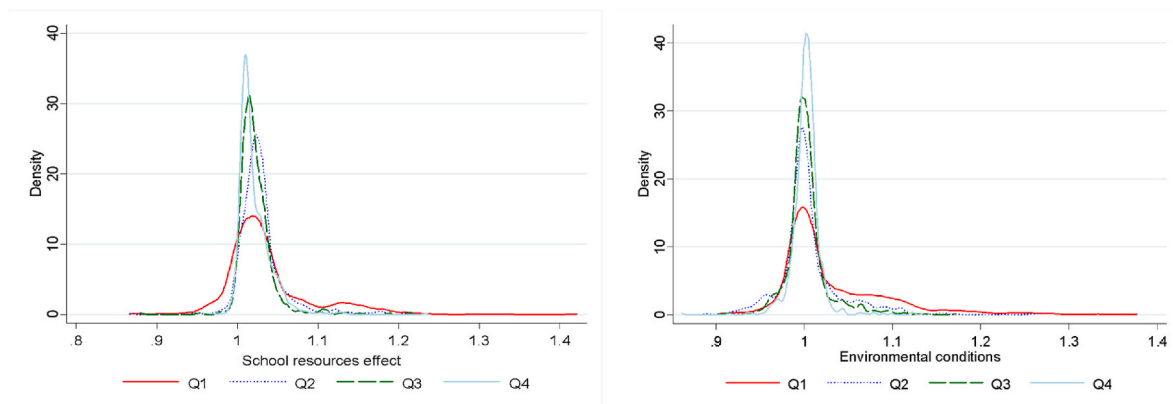


Fig. 9. Kernel density distribution of REE (left panel) and EEE (right panel) by quartiles of pupils' socioeconomic condition. **Note:** The efficiency score attached to the EEE emerges after we also control for the resource endowment.

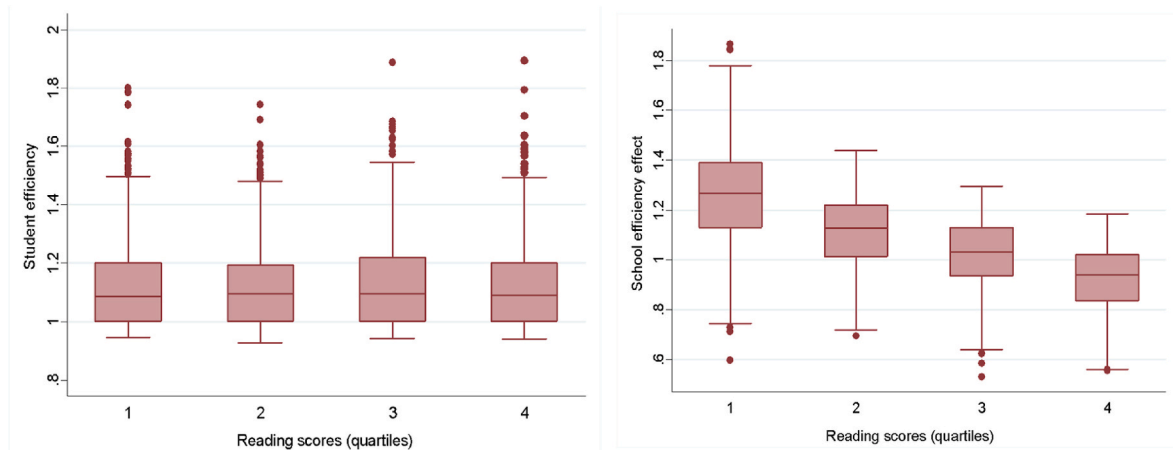


Fig. 10. Student effect (STE)-left panel- and net school effect (SCE3)-right panel-by quartiles of pupils' reading scores.

outcomes by 12%. Indeed, differences across school-types in this component are not statistically significant (Table 7).

Regarding school-effects, the initial SCE sharply decreases for all school-types after discounting the resource and environment endowment, particularly at *Full-Time* schools (a 53% drop). Differences are only statistically significant between *Full-Time* and *Practice* and *Full-Time* and *Regular* schools. The median school managerial inefficiency at *Full-Time* schools is two times higher than that at *Practice* schools and almost 1.2 times higher compared to *Regular* schools (Table 7).

Besides, the REE represents, on average, almost half of the original SCE effect across all school-types. The result is lower just for *Practice* schools. Differences are significant, except for the comparison between *Full-Time* and *Regular* schools, even though their initial resource endowment clearly favors the former (Table 3). Alternatively, the EEE shows a comparatively modest influence.

A last piece of information comes from exploring the individual performance of each of our 144 schools. We aggregate each of the pupils' efficiency scores to their individual school and then organize these

**Table 6**

Decomposition of overall efficiency effect in three levels by school type.

| School type                     | Learning |        |       | Practice |        |       | Full Time |        |       | Regular |        |       |
|---------------------------------|----------|--------|-------|----------|--------|-------|-----------|--------|-------|---------|--------|-------|
| Efficiency decomposition        | Mean     | Median | SD.   | Mean     | Median | SD.   | Mean      | Median | SD.   | Mean    | Median | SD    |
| Overall efficiency (OE)         | 1.23     | 1.22   | 0.169 | 1.162    | 1.156  | 0.149 | 1.202     | 1.187  | 0.173 | 1.175   | 1.170  | 0.158 |
| Student efficiency (STE)        | 1.121    | 1.10   | 0.136 | 1.121    | 1.094  | 0.134 | 1.121     | 1.090  | 0.148 | 1.121   | 1.090  | 0.144 |
| Level 1                         |          |        |       |          |        |       |           |        |       |         |        |       |
| School efficiency (SCE1)        | 1.103    | 1.108  | 0.197 | 1.036    | 1.039  | 0.181 | 1.072     | 1.069  | 0.198 | 1.048   | 1.061  | 0.187 |
| Level 2                         |          |        |       |          |        |       |           |        |       |         |        |       |
| Resource endowment effect (REE) | 1.049    | 1.035  | 0.050 | 1.016    | 1.012  | 0.018 | 1.021     | 1.022  | 0.022 | 1.024   | 1.019  | 0.030 |
| School efficiency (SCE2)        | 1.051    | 1.049  | 0.189 | 1.020    | 1.021  | 0.177 | 1.050     | 1.050  | 0.194 | 1.023   | 1.032  | 0.181 |
| Level 3                         |          |        |       |          |        |       |           |        |       |         |        |       |
| Env. endowment effect (EEE)     | 1.019    | 1.001  | 0.052 | 1.006    | 1.003  | 0.017 | 1.012     | 1.000  | 0.053 | 1.005   | 1.000  | 0.029 |
| Resource endowment effect (REE) | 1.049    | 1.035  | 0.050 | 1.016    | 1.012  | 0.018 | 1.021     | 1.022  | 0.022 | 1.024   | 1.019  | 0.030 |
| Net school efficiency (SCE3)    | 1.031    | 1.019  | 0.184 | 1.014    | 1.016  | 0.175 | 1.038     | 1.032  | 0.193 | 1.018   | 1.026  | 0.179 |
| Number of students              | 816      |        |       | 798      |        |       | 1359      |        |       | 1147    |        |       |

Source: own computation based on ARISTAS database.

**Table 7**

Mann-Whitney distribution tests by school type.

| Variable                 | Mann-Whitney test | Learning vs Practice school. | Learning vs Full Time school | Learning vs Regular school | Practice vs Full time school | Practice vs Regular school | Full time vs Regular school |
|--------------------------|-------------------|------------------------------|------------------------------|----------------------------|------------------------------|----------------------------|-----------------------------|
| Overall efficiency (OE)  | z-statistic       | 8.133                        | 4.123                        | 7.22                       | -4.886                       | -1.646                     | 3.581                       |
|                          | p-value           | 0.0000                       | 0.0000                       | 0.0000                     | 0.0000                       | 0.0997                     | 0.0003                      |
| Student efficiency (STE) | z-statistic       | -0.892                       | -0.325                       | -0.492                     | 0.575                        | 0.384                      | -0.184                      |
|                          | p-value           | 0.3727                       | 0.7453                       | 0.6229                     | 0.5652                       | 0.701                      | 0.8538                      |
| School effect (level 1)  | z-statistic       | 6.899                        | 3.725                        | 6.002                      | -3.983                       | -1.556                     | 2.698                       |
|                          | p-value           | 0.0000                       | 0.0002                       | 0.0000                     | 0.0001                       | 0.1198                     | 0.007                       |
| School resources effect  | z-statistic       | 20.174                       | 15.396                       | 14.674                     | -9.781                       | -10.438                    | 0.002                       |
|                          | p-value           | 0.0000                       | 0.0000                       | 0.0000                     | 0.0000                       | 0.0000                     | 0.9982                      |
| Cont. and educ. effect   | z-statistic       | 1.217                        | 4.483                        | 4.811                      | 3.687                        | 4.486                      | -0.09                       |
|                          | p-value           | 0.2235                       | 0.0000                       | 0.0000                     | 0.0000                       | 0.0000                     | 0.928                       |
| School efficiency (SCE3) | z-statistic       | 1.538                        | -0.879                       | 1.254                      | -2.578                       | -0.442                     | 2.372                       |
|                          | p-value           | 0.1241                       | 0.3794                       | 0.2098                     | 0.0099                       | 0.6583                     | 0.0177                      |

Note: tests on the statistical significance of the equality of medians of each distribution.

Source: own computation based on ARISTAS database.

school-level data by school-type. This alternative presentation allows us to learn about the variation in the effective capacity of the different types of schools to support the performance of different groups of pupils. We focus the analysis on the residual school effect (SCE3).

According to Fig. 11, there is a negative correlation between the net school managerial efficiency and the socioeconomic status of children, except for *Learning* schools. Hence, most school-types seem to perform better with their richest students. Indeed, the potential improvement in the education attainments of these children merely rely on their own individual effort. Interestingly, *Learning* schools exhibit the opposite pattern. While this result might underline that their focus on compensatory education programs and teaching strategies have successfully impacted on the most disadvantaged pupils, these schools are still not effective enough to support the rest of their students.

Alternatively, the trends in Fig. 12 show that the net school effect decreases as the scoring tests raise. This means that schools seem to be more effective with their more dedicated students. This result reinforces the general finding of the previous section. In this sense, no school-type seems to clearly compensate the underperformance of their less able students. The graphs also suggest that this general trend is particularly strong at *Full-Time* schools and not so remarkable at *Learning* schools.

On the whole, this last set of results show that all school-types are differentially effective to affect the performance of their students. Both, because of the characteristics of each school-type and the dynamics within each institution, the schools should improve their performance to effectively contribute with students from the lowest socioeconomic contexts and academic records.

## 6. Concluding remarks

Frontier techniques, aiming to identify whether resources are used at

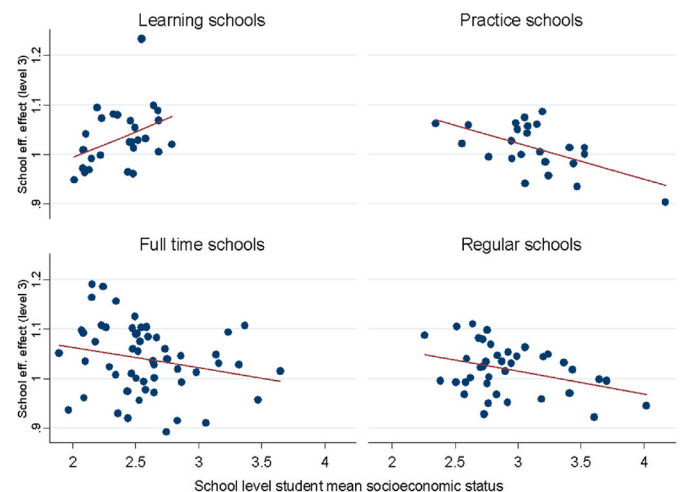


**Fig. 11.** Net school efficiency effect (SCE3) by pupils' average socioeconomic status across school types. **Note:**  $R^2$  of the fitted linear regression is 0.15 at *Learning* schools, 0.32 at *Practice* schools, 0.05 at *Full-Time* schools. and 0.13 at *Regular* schools.

their full potential, have increasingly been recognized as a relevant tool to design school funding policies. However, the metafrontier and multilevel approaches, that is important to untangle pupils' from schools' improvement efforts, have been rarely applied in the non-parametric literature. In this paper, we resort to this methodology to discuss the role that Uruguayan education authorities have granted to efficiency as a central strategy to improve the stagnant academic

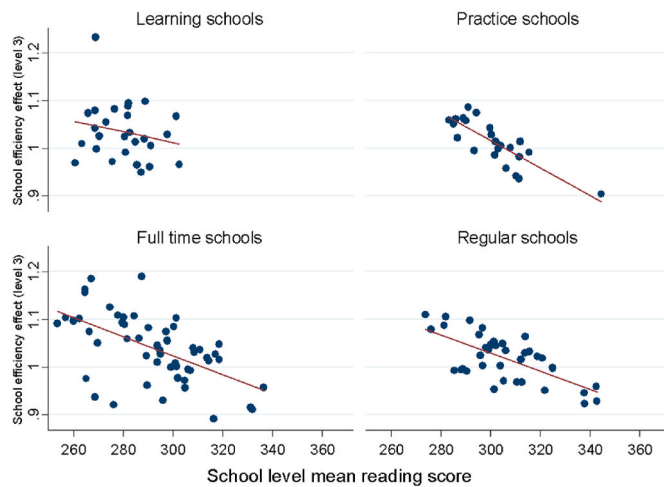


**Fig. 12.** Net school efficiency effect (SCE3) by pupils' average reading test score across school types. **Note:**  $R^2$  of the fitted linear regression is 0.04 at *Learning schools*, 0.08 at *Practice schools*, 0.33 at *Full Time schools* and 0.46 at *Regular schools*.

performance of students.

Our analysis is focused on the performance of public primary school students. This educational system has been characterized by a structural lack of adequate funding and high inequities in terms of academic results due to a remarkable socio-spatial school segregation. Based on a recently released dataset, we apply a non-parametric metafrontier approach to identify student and school level inefficiencies. Besides, we consider a multi-level decomposition to acknowledge that school efficiency might be affected by differences in the input allocation and the context conditions in which it operates.

We show that the range of potential improvement in educational attainment is relevant (on average, the overall efficiency gains reach 19.2%). However, against what the authorities seem to interpret, this inefficiency is mainly related to students not to schools' under-performance. Pupils across public schools are not behaving according to their full potential, no matter their socioeconomic status or academic performance. In all cases, they could still obtain 12% higher test scores using the same resources. A similar pattern replicates across all the public school-types under study.

We also find that the school's performance could be misunderstood if its resources and environmental conditions are overlooked in the assessment. In fact, over half of the initial school inefficiencies are caused by the characteristics of the enrolled students and the resource endowment at the school level. This conclusion arises when we look at individual schools as well as when we aggregate the pupils' data into school-types. This finding suggests that public policies should be more

focused on allocating increasing resources to those schools with structural under-endowment than on removing managerial inefficiencies.

Additionally, our efficiency decomposition identifies the gaps in the performance of schools which deserve particular attention. Specifically, we find a concentration of efficiency constraints at those schools and students characterized by low academic and socioeconomic status. The evidence highlights serious efficiency restrictions for the worst achievers and the economically disadvantaged.

Finally, we show that all school-types tend to be differentially effective with certain groups of students. In general, they take the most of their highest-achievers and the most economically favored pupils. Therefore, instead of mitigating original inequities, schools are just reproducing them. In this sense, it is crucial to promote focused policies to provide managerial support to schools with a high concentration of disadvantaged students. In terms of the present debate in Uruguay, this research suggests that enhancing school management is a necessary condition, but it is not enough. Improving the quality of the public primary education system requires a genuine revision of the optimal level of the current school funding system, which should also imply additional resource allocations.

Beyond the Uruguayan case, this paper highlights the relevance and practical implications of accurately assessing the potential improvement that schooling efficiency policies may attain. In developing countries, frequently constrained by tight budgets and infrastructure and socioeconomic needs of pupils, an efficiency decomposition analysis avoids to unfairly assign a responsibility to schools for those inefficiencies they are not able to improve. However, it might help to identify relevant educational policy challenges to provide equitable learning opportunities for all students.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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## Appendix

**Table A1**  
Correlation between outputs and inputs.

| Inputs                                           | Outputs            |                |
|--------------------------------------------------|--------------------|----------------|
|                                                  | Mathematics scores | Reading scores |
| Socioeconomic and cultural status of students    | 0.392              | 0.388          |
| School facilities                                | 0.049              | 0.043          |
| Mean socioeconomic and cultural status at school | 0.324              | 0.303          |
| Crime and violence in school's surroundings      | 0.217              | 0.202          |
| Learning motivations                             | 0.126              | 0.116          |

Note: all correlations are statistically significant at 1%.

Source: own computation based on ARISTAS database

**Table A2**  
Decomposition of overall efficiency effect in levels by sex.

| Efficiency decomposition              | Boys   |           | Girls  |           |
|---------------------------------------|--------|-----------|--------|-----------|
|                                       | Mean   | Std. Dev. | Mean   | Std. Dev. |
| Overall efficiency (OE)               | 1.204* | 0.171     | 1.182* | 0.159     |
| Student efficiency (STE)              | 1.116  | 0.138     | 1.124  | 0.145     |
| Level 1                               |        |           |        |           |
| School efficiency (SCE1)              | 1.078* | 0.196     | 1.052* | 0.190     |
| Level 2                               |        |           |        |           |
| School resources effect               | 1.027  | 0.138     | 1.027  | 0.033     |
| School efficiency (SCE2)              | 1.050* | 0.189     | 1.024* | 0.184     |
| Level 3                               |        |           |        |           |
| Context and education concerns effect | 1.009* | 0.042     | 1.012* | 0.042     |
| School resources effect               | 1.027  | 0.138     | 1.027  | 0.033     |
| School efficiency (SCE3)              | 1.041* | 0.188     | 1.013* | 0.180     |
| Observations                          | 1980   |           | 2140   |           |

\* 1% statistical significance of the Mann-Whitney tests.

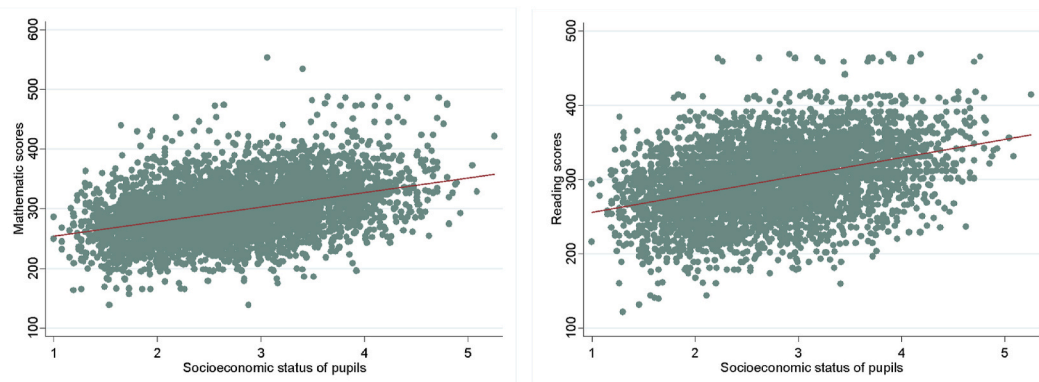


**Fig. A1.** Test performance and pupils' socioeconomic status correlation (public primary 6th grade students) **Note:** Socioeconomic status according to ARISTA's scale.

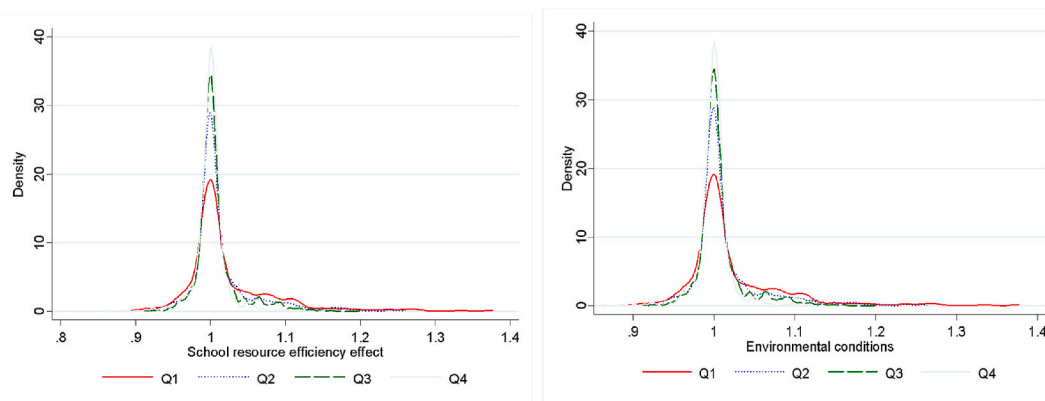


**Fig. A2.** School resources (left panel) and environmental conditions efficiency effects (right panel) by quartiles of student's reading scores.

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