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# Flexoelectric Enhancement of Strain Gradient Elasticity Across a Ferroelectric-to-Paraelectric Phase Transition

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**Abstract:** We study the temperature dependent elastic properties of  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  freestanding membranes across the ferroelectric-to-paraelectric phase transition using an atomic force microscope. The bending rigidity of thin membranes can be stiffer compared to stretching due to strain gradient elasticity (SGE). We measure the Young's modulus of freestanding  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  drumheads in bending and stretching dominated deformation regimes on a variable temperature platform, finding a peak in the difference between the two Young's moduli obtained at the phase transition. This demonstrates a dependence of SGE on the dielectric properties of a material and alludes to a flexoelectric origin of an effective SGE.

**Keywords:** Oxides, Ferroelectric, Nanomechanics, Flexoelectricity, Non-linear elasticity

Conventional mechanical response of materials is usually described using linear elastic theory which is a local theory and does not include any nonlinear or non-local contributions. In this framework, the Young's modulus  $E$  and the Poisson's ratio  $\nu$  are used to calculate the deformation response of a material geometry upon loading. Strain gradient elasticity (SGE) is a non-linear and non-local extension of elastic theory by including energetic contributions from strain gradients<sup>1,2</sup>, which can become important while considering nanomaterials and nanodevices where strain gradients can be large upon deformation<sup>2</sup>. SGE thus extends the notion of elasticity by not only adding nonlinearity, but also non-locality as strain gradients are intrinsically non-local in nature. For a material with SGE, the total elastic energy density  $\phi$  upon deformation is given by:

$$\phi = \frac{1}{2} E \varepsilon^2 + \frac{1}{2} K |\nabla \varepsilon|^2 \quad (1)$$

where  $E$  is the Young's modulus,  $\varepsilon$  is the strain and  $K$  is the strain gradient elastic coupling coefficient.

The origin of SGE is still an open question and conventional materials have been argued to have a diminishingly small strain gradient response<sup>3</sup>. In our previous work, however, while measuring the elastic response in sub-100-nm-thick SrTiO<sub>3</sub> membranes, it was observed that at the length scales of  $\sim 10$  nm, the SGE contribution to elasticity is not just perturbative, but on par with the linear elastic response<sup>4</sup>. Deducing the origin of the SGE response still remained elusive. One of the proposed mechanisms to make SGE relevant for geometries at the nanoscale is flexoelectricity,<sup>5,6</sup> [add Ref. J. Appl. Mech. 81, 081004 (2014)] which refers to the induced polarization in a material subjected to a strain gradient<sup>7-11</sup>. Using this connection between SGE and flexoelectricity, the flexoelectric response coefficients for SrTiO<sub>3</sub> were inferred from the mechanical response<sup>4</sup>. Naturally, this leads to the question, is there a more direct way to measure the connection between flexoelectricity and SGE? To answer this, we scrutinize flexoelectricity in materials and how it links to an effective SGE more closely.

Since strain gradients break centrosymmetry, flexoelectric polarization is universal irrespective of the underlying lattice structure (*i.e.*, flexoelectricity is governed by a fourth-rank tensor and is thus present in all materials, but it is significantly stronger in high-K dielectrics<sup>10</sup>). Flexoelectricity has been measured in a variety of materials<sup>12-17</sup>, but in the last decade, with advances in nanofabrication, flexoelectricity in at the nanoscale has been well characterized<sup>18-20</sup>, where owing to the small dimensions, strain gradients can be large. In oxide membranes, flexoelectricity has been observed to affect the polarization profile of BaTiO<sub>3</sub><sup>21</sup> and BiFeO<sub>3</sub><sup>22</sup> upon bending, domain-wall motion in PbZr<sub>0.2</sub>Ti<sub>0.8</sub>O<sub>3</sub><sup>23</sup> and even photoconductance in BiFeO<sub>3</sub><sup>24</sup>. To characterize flexoelectricity, the canonical parameter is  $\mu$ , the flexoelectric coefficient which is defined as the change in polarization  $P$  of a material when exposed to a unit strain gradient. The parameter  $\mu$  is directly related to the flexocoupling coefficient of the material  $f$  via the dielectric constant  $\chi$  by  $\mu = \chi f$ , as  $f$  is the proportionality coefficient between the strain gradient and the electric field introduced in the material,. This dependency of the flexoelectric effect on the dielectric susceptibility has been previously established in many materials<sup>16,17,25,26</sup>. Thus, relating the polarization energy density upon bending to SGE<sup>4</sup>, we find that

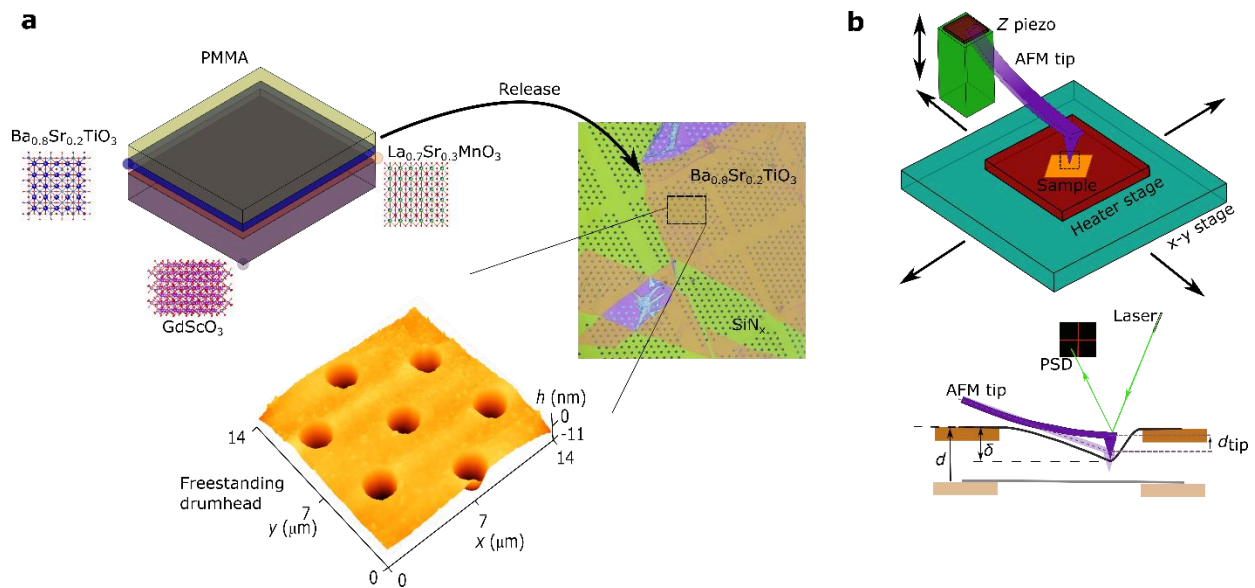
$$K = \chi f^2. \quad (2)$$

This shows that the SGE coefficient  $K$  is proportional to the dielectric constant of the material as long as the flexocoupling coefficient  $f$  remains the same.

Recent work has shown that the stiffness of a ferroelectric material can be altered by a combination of polarization and the flexoelectric effect<sup>27</sup>. As discussed above, in SrTiO<sub>3</sub> membranes, an SGE response was observed with an implied coupling to the flexoelectricity<sup>4</sup>. The flexoelectric contribution to deformation energy results in a higher inferred elastic modulus from a bending dominant deformation ( $E_{lin}$ ) as compared to that inferred from a stretching dominant deformation ( $E_{cub}$ )<sup>4,5</sup>. This difference between bending and stretching elasticity can be quantified by

$$E_{lin} - E_{cub} = \frac{12\chi f^2}{t^2}. \quad (3)$$

This relies on the fact that a uniform strain gradient is present across the material as it is bent, which is proportional to the maximum strains at the free boundaries and inversely proportional to the thickness. Consequently, as long as the sample is experiencing pure bending deformation (i.e. neutral axis has 0 strain), this relation will hold regardless of the actual magnitude of the strain and strain gradients. Furthermore, the difference between the two moduli is also linearly dependent on the dielectric susceptibility of the material for SGE rooted in flexoelectricity. This stiffening of the gradient elastic response rooted in a non-local strain driven polarization draws parallels to the stiffening of the elastic response in piezoelectric materials through a local strain driven polarization [add Ref. J. Acoust. Soc. Am. 42, 1223–1231 (1967)].



**Figure 1.** Making freestanding drumheads and measuring them. (a) A schematic of the as-grown heterostructure coated with PMMA is shown before release. After release, an optical micrograph and a 3D rendered AFM topography image are shown to

clearly display the freestanding drumheads. (b) A representation of the AFM measurement on a heater stage and the bottom panel shows all relevant parameters to measure the mechanical response of a freestanding drumhead.

Thus, to measure the direct effect of dielectric properties and flexoelectricity on SGE, we need a material where the dielectric properties change significantly as a function of some controllable parameter. When a ferroelectric material goes through a ferroelectric-to-paraelectric phase transition, the dielectric permittivity diverges at the transition temperature in the bulk limit. Therefore, such materials are ideal for exploring the effect of flexoelectricity on the bending rigidity through a temperature dependent elastic measurement. Such materials should also be thin enough to have a significant effect of SGE on the mechanical properties, as that contribution is inversely dependent on the square of the thickness. Recent developments in thin-film growth have made it possible to grow high quality single-crystalline oxide thin-film heterostructures and membranes with a variety of physical properties<sup>21,28-31</sup>. In this work we use freestanding  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  membranes due to the proximity of its ferroelectric-to-paraelectric phase transition above room temperature<sup>32,33</sup>. These membranes were prepared by growing a thin film of  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  using pulsed laser deposition (PLD) with a buffer layer of  $\text{La}_{0.7}\text{Sr}_{0.3}\text{MnO}_3$  on a  $\text{GdScO}_3$  (110) substrate (Figure 1a). The buffer layer was then dissolved using a solution of potassium iodide in hydrochloric acid and water (Figure 1) <sup>33,34</sup>. After the buffer layer was etched, the  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  membrane was transferred onto a  $\text{SiN}_x$  grid with an array of 2  $\mu\text{m}$  diameter holes to form freestanding drumheads. This is similar to the drumhead geometry used to study several other materials including graphene<sup>35,36</sup>,  $\text{MoS}_2$  <sup>37</sup>, and  $\text{SrTiO}_3$  <sup>4</sup>. An atomic force microscope (AFM) topography scan of these drumheads is shown (Figure 1a).

Here, we use the AFM to measure the elastic properties of these freestanding  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  drumheads. Such a measurement protocol was developed to measure the elastic properties of 2D materials <sup>35-37</sup>, particulars of the measurement used in this paper have been described in detail elsewhere with a similar measurement setup on  $\text{SrTiO}_3$ <sup>4</sup>. The radius of the tip used is  $\sim 15$  nm (Multi75DLC from BudgetSensors), which is much smaller compared to the radius of the drumhead (1  $\mu\text{m}$ ) and a diamond-like-carbon coated tip is used for the measurement to prevent it from wearing during the experiment. Different from that previous work, this measurement was conducted using a variable temperature stage on the AFM (Figure 1b). This enables elastic measurements across a range of temperatures ranging from 40-200  $^\circ\text{C}$ , which encompasses the phase-transition temperature established previously<sup>33</sup>. Once the drumhead of choice has been identified, we produce a force-distance ( $F - d$ ) map on a 32x32 pixel array on and around the drumhead area. Note that for all measurements, the force constant of the AFM tip being used is measured using the thermal method<sup>38</sup>. This  $F - d$  map can be turned into a force-deflection ( $F - \delta$ ) map by subtracting the response of the cantilever and the  $\text{SiN}_x$  film. The details on the exact steps for doing this have been published

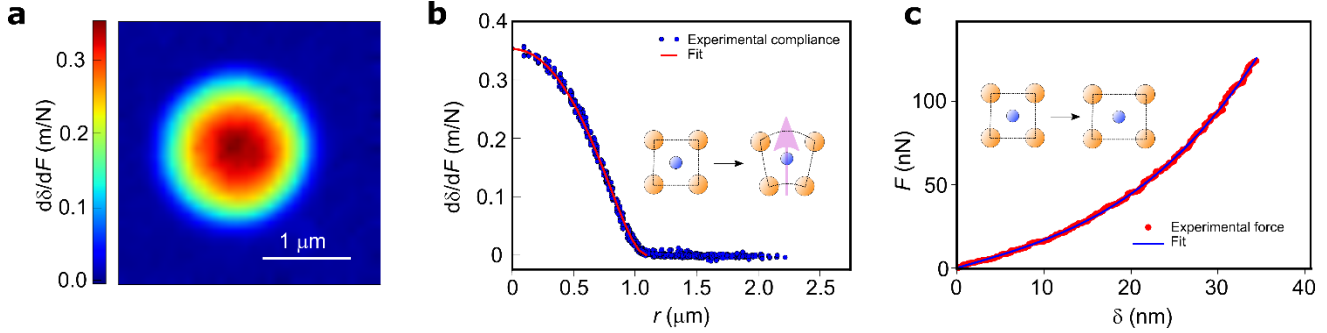
previously [put ref 4]. Once we have the  $F - \delta$  map, we can map the compliance of the drumhead ( $\partial\delta/\partial F$ ) as a function of position, giving us a compliance map (Figure 2a). For a 13-nm-thick membrane of  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$ , we can probe both the linear and nonlinear  $F - \delta$  response of the drumheads by varying the applied force. The linear  $F - \delta$  response regime for lower forces (below 10 nN) captures the perturbative bending dominated deformation of the membrane from its equilibrium position. Similarly, the nonlinear regime of the  $F - \delta$  response arises due to geometric nonlinearities in the membrane deformation implying a more stretching dominated deformation<sup>4</sup>. To extract the Young's modulus  $E_{\text{lin}}$  from the linear  $F - \delta$  response, we use numerical solutions of the Euler-Bernoulli bending equation for a clamped plate with uniform cross section, the general form of which is given by

$$\frac{E_{\text{lin}}t^3}{12(1-\nu^2)}\nabla^4u(r,\theta) - T\nabla^2u(r,\theta) = \rho(r,\theta) \quad (4)$$

to fit to the experimental compliance as a function of radial distance from the center of the drumhead (Figure 2b). Here,  $E_{\text{lin}}$  captures the elastic response of the material upon bending, as defined above,  $t$  is the thickness,  $T$  is the pretension,  $u$  is the deflection profile of the drumhead, and  $\rho$  is the pressure profile of the load applied. The free parameters for this fit are  $E_{\text{lin}}$  and  $T$ . For nonlinear response, the Young's modulus for a stretching dominated response,  $E_{\text{cub}}$ , is obtained by fitting the  $(F - \delta)$  curve to a cubic response upon central point loading (Fig. 2c), given by

$$F(\delta) = \frac{4\pi E_{\text{lin}}t^3}{3(1-\nu^2)R^2}\delta + \pi T\delta + \frac{cE_{\text{cub}}t}{R^2}\delta^3. \quad (5)$$

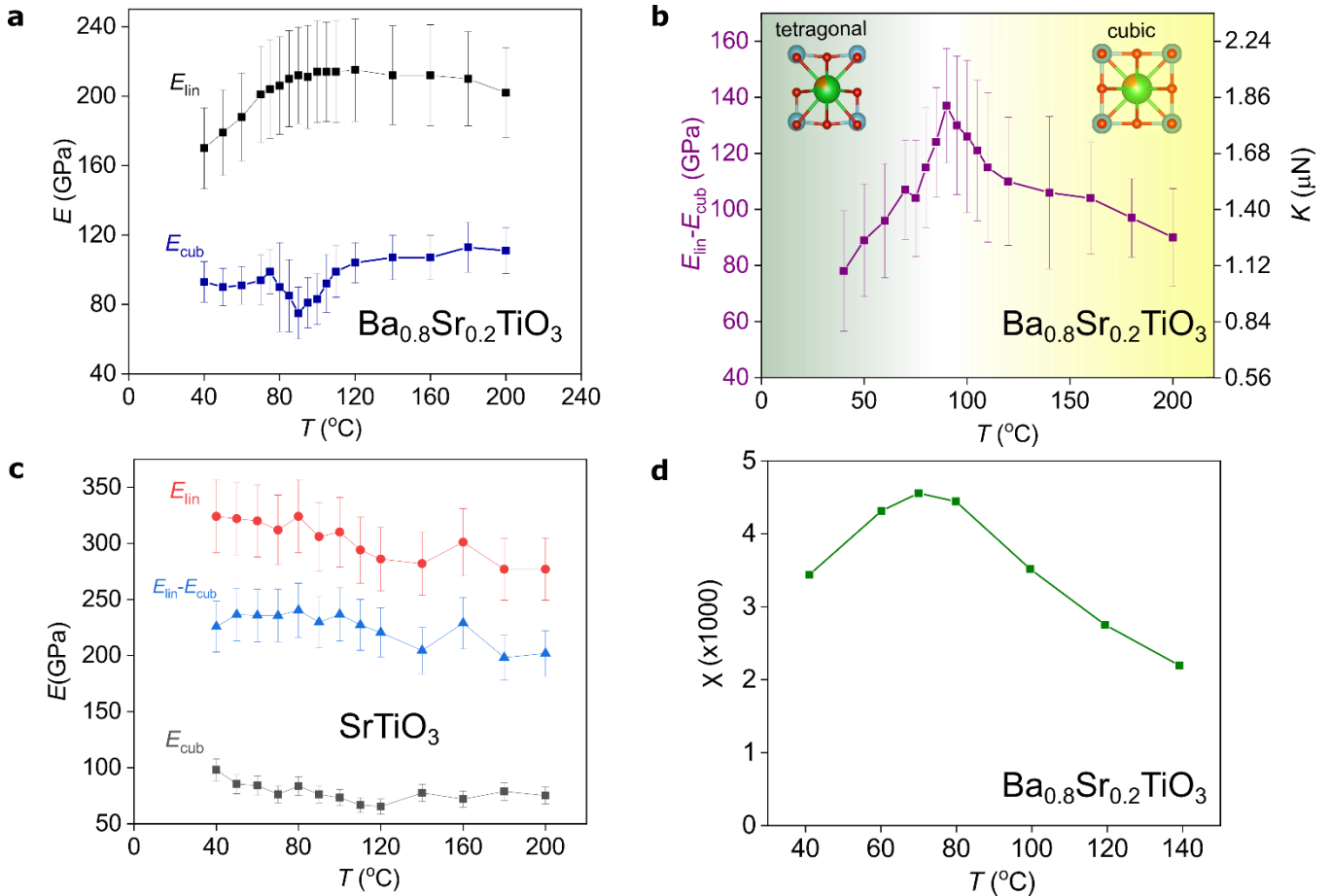
The lattice deformation mechanisms that  $E_{\text{cub}}$  and  $E_{\text{lin}}$  represent in terms of rigidity are different. More details on handling the raw data and its analysis are shown in Supplementary Figure S2. Since  $E_{\text{lin}}$  captures the bending response, there is a built-in strain gradient and thus this modulus is affected by SGE. For a stretching deformation, there is no significant strain gradient and thus  $E_{\text{cub}}$  is nominally not affected by SGE. Note that  $E_{\text{lin}}$  and  $E_{\text{cub}}$  are not calculated from a model completely capturing the free energy minimization of Equation 1, but rather by appropriate extension of local elastic theories in bending and stretching dominated deformation regimes, the details of which are discussed in our previous work on  $\text{SrTiO}_3$  [add ref 4].



**Figure 2.** Extracting the bending and stretching rigidity of freestanding drumheads. (a) A 32 x 32 pixel force map taken on and around a drumhead and the compliance from the perturbative linear response is plotted as a function of spatial position, after subtracting out the compliance of the neighboring SiN<sub>x</sub> membrane. (b) The compliance from linear response plotted as a function of distance from the center of the drumhead (blue dots) and a fit to the Euler-Bernoulli compliance obtained by solving and iterating over Equation 4. (c) The nonlinear nature of the  $F - \delta$  curve due to the predominance of stretching at higher forces. The fit is to the cubic force response equation (Equation 5) and the coefficient of the cubic response is used to evaluate  $E_{\text{cub}}$ .

We studied how the mechanical stiffness for the two different stretching- and bending-dominated deformations evolve with temperature by looking at the moduli inferred from linear ( $E_{\text{lin}}$ ) and cubic ( $E_{\text{cub}}$ ) responses, separately, across the studied temperature range. The temperature traces are taken over seven different holes. At each temperature, we let the sample equilibrate for more than 20 minutes which is much longer than the time constant of equilibration of our sample and then measure the mechanical response (Figure 3a) (See Supporting Information Figure S1). We first observe that the  $E_{\text{lin}} > E_{\text{cub}}$  at all measured temperatures. Generally, for materials undergoing a ferroelectric-to-paraelectric phase transition, as they also go through a structural phase transition changing the lattice symmetry, the material becomes less stiff at the transition temperature as the lattice has to accommodate a structural change<sup>39,40</sup>. We observe a similar drop in  $E_{\text{cub}}$  around 85 °C showing a softening of the elastic response for stretching near the phase transition (Figure 3a). However, such softening is absent on the temperature trace of  $E_{\text{lin}}$ , signifying a smooth variation of the bending stiffness across this temperature range. These same trends can be observed in each individual trace of each hole (see Supporting Information Figure S3). Note that another effect, which is the electromechanical coupling coefficient peaking at the paraelectric-to-ferroelectric phase transition<sup>41,42</sup> can possibly be used to consider our data. The electromechanical coupling parameter directly relates how efficiently the mechanical energy of a strained material manifests itself as electrical energy through polarizations. This coupling also peaks near the paraelectric-ferroelectric transition. However, the electromechanical coupling would affect bending and stretching in the same manner, as it is a local effect and would not differentiate between different deformation regimes. Thus, we can rule this out as a dominant contribution. This implies that the intrinsic bending rigidity varies smoothly across the phase transition of Ba<sub>0.8</sub>Sr<sub>0.2</sub>TiO<sub>3</sub> while the stretching rigidity response shows a dip. The difference between bending and stretching rigidity is governed directly by SGE [see Eq. 3],

thus showcasing that SGE contribution is essentially being enhanced across the phase transition. This is apparent when we plot the difference  $E_{lin} - E_{cub}$  as a function of temperature (Figure 3b) and a peak can be seen at the phase-transition temperature. We note here that across the phase transition, the relative change in the flexocoupling coefficient  $f$  would be of the order of the relative change in the lattice parameter  $a$  ( $\Delta f/f \sim \Delta a/a$ ), which means for the change in the flexoelectric coefficient  $\mu$  across the phase transition,  $f$  is effectively constant as compared to the change in the dielectric constant. We also performed a similar experiment on  $SrTiO_3$  as a control, as it does not have a phase transition in this temperature range. We did not observe any dip-like features and both  $E_{lin}$  and  $E_{cub}$  decrease monotonically with increase in temperature (Figure 3c). Comparing our results for  $Ba_{0.8}Sr_{0.2}TiO_3$  to the connection between SGE and flexoelectricity and knowing that the dielectric response peaks at the ferroelectric-to-paraelectric phase transition<sup>(add ref. 33)</sup> (Figure 3d), this finding clearly demonstrates a dependence of SGE on the dielectric response of the material and hence flexoelectricity. The slight difference between the phase transition temperature observed in the dielectric measurement and through our mechanical measurement can be attributed to the different thicknesses of membranes used (100 nm in the capacitive measurement versus 13 nm for mechanical measurement).



**Figure 3.** Measured elastic moduli for freestanding drumheads. (a) Average  $E_{\text{lin}}$  and  $E_{\text{cub}}$  of seven  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  freestanding drumheads with temperature varying from 40 to 200 °C.  $E_{\text{cub}}$  shows a clear dip at the phase transition temperature, as expected at a structural phase transition. By contrast  $E_{\text{lin}}$  varies smoothly across this temperature range. Error bars indicate the error in measuring the spring constant of the AFM tip by the thermal method<sup>38</sup> and the standard deviation over the 7 datasets. (b) The difference between the two measured moduli shows a peak at the transition temperature. This peak also corresponds to a peak in the dielectric response of the material as it goes through a paraelectric-ferroelectric phase transition. This shows a dependence of the SGE coefficient  $K$  and the dielectric properties of  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  and hence flexoelectricity. (c) Control experiment on the same temperature range as that for  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  but on  $\text{SrTiO}_3$  which does not have a phase transition. We can see no dip-like features in either  $E_{\text{lin}}$  or  $E_{\text{cub}}$  nor a peak appearing in the difference  $E_{\text{lin}} - E_{\text{cub}}$  as was seen in the case of  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$ . The error bars represent the same errors as in (a). (d) The permittivity measurement of a 100 nm thick  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  showing the peak in dielectric response, cutoff by finite thickness effects, at the phase transition, measured on circular capacitor structures defined on the thin film via dry etching. Data adapted with permission from Ref.<sup>33</sup> where more details can be found about this measurement.

With the interest in flexoelectricity growing recently, this work showcases the flexoelectric influence on the elastic moduli of materials, not just on the polarization response of the material. This finding provides significant insights for the future studies and applications of oxide materials in nanoelectromechanical/microelectromechanical systems (NEMS/MEMS). Our finding shows that at the nanoscale, the mechanical properties for certain materials can be tuned via the dielectric response, opening possibilities for fine control over material rigidity using dielectric tuning parameters such as temperature and electric field. Furthermore, our technique can probe a phase transition in a thin membrane material where a capacitive measurement of the dielectric constant is extremely challenging and perturbative. To conclude, we measured the temperature dependent elastic properties of freestanding  $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$  membranes in bending and stretching dominated deformation regimes and found that the enhancement of the bending rigidity with respect to the stretching rigidity was highly temperature dependent. Across a ferroelectric-to-paraelectric phase transition, the SGE response is peaked at the transition temperature, showing a clear dependence of SGE on the dielectric properties of the material. This strongly indicates that the origin of SGE is flexoelectric in nature.

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