# Education choices and job market characteristics 

Qianshuo Liu ${ }^{\text {a,b,c }, ~ I n e ́ s ~ M a c h o-S t a d l e r ~}{ }^{\text {a,c,* }}$<br>${ }^{\text {a }}$ Universitat Autònoma de Barcelona, Spain<br>${ }^{\mathrm{b}}$ IAE-CSIC, Spain<br>${ }^{\text {c }}$ BSE, Spain

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#### Abstract

We propose a simple three-stage model where heterogeneous schools compete via tuition fees, individuals with the ex-ante unknown ability make their education choices to (eventually) get a diploma and reveal their ability, and finally the job market determines the assignment of workers to firms and the equilibrium wages. In equilibrium, wages in the labor market and schools' fees and individuals' school choices are strongly related. We also analyze the effects of the existence of a public school or a subsidy on social welfare.


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## 1. Introduction

Education decisions determine the workers' characteristics in the labor market and the economy's output. Also, future wages influence the schools' competition and the individuals' education choices.

We propose a three-stage model. First, two schools differing in terms of how demanding they are, compete for students through their tuition fees. Then individuals with ex-ante unknown ability select which school to attend, if any. Finally, a one-to-one labor market matching between firms and workers occurs. The equilibrium wages (hence, the gains from education) are a function of the workers' and the firms' productivity.

Among our results, we highlight that subsidizing high-demanding schools is especially helpful when the workers' ability is very valuable and the dispersion of skills in the population is large. Introducing a public school can also increase welfare.

We contribute to the literature that relates education decisions and job market conditions. Flinn and Mullins (2015) study the effects of minimum wages and college costs in a search and matching model where heterogeneous individuals make ex-ante

[^0]schooling choices. MacLeod and Urquiola (2019) consider two identical schools and study the school choice when each firm recruits from only one school or schools face capacity constraints. Boleslavsky and Cotton (2015) consider one firm and two schools of a given capacity, competing through investment and grading systems. Hatfield et al. (2014) show how the labor market's design and frictions affect incentives for human capital acquisition. We complement the literature by considering heterogeneous schools competing via tuition fees and the relationship between schools' competition and job market outcomes.

## 2. Model

We consider three sets of (risk-neutral) players: individuals, schools, and firms.

There is a continuous of individuals of size one. They have the same ex-ante ability to perform a specialized task if they obtain a diploma. It is public knowledge that ex-post a proportion $\alpha$ has an ability $k>0$ in the specialized task, and the remaining individuals have ability 0 . Hence, an individual's ex-ante expected ability is $\alpha k$. Individuals can pay for education, or there is a perfect credit market.

Two schools train and provide the diploma to perform the specialized task. The schools' type is public information; they have no capacity constraints and bear no costs. An individual graduates at the high-demanding school $S_{H}$ if (and only if) he has ability $k$. Hence, the ability of $S_{H}$ 's graduates is $k_{h}=k$. An individual always graduates at the low-demanding school $S_{L}$, with
the expected ability $k_{e}=\alpha k$. We denote type $k_{\emptyset}$ an individual without a degree.

There is a continuum of firms. Firm $i$ is characterized by its productivity $\gamma_{i}$, uniformly distributed in $[0,1]$. We assume that there is a mas-point of firms with $\gamma_{i}=0$, so that the population of firms is "larger" than that of individuals. ${ }^{1}$

Each firm needs one worker. A contract between firm $i$ and worker $j$ specifies the wage $w_{i j}$. Individuals without a degree can only perform routine tasks. Their output in firm $i$ is $R_{i \emptyset}=\beta$. The outcome of a graduated worker of ability $k_{j}$, with $j=h, e$, is $R_{i j}=\beta+\gamma_{i} k_{j}{ }^{2}$

Decisions are made in three stages. First, schools $S_{H}$ and $S_{L}$ compete in fees, $F_{H}$ and $F_{L}$. Second, individuals decide their education, given $\left(F_{H}, F_{L}\right)$. We denote $Q_{H}$ (resp., $Q_{L}$ ) the amount of individuals attending school $S_{H}$ (resp., $S_{L}$ ), and $Q_{\varnothing}$ the amount not attending any school. The school selection and the education outcome are public information.

In the third stage, individuals (workers) and firms match in the labor market. We formalize it as a one-to-one assignment game with transferable utility. The equilibrium determines the matching between firms and workers and wages.

## 3. Equilibrium decisions

We solve by backward induction.

## Stage 3: Labor market

We denote workers by their productivity $k_{j}$, with $j=h, e, \emptyset$. If the distribution of the individuals in schools at $T=2$ is $\left(Q_{H}, Q_{L}, Q_{\emptyset}\right)$, with $Q_{H}+Q_{L}+Q_{\varnothing}=1$, then there are $\alpha Q_{H}$ $k_{h}$-workers, $Q_{L} k_{e}$-workers, and $Q_{\emptyset}+(1-\alpha) Q_{H} k_{\emptyset}$ workers.

The total surplus generated by the partnership $(i, j)$ coincides with the outcome $R_{i j}$. Given the complementary between firms' and workers' productivity, the equilibrium matching is positive assortative. This implies two cut-offs, $\gamma_{e}$ and $\gamma_{h}$, with $0 \leq \gamma_{e} \leq$ $\gamma_{h} \leq 1$, such that (a) firms in [ $\left.\gamma_{h}, 1\right]$ match with the $k_{h}$-workers, (b) firms in $\left[\gamma_{e}, \gamma_{h}\right]$ hire $k_{e}$-workers, (c) firms in ( $0, \gamma_{e}$ ] hire $k_{\varnothing}$-workers, and (d) firms with $\gamma=0$ are unmatched. The equilibrium cut-offs are $\gamma_{e}=Q_{\emptyset}+(1-\alpha) Q_{H}$ and $\gamma_{h}=1-\alpha Q_{H}$.

In equilibrium, workers with the same productivity get the same wage. To explain the wages, suppose that the three types of workers are in the market. First, the competition of firms with $\gamma_{i}=0$ to hire $k_{\emptyset}$-workers leads to $w_{\emptyset}=\beta$. Second, the marginal firm $\gamma_{e}$ pays a $k_{e}$-worker a salary that leaves it indifferent to hiring a $k_{\emptyset}$-worker: $w_{e}=\beta+\gamma_{e} \alpha k$. Finally, $k_{h}$-workers obtain the most firm $\gamma_{h}$ is ready to pay for them (the alternative is a $k_{e}$-worker): $w_{h}=\beta+\gamma_{h} k-\left(\gamma_{h}-\gamma_{e}\right) \alpha k$. A similar argument leads to the equilibrium wages for all possible configurations:

Lemma 1. Given $\left(\alpha Q_{H}, Q_{L}, Q_{\emptyset}+(1-\alpha) Q_{H}\right)$ :
(1) If $Q_{H}>0$ and $Q_{L}>0: w_{\emptyset}=\beta, w_{e}=\beta+\gamma_{e} \alpha k$, $w_{h}=\beta+\gamma_{h} k-\left(\gamma_{h}-\gamma_{e}\right) \alpha k$.
(2) If $Q_{H}>0$ and $Q_{L}=0: w_{\emptyset}=\beta, w_{h}=\beta+\gamma_{h} k$.
(3) If $Q_{H}=0, Q_{L}>0$, and $Q_{\emptyset}>0: w_{\emptyset}=\beta, w_{e}=\beta+\gamma_{e} \alpha k$.
(4) If $Q_{H}=0, Q_{L}=1$ or $Q_{\emptyset}=0$ : $w_{e}=\beta$.

## Stage 2: Individual's education decision

An individual's utility is the difference between his expected salary and the school's fee: $U(\emptyset)=\beta, U\left(S_{L}\right)=w_{e}-F_{L}$, and

[^1]$E U\left(S_{H}\right)=\alpha w_{h}+(1-\alpha) \beta-F_{H}$. The individual selects the school solving $\operatorname{Max}\left\{U(\emptyset), U\left(S_{L}\right), E U\left(S_{H}\right)\right\}$.

## Stage 1: Schools choose $F_{H}$ and $F_{L}$

Anticipating the effect on the individuals' choice, the revenuemaximizing schools set their fees simultaneously and non-cooperatively. Proposition 1 states, in particular, that all individuals attend school at equilibrium.

Proposition 1. In equilibrium,
(i) schools' fees: $F_{H}=\frac{2}{3} \alpha(1-\alpha) k, F_{L}=\frac{1}{3} \alpha(1-\alpha) k$,
(ii) demands for schools: $Q_{H}=\frac{2}{3}, Q_{L}=\frac{1}{3}$,
(iii) wages: $w_{\emptyset}=\beta, w_{e}=\beta+\frac{2}{3} \alpha(1-\alpha) k, w_{h}=\beta+(1-\alpha) k$.

Corollary 1. In equilibrium,
(i) schools' total revenue: $R=\frac{5}{9} \alpha(1-\alpha) k$,
(ii) individuals' total surplus: $U=\beta+\frac{1}{3} \alpha(1-\alpha) k$,
(iii) firms' total profit: $\Pi=\frac{1}{18} \alpha(8 \alpha+1) k$,
(iv) total welfare: $W=\beta+\frac{1}{18} \alpha(17-8 \alpha) k$.

Education fulfills two roles in our model. First, graduating from any school allows performing the specialized task. Second, school $S_{H}$ reveals an individual's productivity on this task. The market compensates this second role with higher expected salaries to students attending $S_{H}: \alpha w_{h}+(1-\alpha) w_{\emptyset}>w_{e}$. It also explains that $F_{H}>F_{L}$.

As expected, a higher $k$ leads to higher wages, fees, and participants' surplus. The influence of $\alpha$ is more complex. A higher $\alpha$ reflects a better pool of individuals. However, the population heterogeneity depends on $\alpha$ : it increases until $\alpha=1 / 2$ and then decreases. This explains the non-monotonicity of the equilibrium fees ( $F_{H}, F_{L}$ ) (and schools' revenue) in $\alpha$. For $\alpha=0$ and $\alpha=$ 1, schools have similar education systems; competition leads to $F_{H}=F_{L}=0$. The maximum schools' differentiation occurs at $\alpha=\frac{1}{2}$, where fees are maximum.

The individuals' surplus is increasing (resp., decreasing) for $\alpha<\frac{1}{2}$ (resp., $\alpha>\frac{1}{2}$ ). An increase in $\alpha$ makes $k_{h}$-workers more abundant and decreases $w_{h}$. In contrast, $w_{e}$ first increases and then decreases in $\alpha$. Indeed, for low $\alpha, k_{e}$-individuals are less productive but also rarer, so the firm $\gamma_{e}$ is more productive and pays them more. For large $\alpha, k_{e}$-workers are more abundant and productive, but $\gamma_{e}$ is smaller, which induces a lower wage. Still, firms' and total welfare increase with $\alpha$.

## 4. Improving the market outcome

We have assumed that there are two schools. However, the social welfare is maximum when all individuals attend school $S_{H}$. Nevertheless, if $S_{H}$ is the only school, it can use its monopoly power. Proposition 2 states its choice and total welfare under monopoly.

Proposition 2. Under monopoly:
(a) For $\alpha \geq \frac{1}{2}: F_{H}=\frac{\alpha k}{2}, Q_{H}=\frac{1}{2 \alpha}$. Total welfare is $W_{H}^{T}=\beta+\frac{3 k}{8}$.
(b) For $\alpha \leq \frac{1}{2}: F_{H}=\alpha(1-\alpha) k, Q_{H}=1$. Total welfare is $W_{H}^{T}=\beta+\frac{\alpha(2-\alpha) k^{2}}{2}$.

Corollary 2. Total welfare is higher when $S_{H}$ is the only school in the market (i.e., $W_{H}^{T} \geq W^{T}$ ) iff $\alpha \leq \frac{17-\sqrt{73}}{16} \approx 0.53$.

When $S_{H}$ is a monopoly and the proportion of high-ability students is low ( $\alpha \leq \frac{1}{2}$ ), then their identification is essential for the top firms. Thus, all the students attend $S_{H}$. This is the firstbest. A social planner also benefits from preventing the entrance of $S_{L}$ if $\alpha$ is not too high. Otherwise, a monopoly $S_{H}$ sets too high a fee; consequently, too few individuals attend, and welfare is lower.

Subsidizing the cost of attending $S_{H}$ can also improve welfare with two schools. Consider a subsidy of $\delta$ to school $S_{H}$ for each student attending. ${ }^{3}$ Proposition 3 states the equilibrium, where $\lambda$ denotes the shadow cost of the public funds.

Proposition 3. In equilibrium with two schools and a subsidy $\delta$ per student to $S_{H}$ :
(i) schools' fees: $F_{H}(\delta)=\frac{2}{3}(\alpha(1-\alpha) k-\delta), F_{L}(\delta)=\frac{1}{3}(\alpha(1-$ $\alpha) k-\delta$ ),
(ii) demands: $Q_{H}(\delta)=\frac{2}{3}+\frac{1}{3 \alpha(1-\alpha) k} \delta, Q_{L}(\delta)=\frac{1}{3}-\frac{1}{3 \alpha(1-\alpha) k} \delta$,
(iii) cost of the subsidy: $C(\delta)=\frac{2}{3} \delta+\frac{1}{3 \alpha(1-\alpha) k} \delta^{2}$,
(iv) total welfare: $W^{T}(\delta)=\beta+\frac{1}{18}(\alpha(17-8 \alpha) k+2 \delta$ $\left.-\frac{1}{\alpha(1-\alpha) k} \delta^{2}\right)-\lambda C(\delta)$.

A subsidy $\delta$ leads to a decrease of $\frac{2}{3} \delta$ in $F_{H}$ and only $\frac{1}{3} \delta$ for $F_{L}$. Hence, demand $Q_{H}$ is higher, which is the policy's objective. Of course, the planner's cost $C(\delta)$ increases with $\delta$.

When $\lambda=0$, total welfare $W^{T}(\delta)$ increases in $\delta$ until $\delta=$ $\alpha(1-\alpha) k$, where all students attend $S_{H}$. When $\lambda>0$, the optimal subsidy is an interior solution.

Corollary 3. The optimal subsidy $\delta$ when $\lambda>0$ is (i) increasing in $k$, and (ii) increasing (decreasing) in $\alpha$ for $\alpha<1 / 2$ (resp., $\alpha>1 / 2$ ).

Corollary 3 suggests that subsidizing high-demanding schools is especially helpful in economies where graduates' ability is crucial for firms (a high $k$ ) and the dispersion of individuals' skills is large.

Finally, consider that one of the two schools is public and the other private. A public school can be defined by: (a) it sets its fee to maximize total welfare; (b) it is free. Under definition (a), the social optimum $\left(Q_{H}=1, Q_{L}=0\right)$ is reached. If school $S_{H}$ is public, it sets $F_{H}=0$, and all individuals attend $S_{H}$. If $S_{L}$ is public, then the Nash equilibrium is $F_{L}=F_{H}=\alpha(1-\alpha) k$, which implies $Q_{H}=1, Q_{L}=0$.

Under definition (b), if $S_{H}$ is public and $F_{H}=0$, then the social optimum is also reached. However, if school $S_{L}$ is public and $F_{L}=0$, then $S_{H}$ sets $F_{H}=\frac{\alpha(1-\alpha) k}{2}$, leading to $Q_{H}=\frac{1}{2}$ and $Q_{L}=\frac{1}{2}$. Total welfare is $\beta+\frac{1}{8} \alpha(7-3 \alpha) k$, which is lower than in the two-private school case because the public school attracts too many students.

## 5. Final comments

Our model opens the door to many potential extensions. We find it particularly enriching to consider differences in the schools' cost structure and capacity or the abilities they teach to students. These and other extensions can shed further light on the implications of the labor market characteristics on a country's school system.

## Data availability

No data was used for the research described in the article.

## Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2023.110985.

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[^2]
[^0]:    Th We thank David Pérez-Castrillo and an anonymous reviewer for their helpful comments, and MINECO and Feder, Spain (PID2021-122403NB-I00), the Generalitat de Catalunya, Spain (2017SGR-711), and the Severo Ochoa, Spain (CEX2019-000915-S) programmes for their financial support.

    * Corresponding author.

    E-mail addresses: qianshuo.liu.qs@gmail.com (Q. Liu), ines.macho@uab.cat (I. Macho-Stadler).

[^1]:    1 This assumption is made only for convenience. It allows us to identify the salary of the lowest-ability workers straightforwardly.
    2 The main characteristic of this production function is that there is complementarity between firms' productivity and workers' ability. Any function that exhibits this complementary leads to similar results.

[^2]:    3 This policy is equivalent to giving a subsidy of $\delta$ to each student attending $S_{H}$.

